The Hidden Geometry of Particle Collisions

Patrick T. Komiske III
Massachusetts Institute of Technology
Center for Theoretical Physics

Based on work with Eric Metodiev and Jesse Thaler
1902.02346 (PRL)
2004.04159 (JHEP)

BSM Pandemic Double Feature

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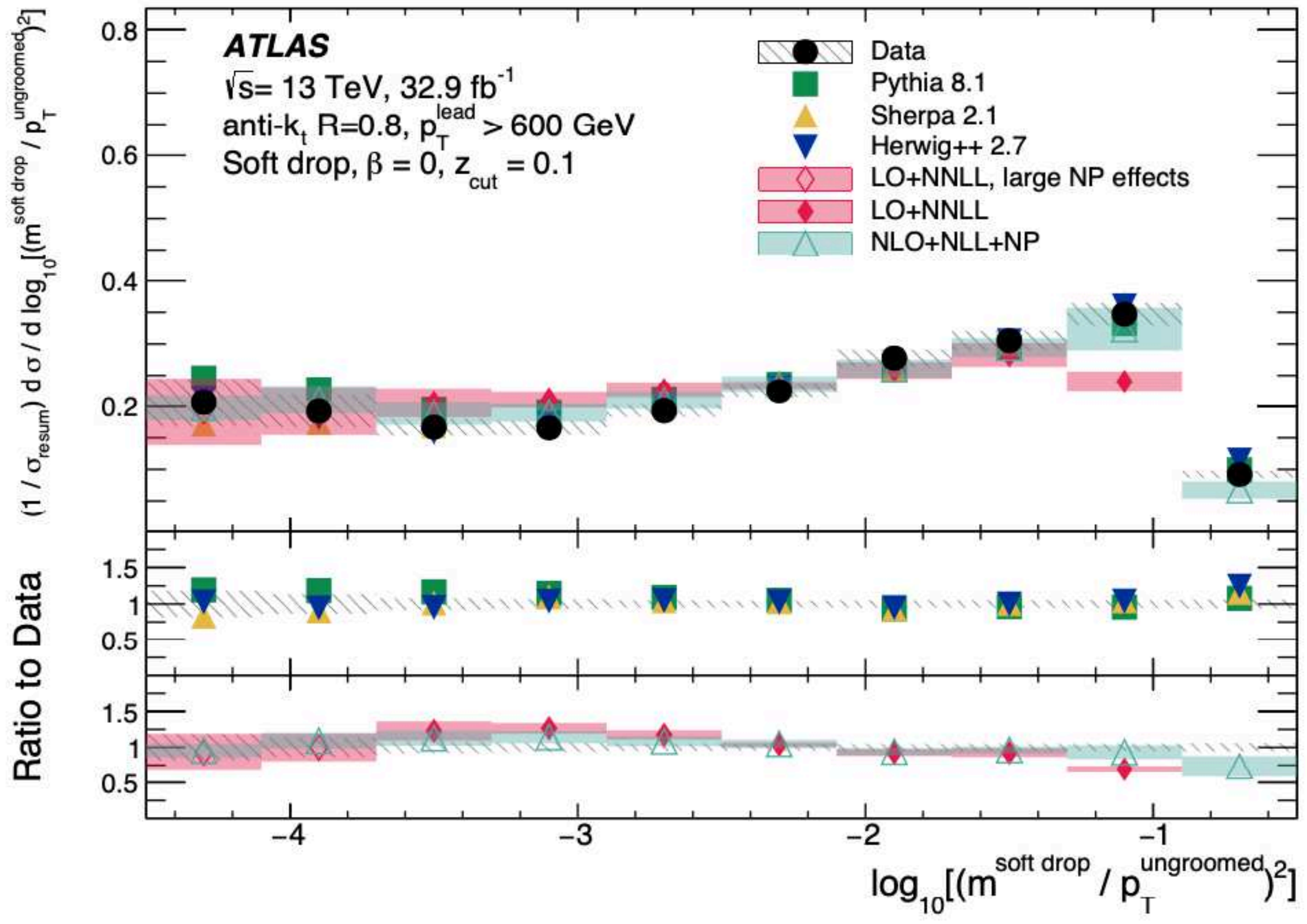
Developing New Analysis Frameworks

Particle theory ***makes predictions*** *and* ***invents new models***

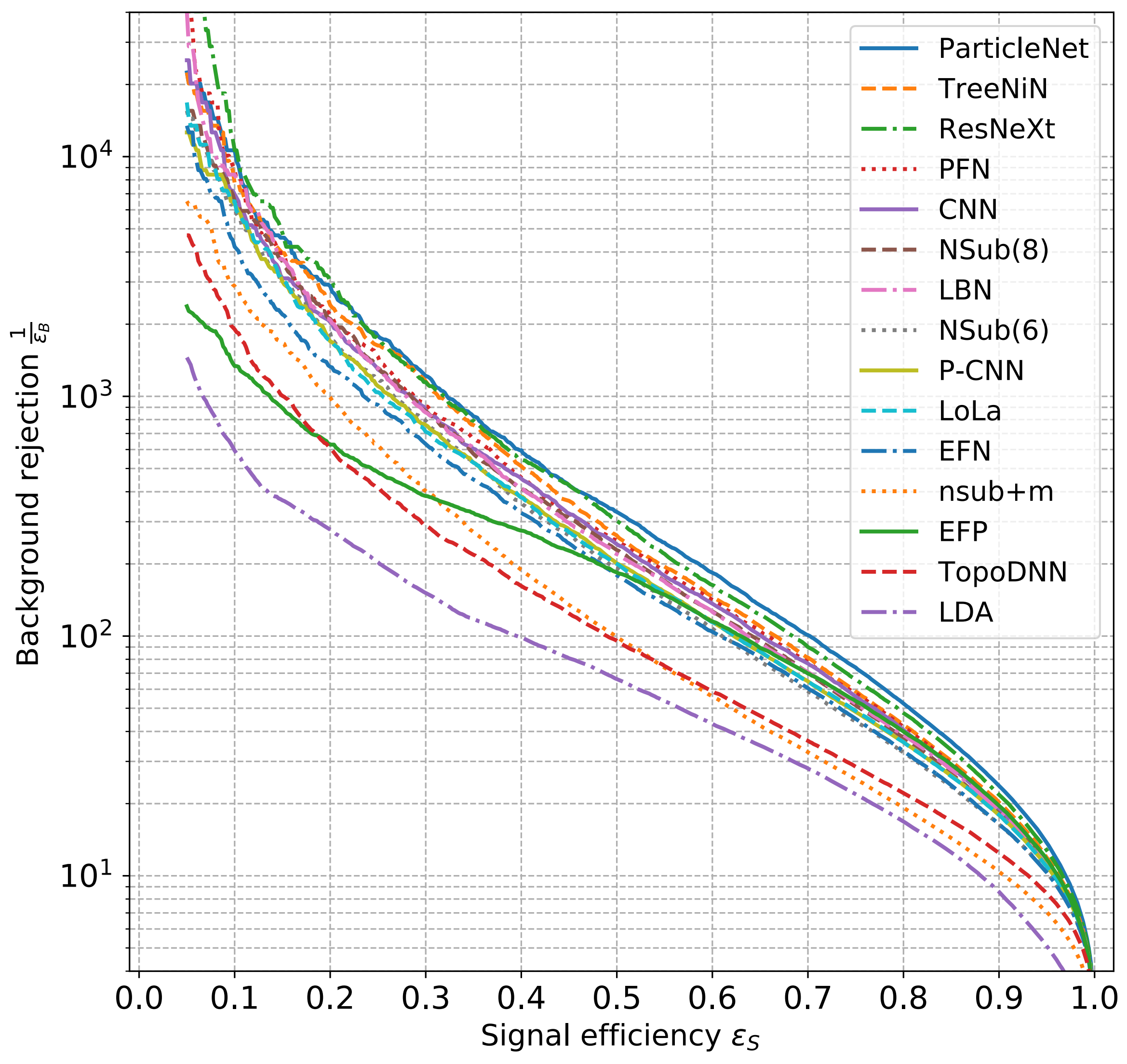
Particle experiment ***conducts measurements*** *and* ***searches for new physics***

Soft Drop jet mass measurement



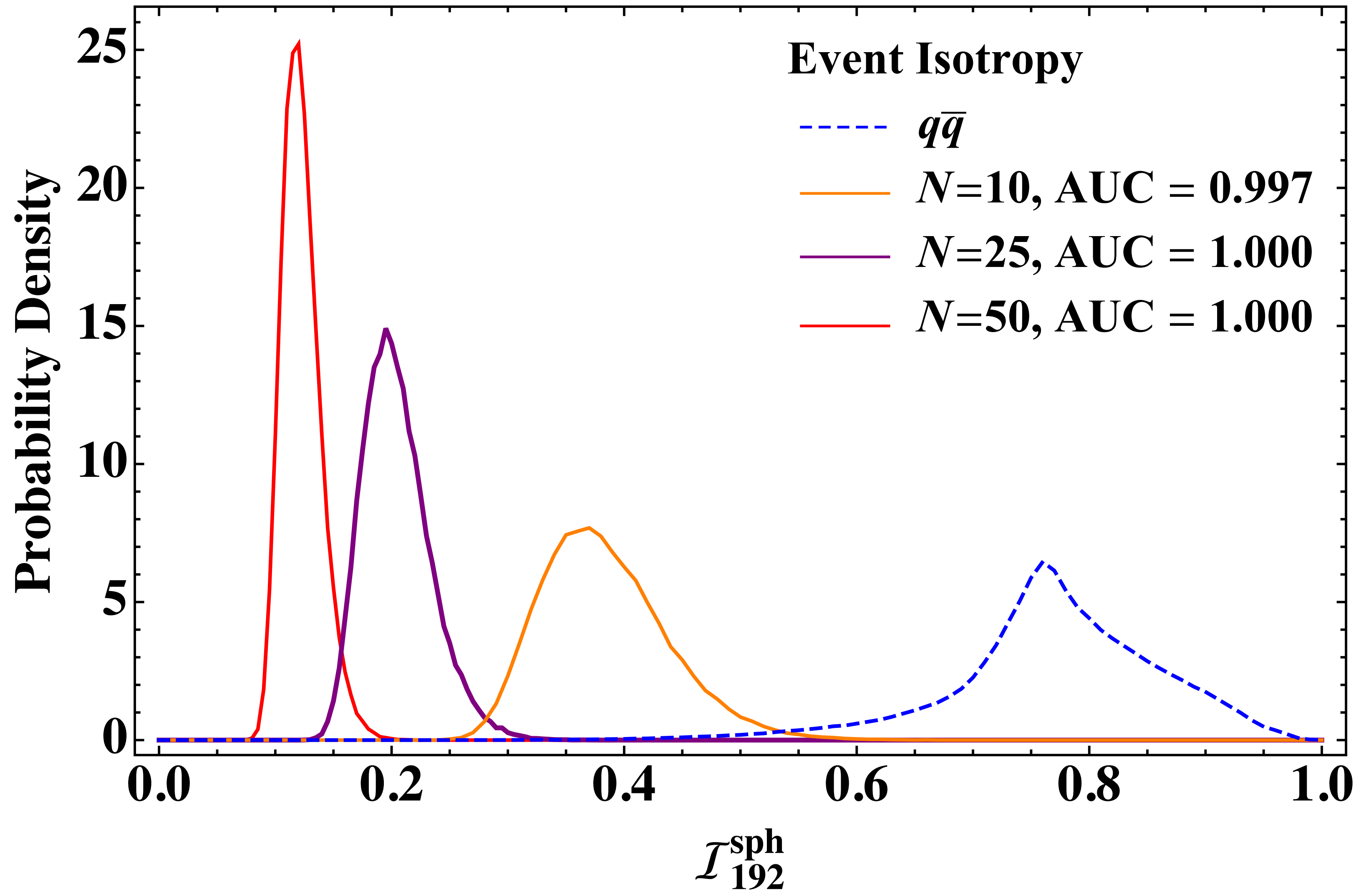
[Larkoski, Marzani, Soyez, Thaler, JHEP 2014;
ATLAS, PRL 2018]

Comparison of ML top taggers



[Kasieczka, Plehn, et al., 1902.09914;
using PTK, Metodiev, Thaler, 1810.05165, and others]

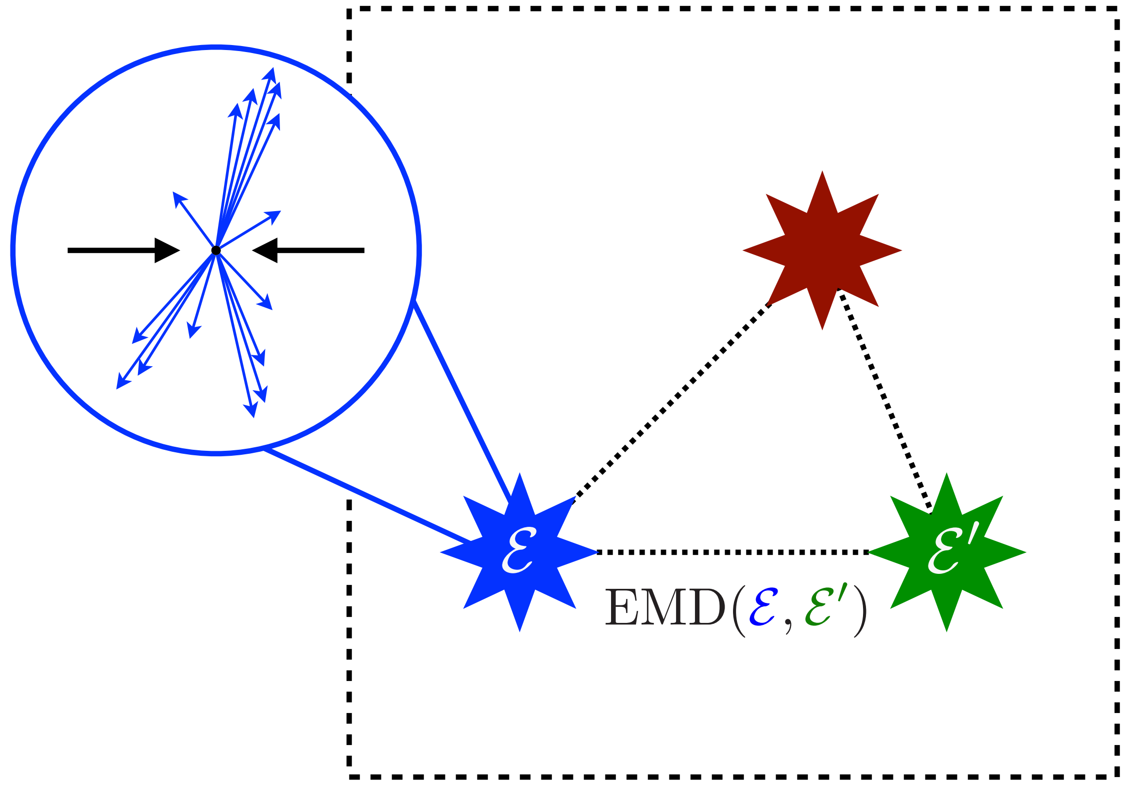
New observables sensitive to new signals

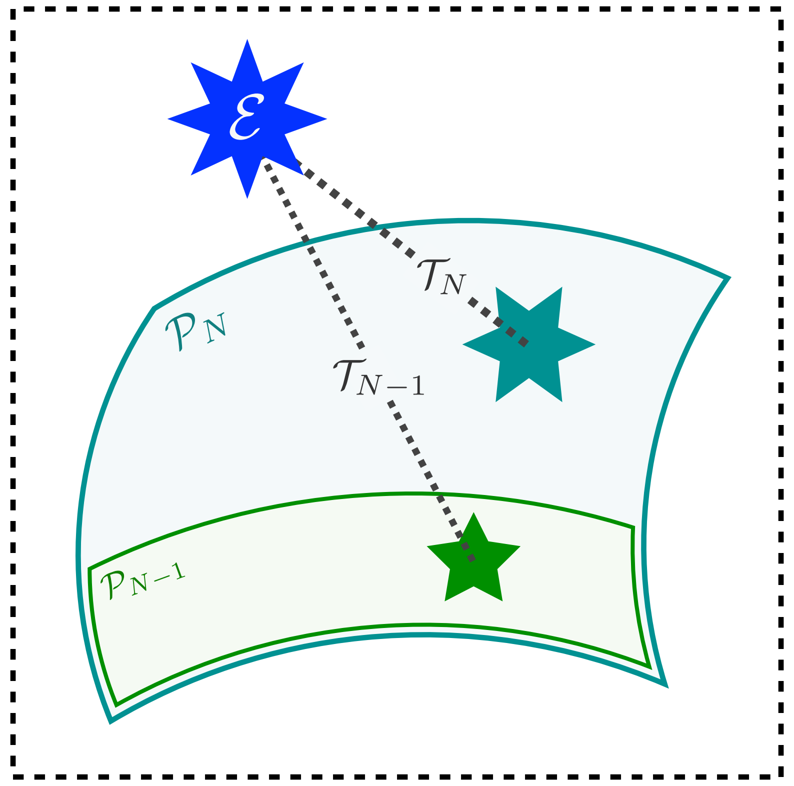


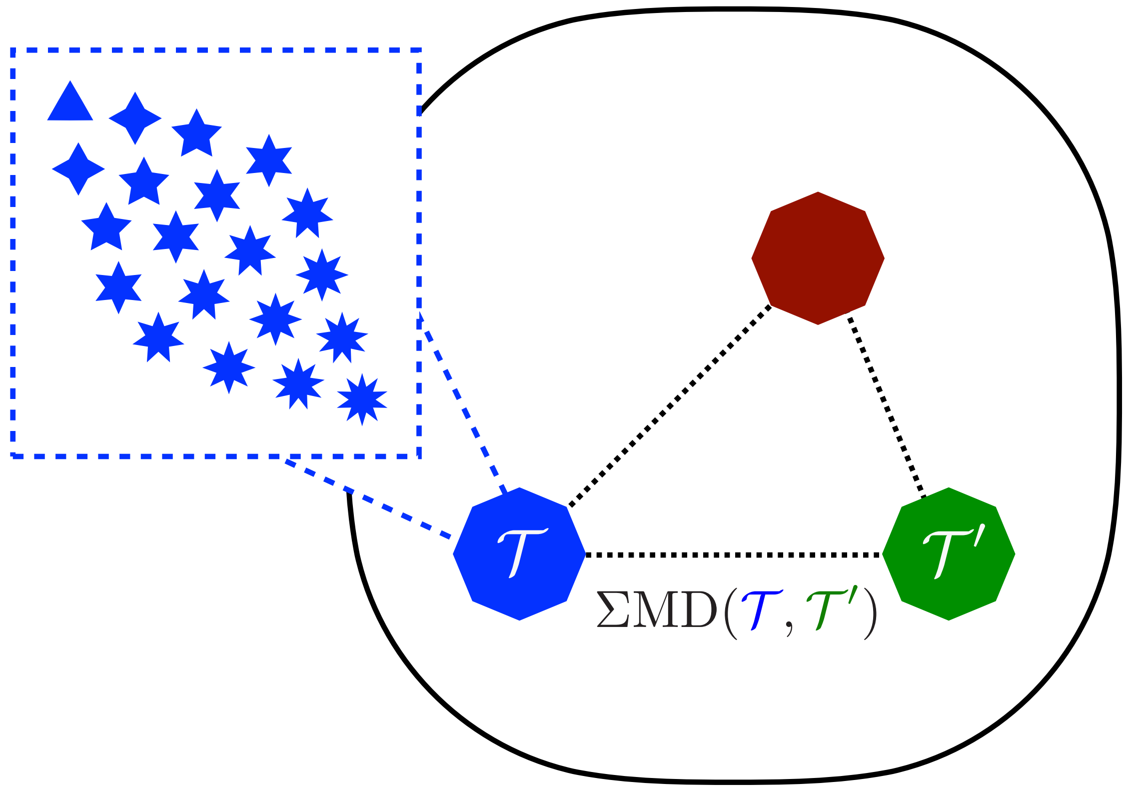
[Cesarotti, Thaler, JHEP 2020;
utilizing PTK, Metodiev, Thaler, PRL 2019]

Twenty more years of the LHC + any future collider(s)

Maximizing physics potential will require insights from data science and ML



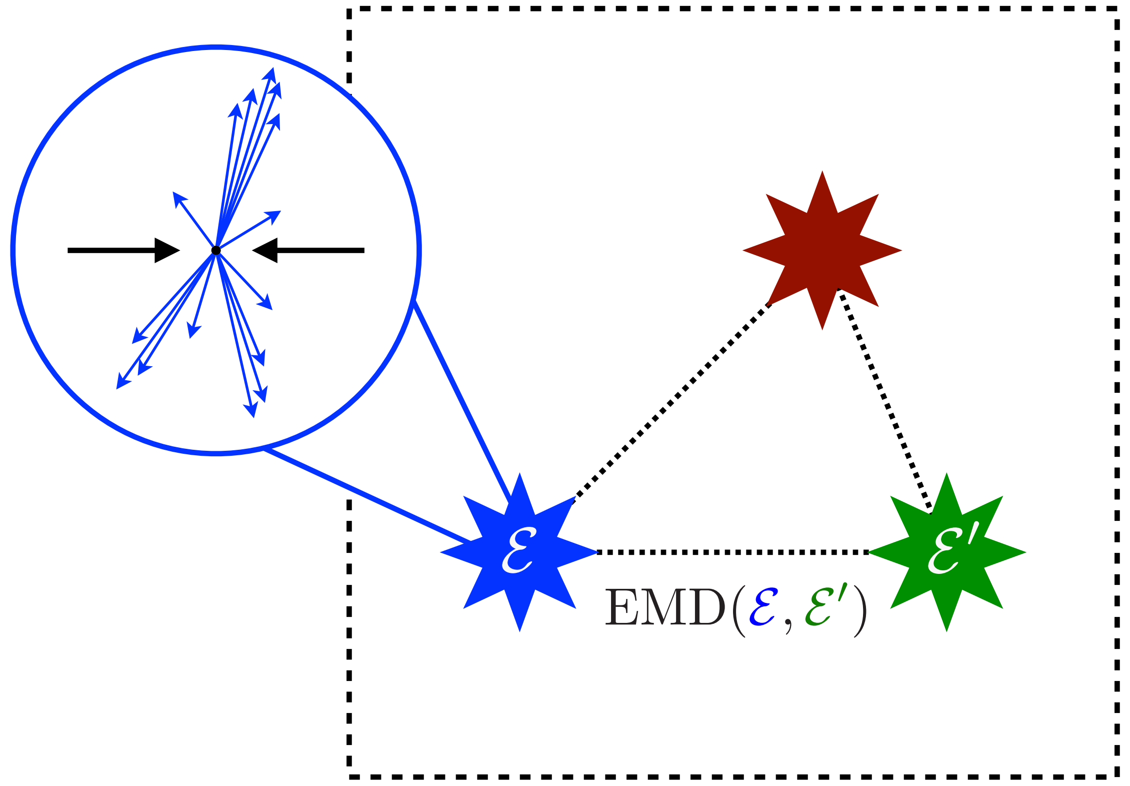




The (Metric) Space of Events

Revealing Hidden Geometry

[Theory Space]



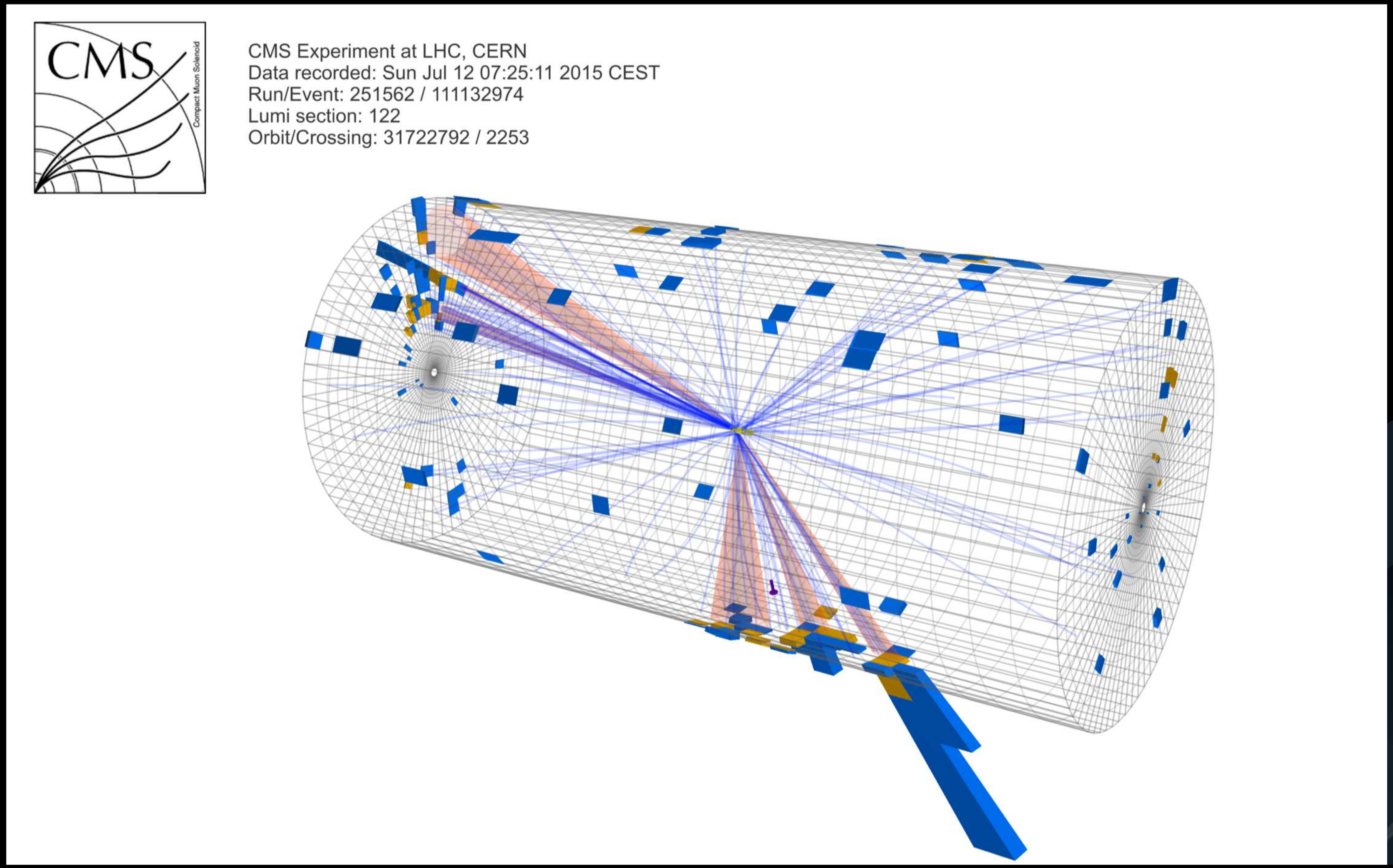
The (Metric) Space of Events

Revealing Hidden Geometry

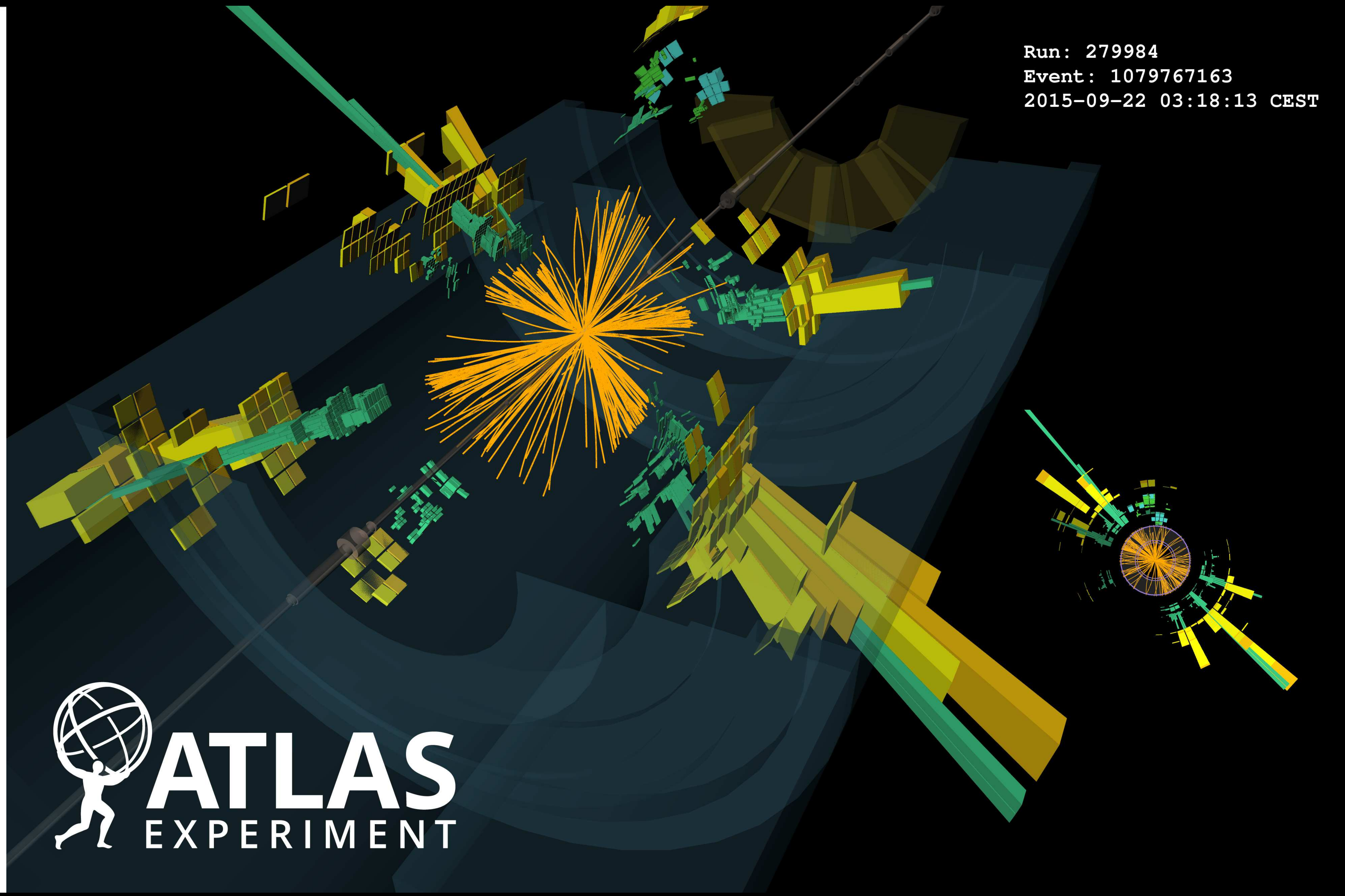
[Theory Space]

Explicit Geometry – Individual Events at the LHC

High-energy collisions produce final state particles with
energy, *direction*, *charge*, *flavor*, and *other quantum numbers*



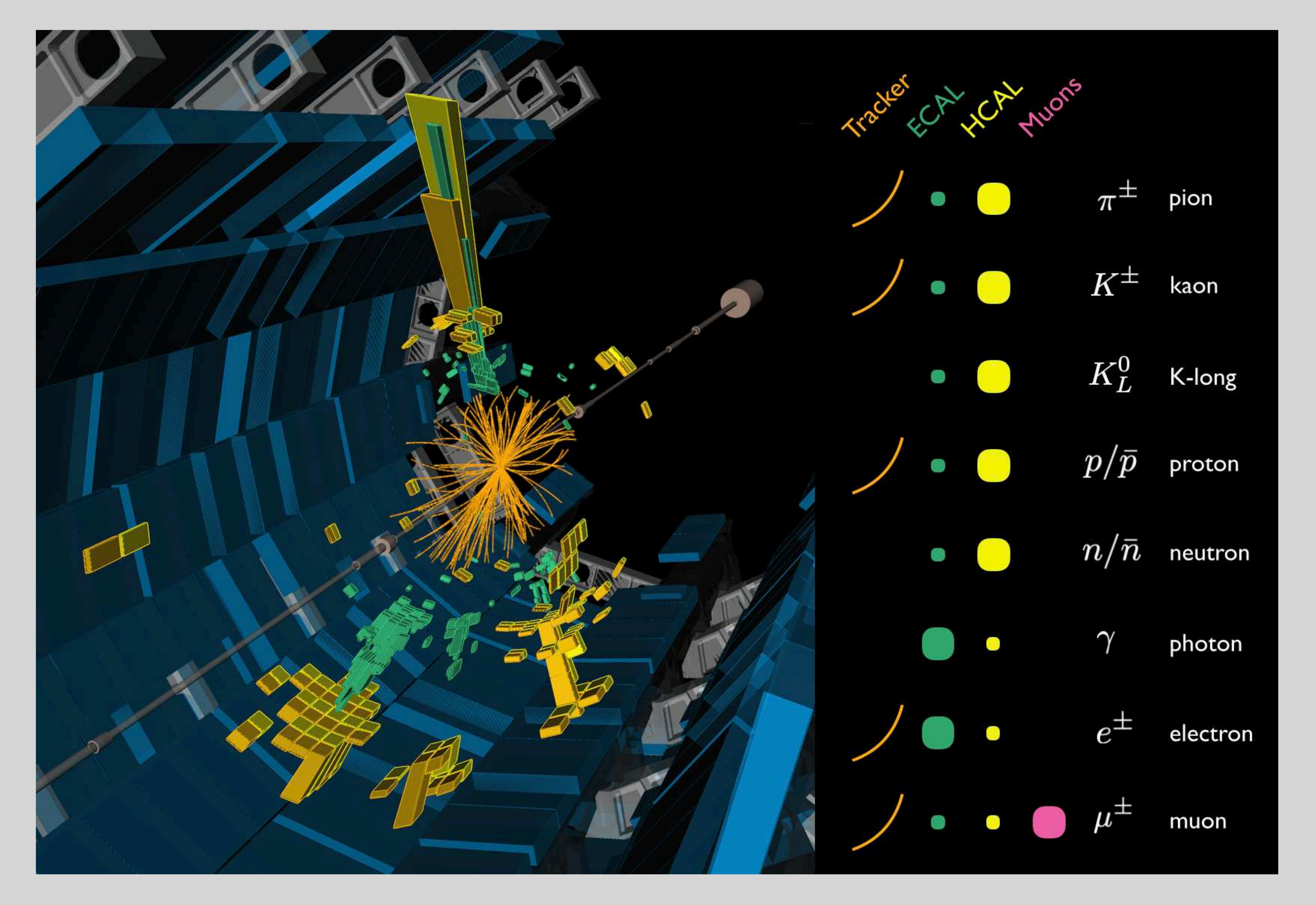
CMS hadronic $t\bar{t}$ event



ATLAS high jet-multiplicity event

Explicit Geometry – Individual Events at the LHC

High-energy collisions produce final state particles with
energy, *direction*, *charge*, *flavor*, and *other quantum numbers*

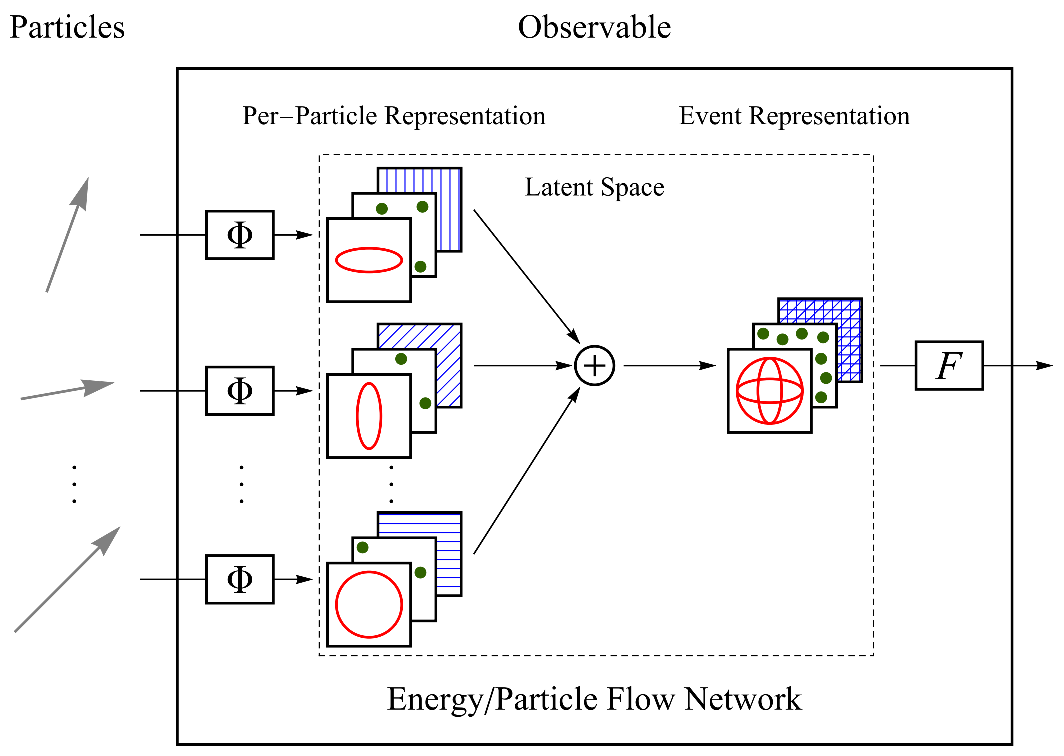


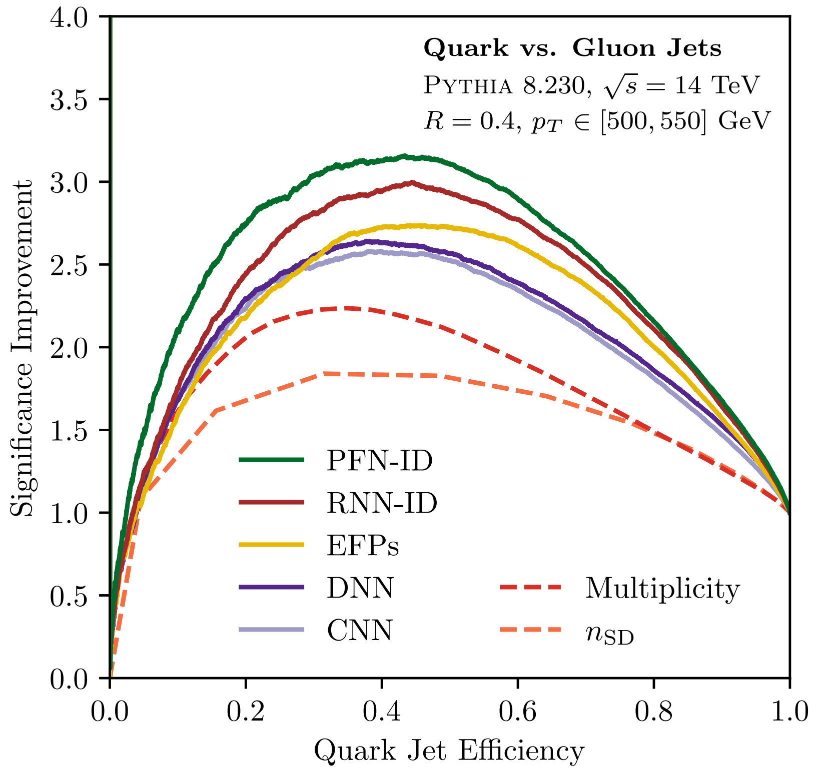
Machine-Learning-Inspired Methods for Particle Physics

Energy/Particle Flow Networks (EFNs/PFNs)

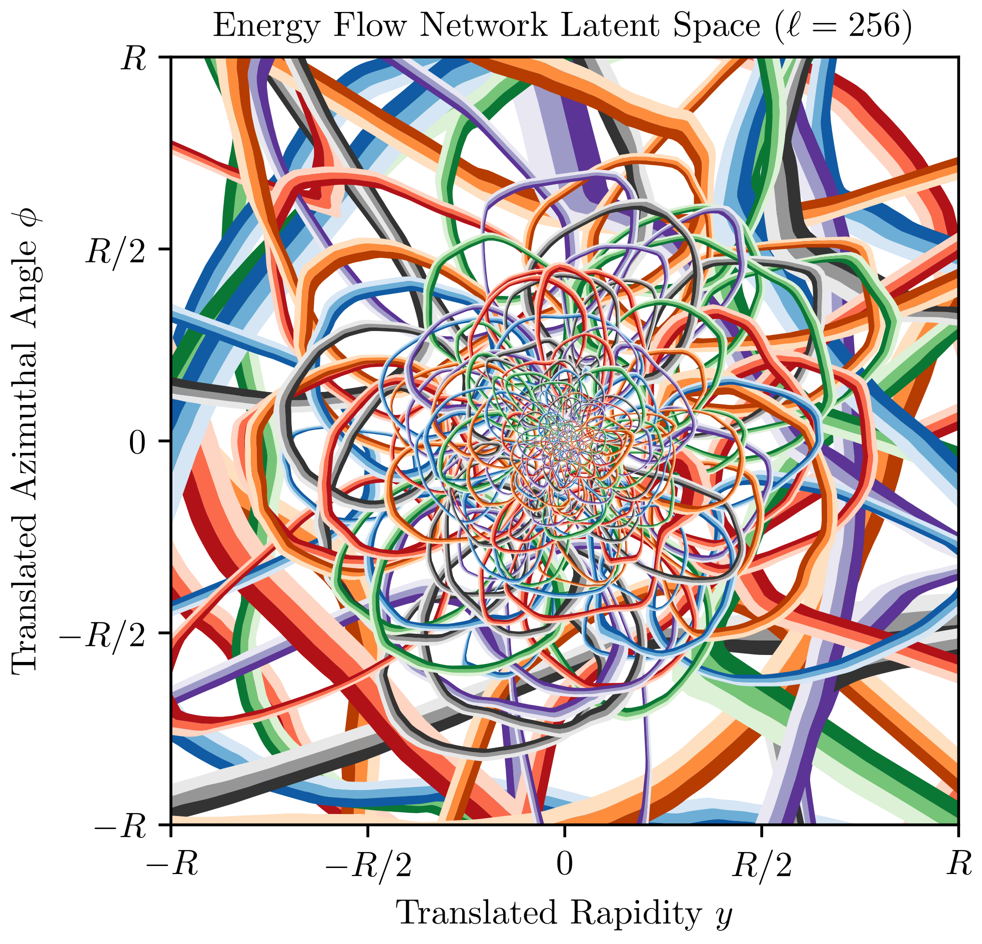
[PTK, Metodiev, Thaler, JHEP 2019]

Permutation symmetric neural network architecture for events with variable numbers of particles





Can be used to build powerful taggers



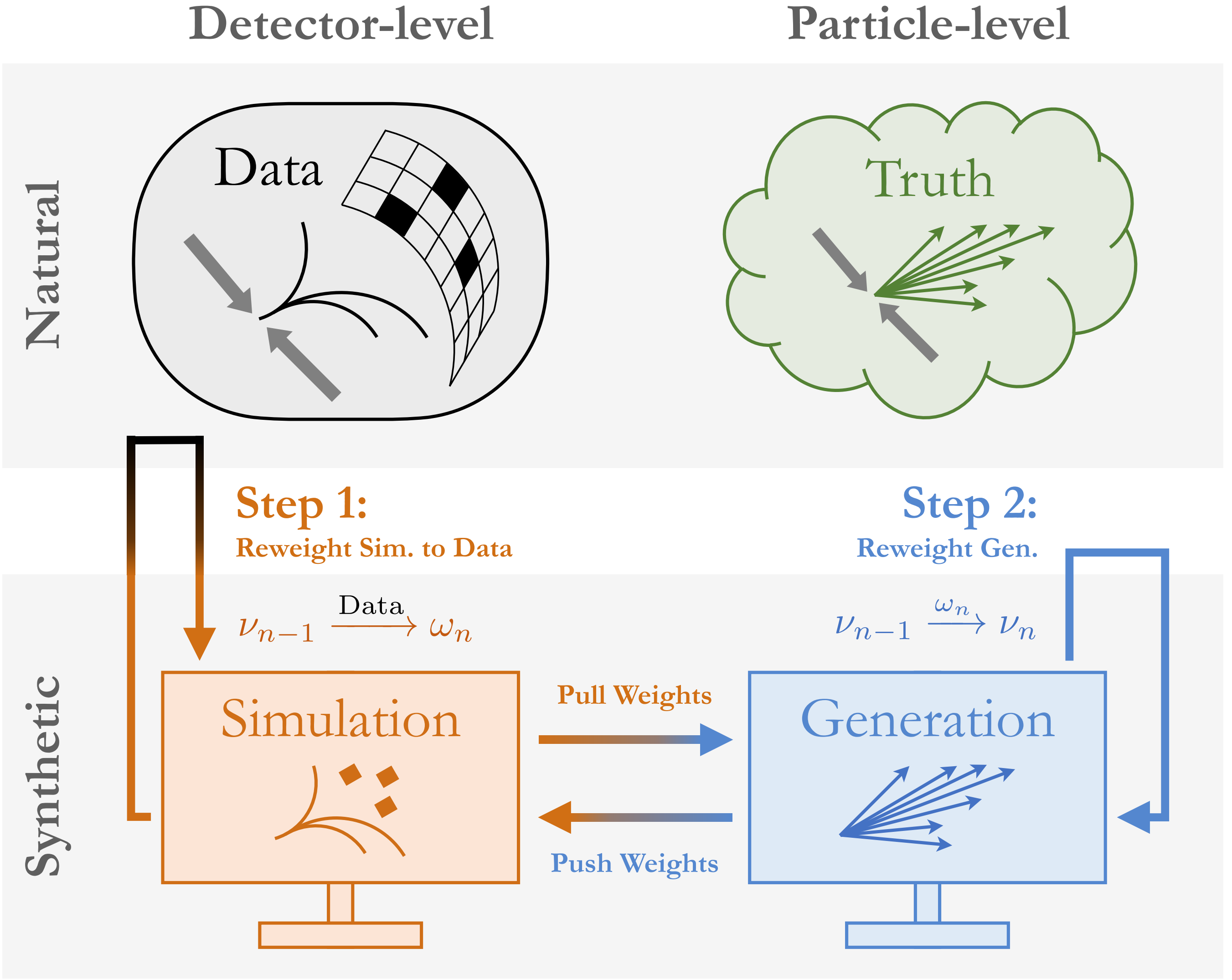
Latent space visualization reveals what the network has learned

Dynamic pixel sizing related to collinear singularity of QCD!

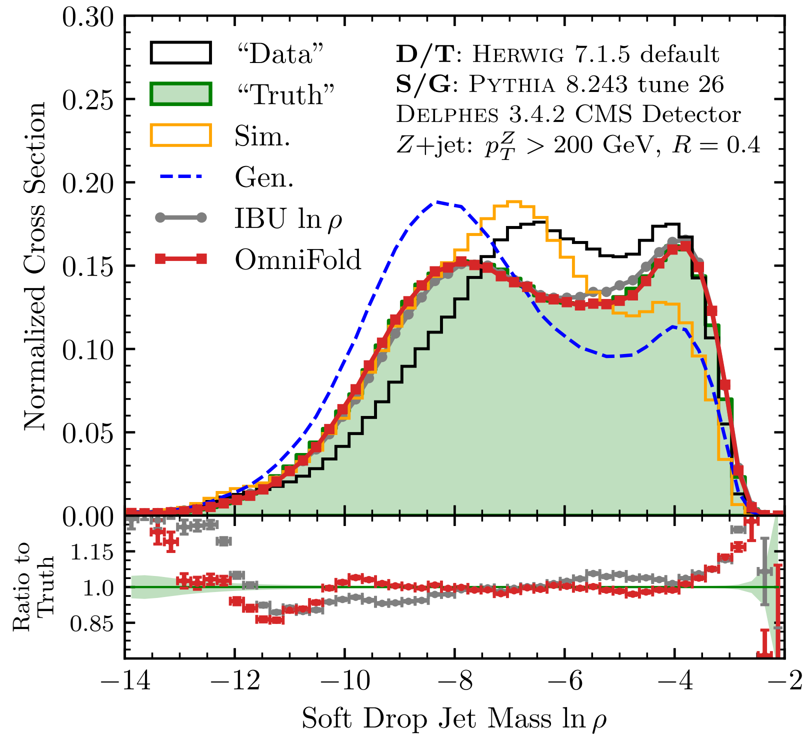
OmniFold

[Andreassen, PTK, Metodiev, Nachman, Thaler, PRL 2020]

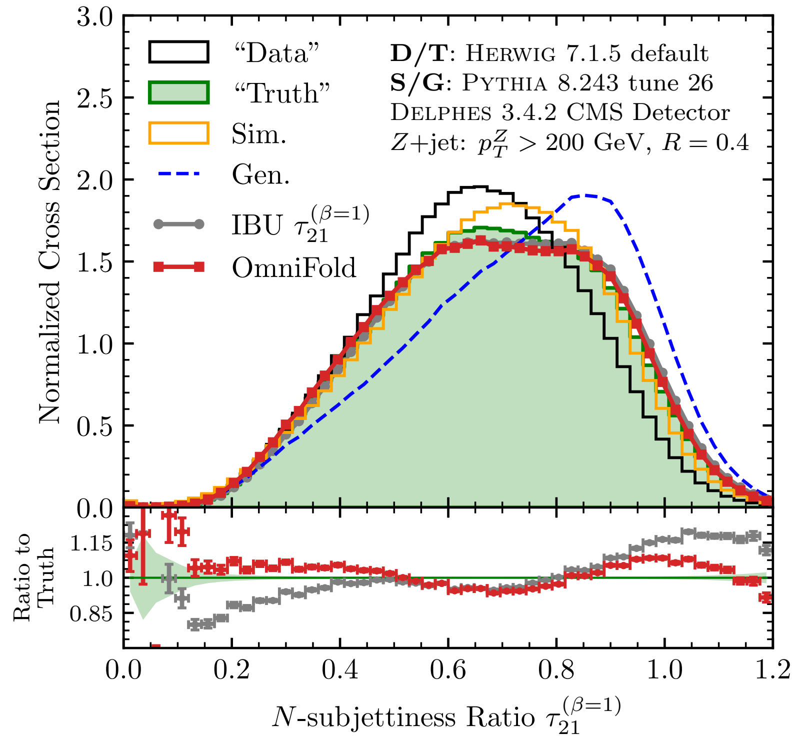
Unbinned, full-phase space unfolding of all observables simultaneously



Single application of OmniFold succeeds where multiple IBUs are required!

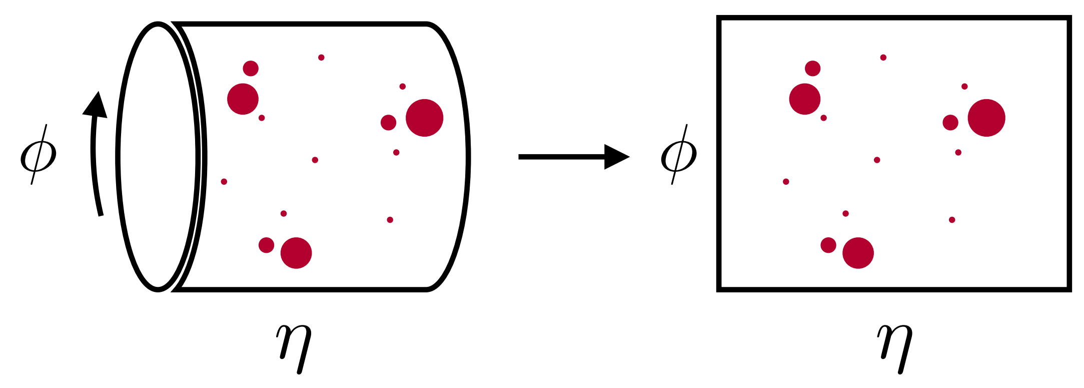


IRC safe



Sudakov safe

Back to Explicit Geometry – Events as Distributions of Energy



[PTK, Metodiev, Thaler, JHEP 2019; PTK, Metodiev, Thaler, JHEP 2020]

Energy flow distribution fully captures IRC-safe information

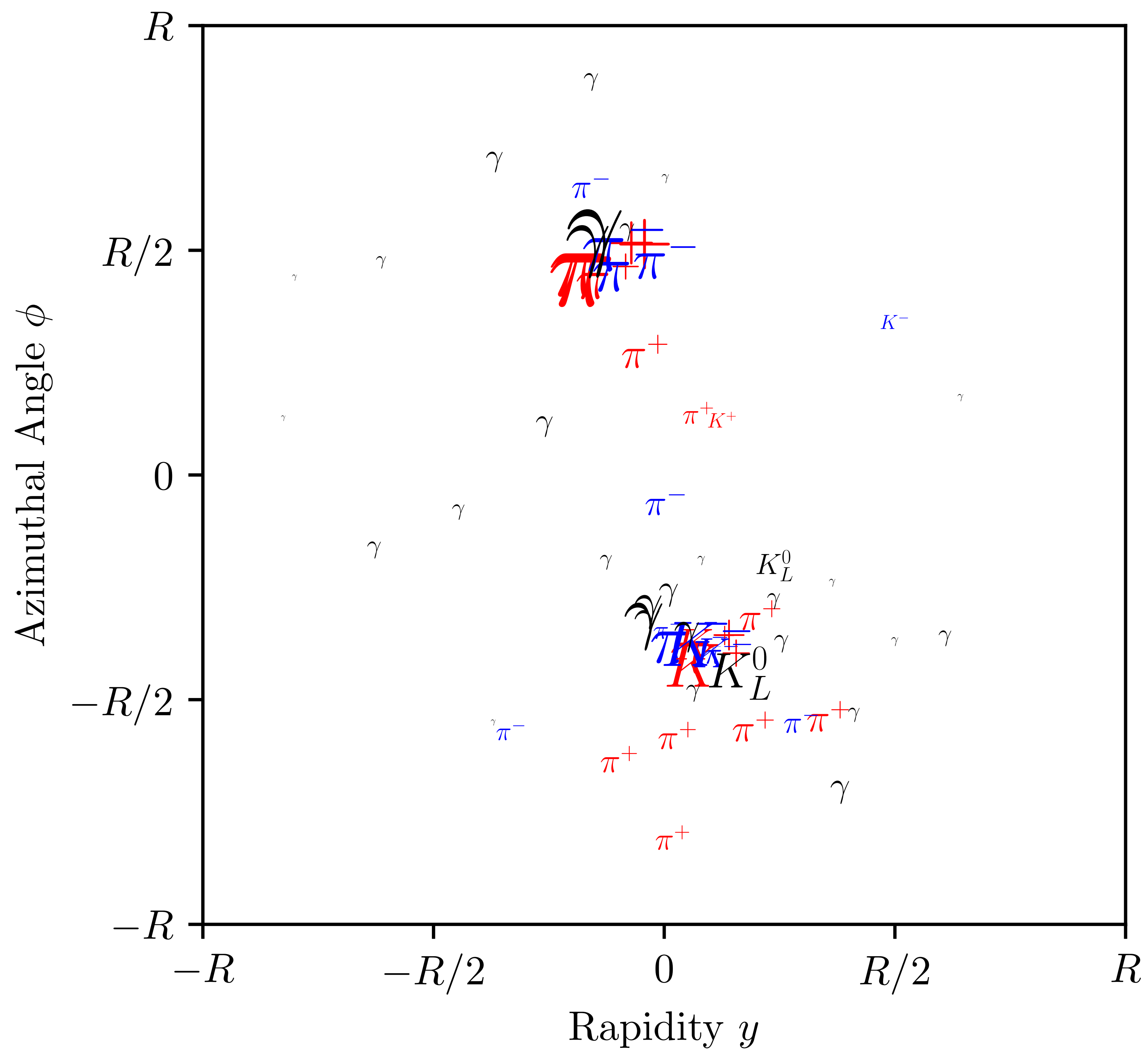
$$\mathcal{E}(\hat{n}) = \sum_{i=1}^{M} E_i\,\delta(\hat{n} - \hat{n}_i)$$

Energy Flow Distribution

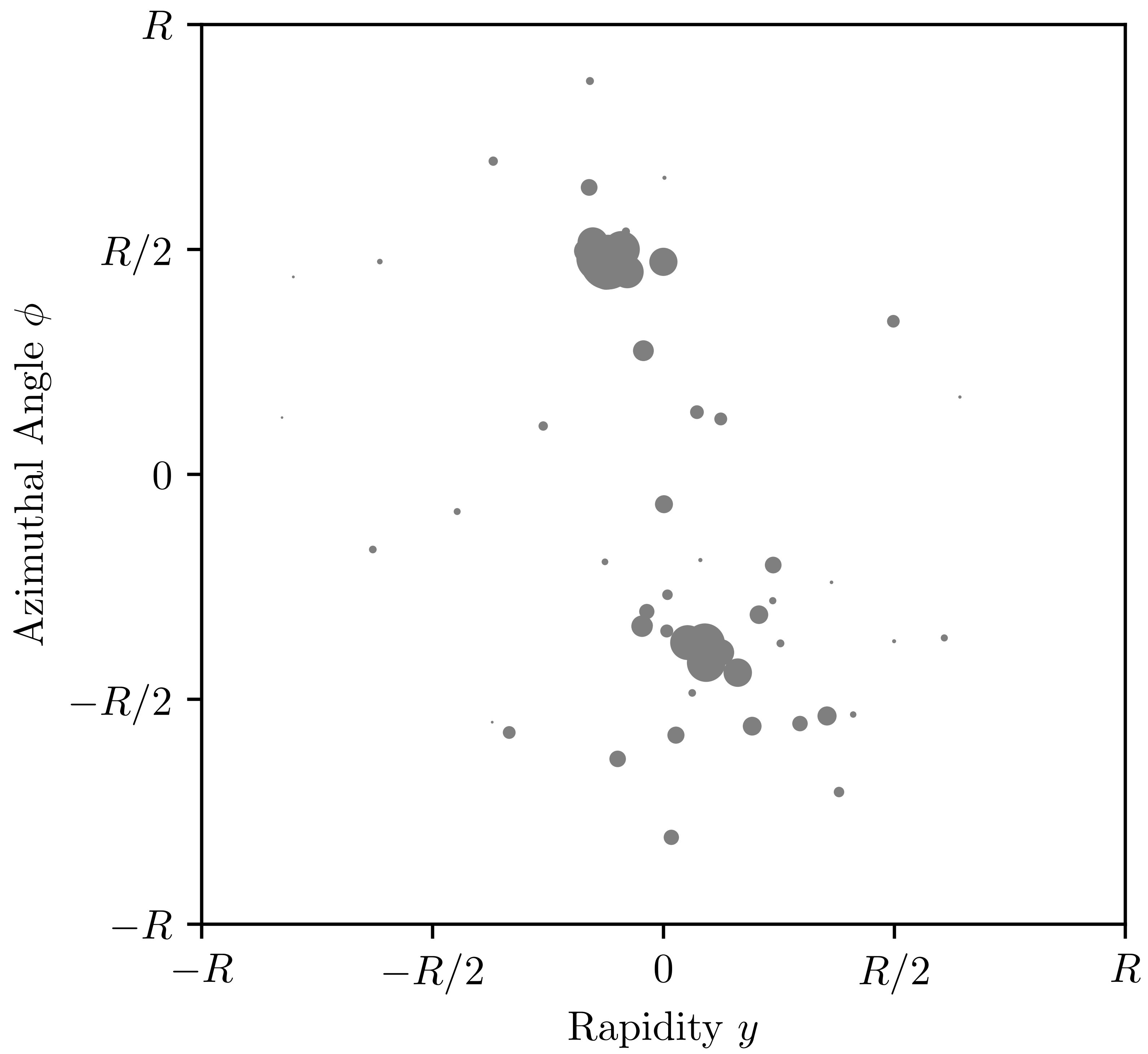
Energy (p_T)

Direction (y,φ)

Full event is a set of particles having momentum and charge/flavor



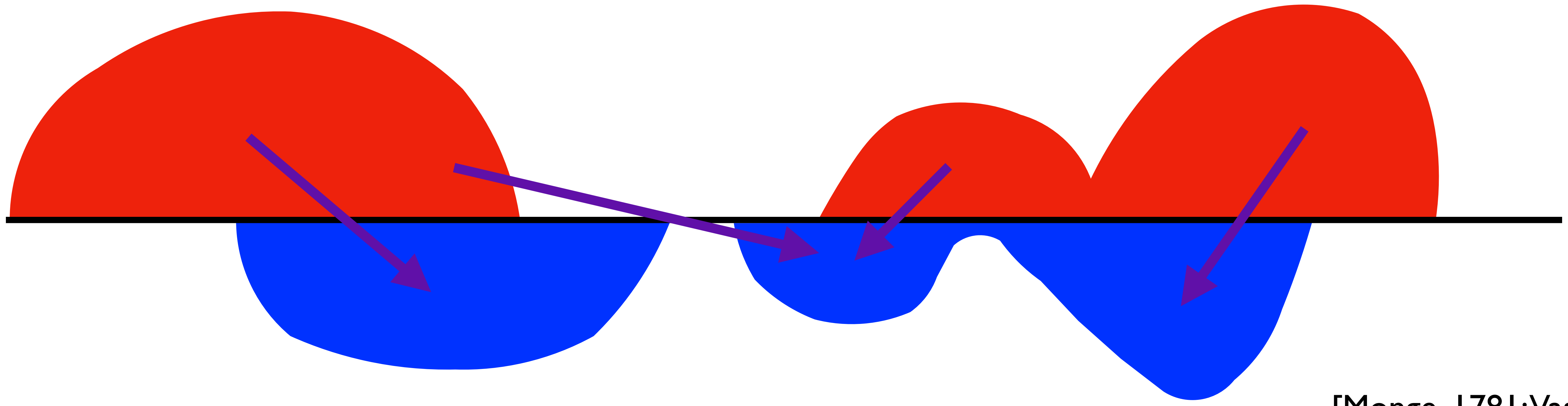
The energy flow is unpixelized and ignores charge/flavor information



Very Basic Question – When are Two Distributions Similar?

Optimal transport minimizes the "work" (stuff x distance) required to transport supply to demand

Supply



Optimal transport cost

Demand

[Monge, 1781; Vaseršteĭn, 1969; Peleg, Werman, Rom, IEEE 1989;
Rubner, Tomasi, Guibas, ICCV 1998, ICJV 2000; Pele, Werman, ECCV 2008; Pele, Taskar, GSI 2013]

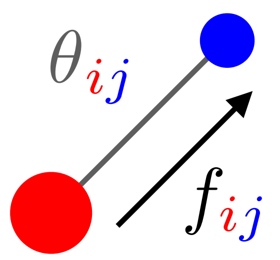
The Energy Mover's Distance (EMD)

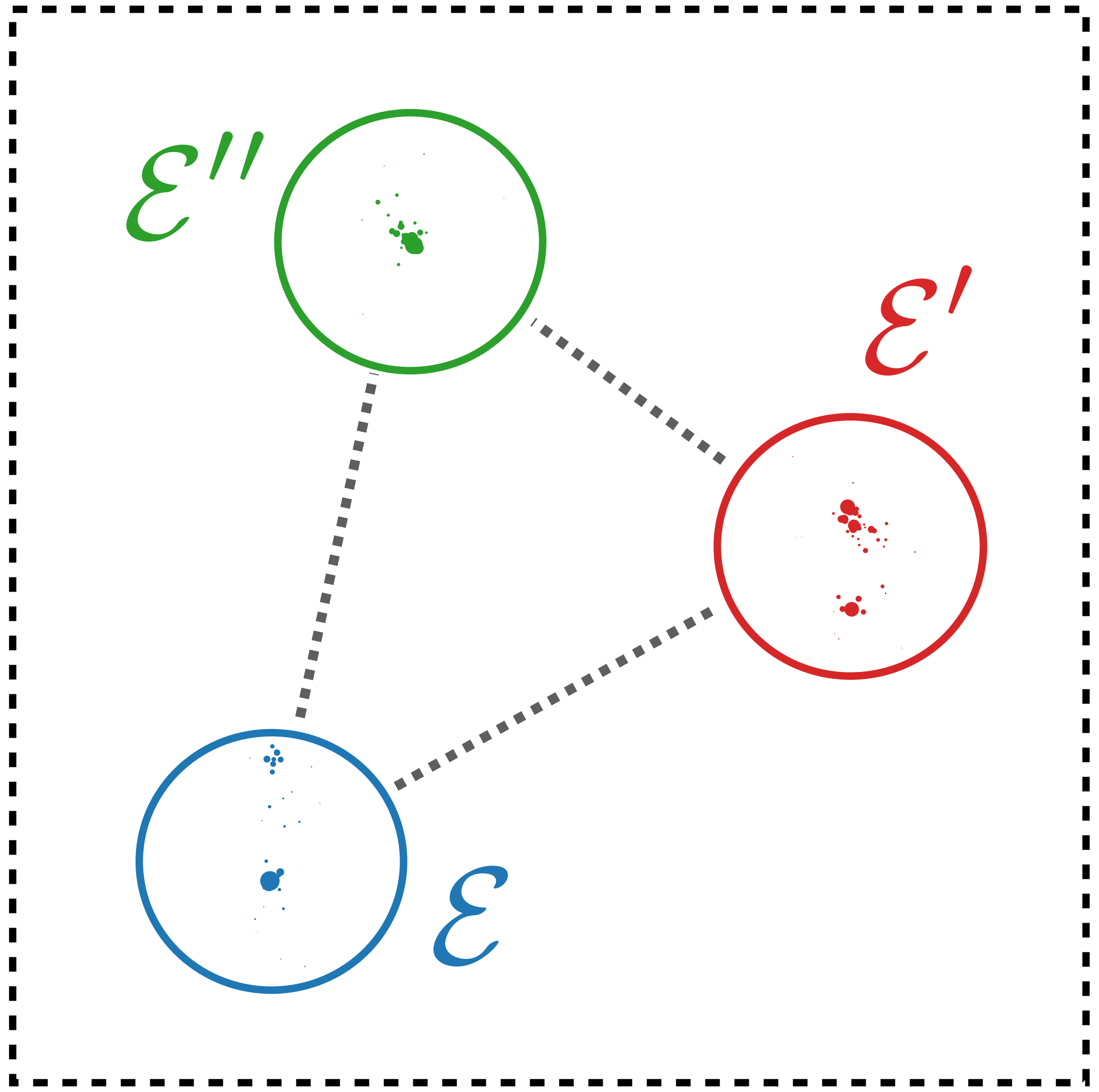
[PTK, Metodiev, Thaler, PRL 2019]

EMD between energy flows defines a metric on the space of events

$$\mathrm{EMD}_{\beta,R}(\mathcal{E},\mathcal{E}') = \underbrace{\min_{\{f_{ij}\geq 0\}} \sum_i \sum_j f_{ij} \left(\frac{\theta_{ij}}{R}\right)^{\beta}}_{\text{Cost of optimal transport}} + \underbrace{\left| \sum_i E_i - \sum_j E'_j \right|}_{\text{Cost of energy creation}}$$

$$\underbrace{\sum_j f_{ij} \leq E_i, \quad \sum_i f_{ij} \leq E'_j, \quad \sum_{ij} f_{ij} = \min\left(\sum_i E_i, \sum_j E'_j\right)}_{\text{Capacity constraints to ensure proper transport}}$$





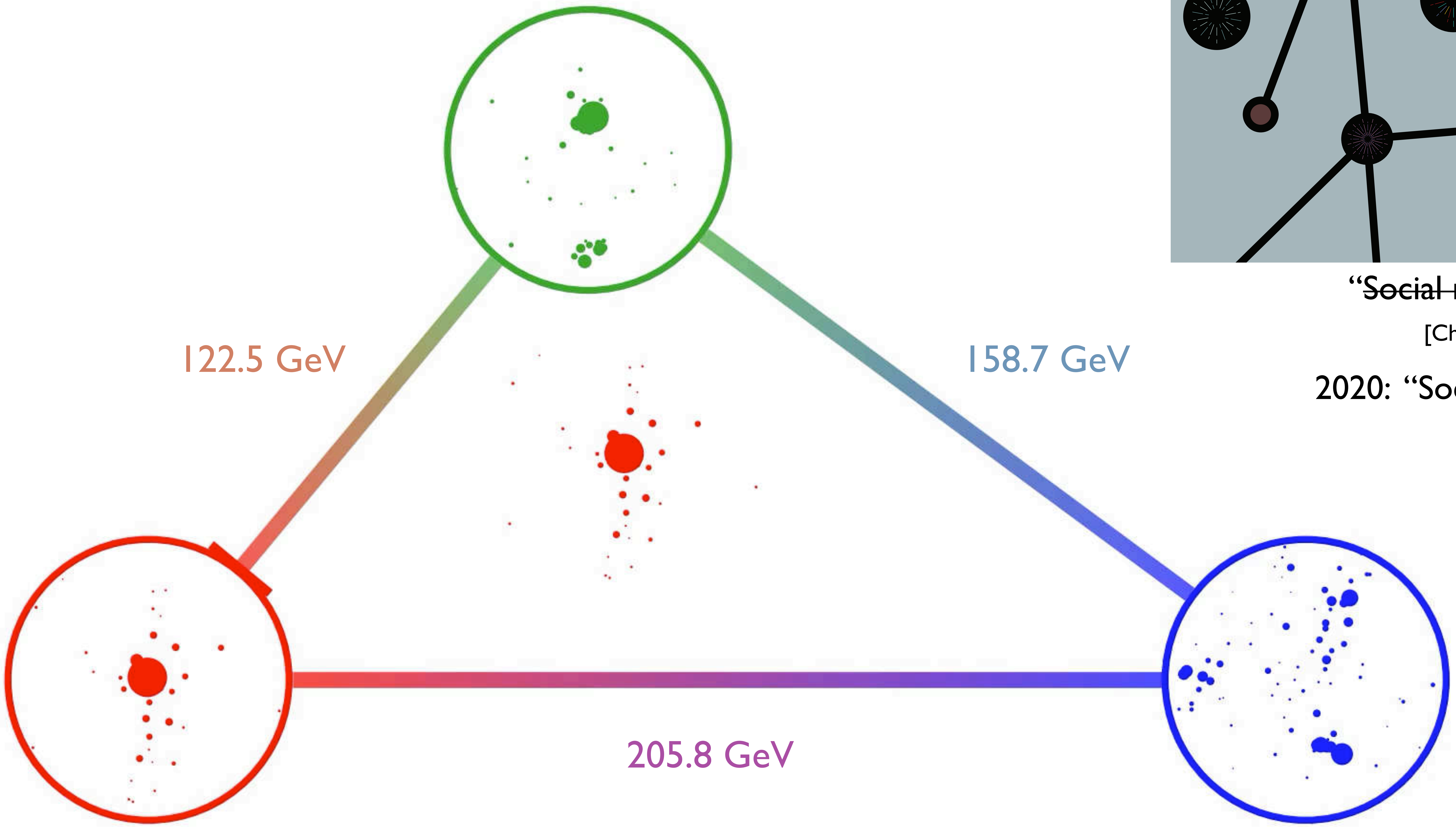
R: controls cost of transporting energy vs. destroying/creating it
β: angular weighting exponent

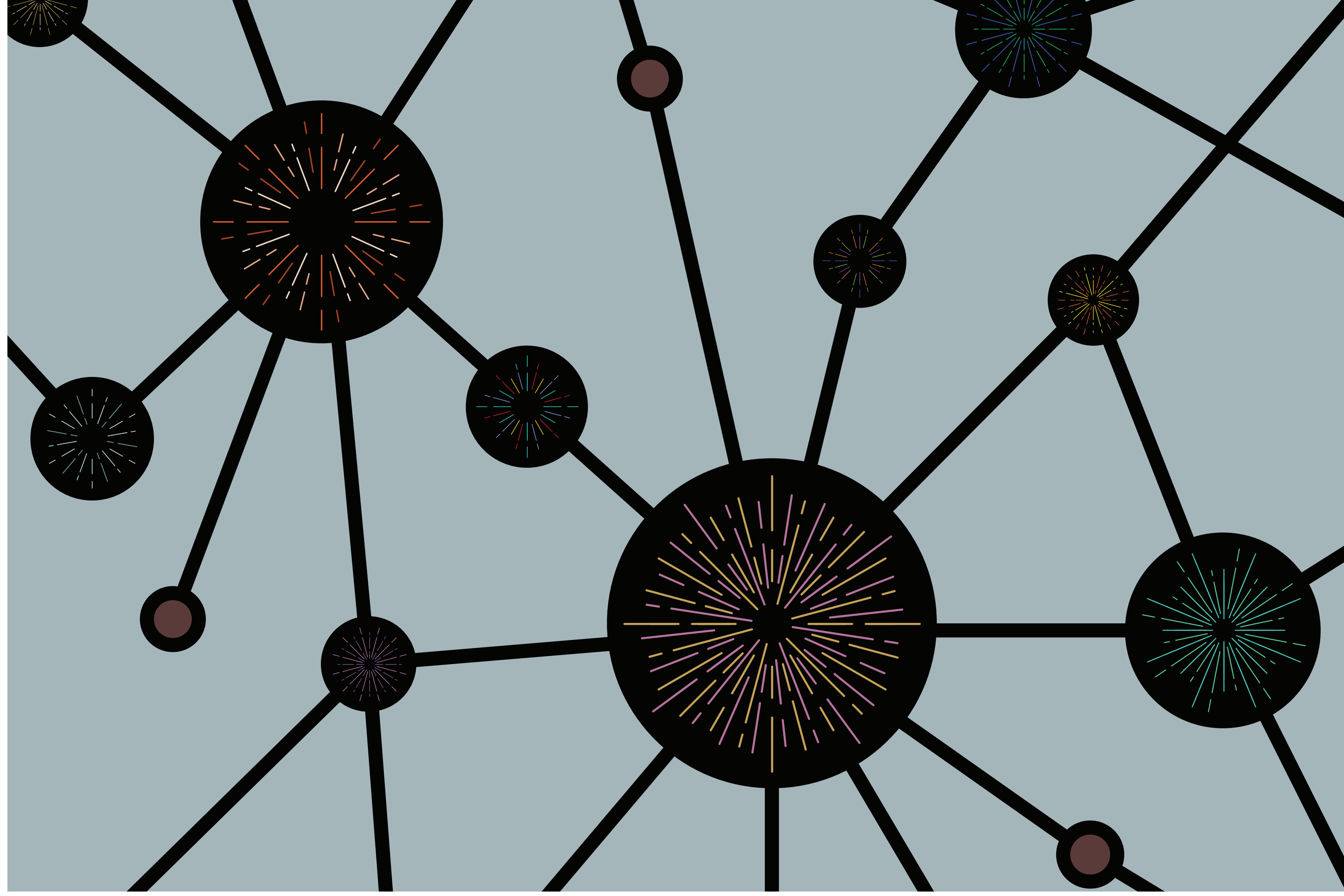
Triangle inequality satisfied for $R \geq d_{\max}/2$

$$0 \leq \mathrm{EMD}(\mathcal{E},\mathcal{E}') \leq \mathrm{EMD}(\mathcal{E},\mathcal{E}'') + \mathrm{EMD}(\mathcal{E}'',\mathcal{E}')$$

i.e. $R \geq$ jet radius for conical jets

Geodesics in the Space of Events

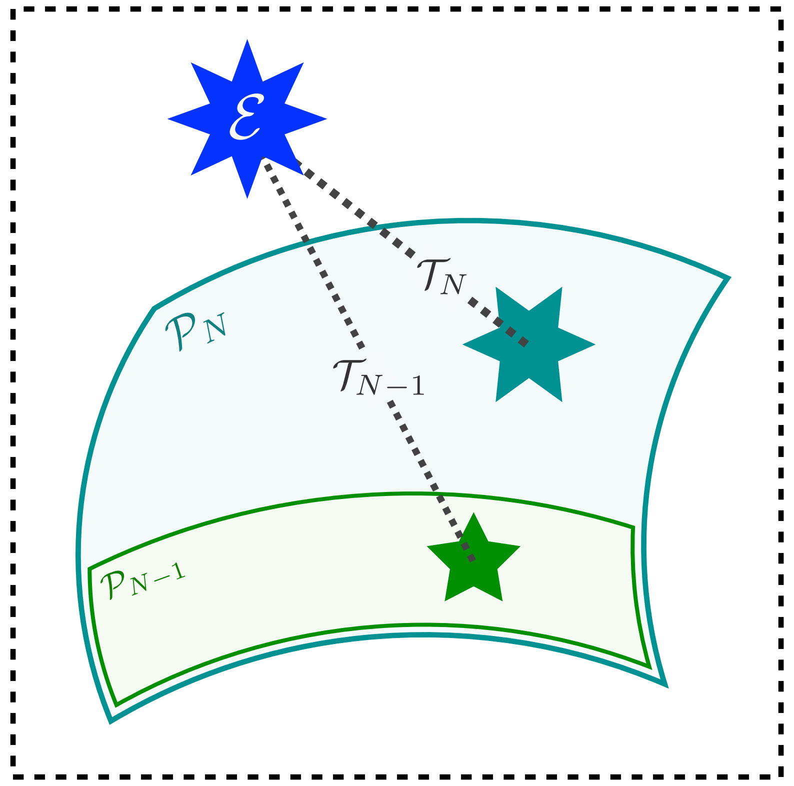




“~~Social networking of jets~~”

[Chu, MIT News 2019]

2020: “Social distancing of jets”



The (Metric) Space of Events

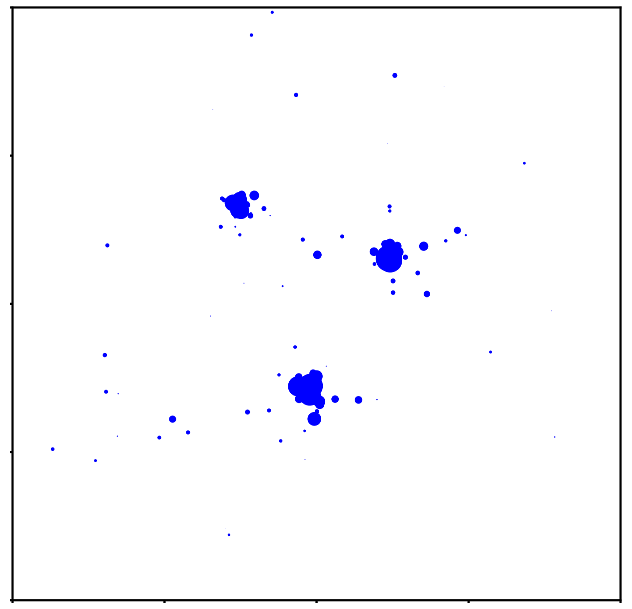
Revealing Hidden Geometry

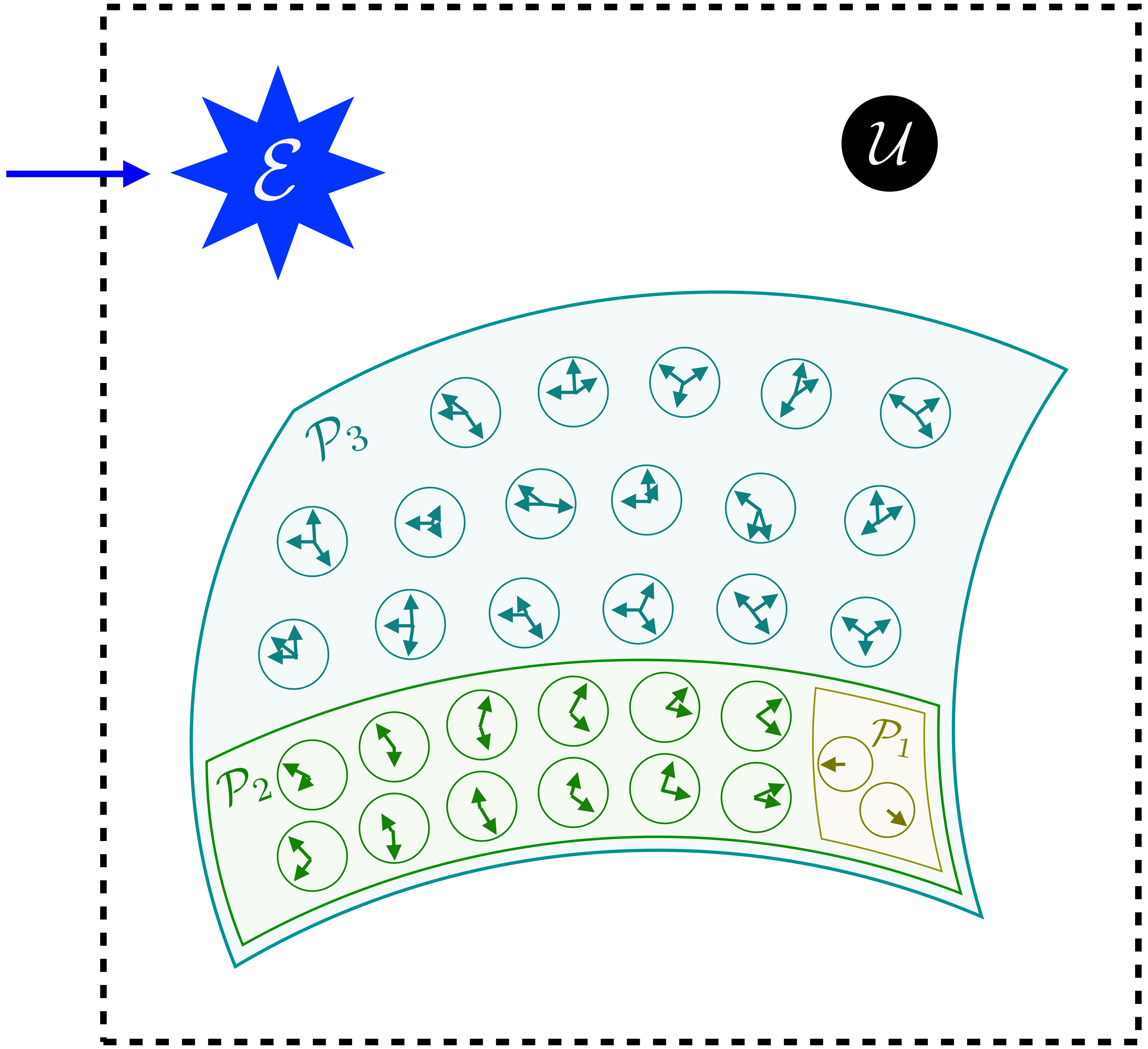
[Theory Space]

N-particle Manifolds in the Space of Events

[PTK, Metodiev, Thaler, JHEP 2020]

$$\mathcal{P}_N = \text{set of all N-particle configurations} = \left\{ \sum_{i=1}^{N} E_i\, \delta(\hat{n} - \hat{n}_i) \;\middle|\; E_i \geq 0 \right\}$$





$\mathcal{P}_1$: manifold of events with one particle

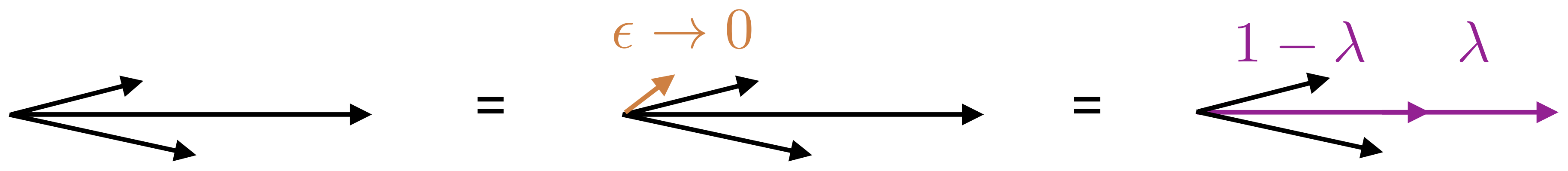
$\mathcal{P}_2$: manifold of events with two particles

$\mathcal{P}_3$: manifold of events with three particles

$\vdots$

$$\mathcal{P}_N \supset \mathcal{P}_{N-1} \supset \cdots \supset \mathcal{P}_3 \supset \mathcal{P}_2 \supset \mathcal{P}_1$$

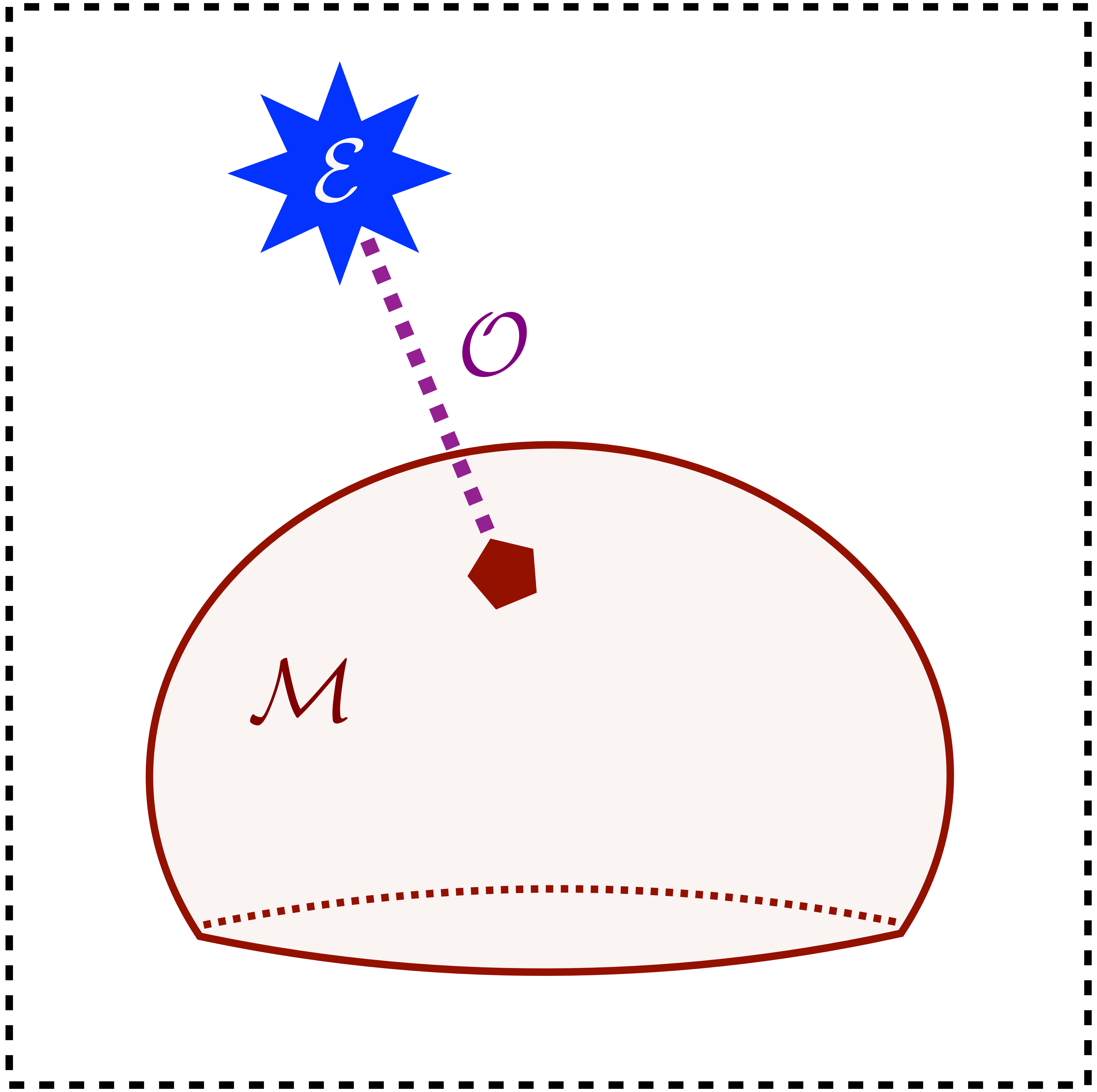
by soft and collinear limits



$\mathcal{U}$ Uniform event, not contained in any P_N

Defining Observables via Event Space Geometry

[PTK, Metodiev, Thaler, JHEP 2020]



Many common observables are distance of closest approach from event to a specific manifold

$$\boxed{\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}'\in\mathcal{M}} \mathrm{EMD}_{\beta,R}(\mathcal{E},\mathcal{E}')}$$

EMD variant for equal-energy events

$$\underbrace{\mathrm{EMD}_{\beta}(\mathcal{E},\mathcal{E}') = \lim_{R\to\infty} R^{\beta}\mathrm{EMD}_{\beta,R}(\mathcal{E},\mathcal{E}')}_{\text{Enforces equal energy (else infinity)}} \underset{\text{on equal-energy events}}{=} \min_{\{f_{ij}\geq 0\}} \sum_{i=1}^{M}\sum_{j=1}^{M'} f_{ij}\theta_{ij}^{\beta}$$

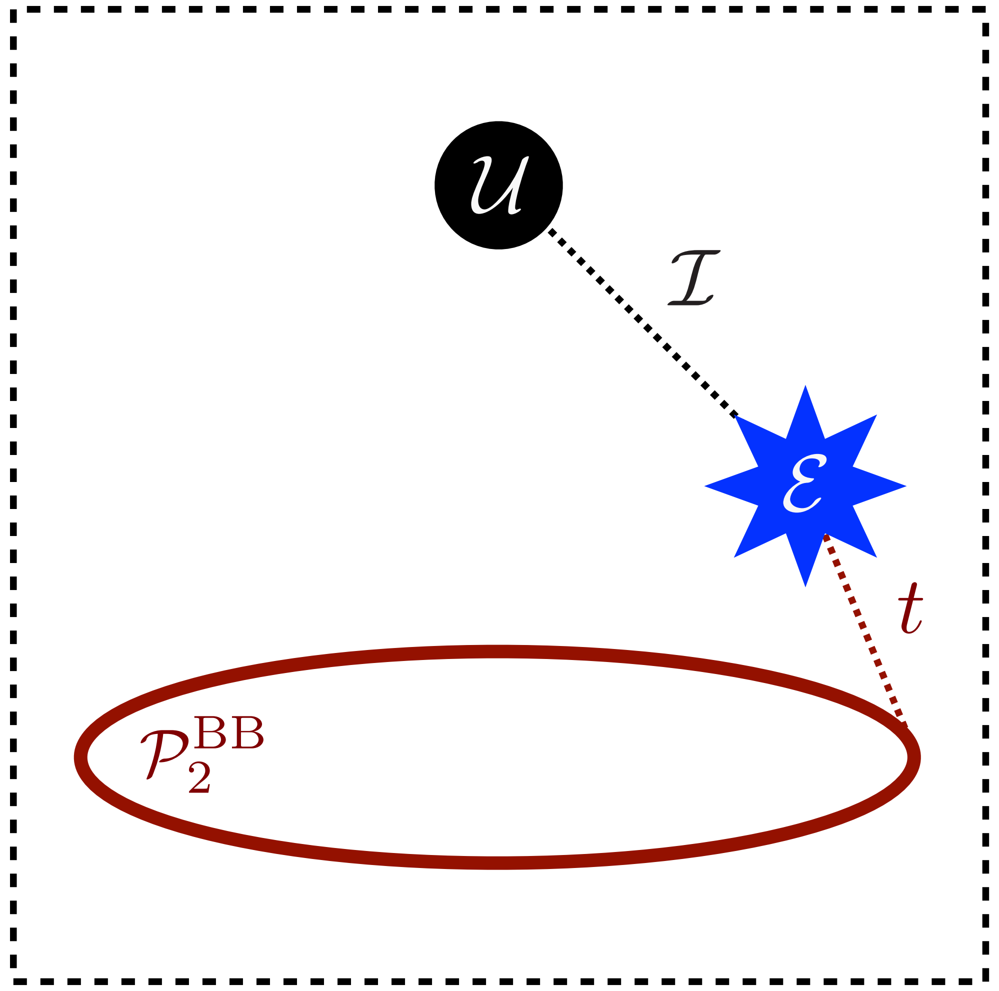
Defining Observables via Event Space Geometry

$$\boxed{\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \mathrm{EMD}_{\beta,R}(\mathcal{E}, \mathcal{E}')}$$

[PTK, Metodiev, Thaler, JHEP 2020]

Thrust, spherocity, isotropy*

Distance of closest approach to a specific manifold



$$t(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{P}_2^{\mathrm{BB}}} \mathrm{EMD}_2(\mathcal{E}, \mathcal{E}')$$

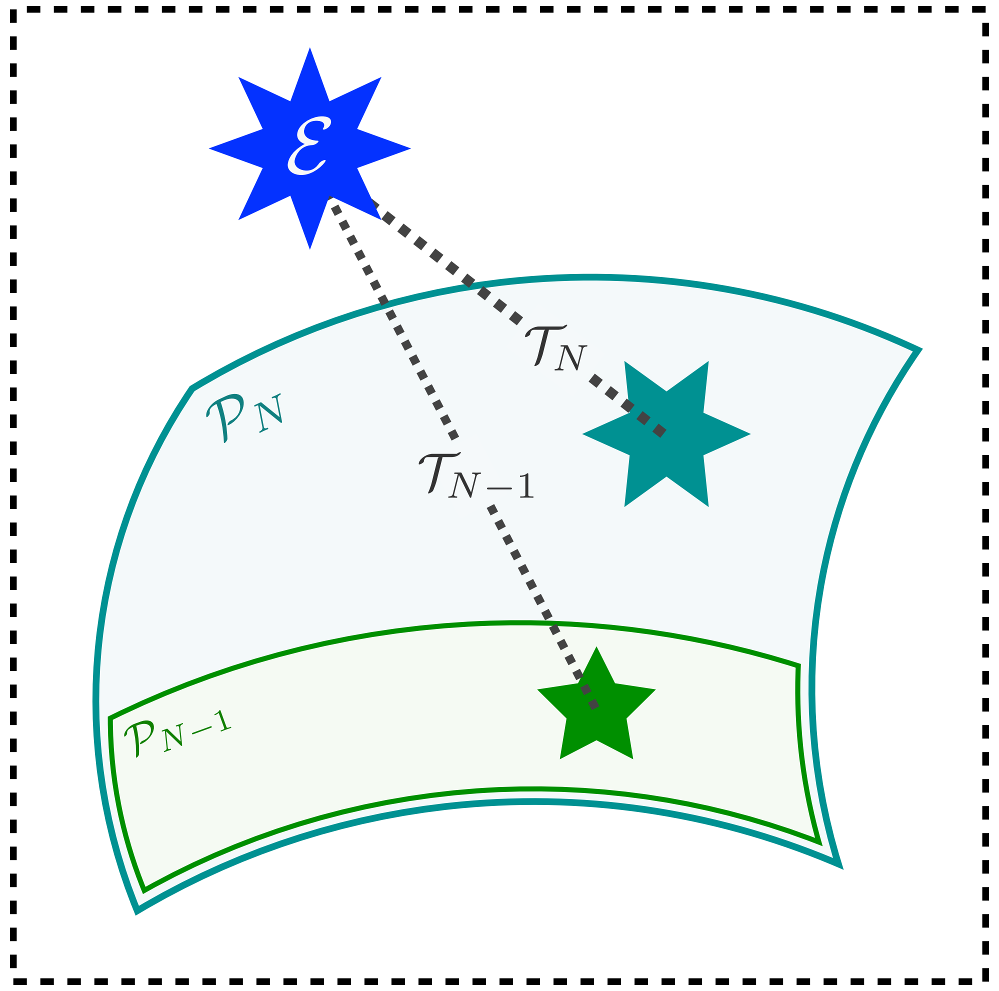
$$\sqrt{s(\mathcal{E})} = \min_{\mathcal{E}' \in \mathcal{P}_2^{\mathrm{BB}}} \mathrm{EMD}_1(\mathcal{E}, \mathcal{E}')$$

$$\mathcal{I}^{(\beta)}(\mathcal{E}) = \min_{\mathcal{E}' \in M_{\mathcal{U}}} \mathrm{EMD}_\beta(\mathcal{E}, \mathcal{E}')$$

[Farhi, PRL 1977; Georgi, Machacek, PRL 1977]
*New! [Cesarotti, Thaler, 2004.06125]

N-jettiness

Minimum distance from event to N-particle manifold



without beam region

$$\mathcal{T}_N^{(\beta)}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{P}_N} \mathrm{EMD}_\beta(\mathcal{E}, \mathcal{E}')$$

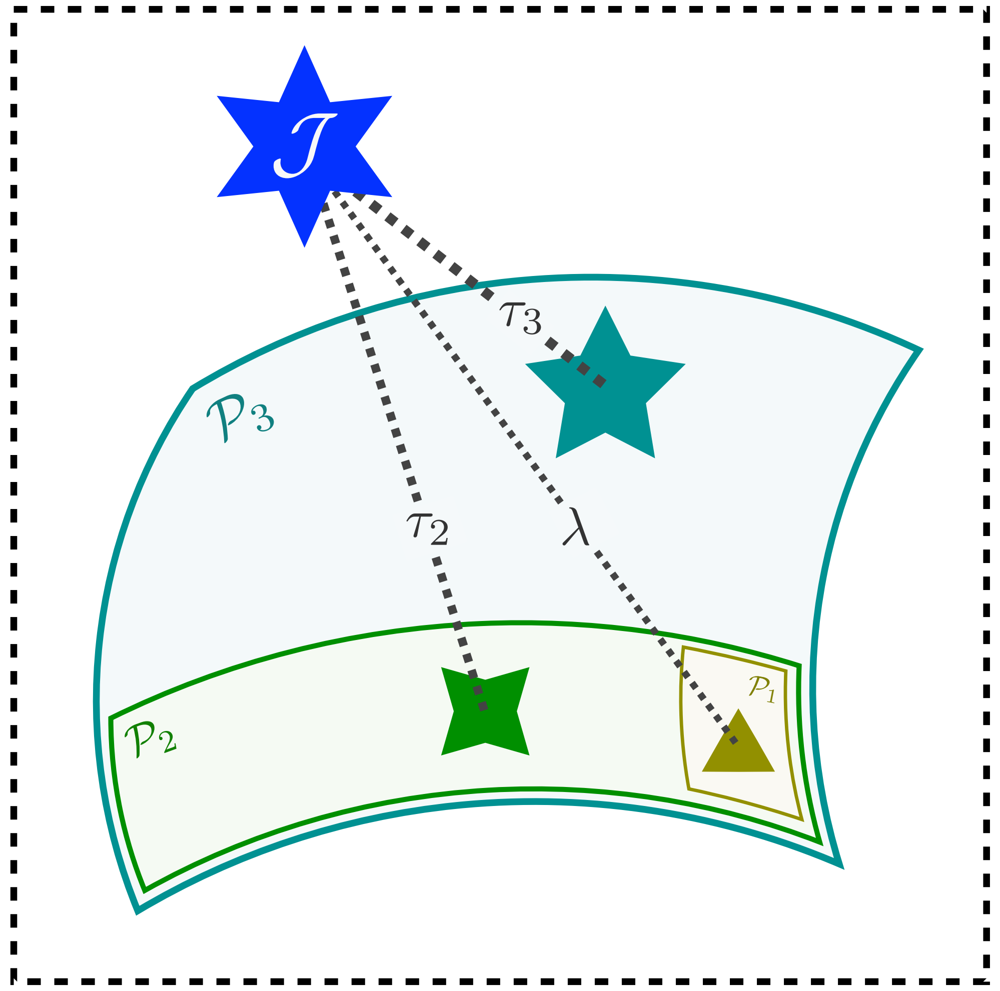
with constant beam distance R^β

$$\mathcal{T}_N^{(\beta,R)}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{P}_N} \mathrm{EMD}_{\beta,R}(\mathcal{E}, \mathcal{E}')$$

[Brandt, Dahmen, Z. Phys 1979;
Stewart, Tackmann, Waalewijn, PRL 2010]

N-subjettiness, angularities

Smallest distance from jet to N-particle manifold



for recoil-free angularity

$$\lambda_\beta(\mathcal{J}) = \min_{\mathcal{J}' \in \mathcal{P}_1} \mathrm{EMD}_\beta(\mathcal{J}, \mathcal{J}')$$

$$\tau_N^{(\beta)}(\mathcal{J}) = \min_{\mathcal{J}' \in \mathcal{P}_N} \mathrm{EMD}_\beta(\mathcal{J}, \mathcal{J}')$$

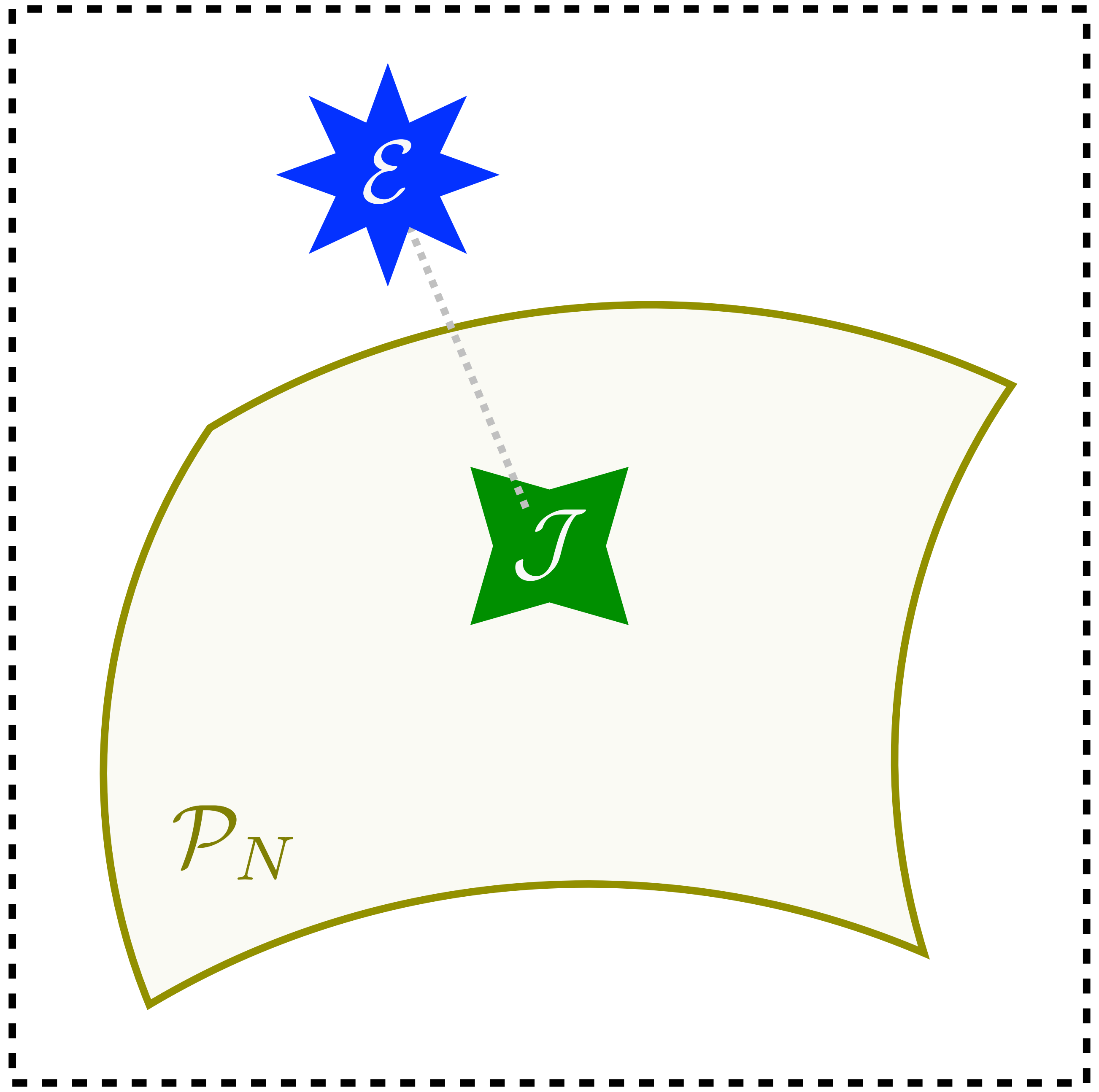
[Ellis, Vermilion, Walsh, Hornig, Lee, JHEP 2010;
Thaler, Van Tilburg, JHEP 2011, JHEP 2012]

Jets in the Space of Events – The Closest N-particle Description of an M-particle Event

[PTK, Metodiev, Thaler, JHEP 2020]

Exclusive cone finding

XCone finds N jets by minimizing N-jettiness

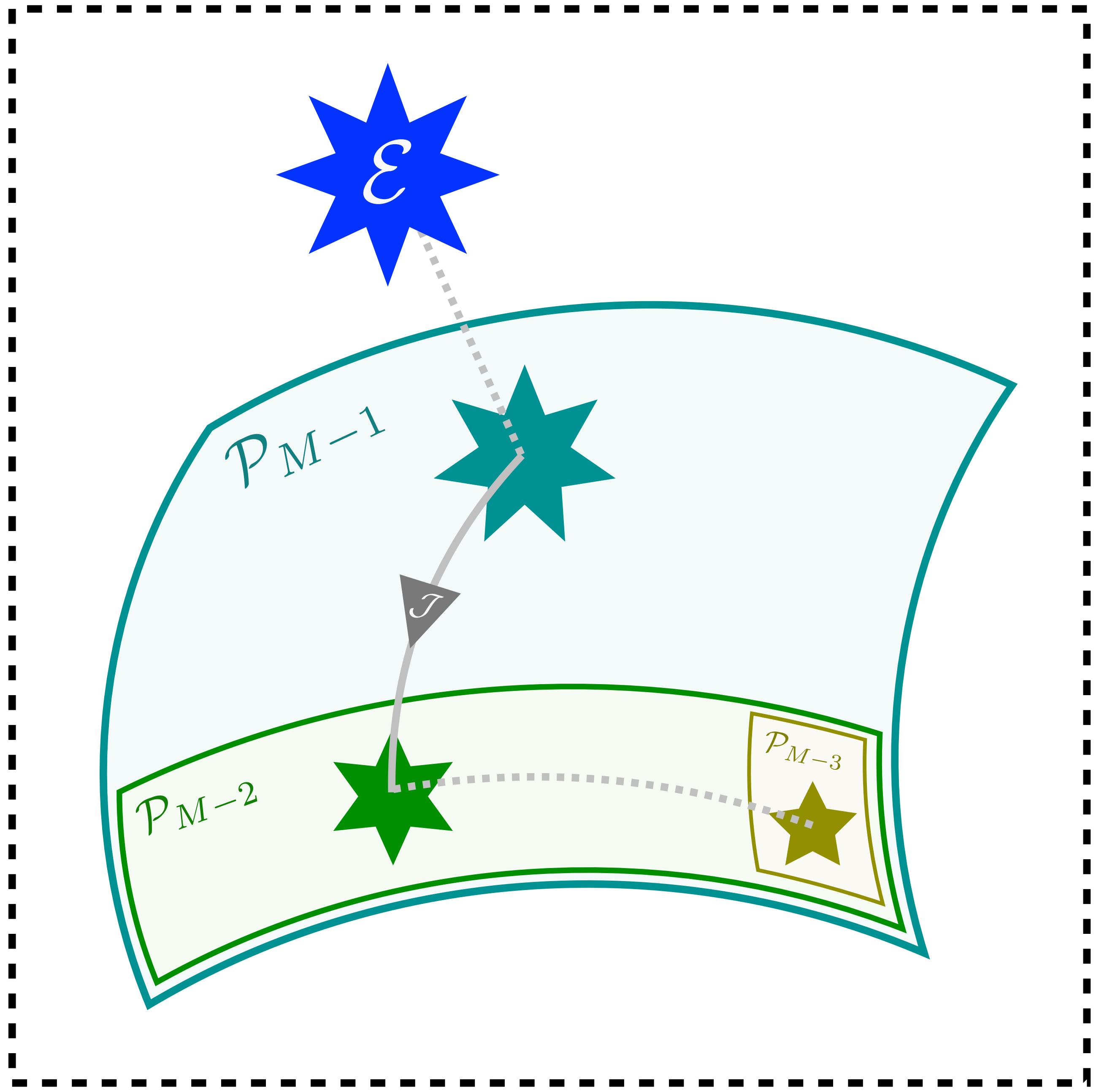


$$\mathcal{J}^{\mathrm{XCone}}_{N,\beta,R}(\mathcal{E}) = \underset{\mathcal{J}\in\mathcal{P}_N}{\arg\min}\, \mathrm{EMD}_{\beta,R}(\mathcal{E},\mathcal{J})$$

[Stewart, Tackmann, Thaler, Vermilion, Wilkason, JHEP 2015; Thaler, Wilkason, JHEP 2015]

Sequential recombination

Iteratively merges particles or identifies a jet



"destroying" energy corresponds to identifying a jet

event with one fewer particle after one step

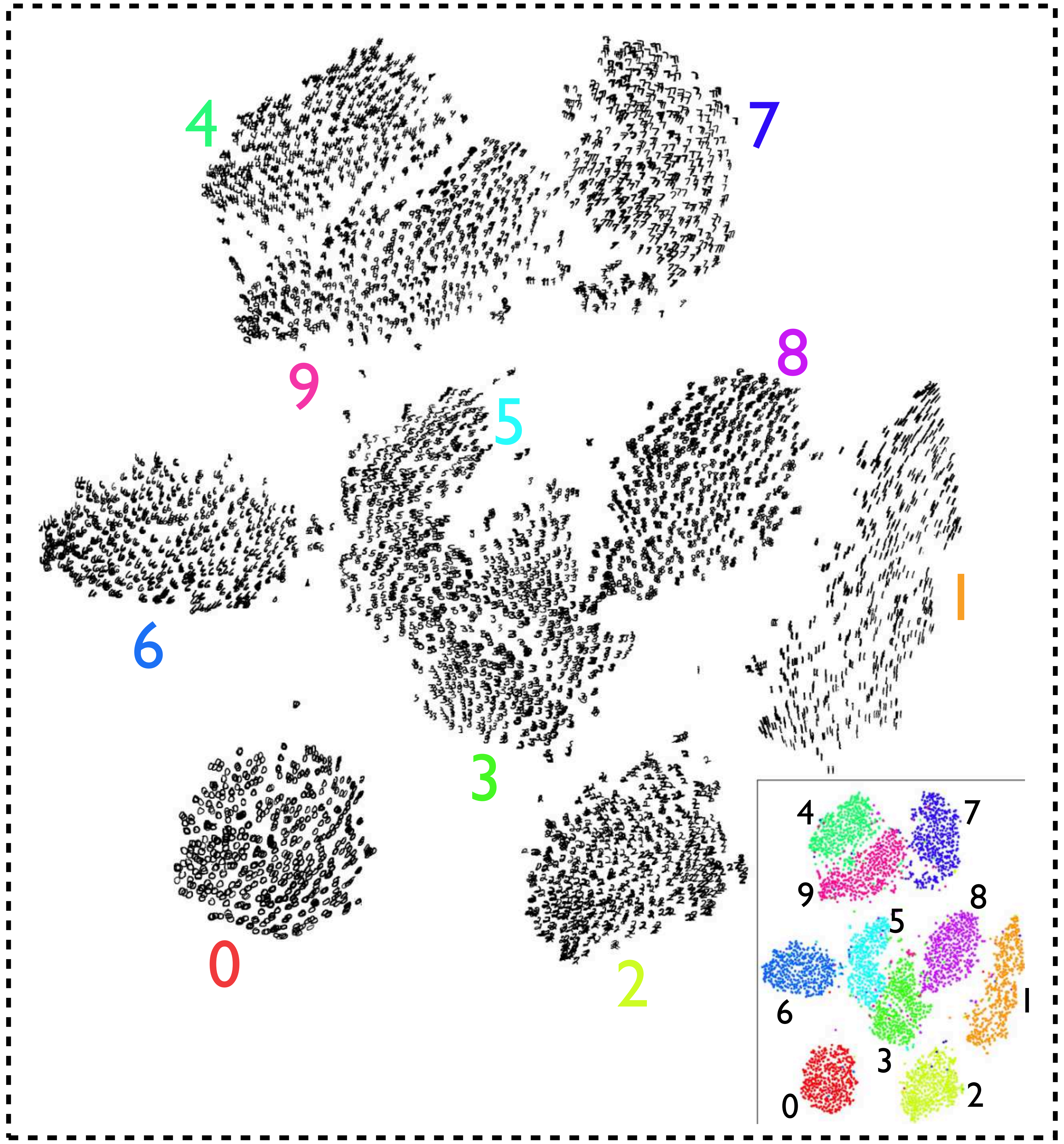
$$\mathcal{E}^{(\beta,R)}_{M-1}(\mathcal{E}_M) = \underset{\mathcal{E}'_{M-1}\in\mathcal{P}_{M-1}}{\arg\min}\, \mathrm{EMD}_{\beta,R}(\mathcal{E}_M,\mathcal{E}'_{M-1})$$

[Catani, Dokshitzer, Seymour, Webber, Nucl. Phys. B 1993; Ellis, Soper, PRD 1993; Dokshitzer, Leder, Moretti, Webber, JHEP 1997; Cacciari, Salam, Soyez, JHEP 2008]

Visualizing Geometry in the Space of Events

t-Distributed Stochastic Neighbor Embedding (t-SNE)

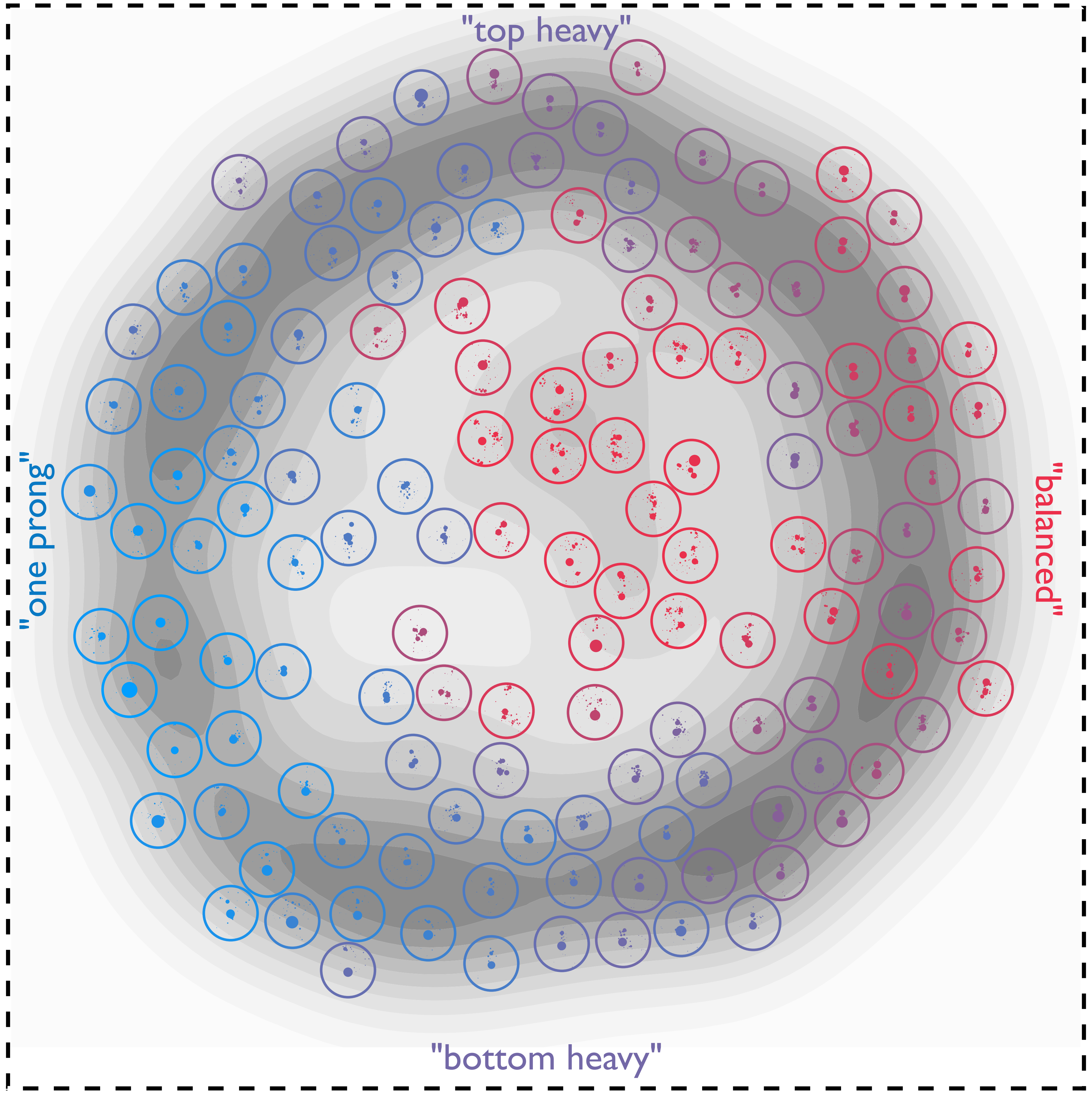
MNIST handwritten digits

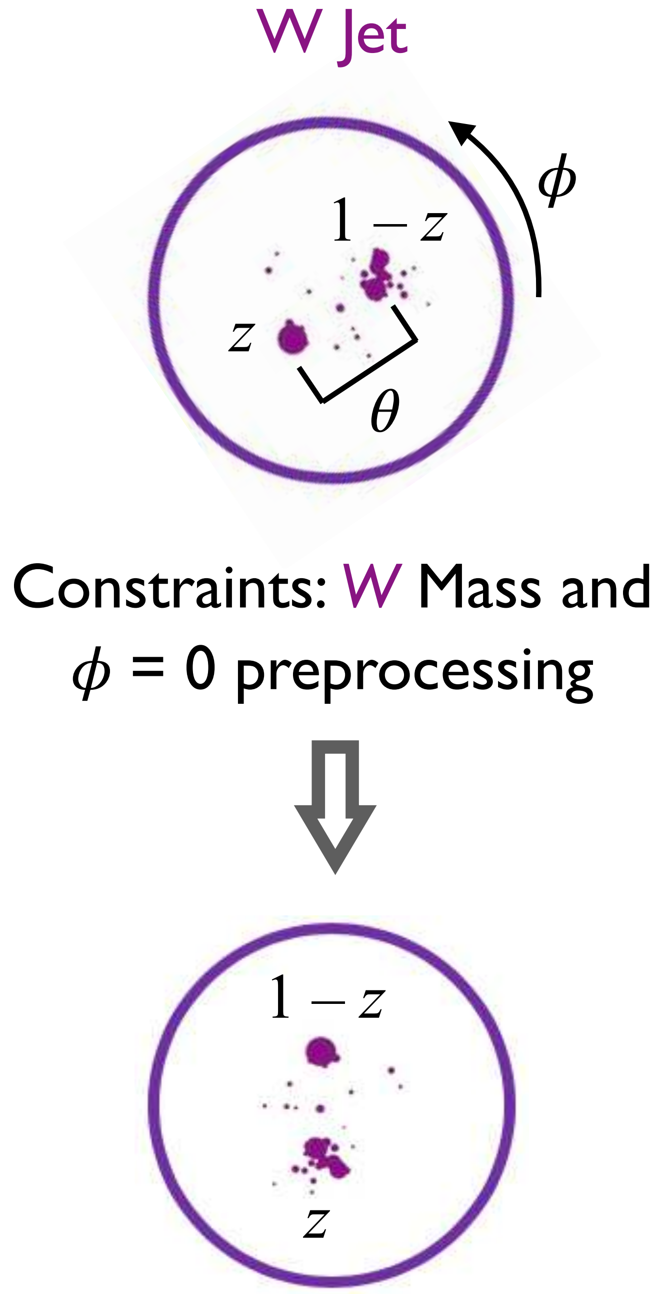


[L. van der Maaten, G. Hinton, JMLR 2008]

[PTK, Metodiev, Thaler, PRL 2019]

Geometric space of W jets



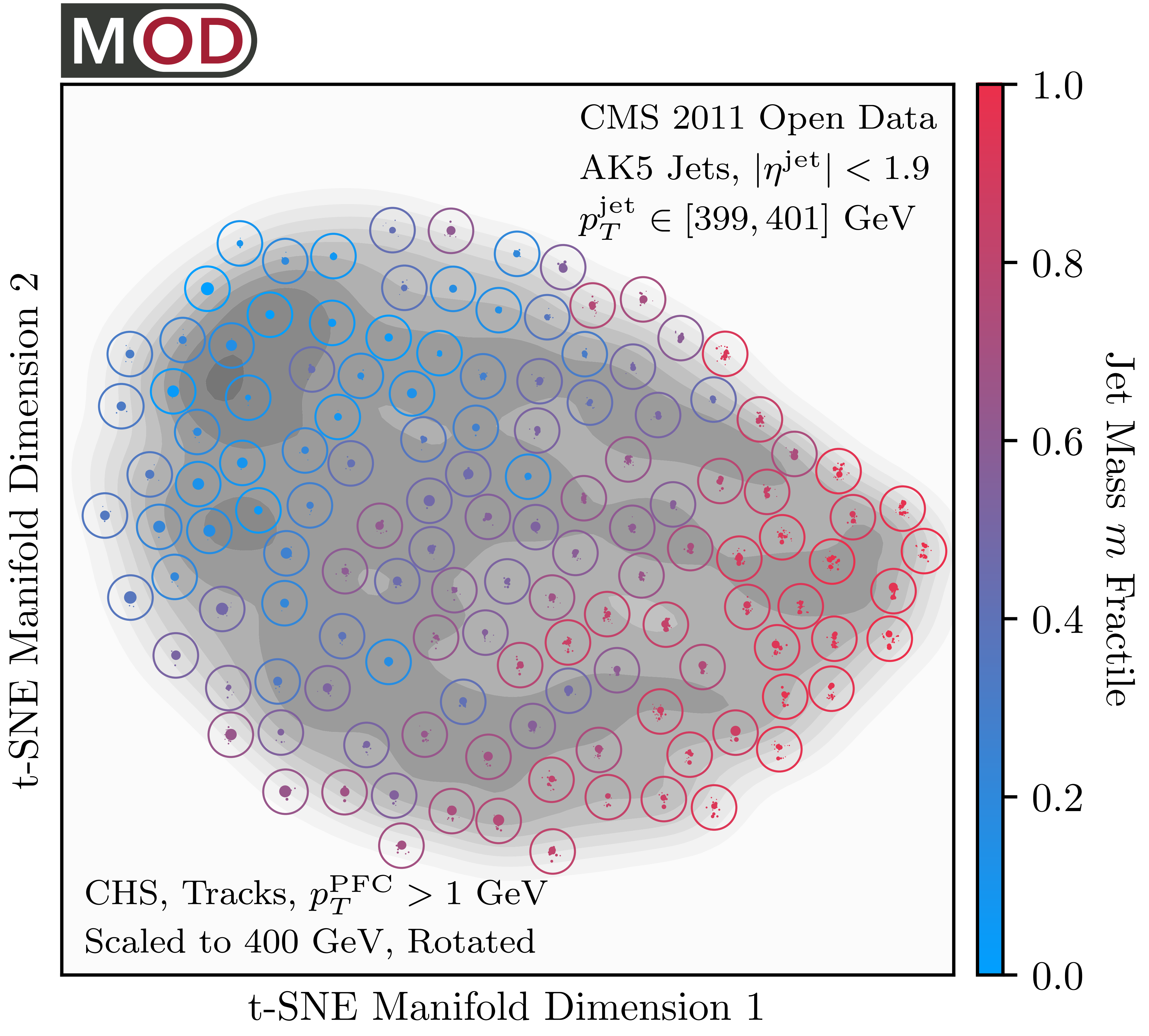


Gray contours represent the density of jets

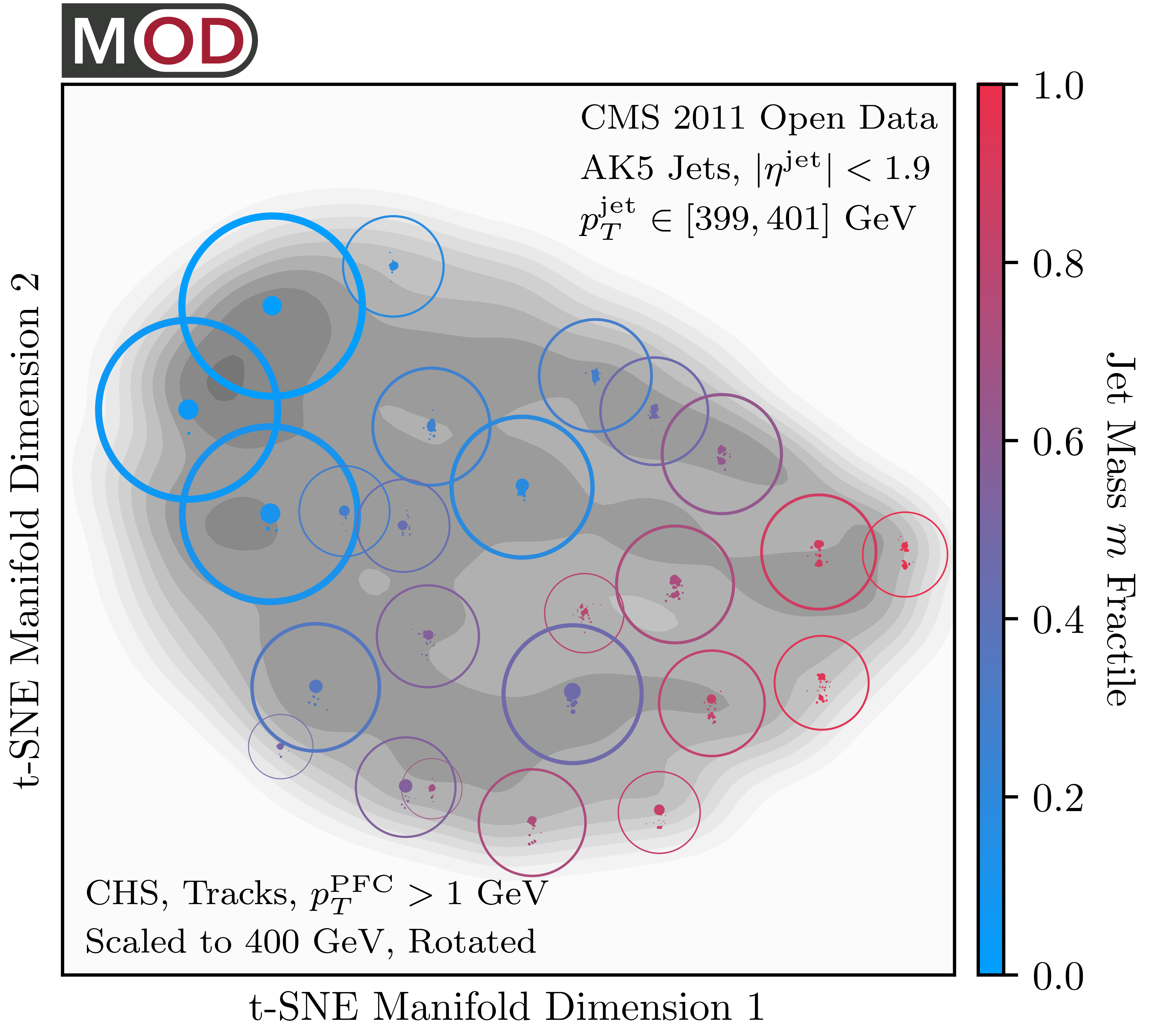
Each circle is a particular *W* jet

Visualizing Geometry in CMS Open Data

[PTK, Mastandrea, Metodiev, Naik, Thaler, PRD 2019; code and datasets at energyflow.network]



Example jets sprinkled throughout

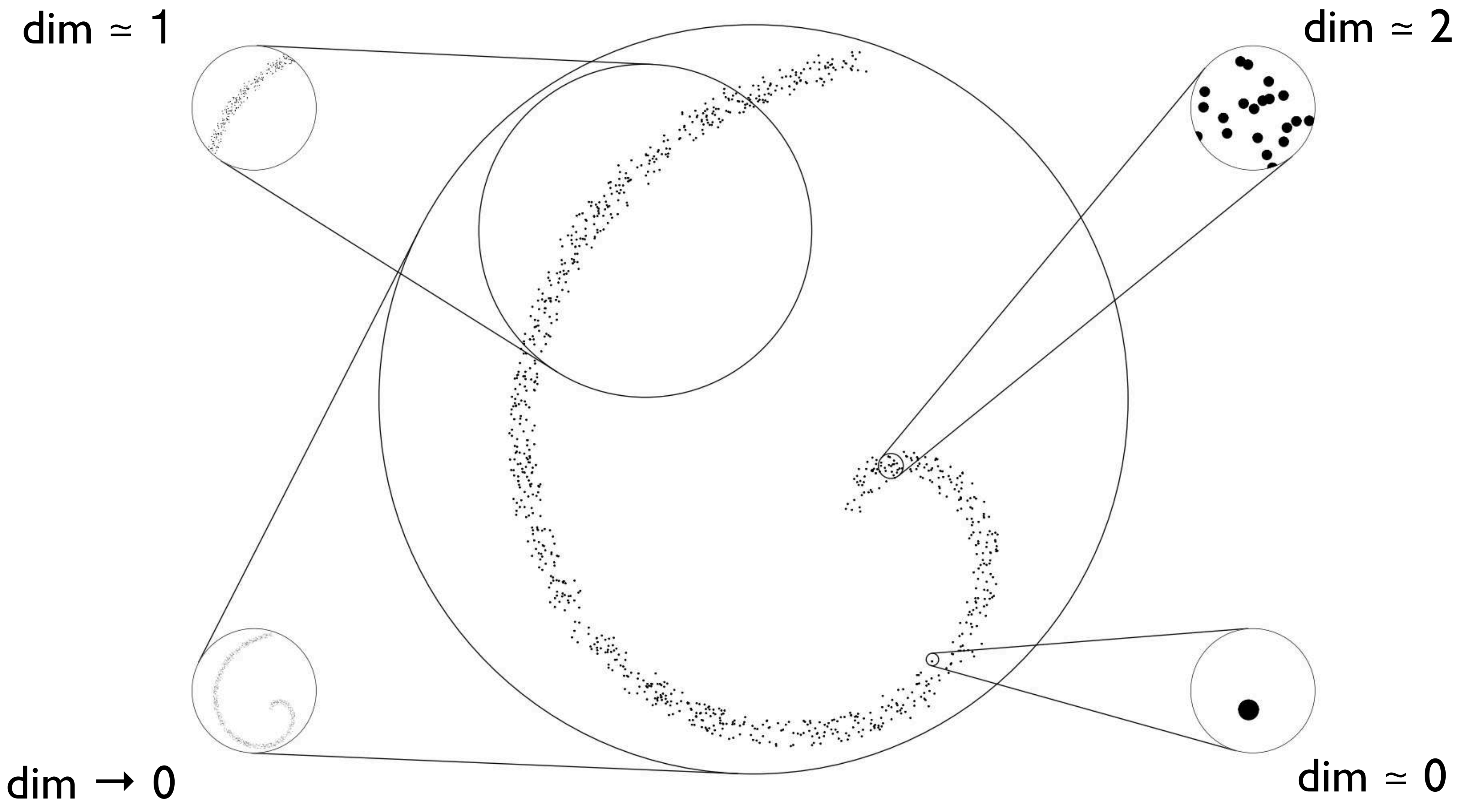


25 most representative jets ("medoids")
Size is proportional to number of jets associated to that medoid

Quantifying Event-Space Manifolds

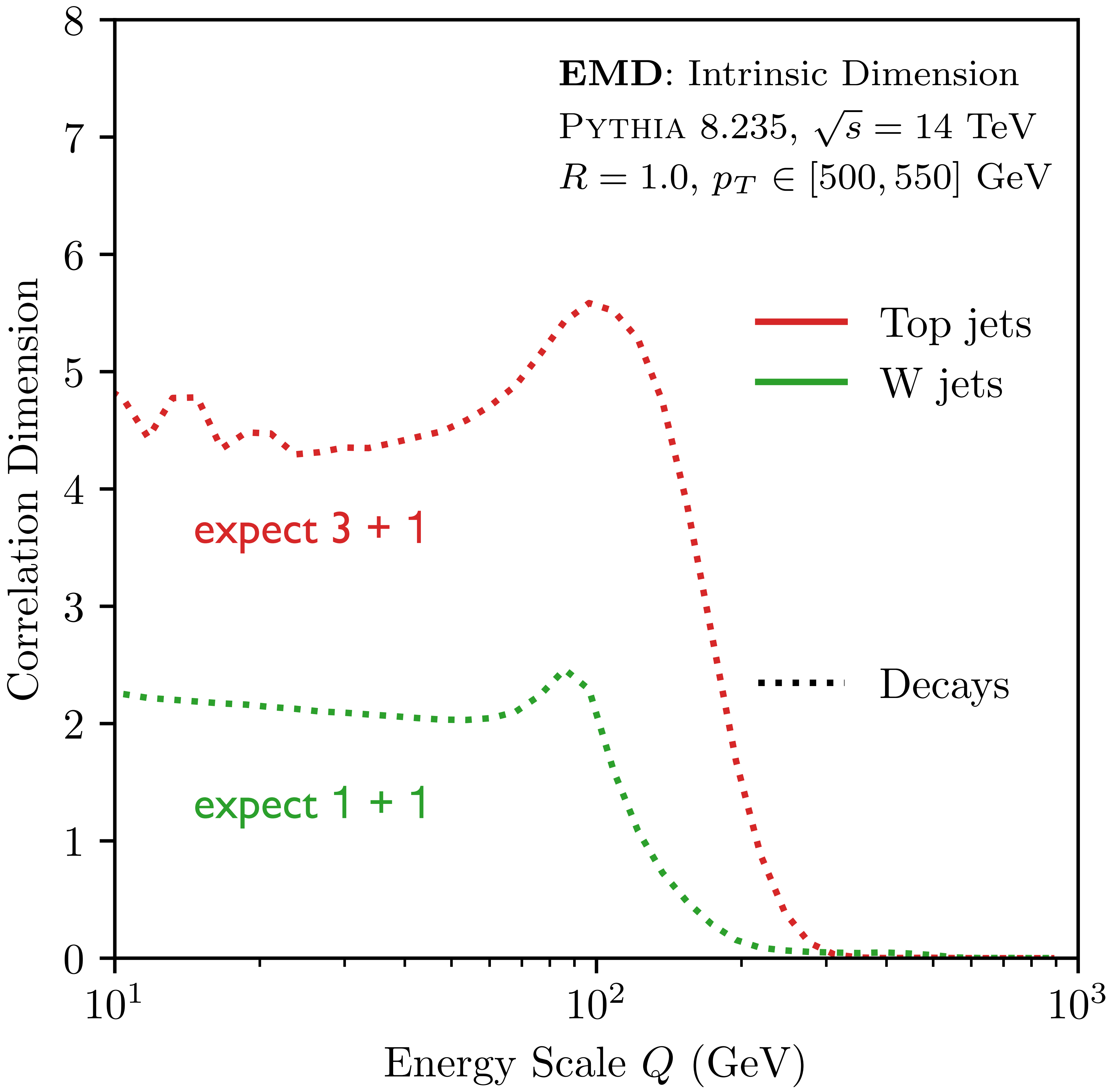
$$\dim(Q) = Q\frac{\partial}{\partial Q}\ln\sum_i\sum_j\Theta(\mathrm{EMD}(\mathcal{E}_i,\mathcal{E}'_j) < Q)$$

Correlation dimension: *how does the # of elements within a ball of size Q change?*



$$N_{\mathrm{neigh.}}(Q) \propto Q^{\dim} \implies \dim(Q) = Q\frac{d}{dQ}\ln N_{\mathrm{neigh.}}(Q)$$

Correlation dimension lessons:
Decays are "constant" dim. at low Q

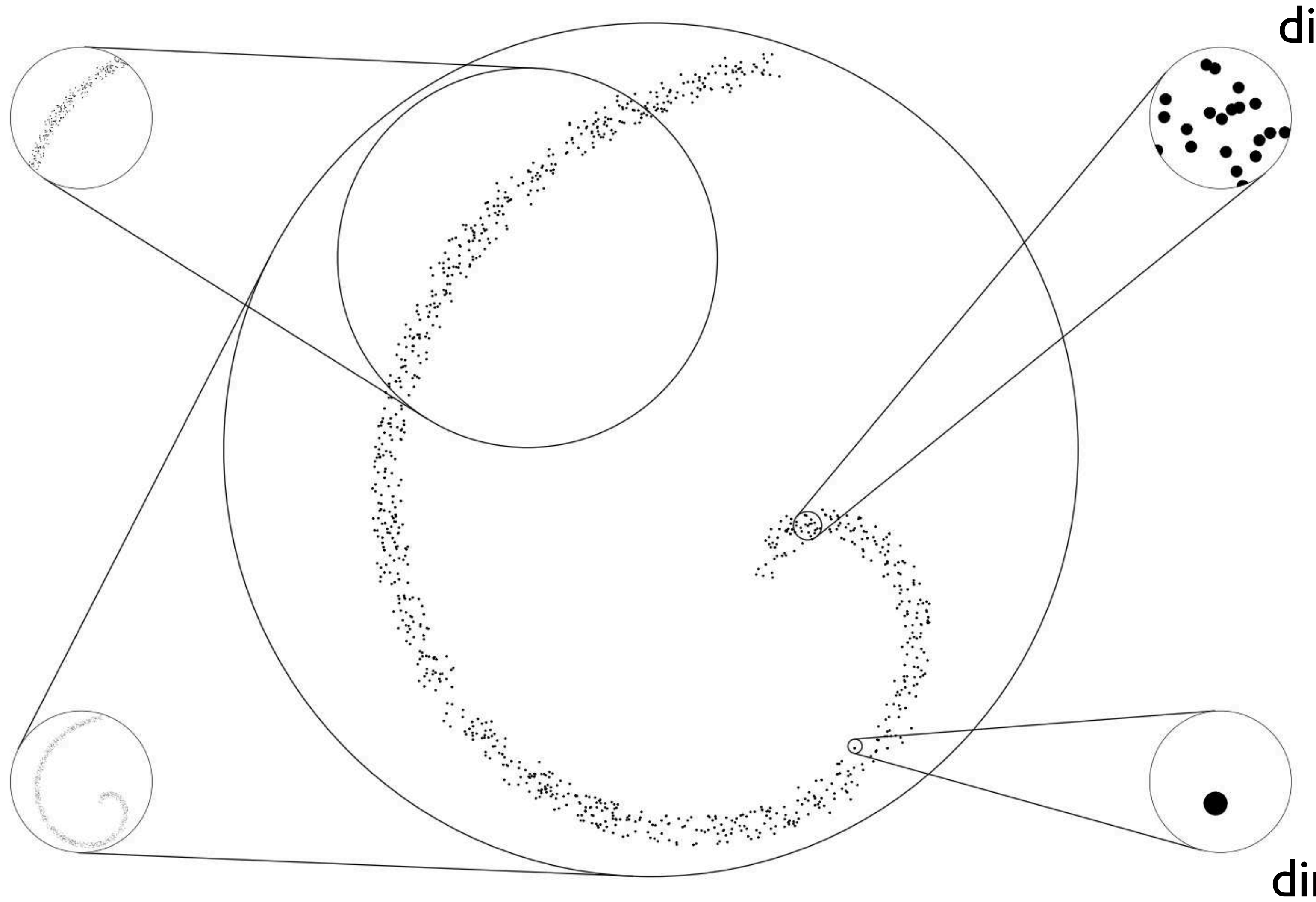


[Grassberger, Procaccia, PRL 1983; PTK, Metodiev, Thaler, PRL 2019]

*Preliminary LL calculation for QG jets in backup

Quantifying Event-Space Manifolds

Correlation dimension: *how does the # of elements within a ball of size Q change?*

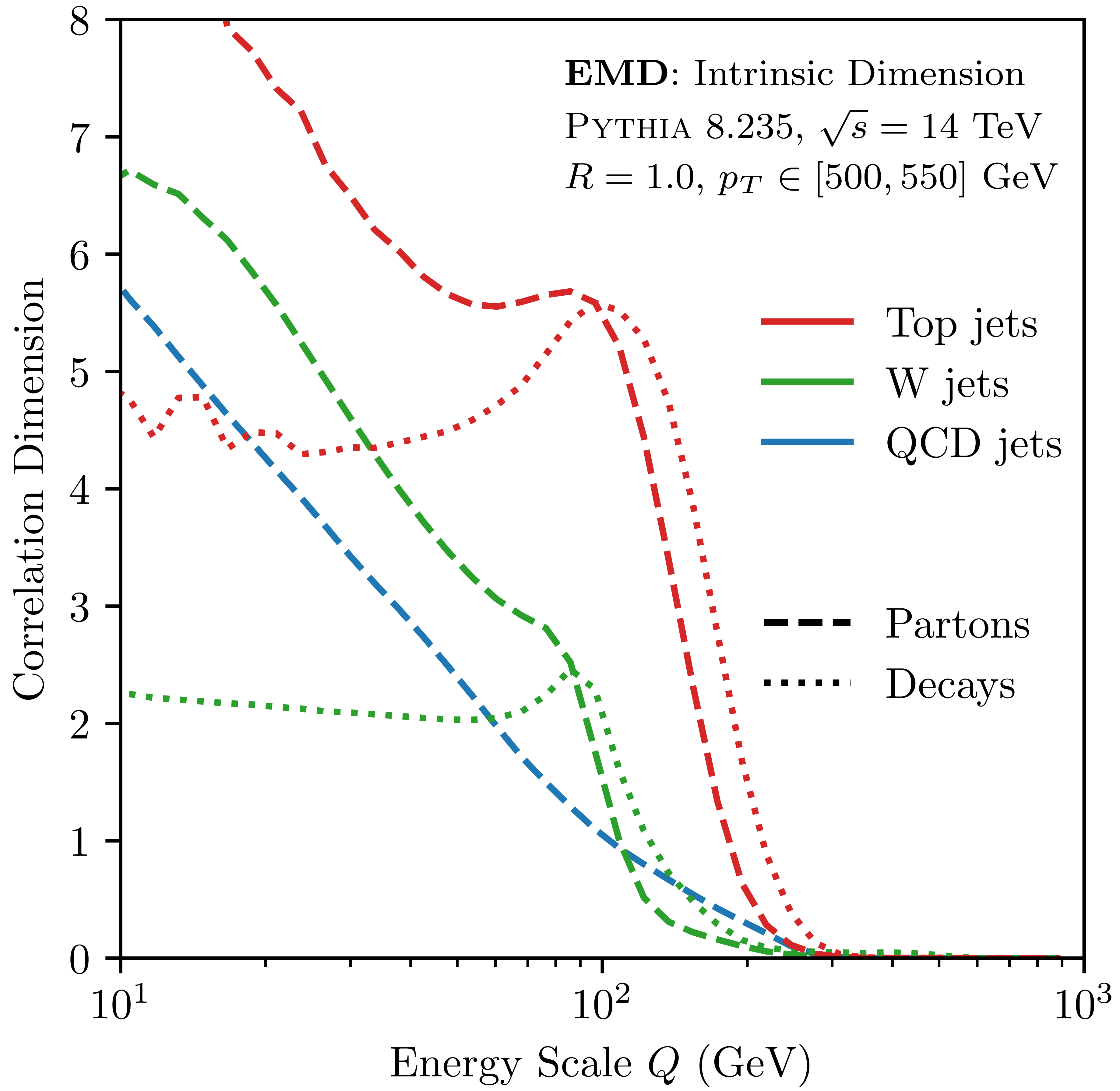


$$N_{\text{neigh.}}(Q) \propto Q^{\text{dim}} \implies \dim(Q) = Q\frac{d}{dQ}\ln N_{\text{neigh.}}(Q)$$

Correlation dimension lessons:

- Decays are "constant" dim. at low Q
- Complexity hierarchy: QCD < W < Top
- Fragmentation increases dim. at smaller scales

$$\boxed{\dim(Q) = Q\frac{\partial}{\partial Q}\ln\sum_i\sum_j\Theta(\text{EMD}(\mathcal{E}_i,\mathcal{E}'_j) < Q)}$$



[Grassberger, Procaccia, PRL 1983; PTK, Metodiev, Thaler, PRL 2019]

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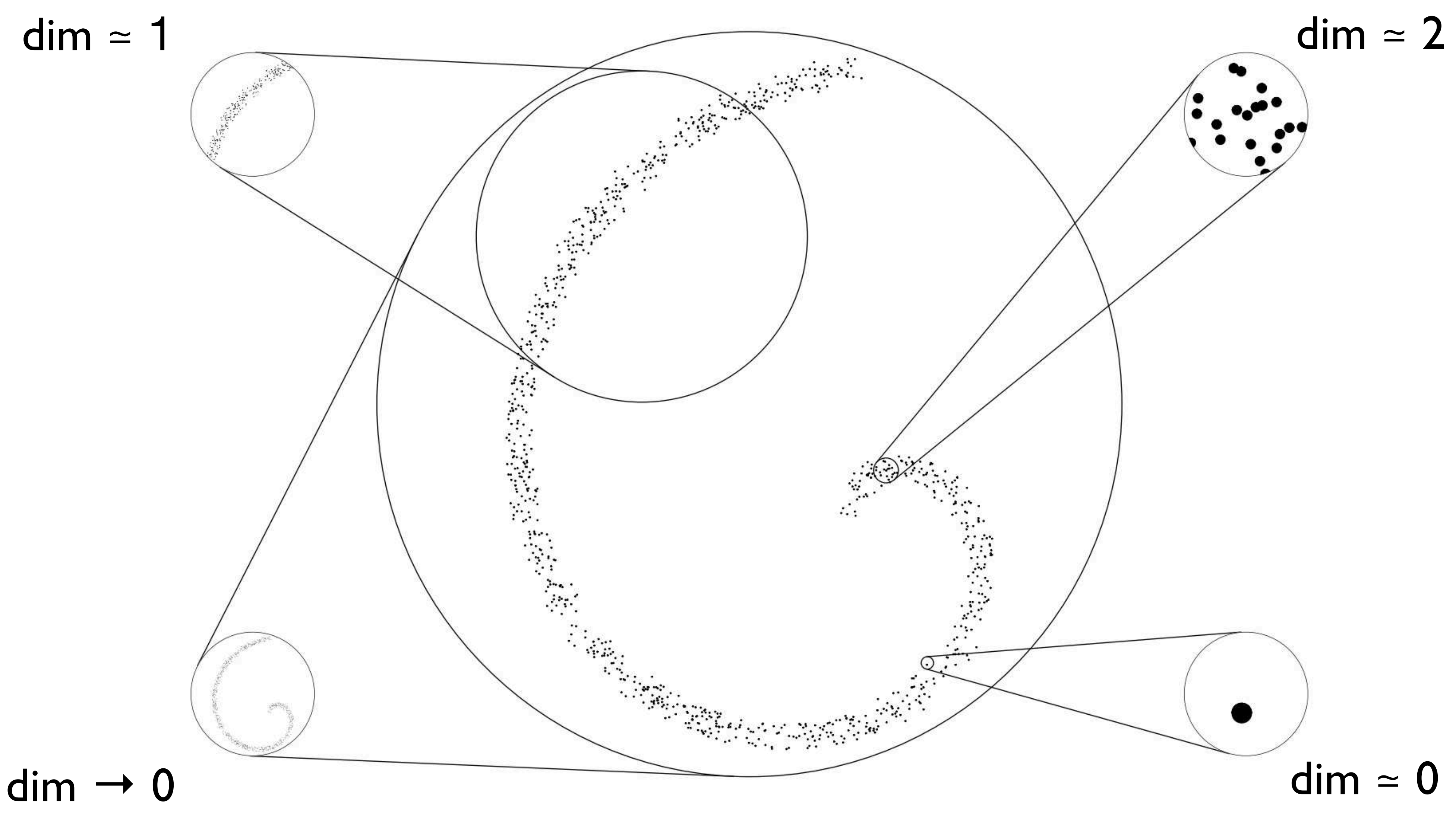

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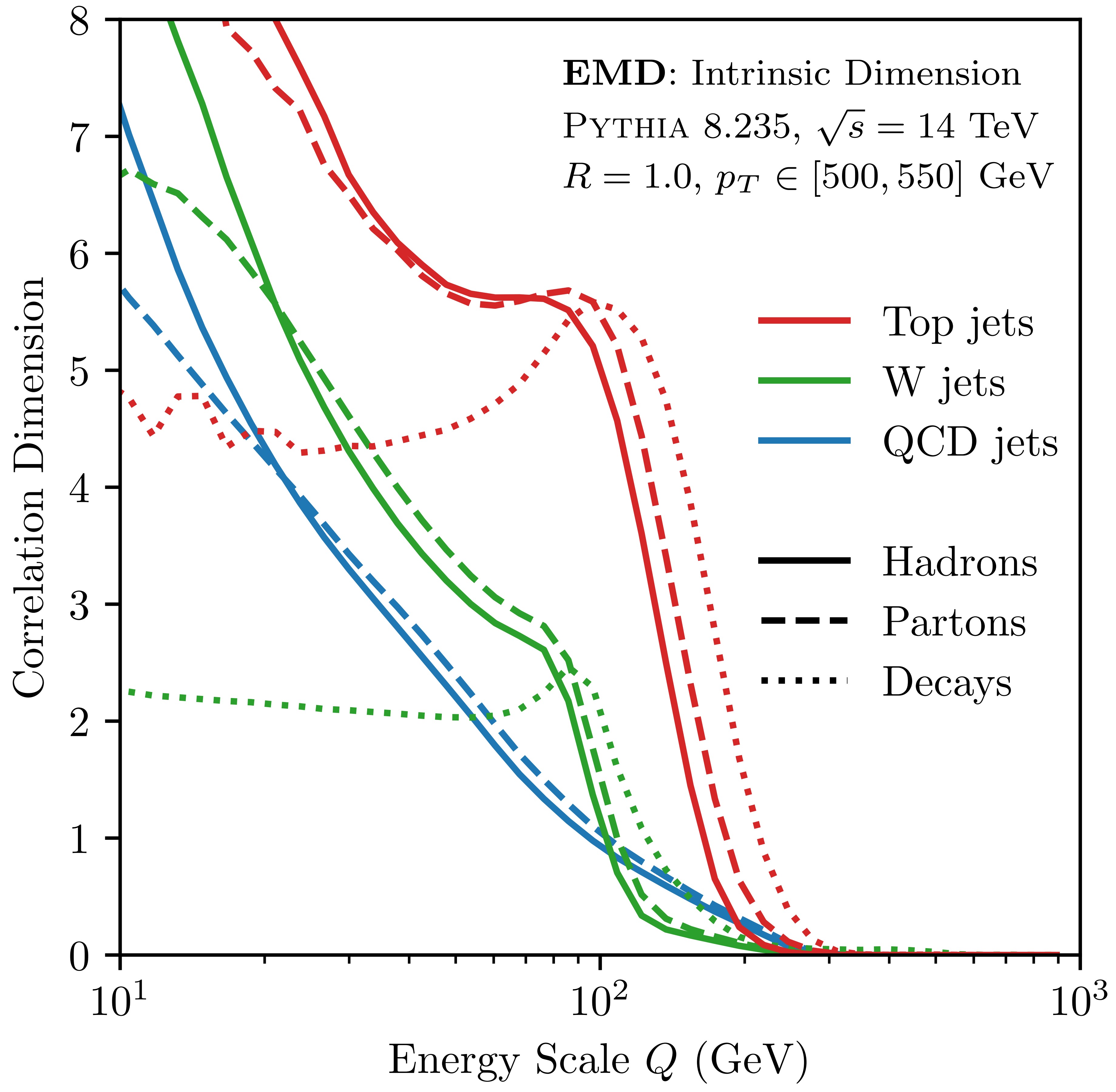


Correlation dimension lessons:

- Decays are "constant" dim. at low Q
- Complexity hierarchy: QCD < W < Top
- Fragmentation increases dim. at smaller scales
- Hadronization important around 20-30 GeV

[Grassberger, Procaccia, PRL 1983; PTK, Metodiev, Thaler, PRL 2019]

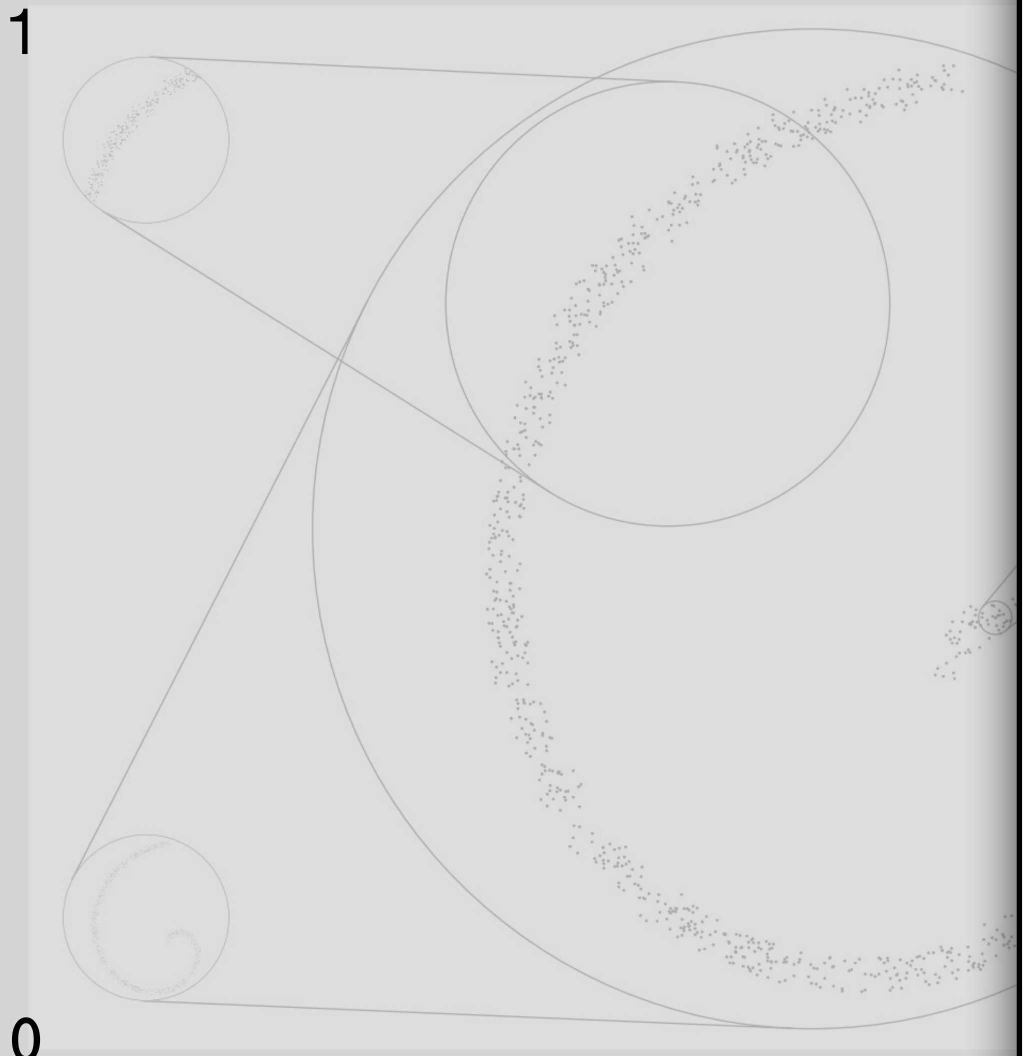
*Preliminary LL calculation for QG jets in backup

Quantifying Event-Space Manifolds

$$\ldots \sum_i \sum_j \Theta(\mathrm{EMD}(\mathcal{E}_i, \mathcal{E}'_j) < Q)$$

Correlation dimension: *how ... elements within a ball of si...*

dim ≃ 1

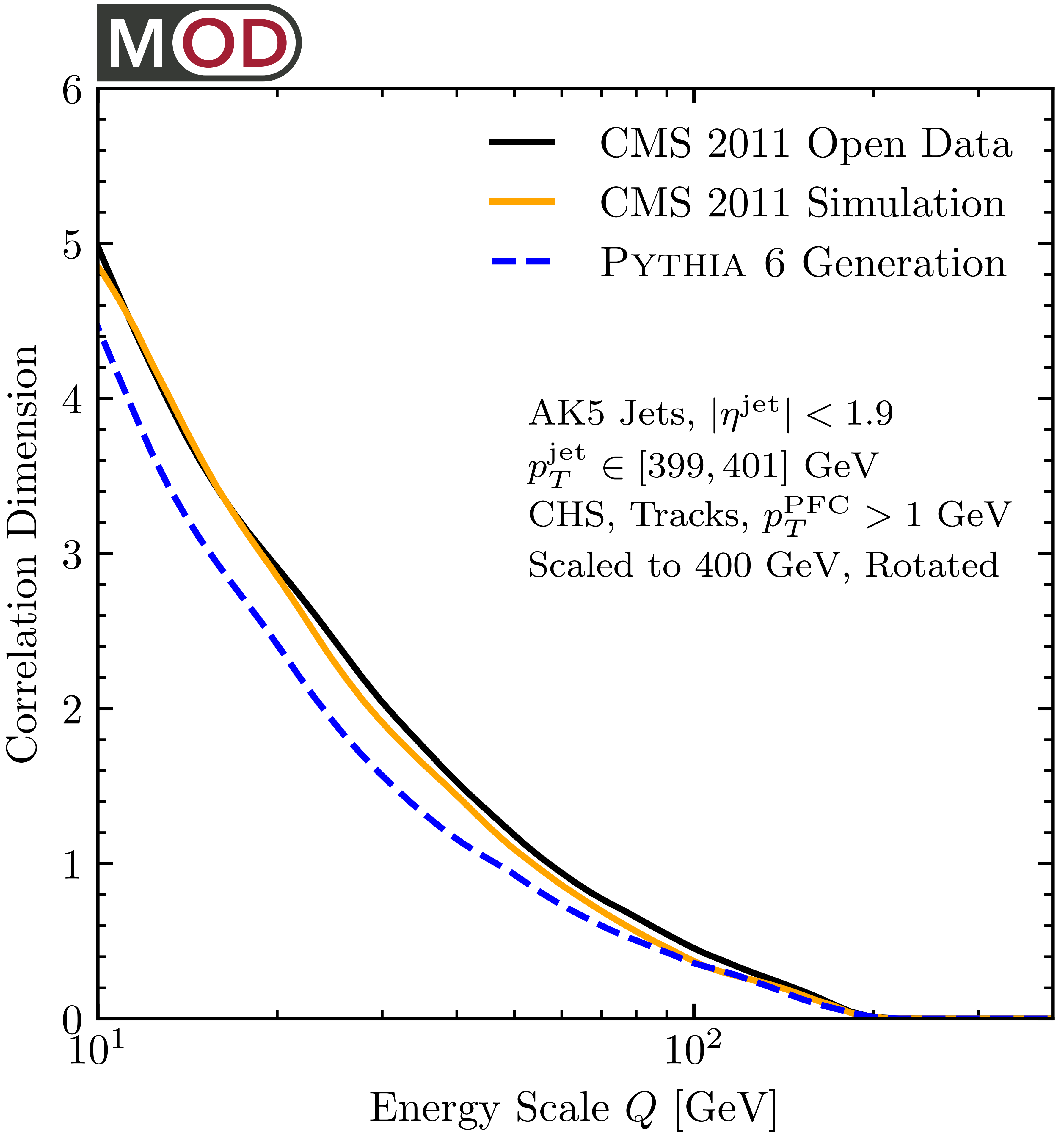


dim → 0

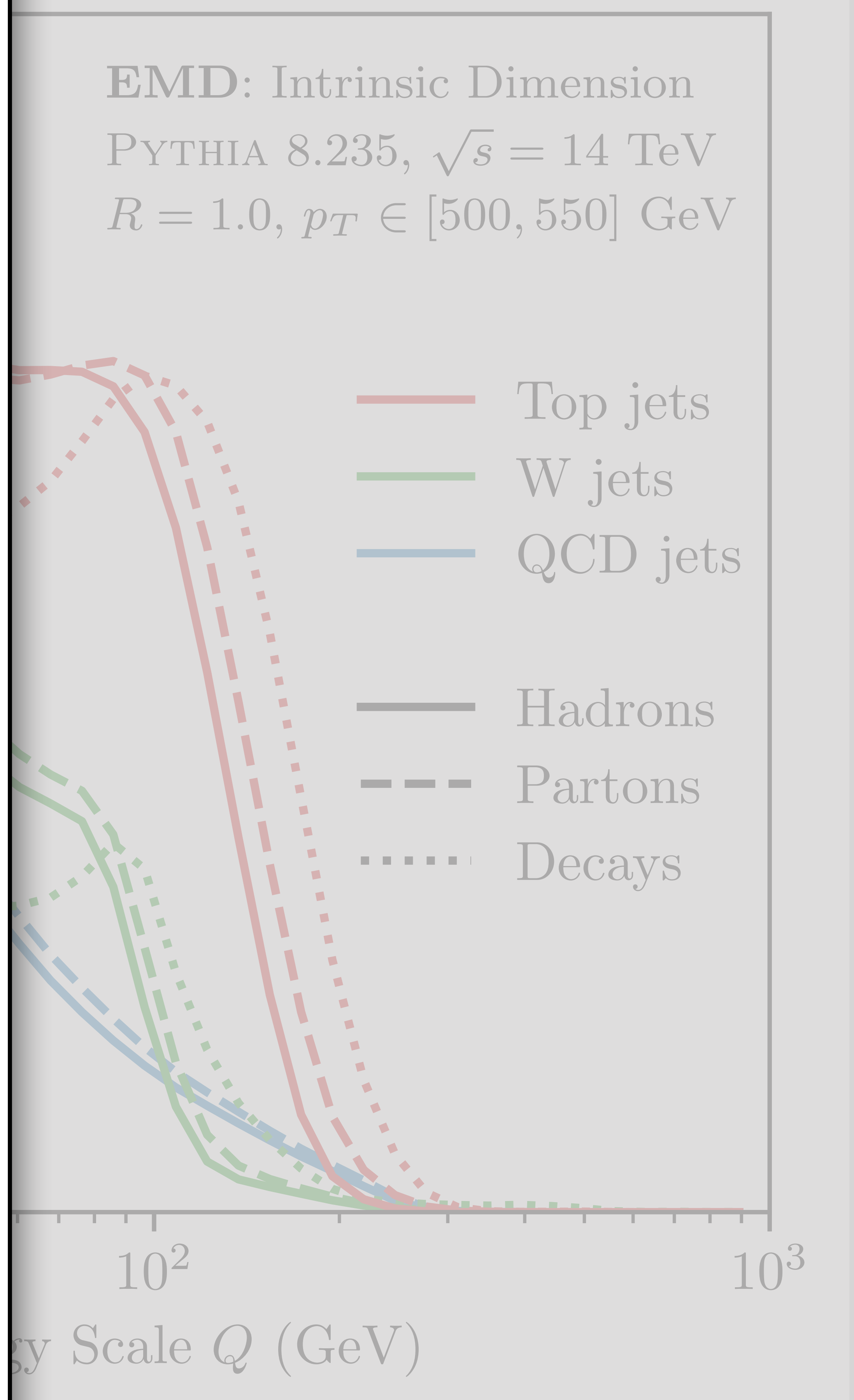
$$N_{\text{neigh.}}(Q) \propto Q^{\text{dim}} \implies \dim(Q \ldots$$

Correlation dimension ...
- Decays are "constant" di...
- Complexity hierarchy: Q...
- Fragmentation increases ...
- Hadronization important ...

... in CMS Open Data



*More in backup

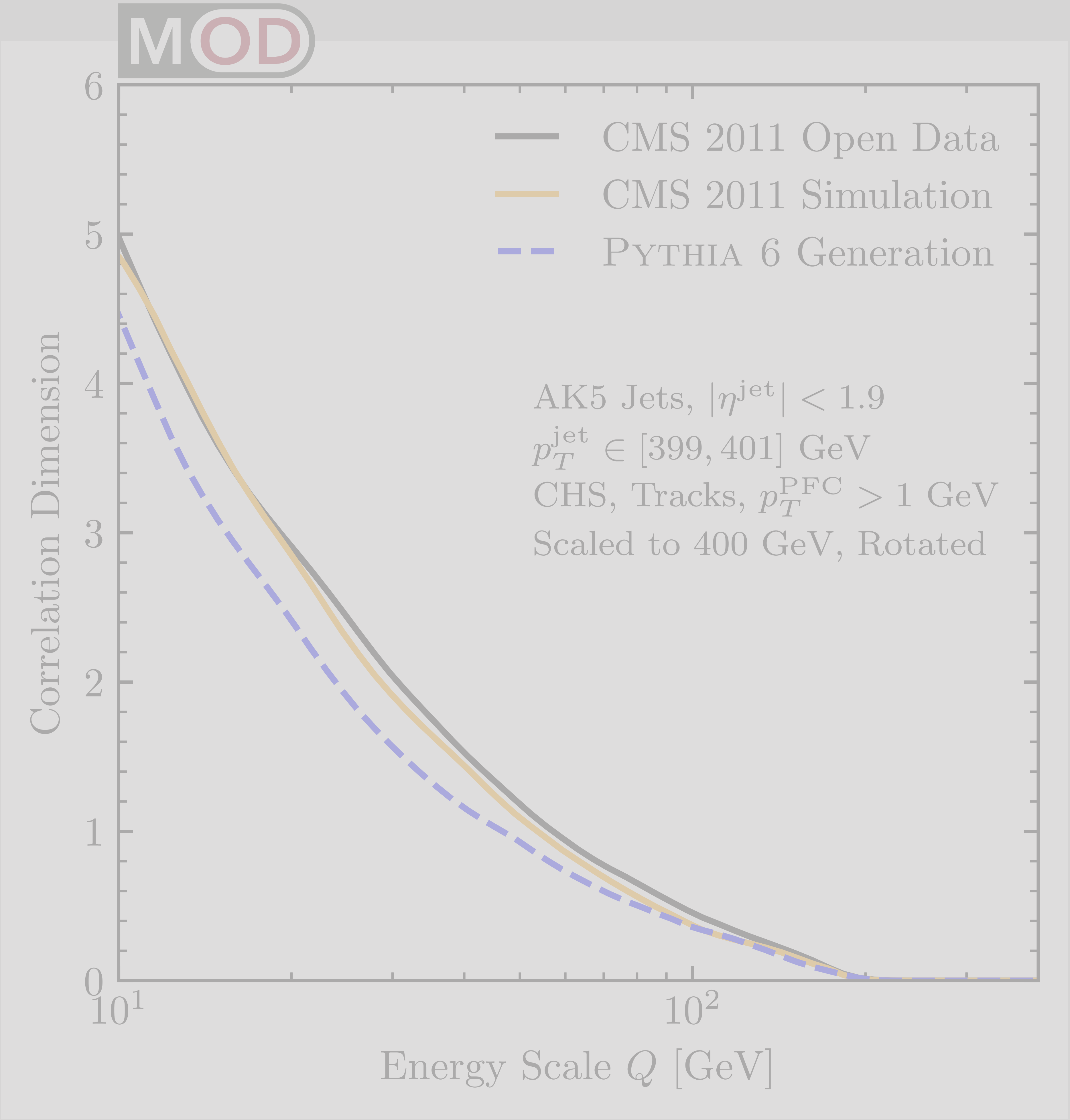


[... Procaccia, PRL 1983; PTK, Metodiev, Thaler, PRL 2019]

*Preliminary LL calculation for QG jets in backup

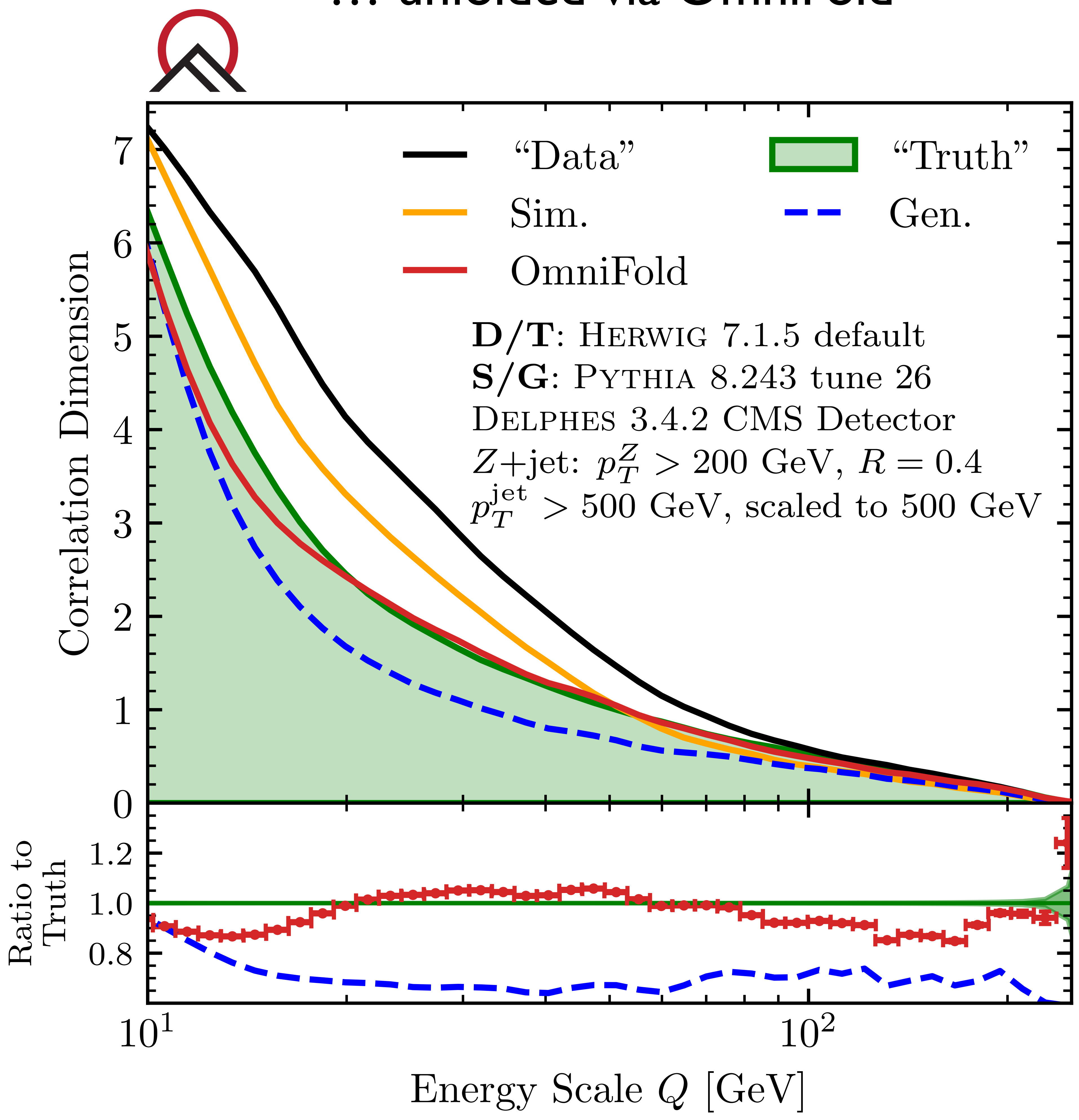
Quantifying Event-Space Manifolds

… in CMS Open Data



*More in backup

… unfolded via OmniFold

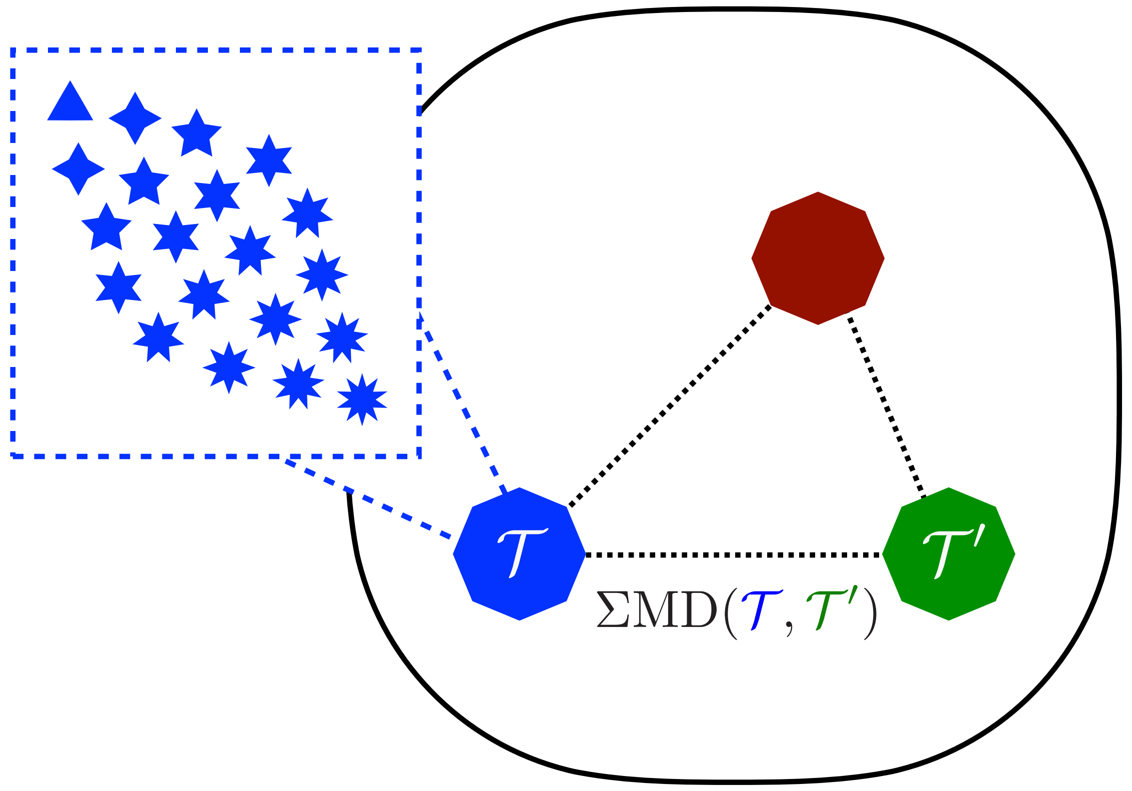


Herwig as "data" and Pythia as MC

*Preliminary LL calculation for QG jets in backup

The (Metric) Space of Events

Revealing Hidden Geometry



[Theory Space]

Templated Metric Construction

Inputs

- “Points” that live in the ground metric space
- “Ground metric” that measures point distances
- “Weights” associated to each point

Output

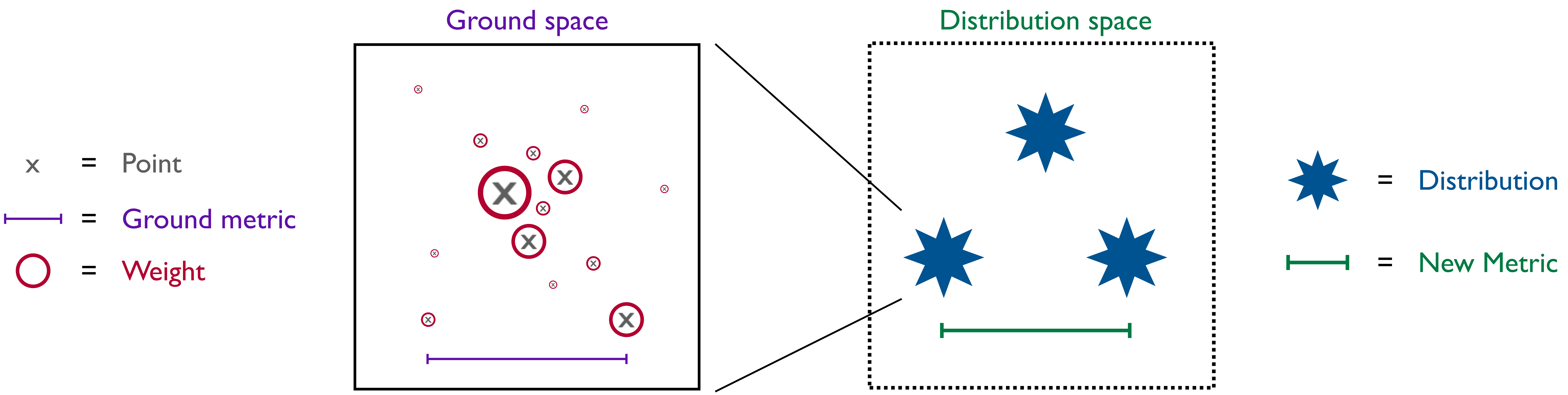
- A new metric for collections of weighted points
- A metric space where these distributions live

p-Wasserstein metric from optimal transport theory

$$W_p(\mu,\nu) = \left(\inf_{J\in\mathcal{J}(\mu,\nu)} \int_{M\times M} d(x,y)^p \, \mathrm{d}J(x,y)\right)^{1/p}$$

$(M,\ d)$, metric space

$\mathcal{J}(\mu,\nu)$, space of joint distributions with marginals $\mu,\ \nu$



Templated Metric Construction – Energy Mover's Distance

[PTK, Metodiev, Thaler, PRL 2019]

Inputs

- "Points" that live in the ground metric space
- "Ground metric" that measures point distances
- "Weights" associated to each point

Output

- A new metric for collections of weighted points
- A metric space where these distributions live

p-Wasserstein metric from optimal transport theory

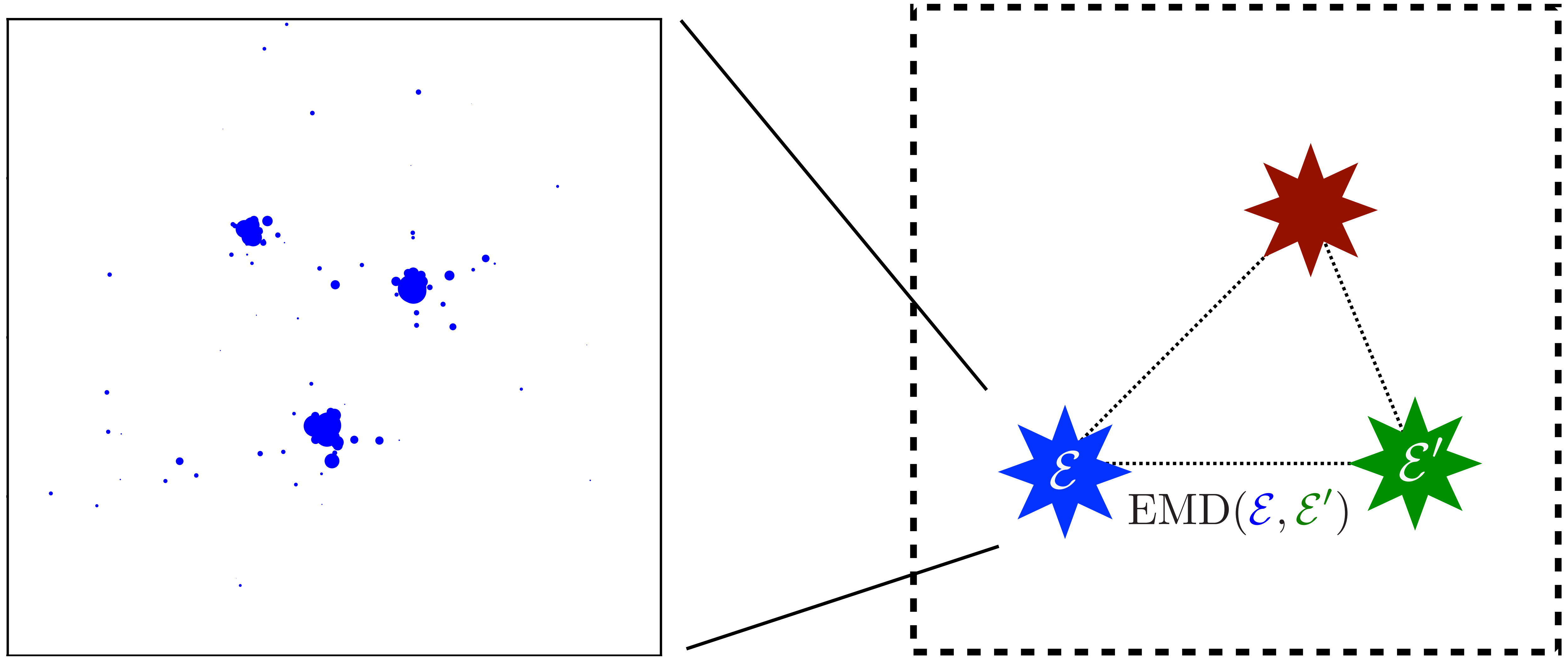
$$W_p(\mu,\nu) = \left(\inf_{J\in\mathcal{J}(\mu,\nu)} \int_{M\times M} d(x,y)^p \, \mathrm{d}J(x,y) \right)^{1/p}$$

$(M,\ d)$, metric space

$\mathcal{J}(\mu,\nu)$, space of joint distributions with marginals $\mu,\ \nu$

Rapidity-azimuth plane

Event space



x = Direction (y, ϕ)

= Angular distance ΔR

= Energy

= Event

= EMD

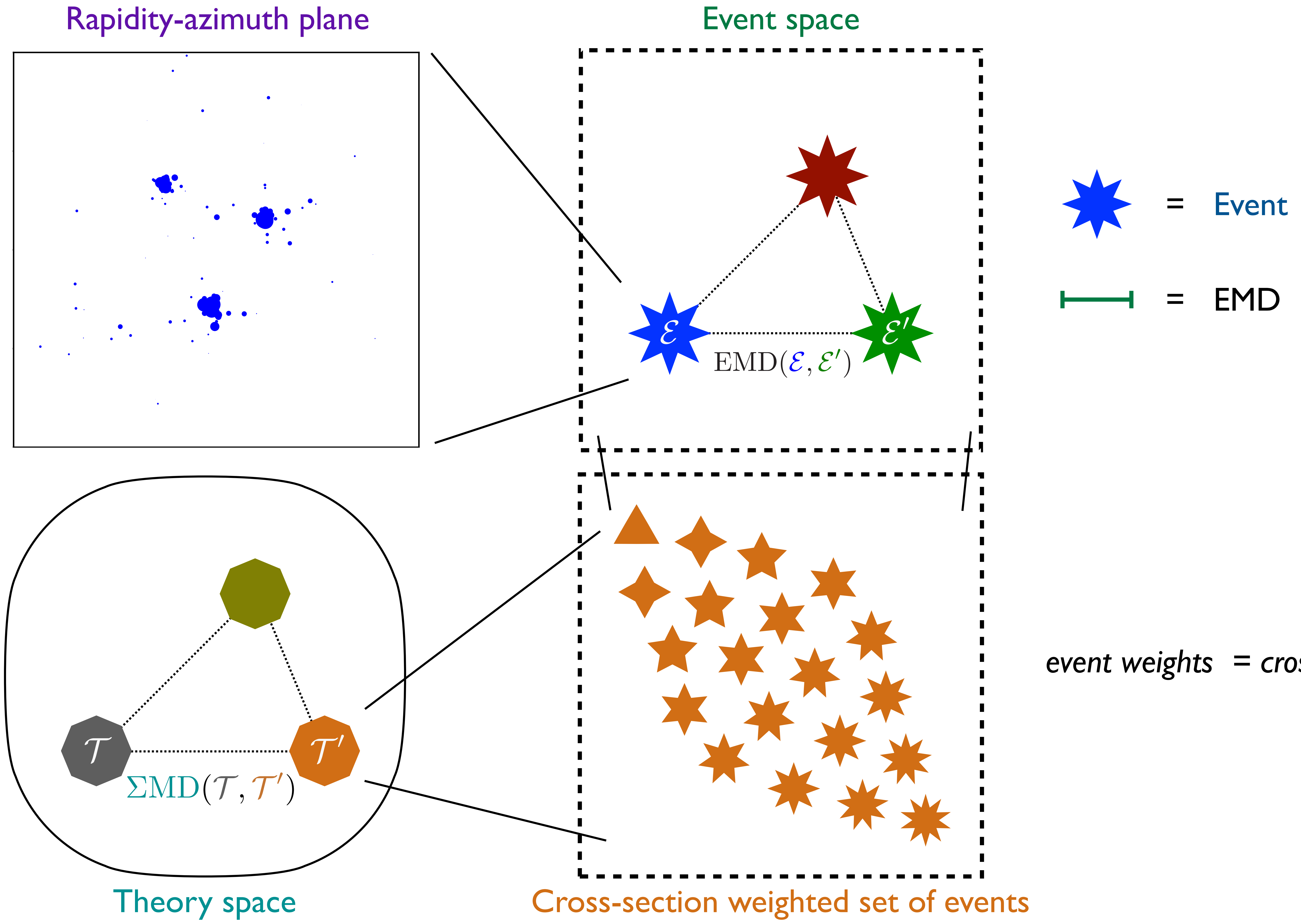
Bootstrapping to the Cross-Section Mover's Distance (ΣMD)

[PTK, Metodiev, Thaler, 2004.04159]

x = Direction (y, ϕ)

= Angular distance ΔR

= Energy



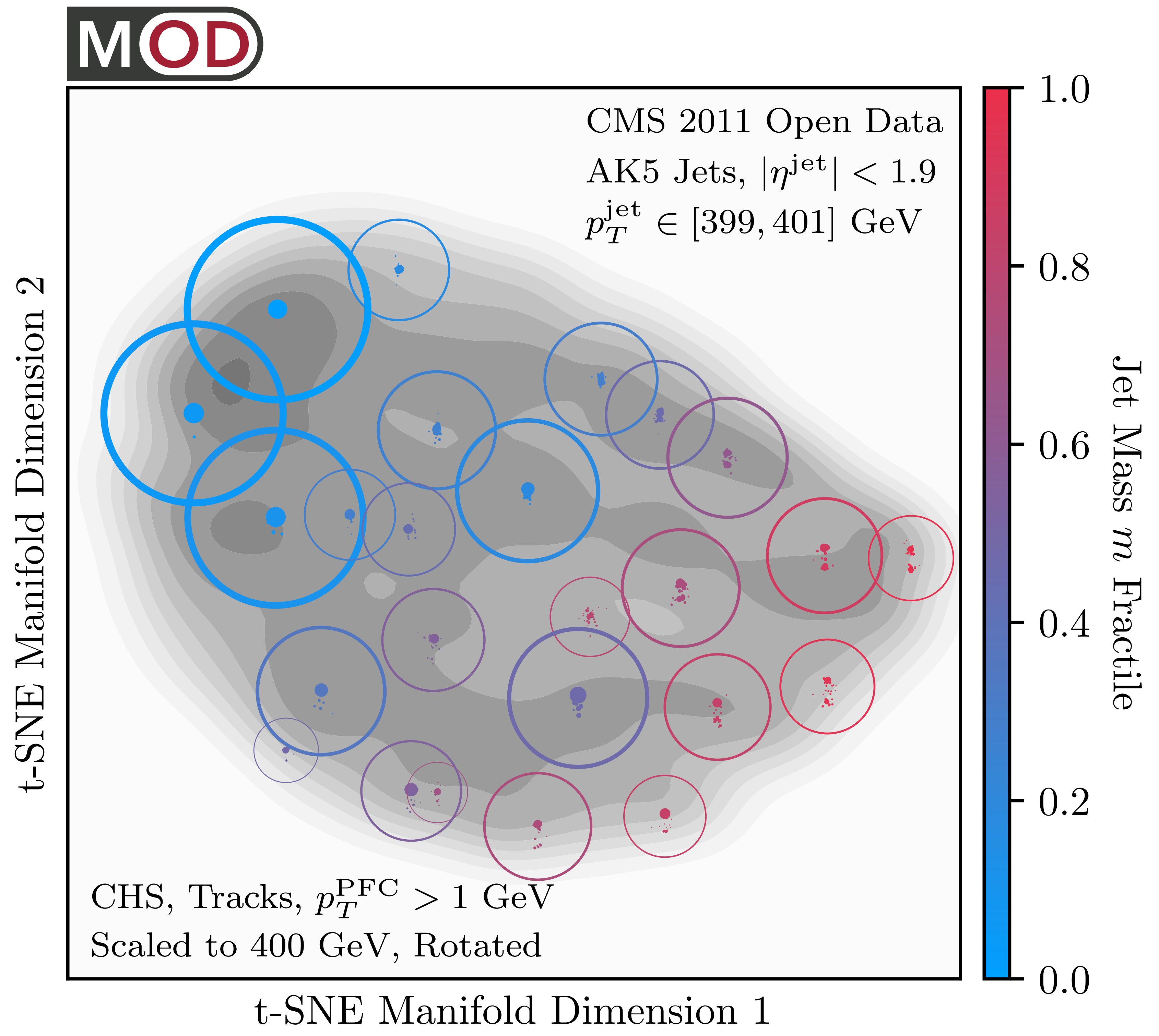
ΣMD is a metric between theories!

event weights = cross section

The Space of Theories

[PTK, Metodiev, Thaler, 2004.04159]

ΣMD provides a rigorous construction of theory space



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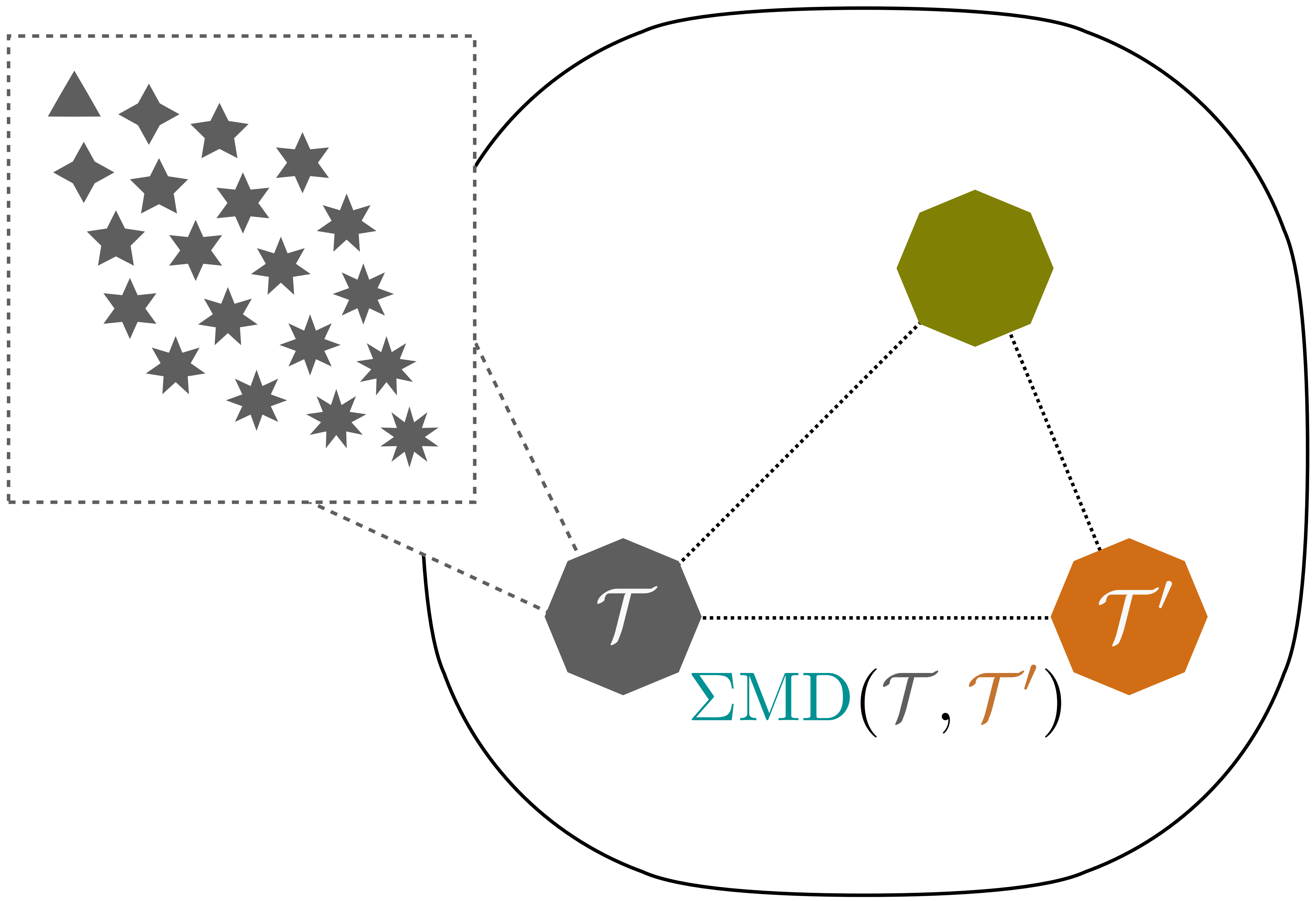


*Theories are distinguished by their energy flows only

Applications of ΣMD and the Space of Theories

N-(sub)jettiness ⇨

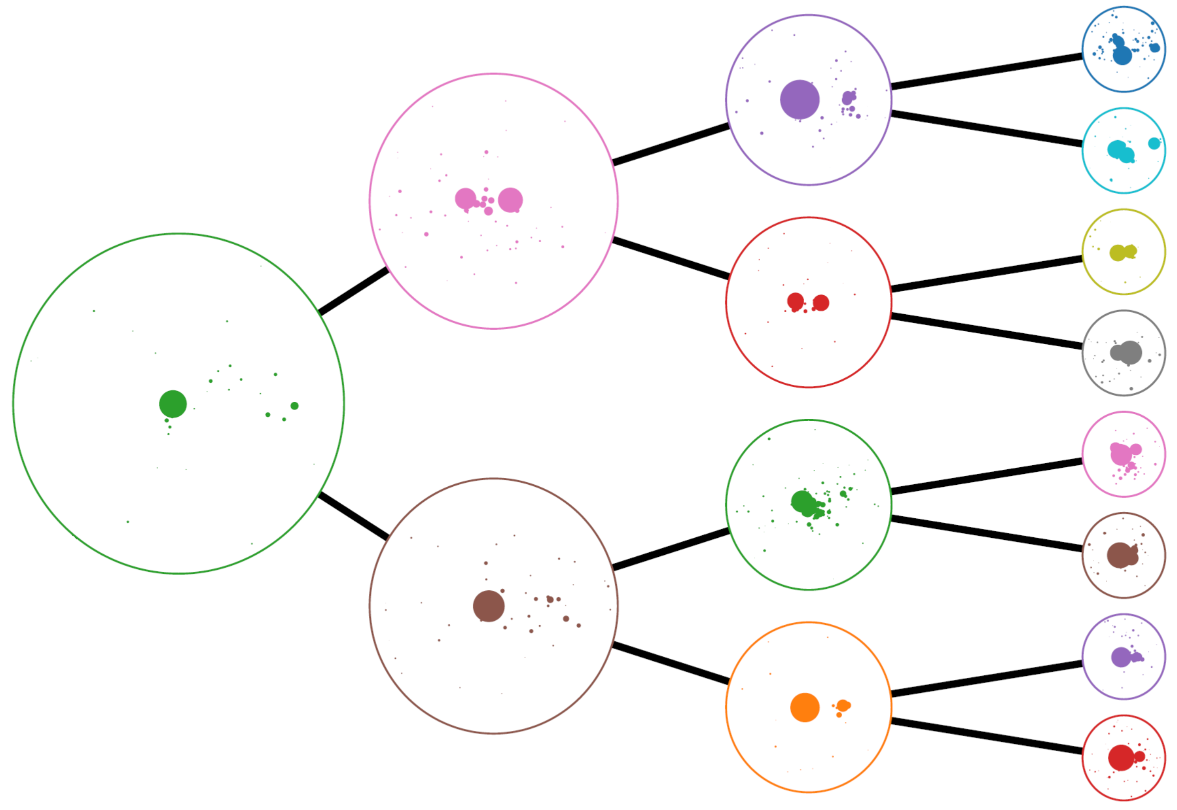
k-eventiness defined

$$\mathcal{V}_k^{(\gamma)}(\{\sigma_i,\mathcal{E}_i\}) = \min_{\mathcal{K}_1,\ldots,\mathcal{K}_k} \sum_{i=1}^{N} \sigma_i \min\{\text{EMD}(\mathcal{E}_i,\mathcal{K}_1),\ldots,\text{EMD}(\mathcal{E}_i,\mathcal{K}_k)\}^\gamma$$

$$\mathcal{V}_k^{(\gamma)}(\mathcal{T}) = \min_{|\mathcal{T}'|=k} \Sigma\text{MD}_\gamma(\mathcal{T},\mathcal{T}')$$

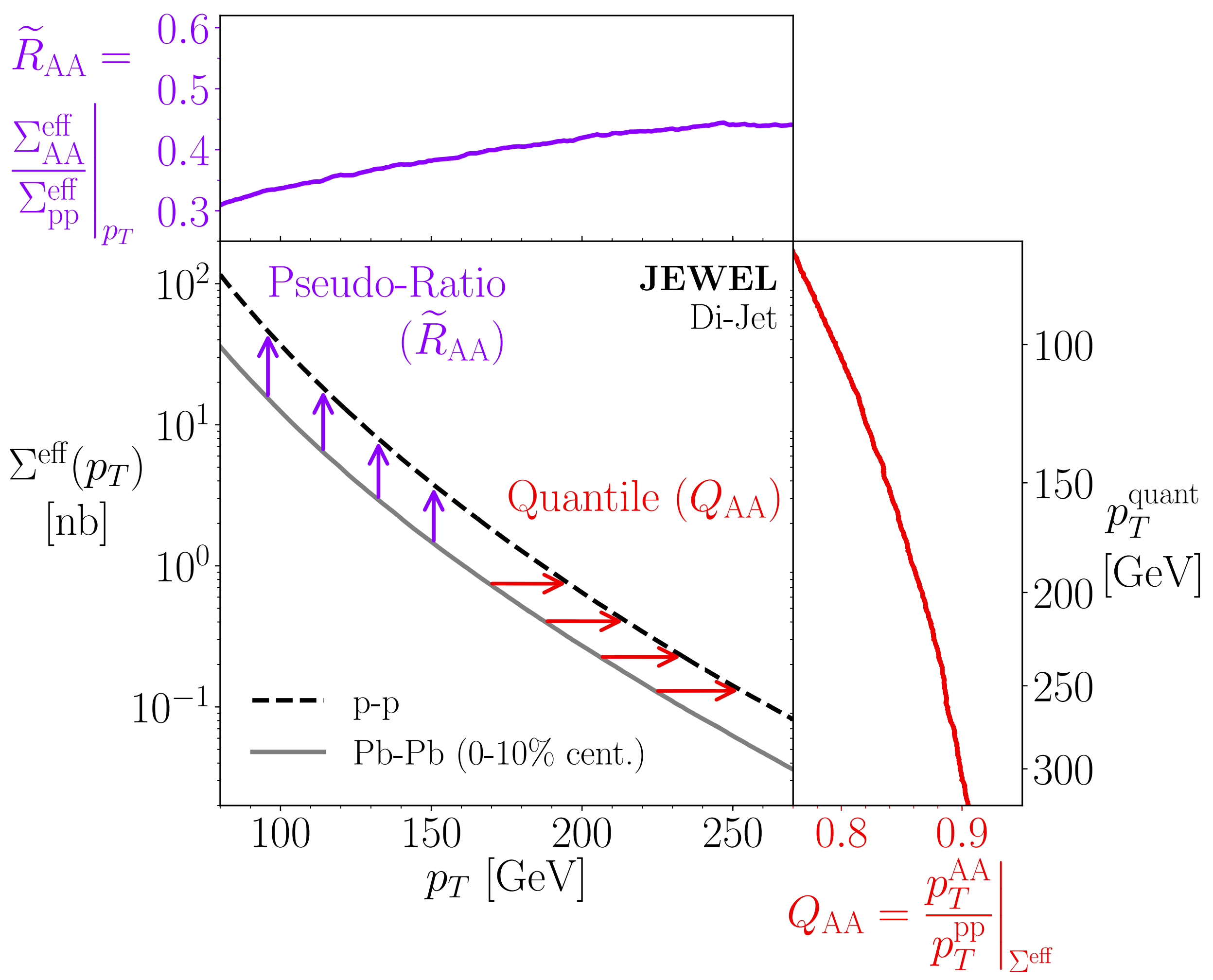
Jet clustering ⇨

Event clustering enabled
- Exclusive cone finding
- Sequential recombination



Jet quenching in HI collisions ⇨

[Brewer, Milhano, Thaler, PRL 2019]



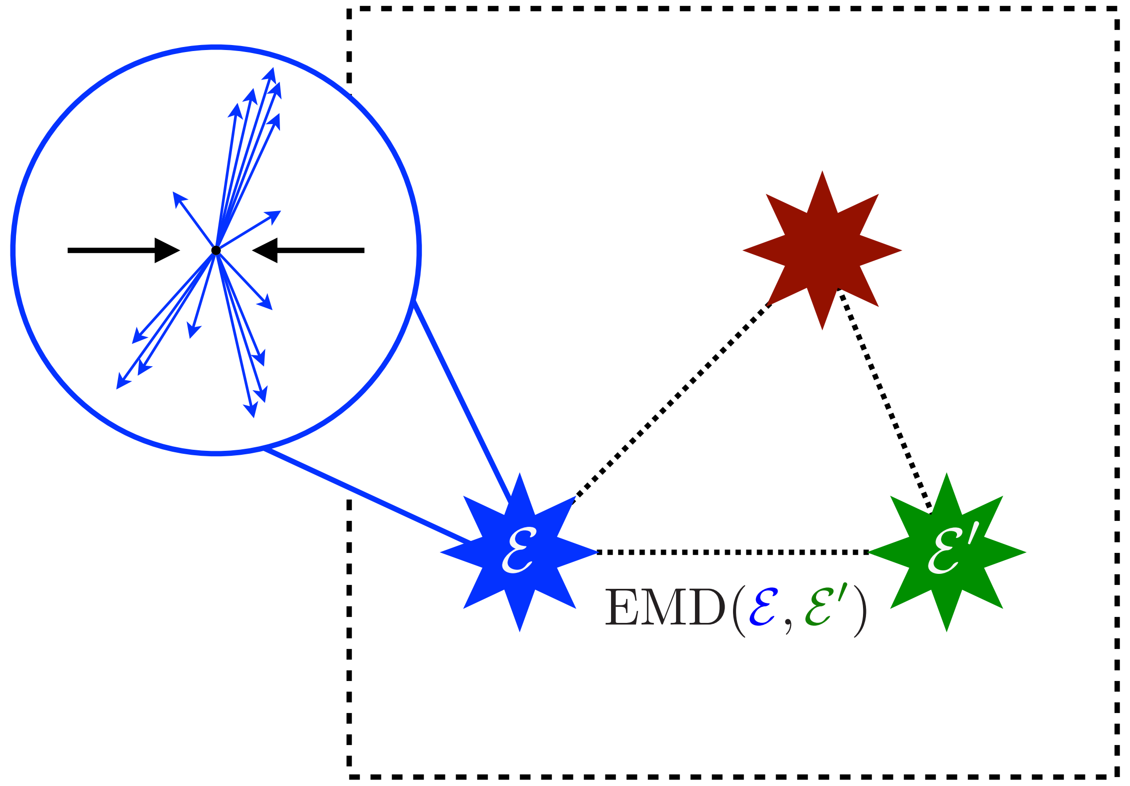
Quantile matching:

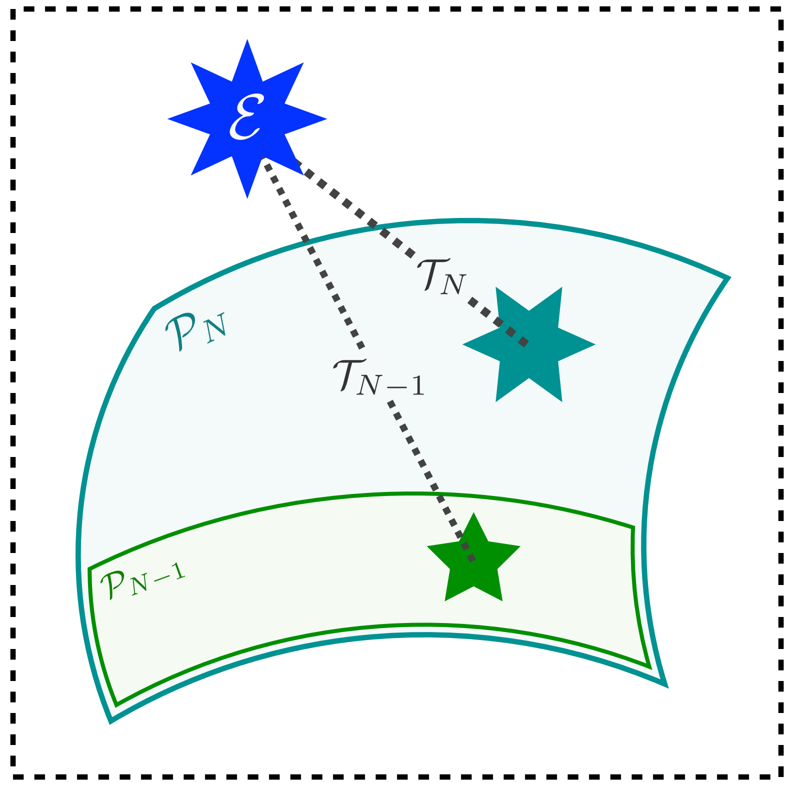
$$\Sigma^{\text{eff}}_{pp}(p_T^{\text{quant}}) \equiv \Sigma^{\text{eff}}_{AA}(p_T^{AA})$$

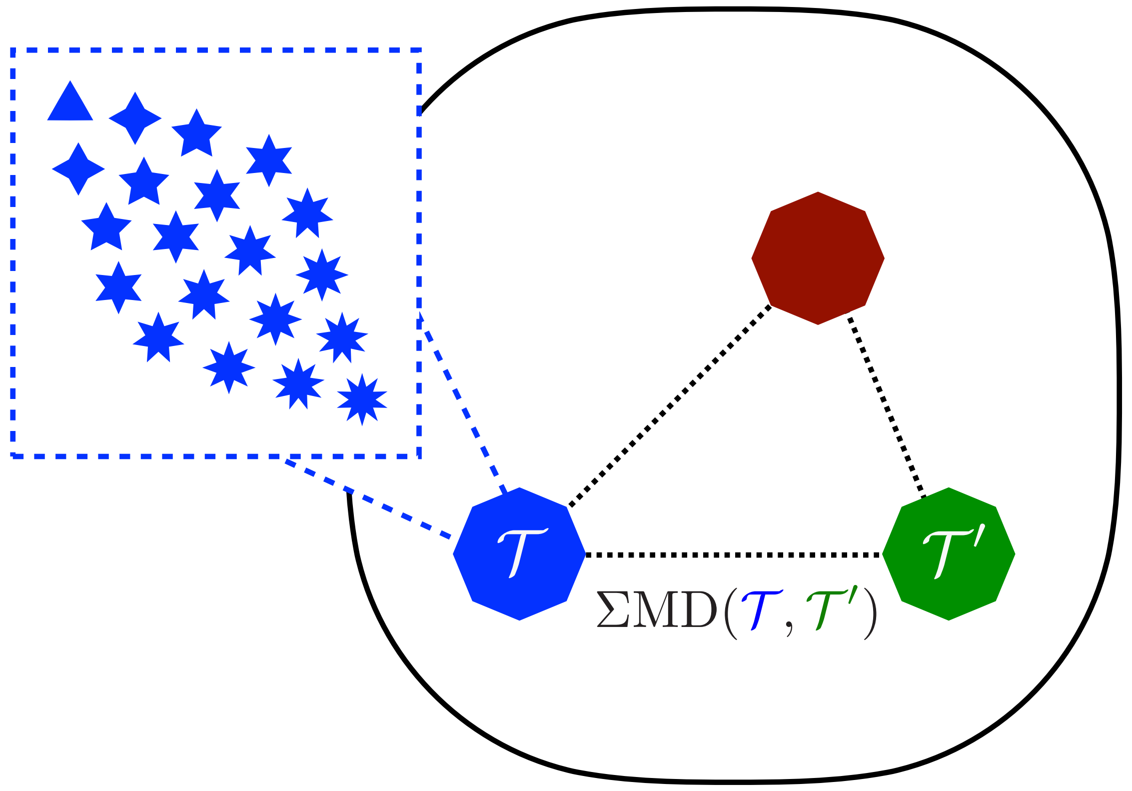
…is exactly a theory moving problem!

$$p_T^{\text{quant}} = \text{TM}(\mathcal{T}_{AA},\mathcal{T}_{pp})[p_T^{AA}]$$

optimal p_T-only theory movement







The (Metric) Space of Events

– Energy flow is theoretically and experimentally robust

– EMD metrizes the space of energy flows (events)

– Manifolds in the space of events can be visualized and quantified

Revealing Hidden Geometry

– Event space exhibits a rich geometry that can be probed using the EMD

– Decades worth of collider techniques are naturally described in this geometry

– Many new techniques are suggested, and new light is shed on old ones

[Theory Space]

– Is rigorously constructed using the cross-section mover's distance ΣMD

– ΣMD uses the EMD as ground metric and cross sections as weights

– Theories can be explored with tools developed for events (e.g. *k*-eventiness)

EnergyFlow Python Package

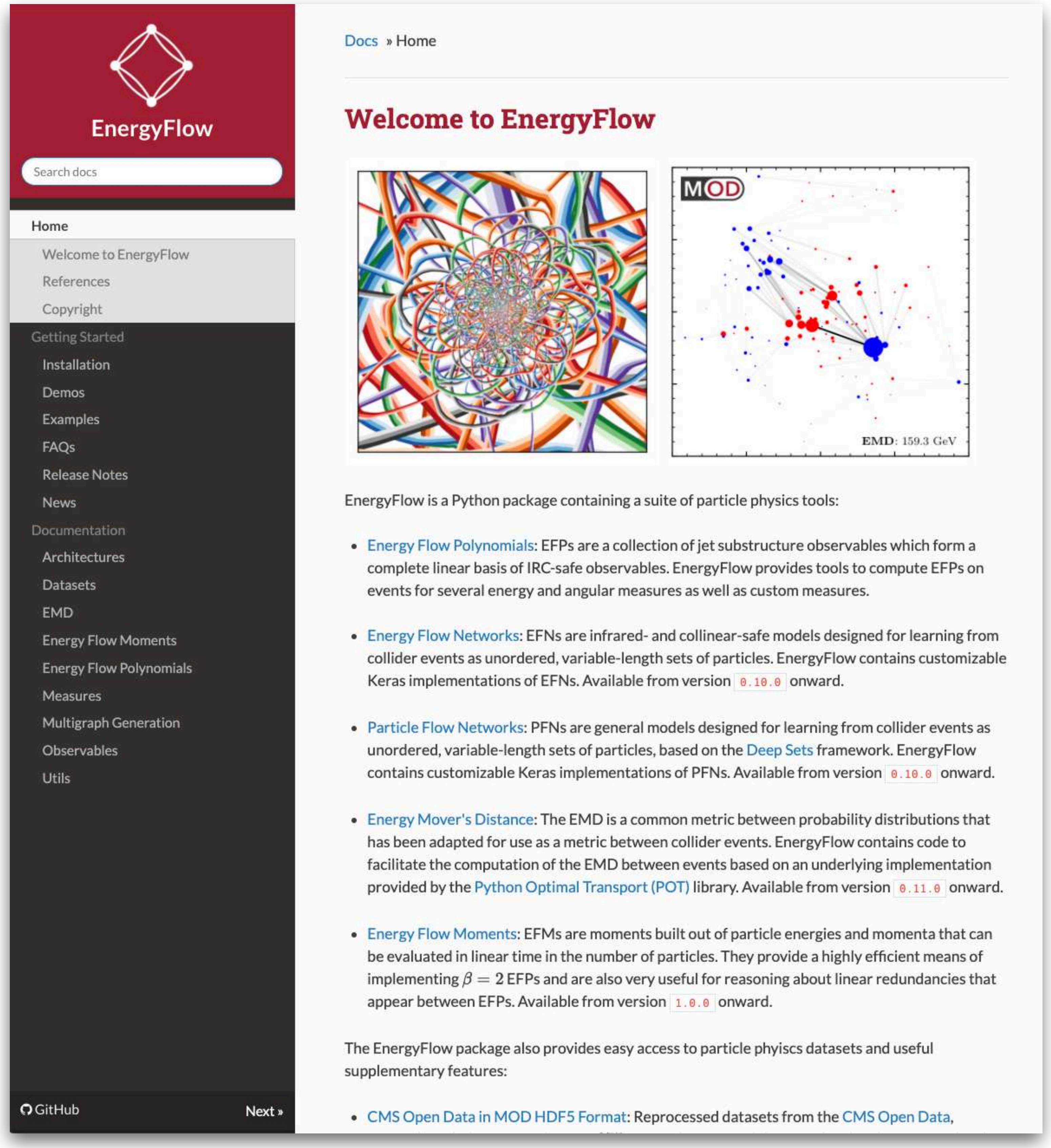
`pip3 install energyflow wasserstein`

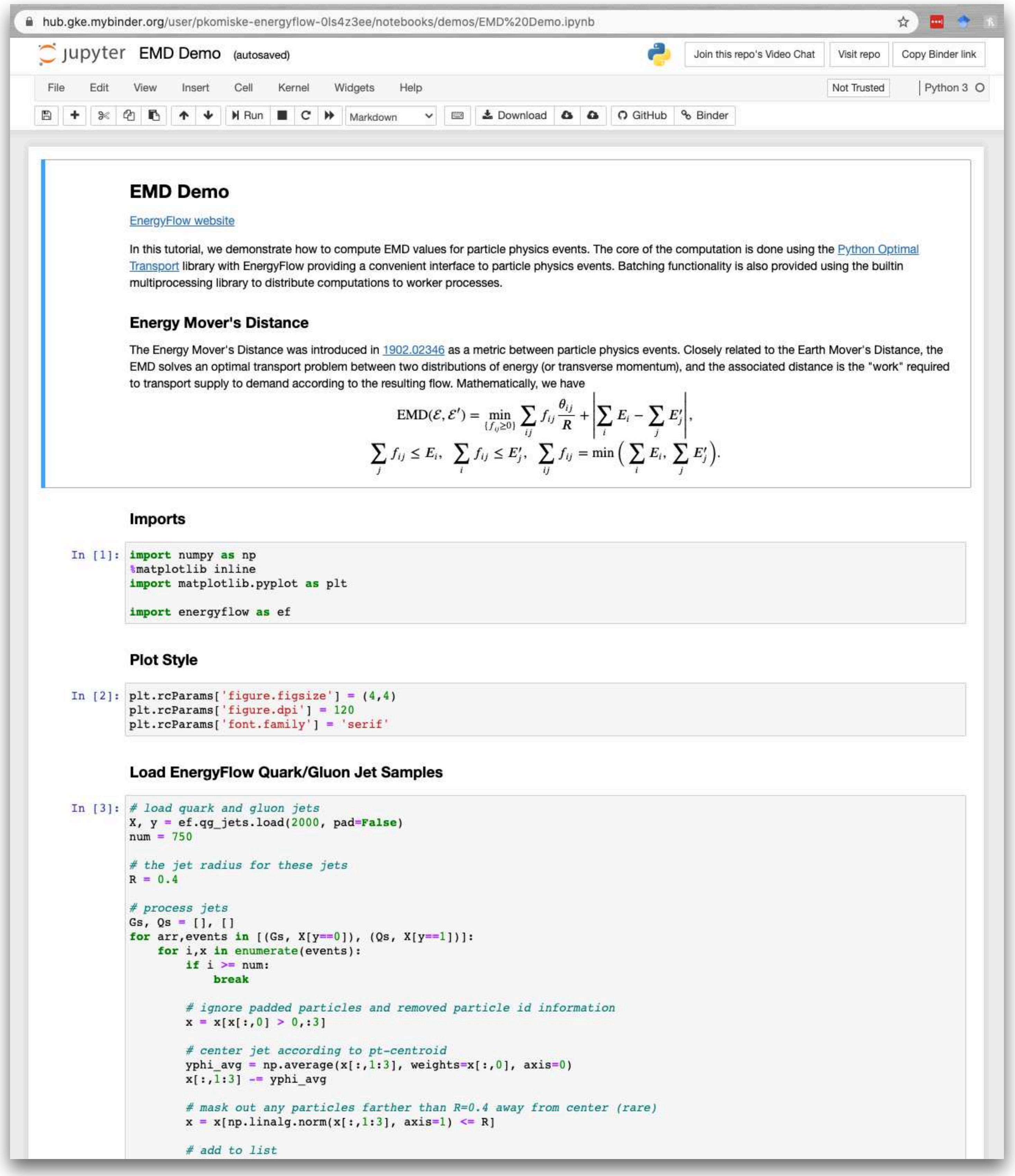
Parallelized EMD calculations via the Wasserstein library

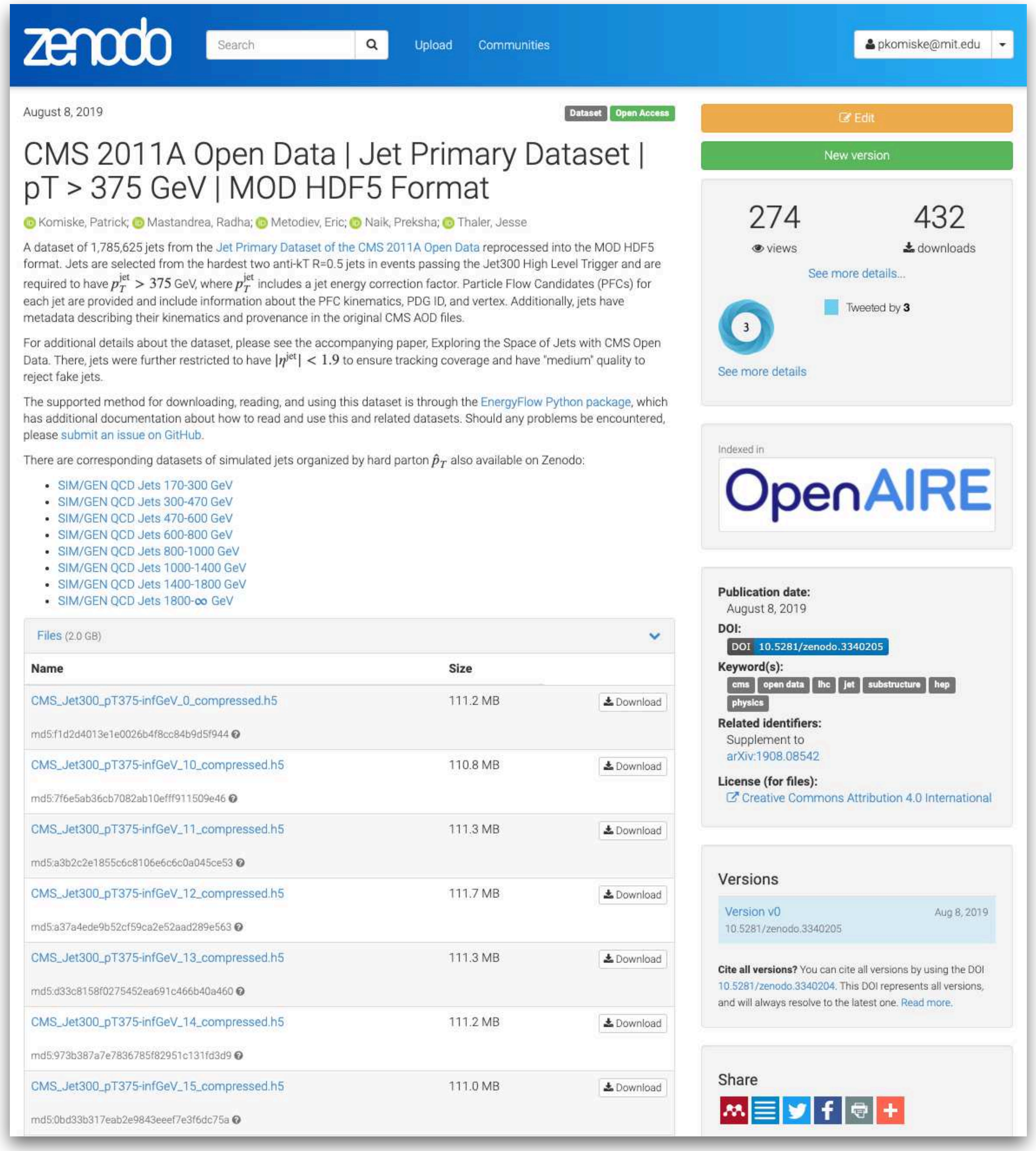
EFN/PFN implementations in TensorFlow/Keras

Detailed examples, demos, and documentation

Interfaces with CMS 2011A Jet Primary Dataset hosted on Zenodo







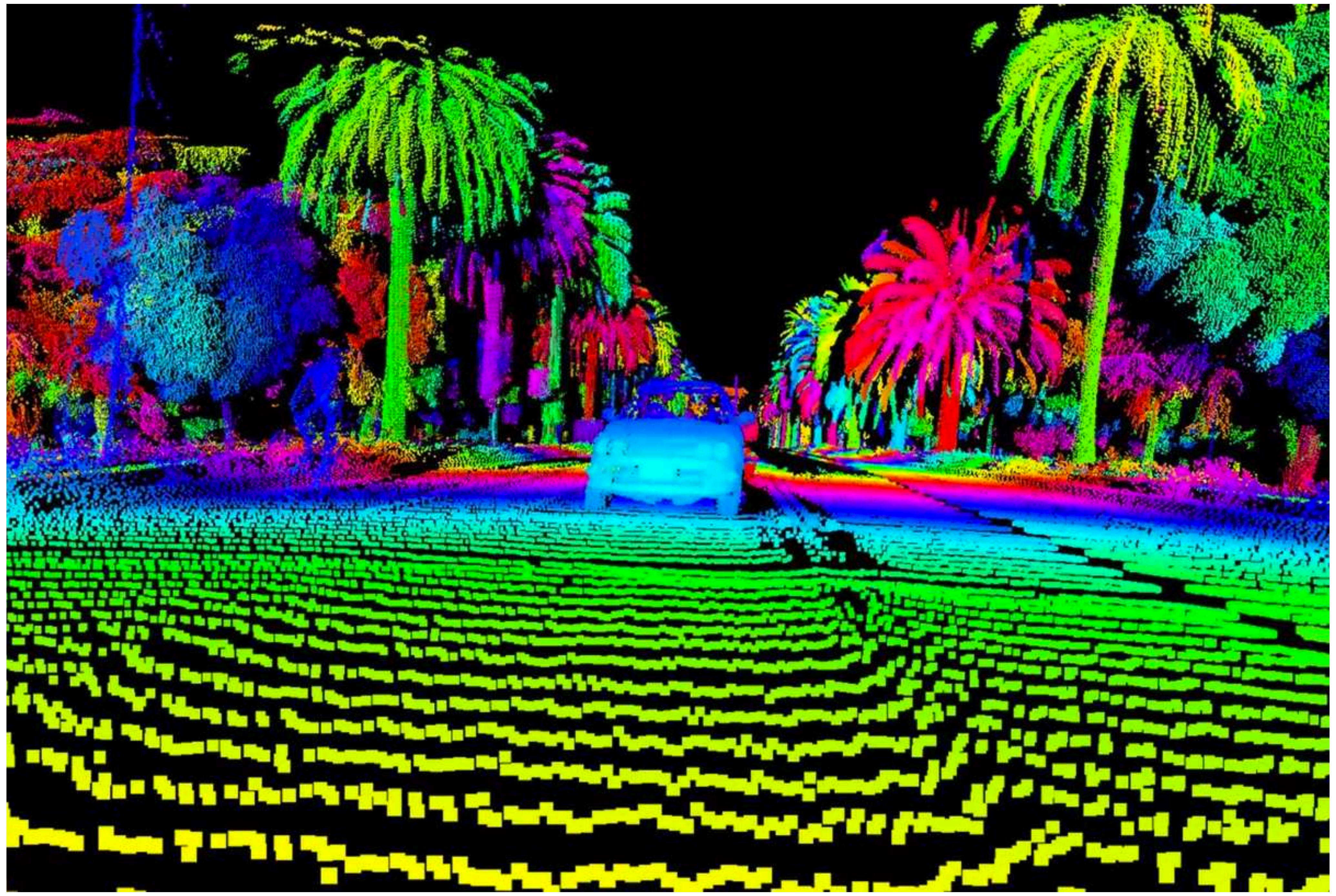
Additional Slides

Neural Network Architectures for Particle Physics

Maximally appropriate ML architectures respect symmetries of the underlying data

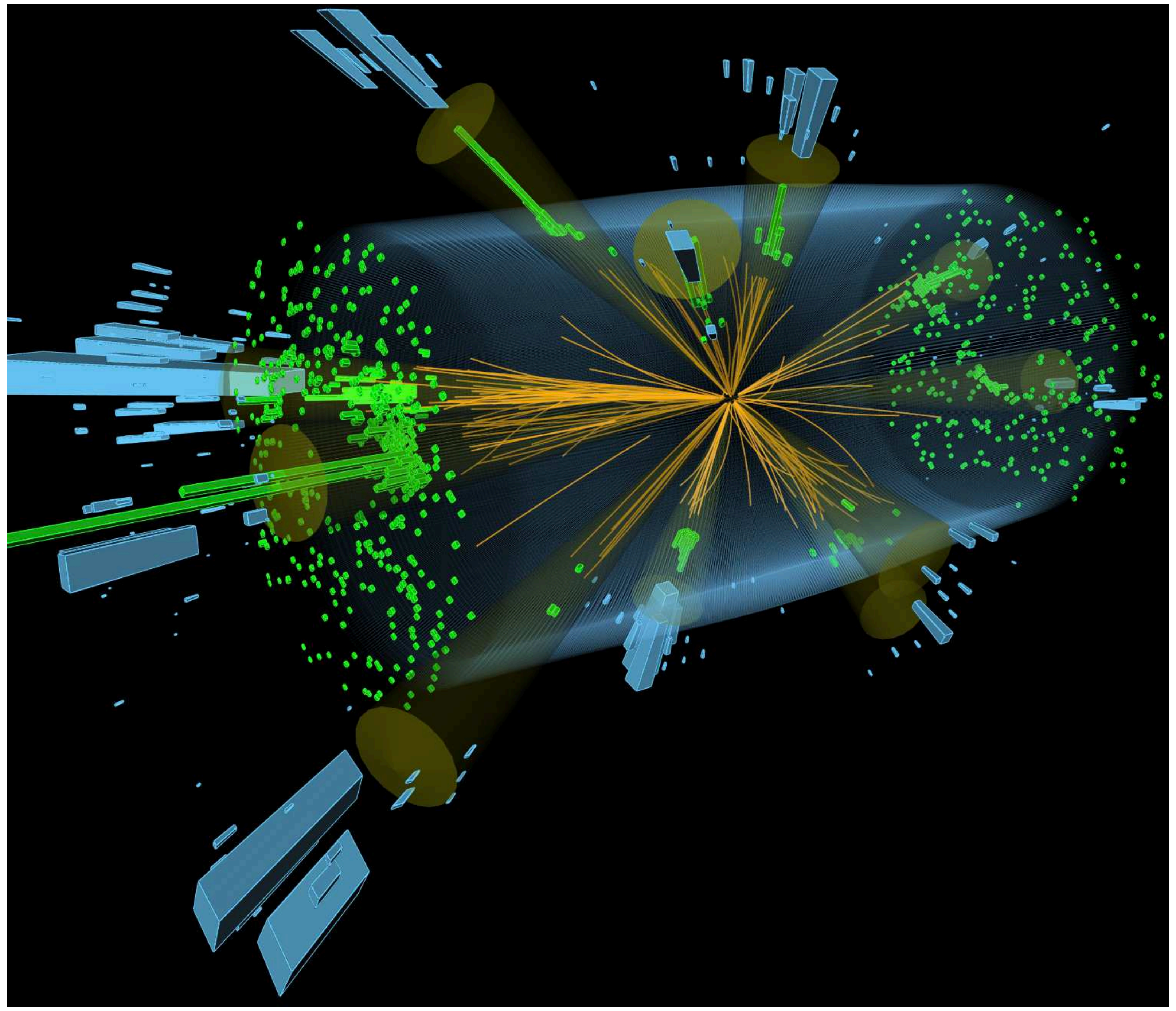
Particle physics events are naturally point clouds (alternatively, images e.g. calorimeters)

Point cloud: "A set of data points in space" –Wikipedia



LIDAR data from self-driving car sensor

An unordered, variable length collection of particles



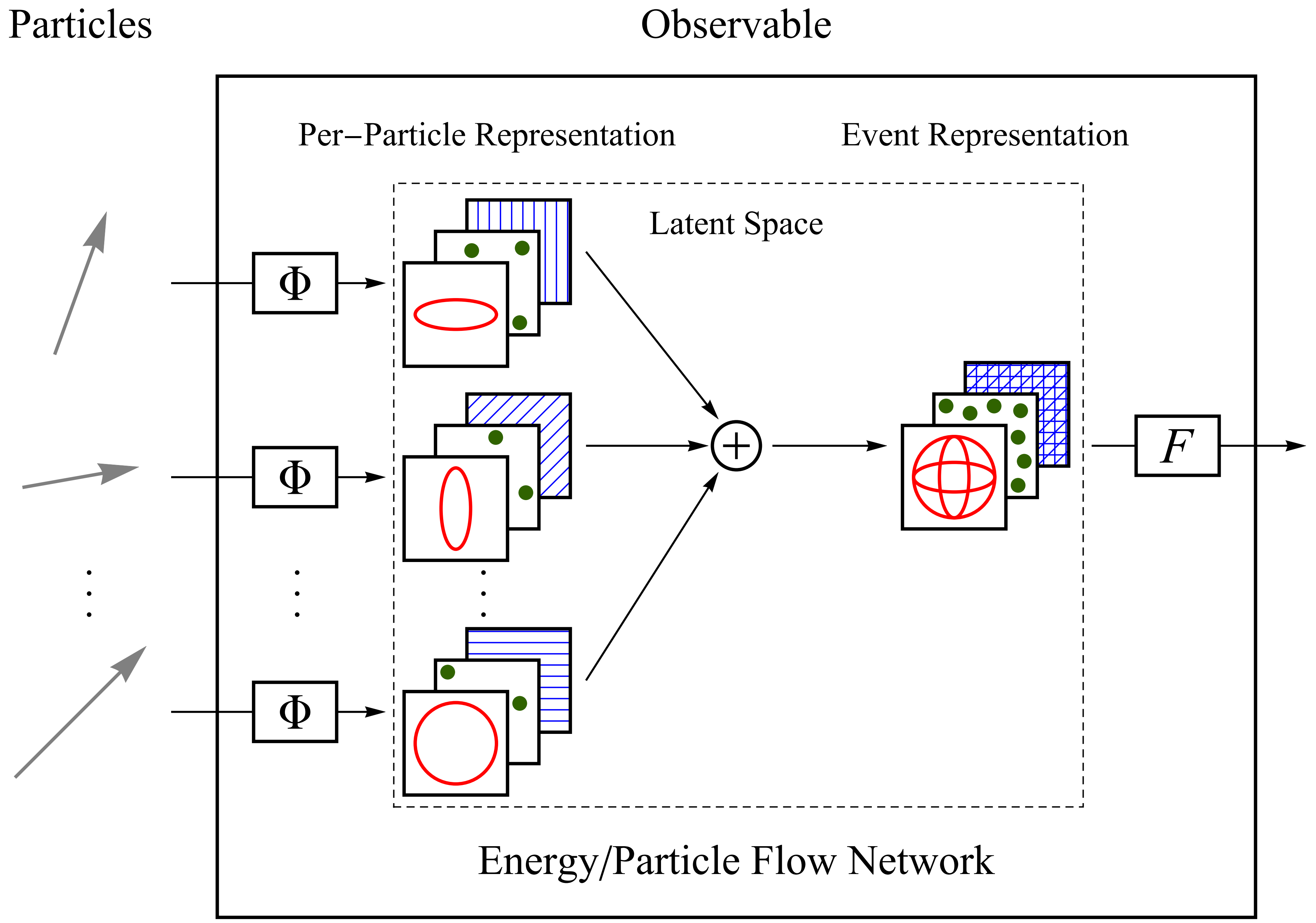
Multi-jet event at CMS

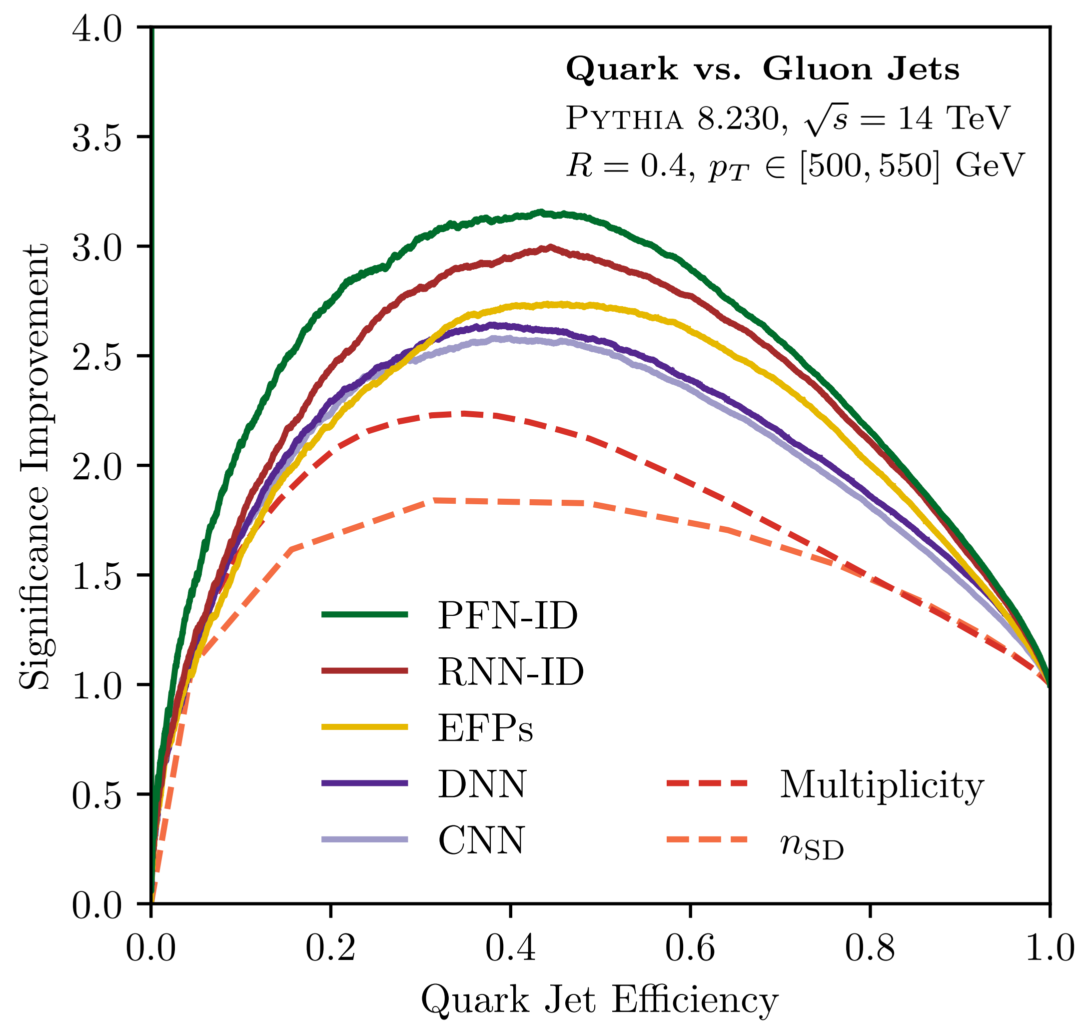
Due to quantum-mechanical indistinguishability

Due to probabilistic nature of event formation

Energy/Particle Flow Networks (EFNs/PFNs)

[Zaheer, Kottur, Ravanbhakhsh, Póczos, Salakhutdinov, Smola, 1703.06114;
PTK, Metodiev, Thaler, 1810.05165;
EnergyFlow Python Package]

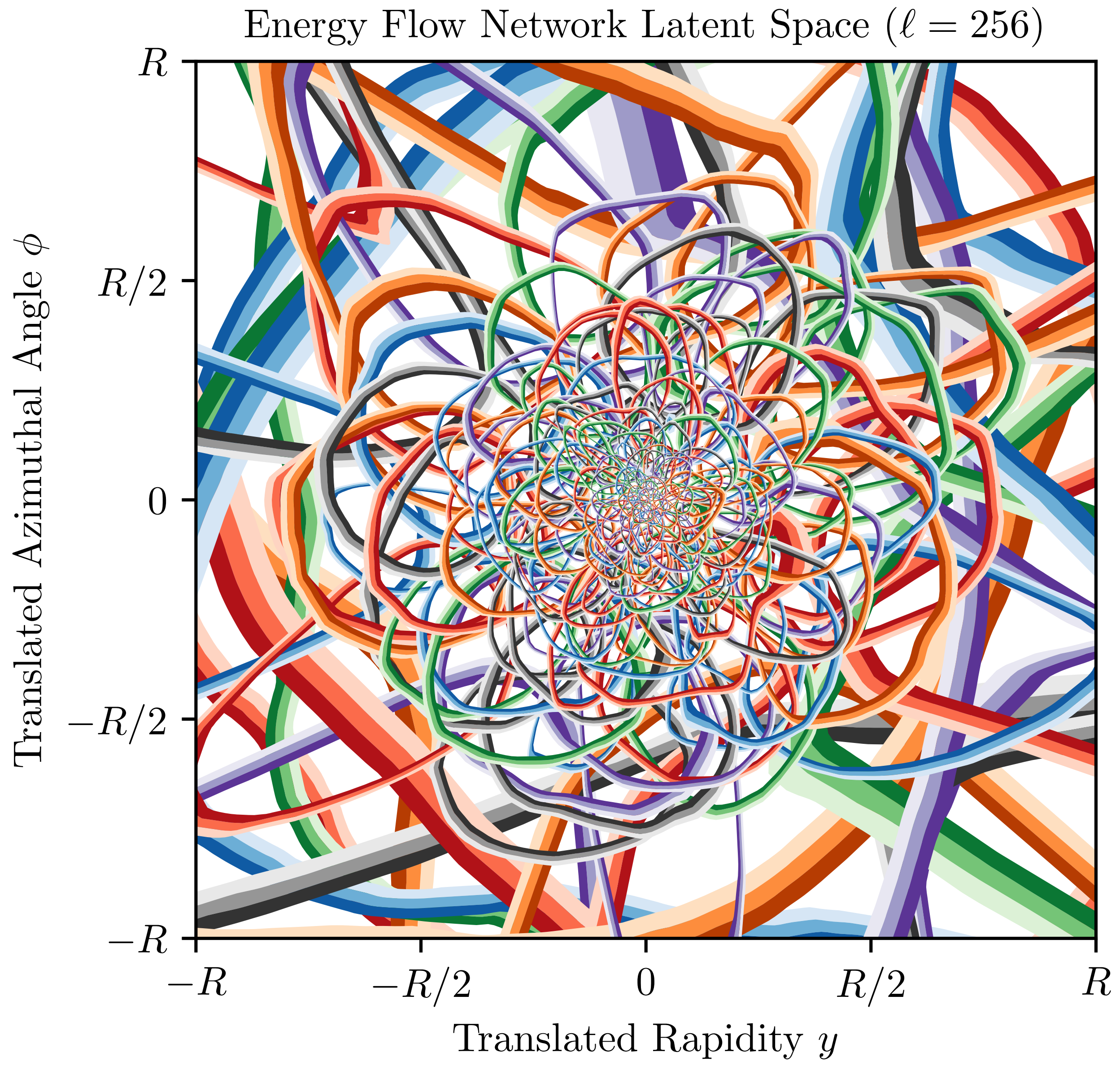




Improved performance (and training) compared to RNN and CNN

Latent space visualization reveals what the network has learned

Dynamic pixel sizing related to collinear singularity of QCD!



Particle Flow Network (PFN)

$$\text{PFN}(\{p_1^\mu, \ldots, p_M^\mu\}) = F\left(\sum_{i=1}^{M} \Phi(p_i^\mu)\right)$$

Fully general latent space

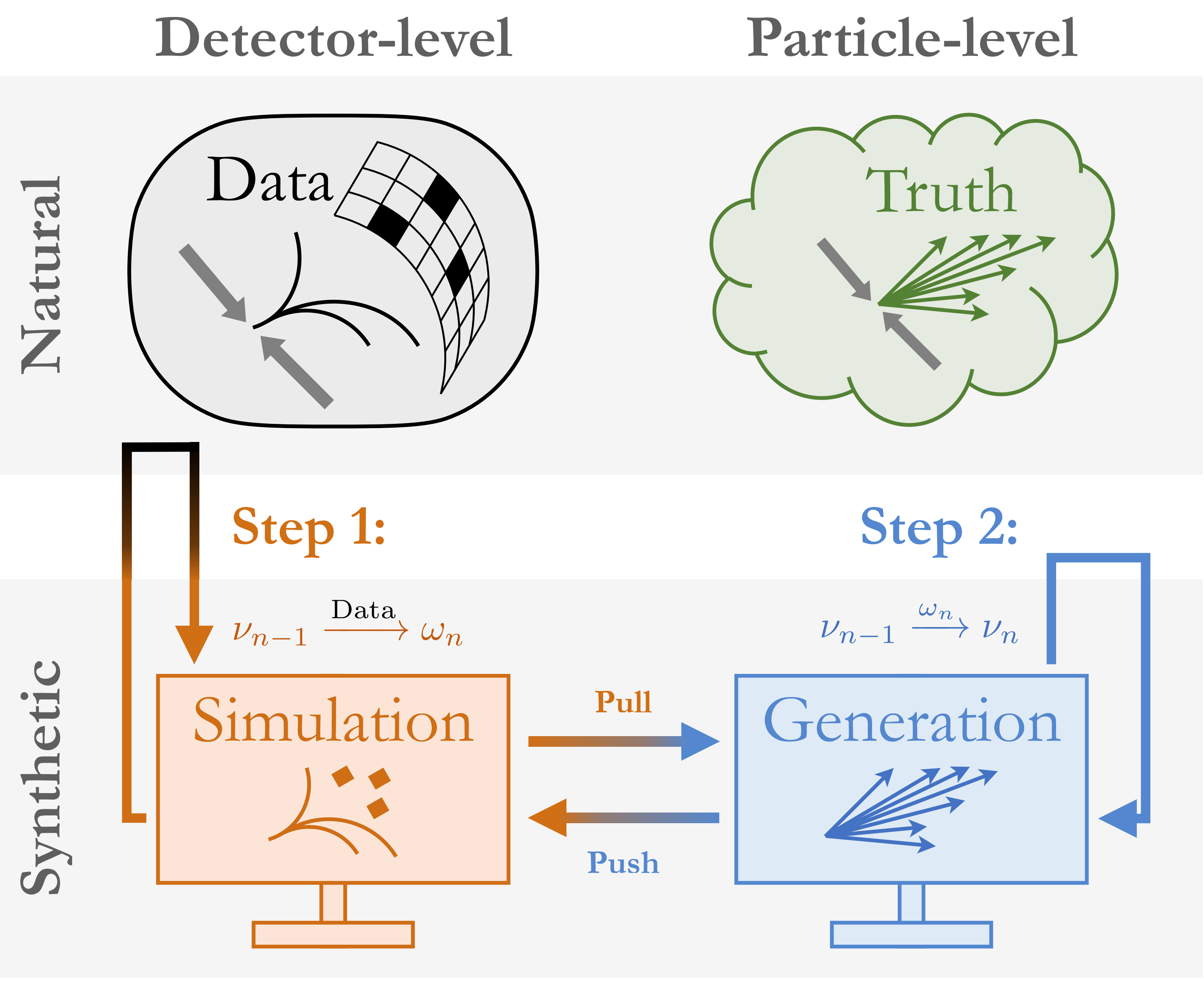
Energy Flow Network (EFN)

$$\text{EFN}(\{p_1^\mu, \ldots, p_M^\mu\}) = F\left(\sum_{i=1}^{M} z_i \Phi(\hat{p}_i)\right)$$

IRC-safe latent space

OmniFold – Unbinned, Full Phase-Space Unfolding

OmniFold weights particle-level Gen to be consistent with Data once passed through the detector



Step 1 – Reweights Sim_{n-1} to data, pulls weights back to particle-level Gen_{n-1}

Step 2 – Reweights Gen_{n-1} to (step 1)-weighted gen_{n-1}, pushes weights to detector-level Sim_n

[Andreassen, PTK, Metodiev, Nachman, Thaler, 1911.09107; PTK talk at ML4Jets 2020]

OmniFold – i.e. continuous IBU

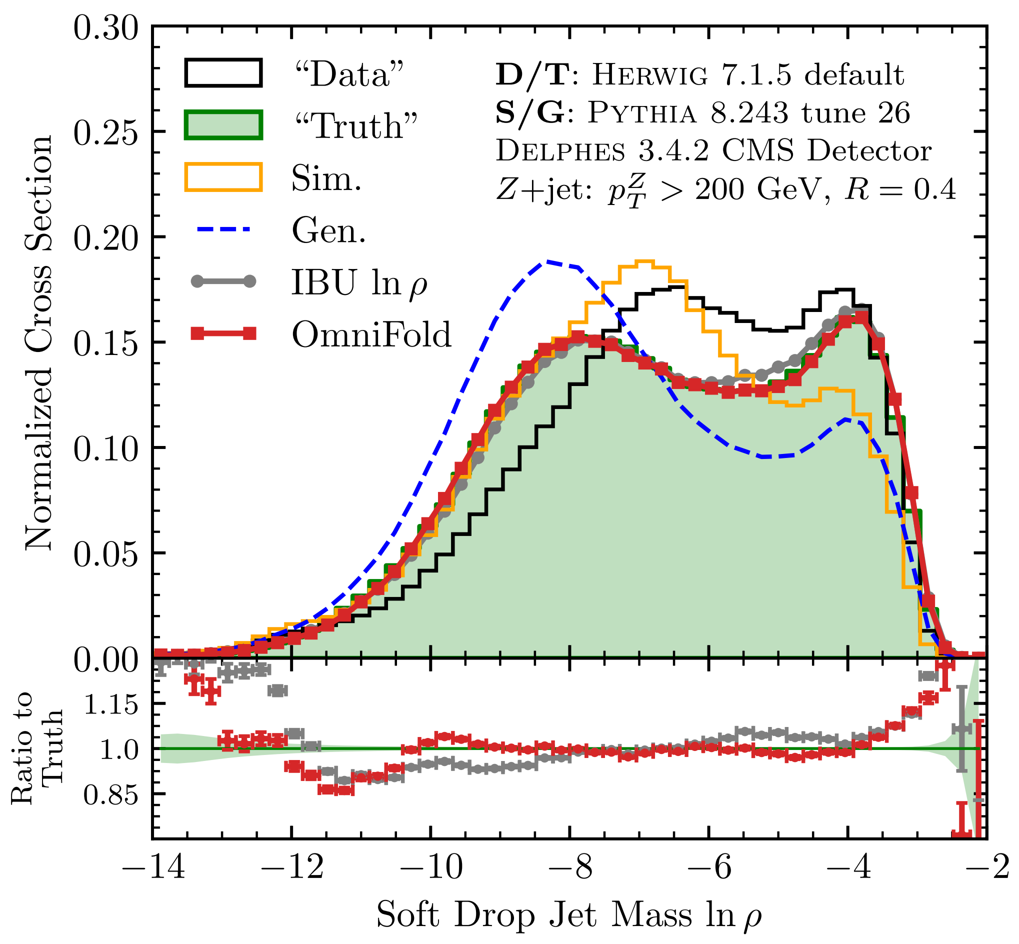
Step 1 – $\omega_n(m) = \nu_{n-1}^{\text{push}} \times L[(1, \text{Data}), (\nu_{n-1}^{\text{push}}, \text{Sim})](m)$

Step 2 – $\nu_n(t) = \nu_{n-1}(t) \times L[(\omega_n^{\text{pull}}, \text{Gen}), (\nu_{n-1}, \text{Gen})](t)$

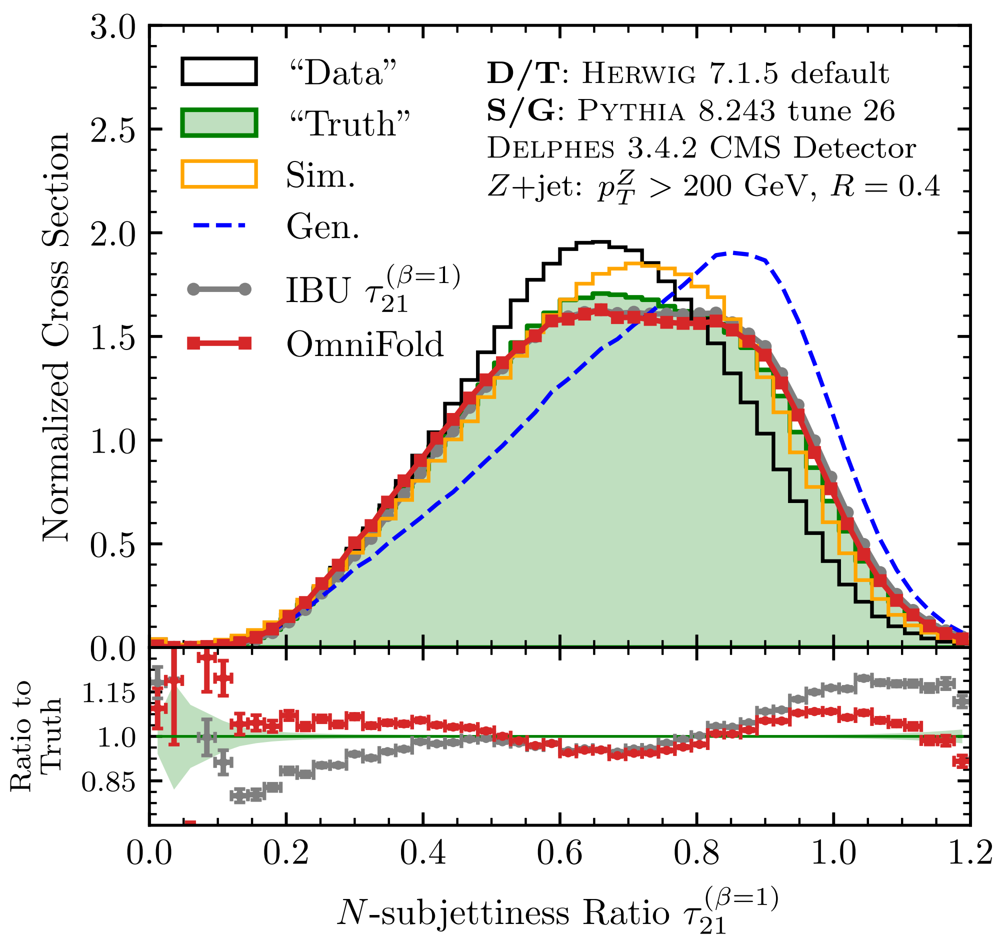
Unfold *any** observable $p_{\text{Gen}}(t)$ using universal weights $\nu_n(t)$

$$\boxed{p^{(n)}_{\text{unfolded}}(t) = \nu_n(t) \times p_{\text{Gen}}(t)}$$

*Observables should be chosen responsibly



IRC safe



Sudakov safe

Explicit Geometry – Individual Events in Theory

Hard collision

Good understanding via perturbation theory

Fragmentation

Semi-classical parton shower, effective field theory

Hadronization

Poorly understood (non-perturbative), modeled empirically

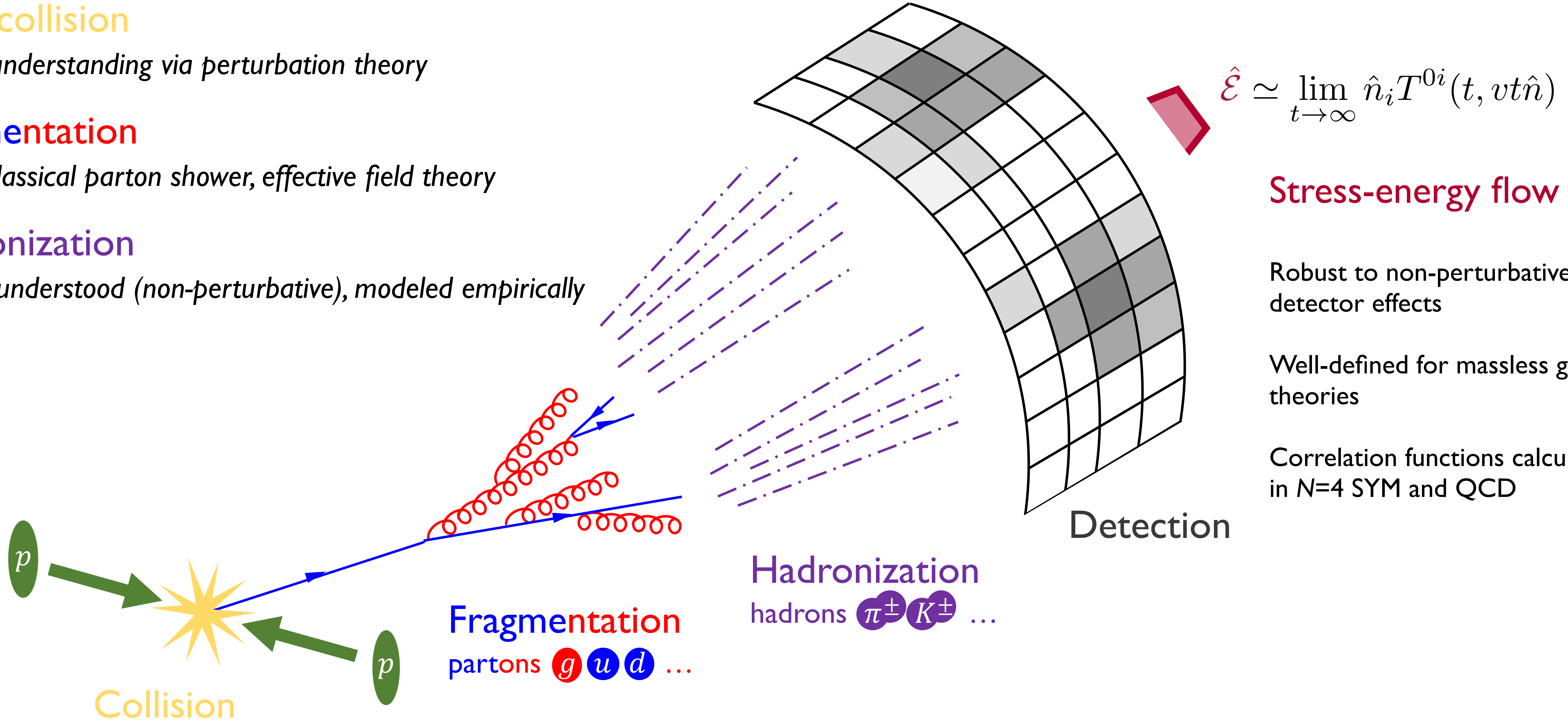


Diagram by Eric Metodiev

Stress-energy flow

Robust to non-perturbative and detector effects

Well-defined for massless gauge theories

Correlation functions calculated in *N*=4 SYM and QCD

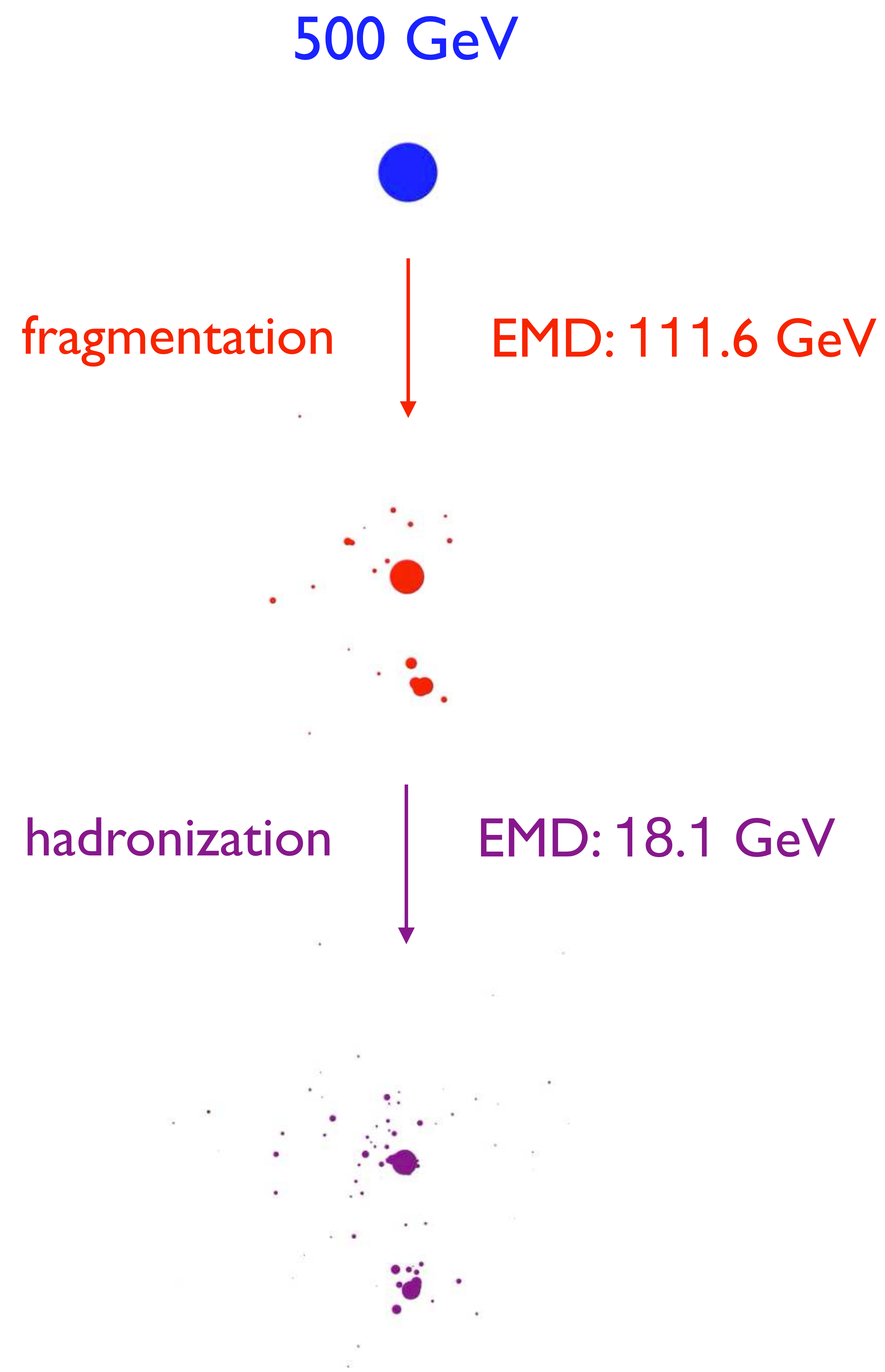
[Sveshnikov, Tkachov, PLB 1996; Hofman, Maldacena, JHEP 2008; Mateu, Stewart, Thaler, PRD 2013; Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, PRL 2014; Chen, Moult, Zhang, Zhu, 2004.11381; Dixon, PTK, Moult, Thaler, Zhu, *to appear soon*, see more here]

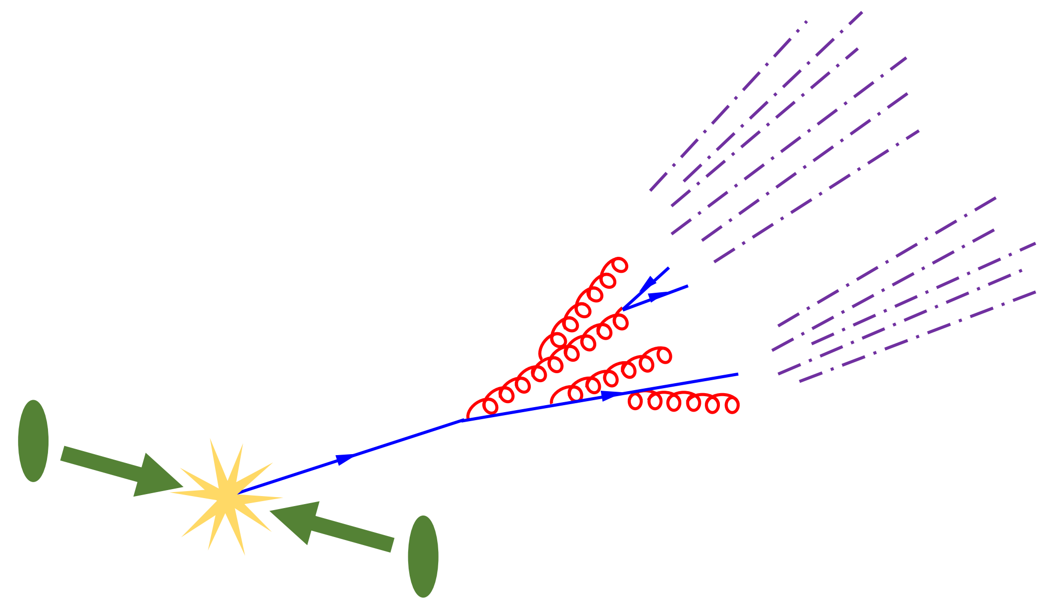
Table of Observables Defined via Event Space Geometry

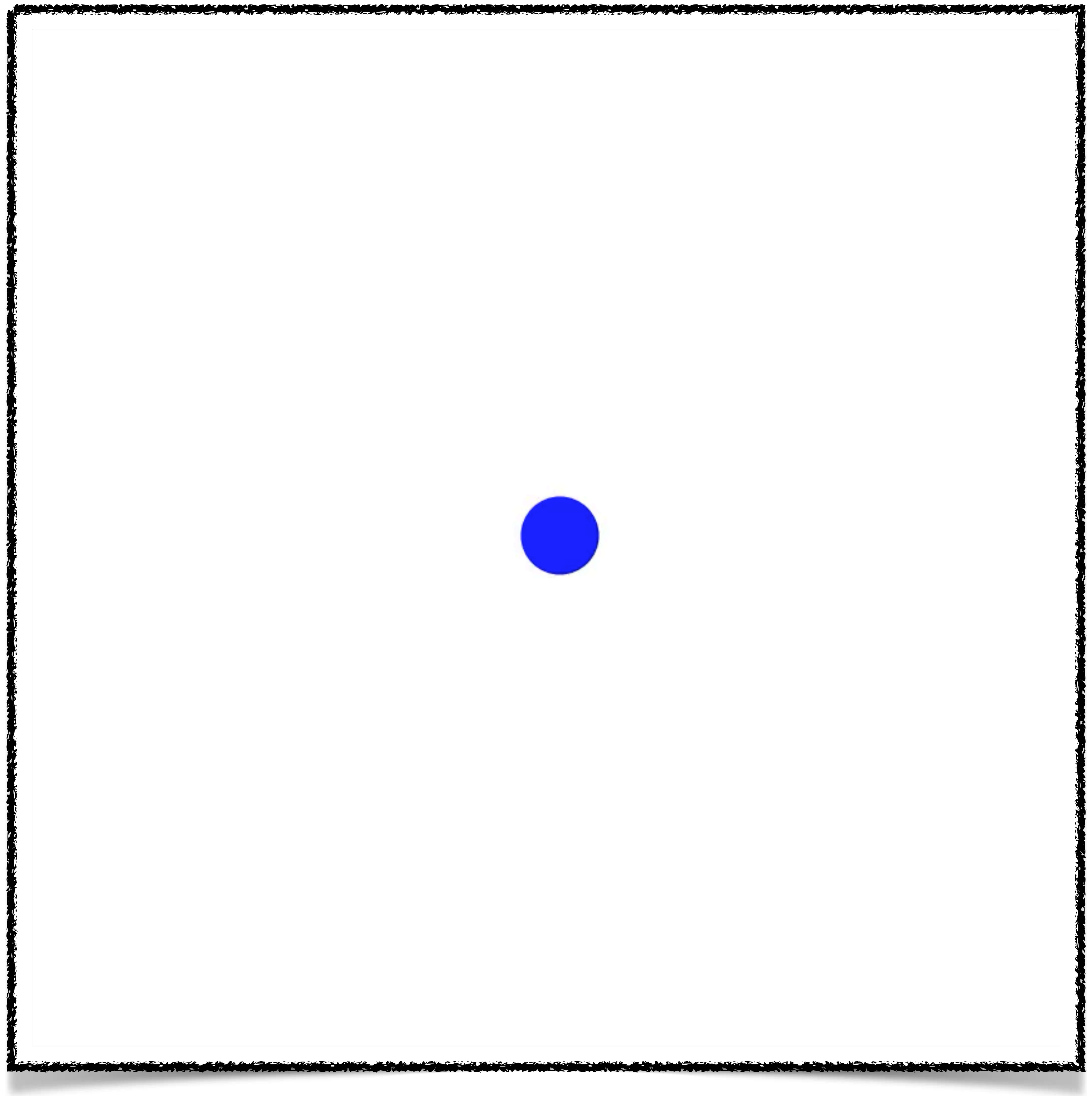
$$\boxed{\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \text{EMD}_{\beta,R}(\mathcal{E}, \mathcal{E}')}$$

$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \text{EMD}_{\beta}(\mathcal{E}, \mathcal{E}')$			
Name		β	Manifold $\mathcal{M}$
Thrust	$t(\mathcal{E})$	2	$\mathcal{P}_2^{\text{BB}}$: 2-particle events, back to back
Spherocity	$\sqrt{s(\mathcal{E})}$	1	$\mathcal{P}_2^{\text{BB}}$: 2-particle events, back to back
Broadening	$b(\mathcal{E})$	1	$\mathcal{P}_2$: 2-particle events
N-jettiness	$\mathcal{T}_N^{(\beta)}(\mathcal{E})$	β	$\mathcal{P}_N$: N-particle events
Isotropy	$\mathcal{I}^{(\beta)}(\mathcal{E})$	β	$\mathcal{M}_{\mathcal{U}}$: Uniform events
Jet Angularities	$\lambda_\beta(\mathcal{J})$	β	$\mathcal{P}_1$: 1-particle jets
N-subjettiness	$\tau_N^{(\beta)}(\mathcal{J})$	β	$\mathcal{P}_N$: N-particle jets

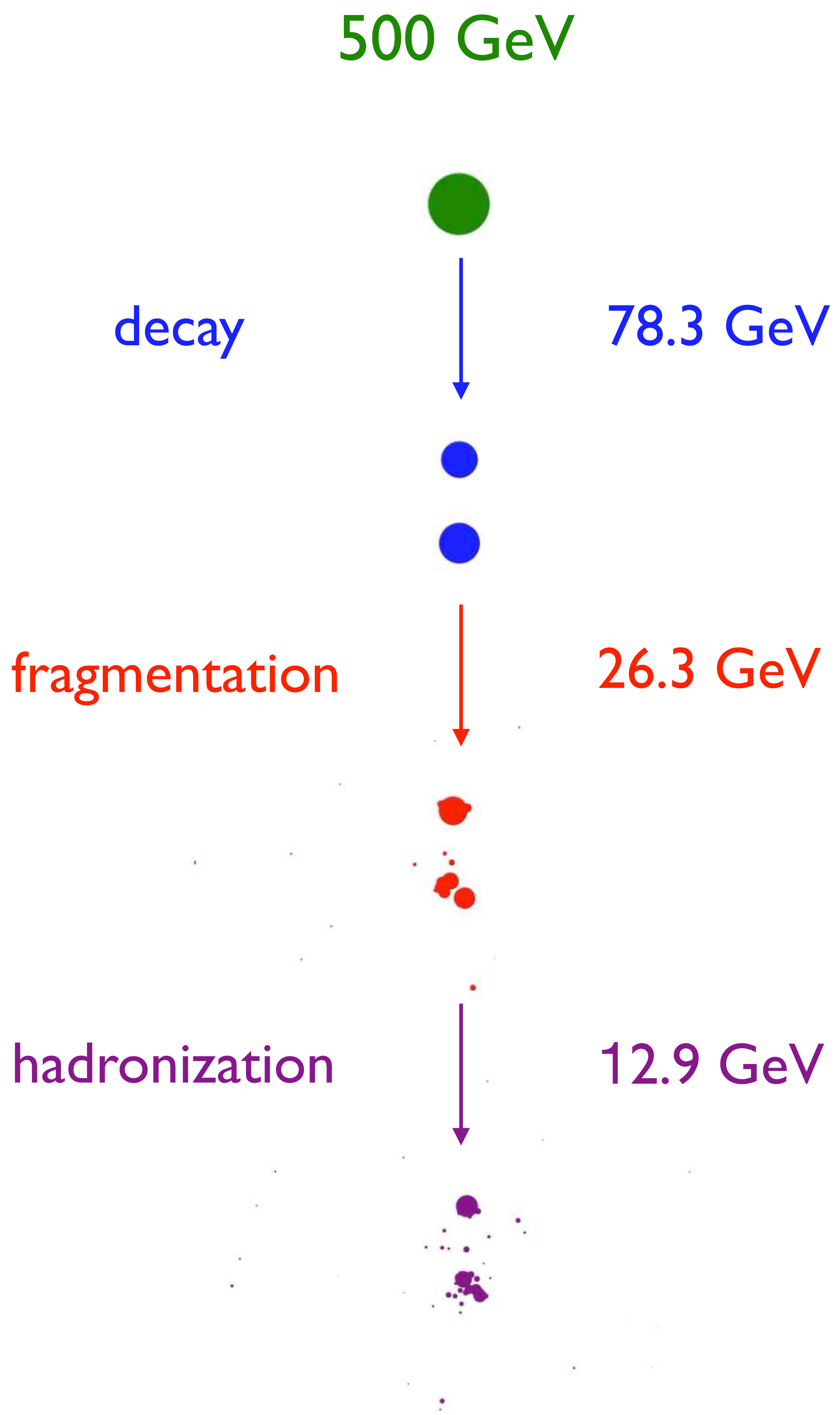
Visualizing Jet Formation – QCD Jets

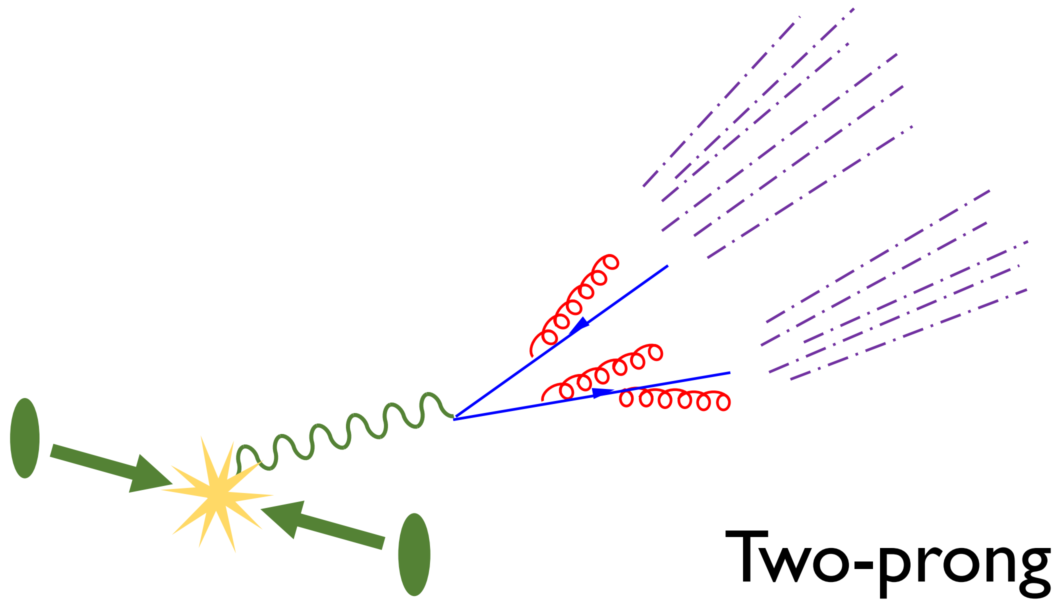




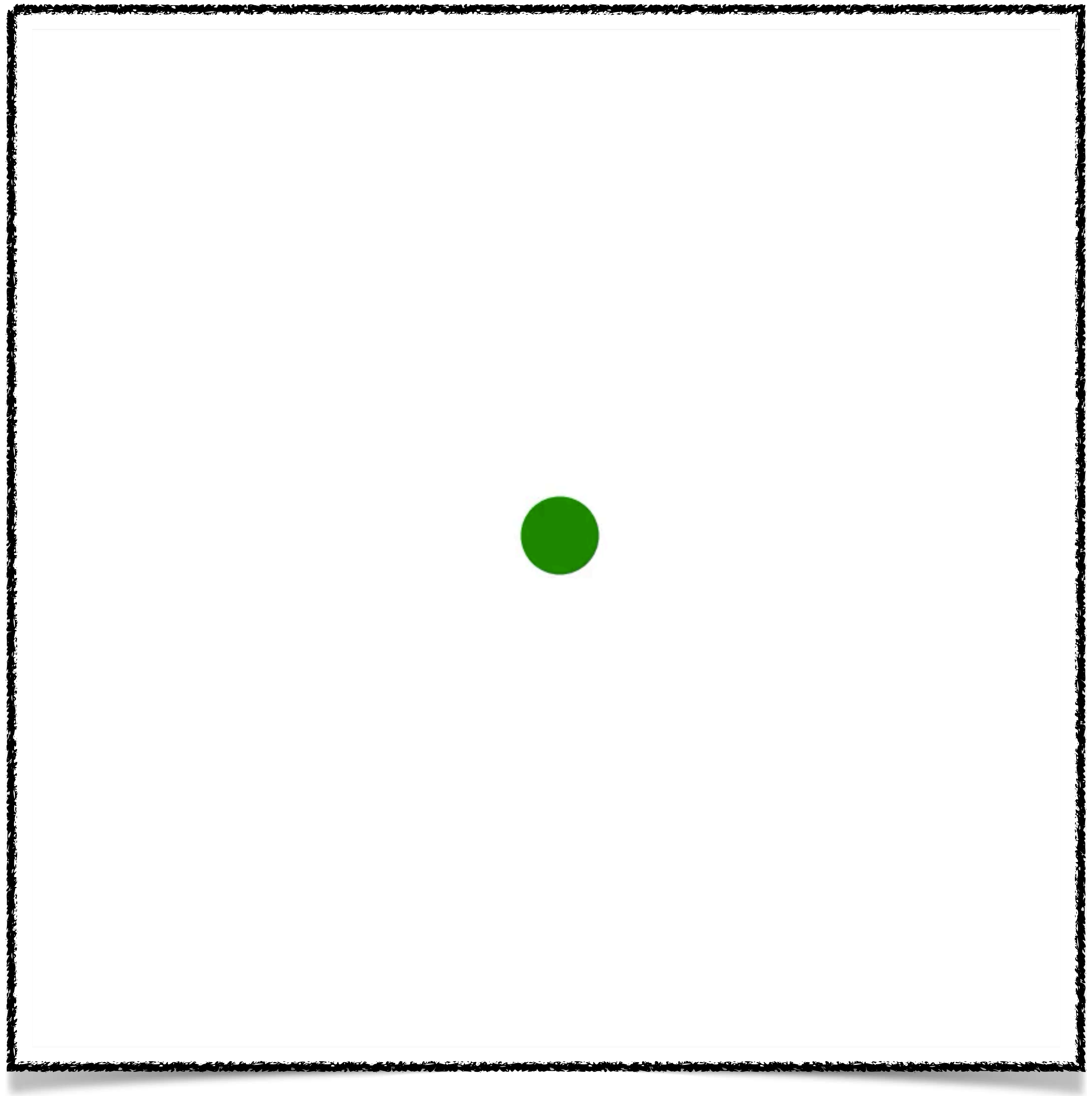


Visualizing Jet Formation – W Jets

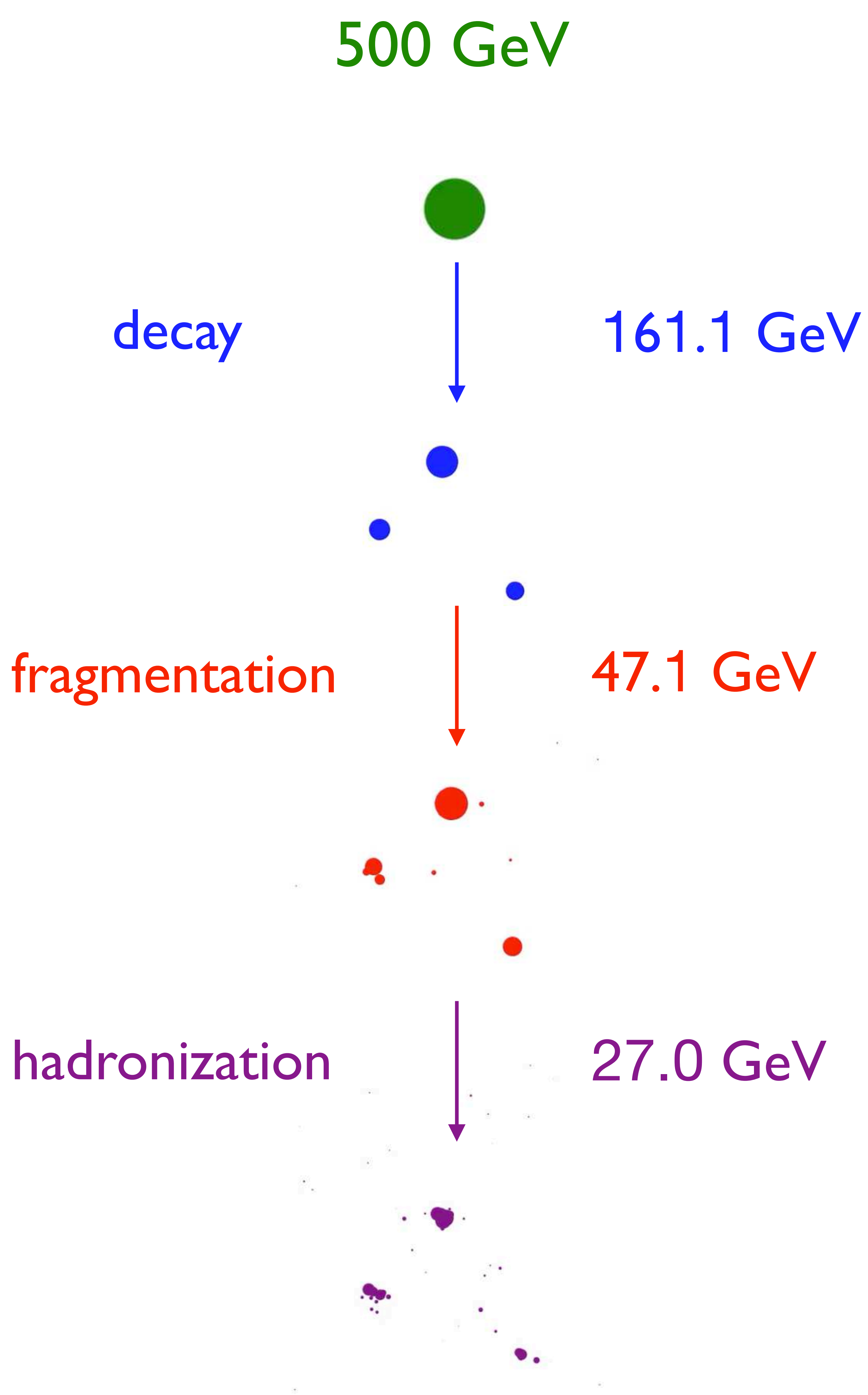


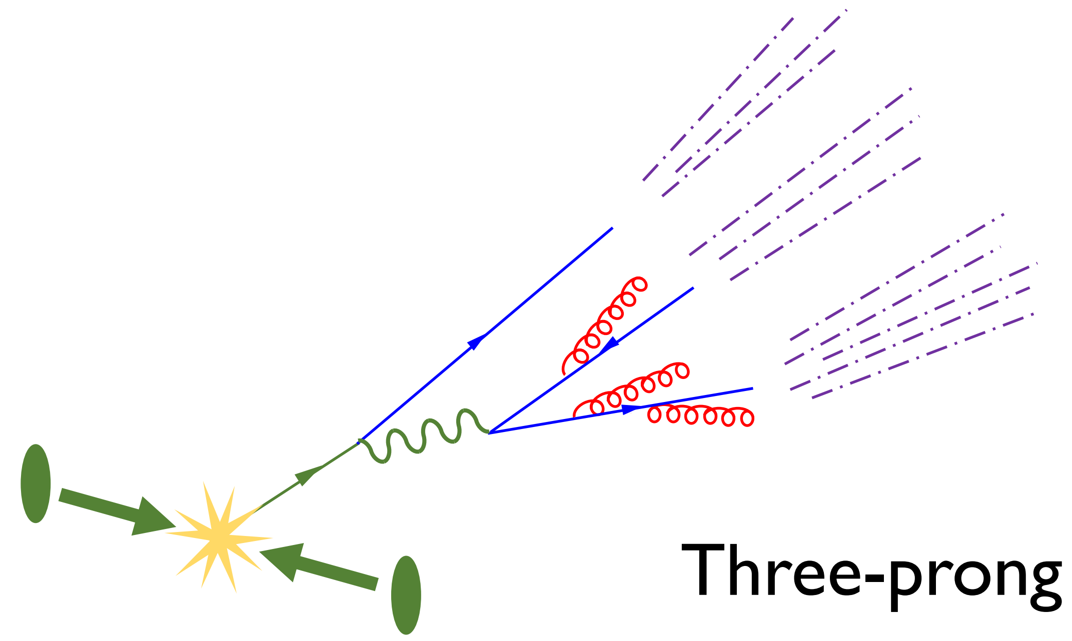


Two-prong decay

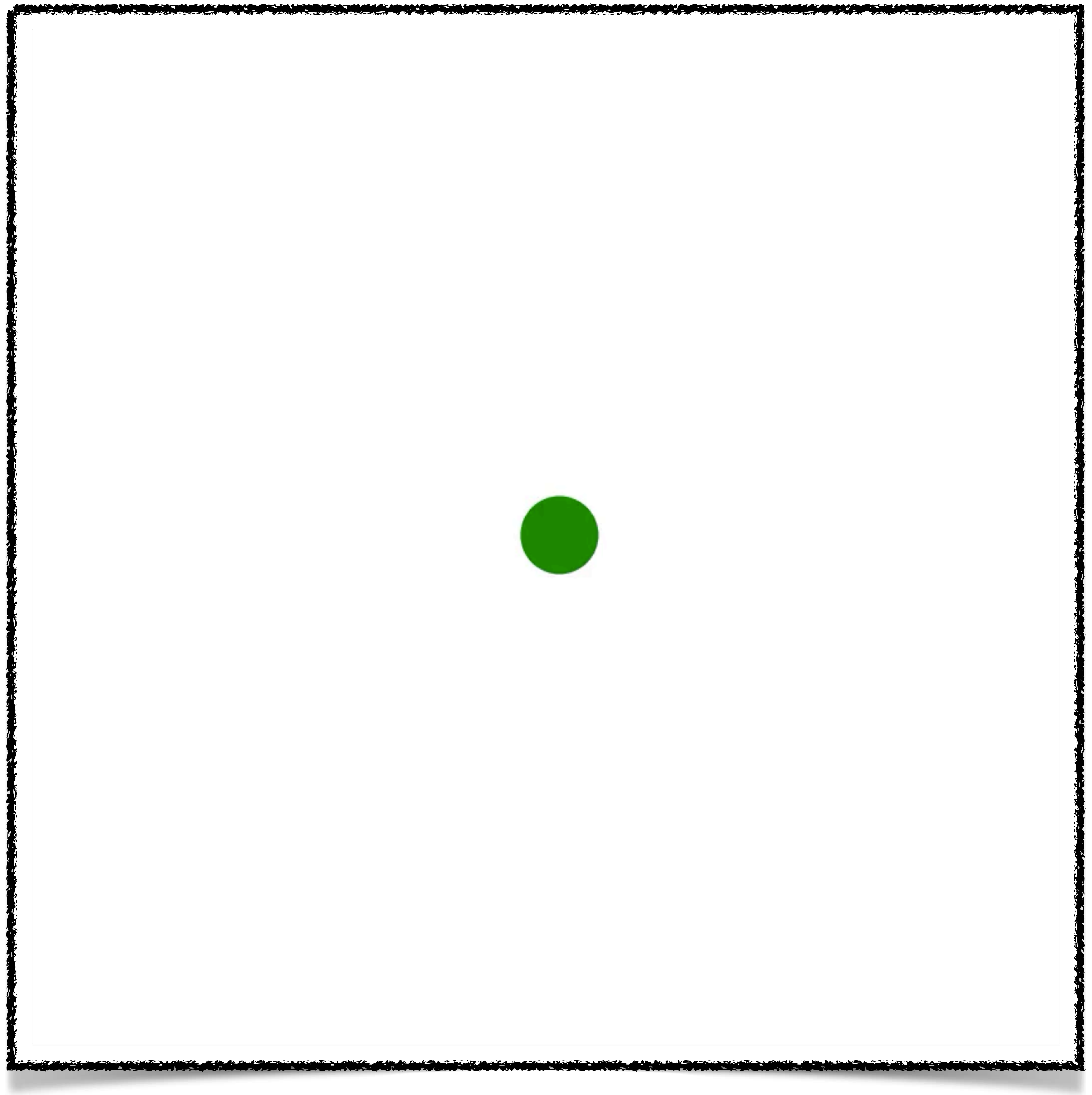


Visualizing Jet Formation – Top Jets





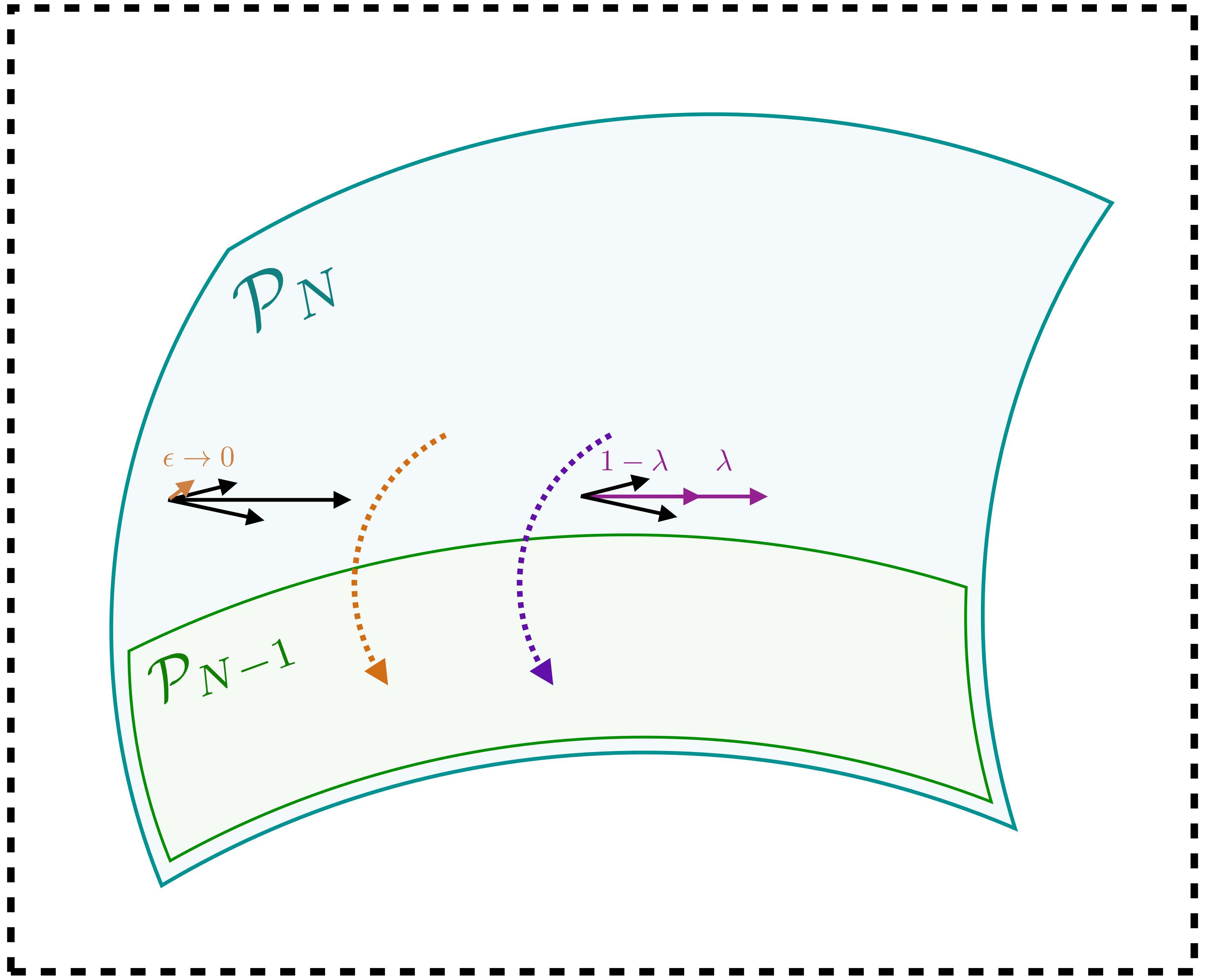
Three-prong decay



N-particle Manifolds in the Space of Events – Infrared Divergences

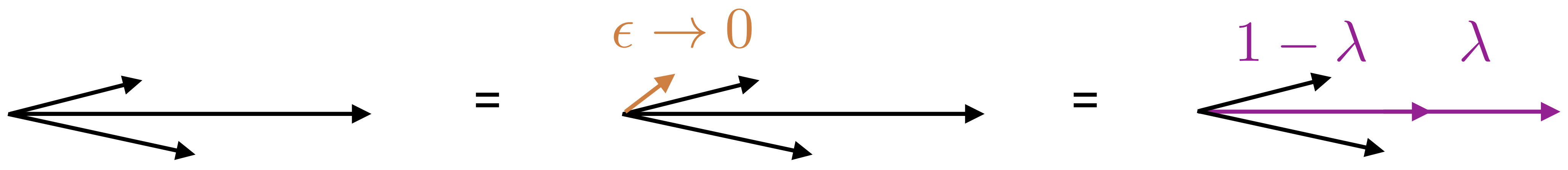
[PTK, Metodiev, Thaler, JHEP 2020]

$$\mathcal{P}_N = \text{set of all N-particle configurations} = \left\{ \sum_{i=1}^{N} E_i\,\delta(\hat{n} - \hat{n}_i) \;\middle|\; E_i \geq 0 \right\}$$



$$dP_{i\to ig} \simeq \frac{2\alpha_s}{\pi} C_a \frac{d\theta}{\theta}\frac{dz}{z}$$

Energy flow is unchanged by exact soft/collinear emissions



Functions of energy flow automatically satisfy exact IRC invariance!

Real and virtual divergences appear naturally together

$$\mathcal{P}_N \supset \mathcal{P}_{N-1} \supset \cdots \supset \mathcal{P}_3 \supset \mathcal{P}_2 \supset \mathcal{P}_1$$

by soft and collinear limits

Defining IRC Safety Precisely

[Sterman, Weinberg, PRL 1997; Sterman, PRD 1978; Banfi, Salam, Zanderighi, JHEP 2005]

Infrared and collinear safety is a proxy for perturbative calculability of an observable

Exact IRC invariance

$$\mathcal{O}(p_1^\mu, \ldots, p_M^\mu) = \mathcal{O}(0p_0^\mu, p_1^\mu, \ldots, p_M^\mu)$$
$$\mathcal{O}(p_1^\mu, \ldots, p_M^\mu) = \mathcal{O}(\lambda p_1^\mu, (1-\lambda)p_1^\mu, \ldots, p_M^\mu)$$

Guarantees observable is well-defined on energy flows

Allows for pathological observables, e.g. pseudo-multiplicity

Smooth IRC invariance

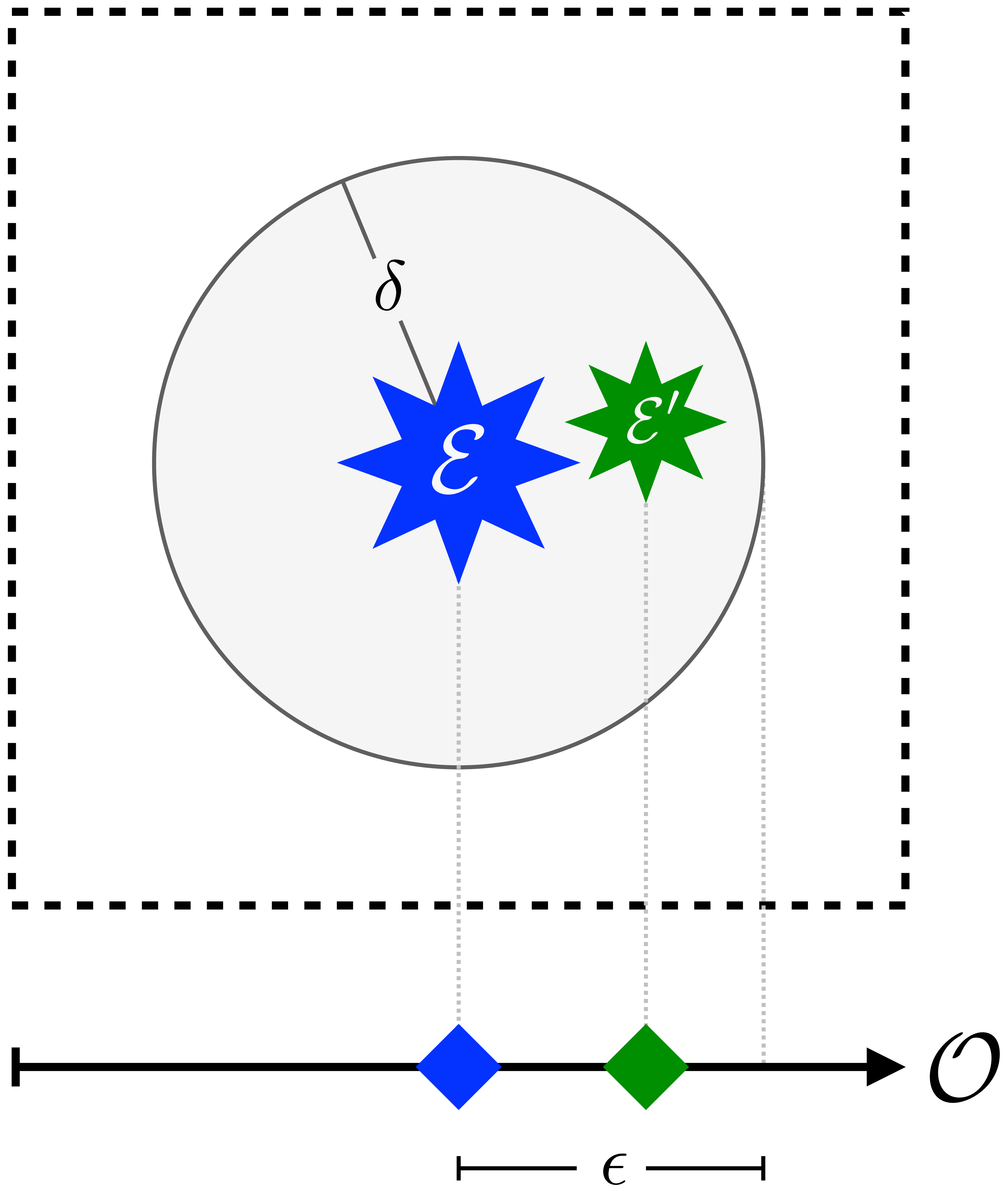
$$\mathcal{O}(p_1^\mu, \ldots, p_M^\mu) = \lim_{\epsilon\to 0} \mathcal{O}(\epsilon p_0^\mu, p_1^\mu, \ldots, p_M^\mu)$$
$$\mathcal{O}(p_1^\mu, \ldots, p_M^\mu) = \lim_{p_0^\mu \to p_1^\mu} \mathcal{O}(\lambda p_0^\mu, (1-\lambda)p_1^\mu, \ldots, p_M^\mu)$$

Eliminates common observables with hard boundaries

All Observables	Comments
Multiplicity ($\sum_i 1$)	IR unsafe and C unsafe
Momentum Dispersion [65] ($\sum_i E_i^2$)	IR safe but C unsafe
Sphericity Tensor [66] ($\sum_i p_i^\mu p_i^\nu$)	IR safe but C unsafe
Number of Non-Zero Calorimeter Deposits	C safe but IR unsafe
Defined on Energy Flows	
Pseudo-Multiplicity ($\min\{N \mid \mathcal{T}_N = 0\}$)	Robust to exact IR or C emissions
Infrared & Collinear Safe	
Jet Energy ($\sum_i E_i$)	Disc. at jet boundary
Heavy Jet Mass [67]	Disc. at hemisphere boundary
Soft-Dropped Jet Mass [38, 68]	Disc. at grooming threshold
Calorimeter Activity [69] (N_{95})	Disc. at cell boundary

More EMD Geometry – Continuity in the Space of Events

[PTK, Metodiev, Thaler, 2004.04159]



Classic $\epsilon - \delta$ definition of continuity in a metric space

An observable $\mathcal{O}$ is EMD continuous at an event $\mathcal{E}$ if, for any $\epsilon > 0$, there exists a $\delta > 0$ such that for all events $\mathcal{E}'$:

$$\text{EMD}(\mathcal{E}, \mathcal{E}') < \delta \quad \implies \quad |\mathcal{O}(\mathcal{E}) - \mathcal{O}(\mathcal{E}')| < \epsilon.$$

Towards a geometric definition of IRC Safety

IRC Safety = EMD Continuity*

*on all but a negligible set‡ of events

‡a negligible set is one that contains no positive-radius EMD-ball

⋮

Perturbation Theory in the Space of Events

[PTK, Metodiev, Thaler, 2004.04159]

Sudakov safety

[Larkoski, Thaler, JHEP 2014; Larkoski, Marzani, Thaler, PRD 2015]

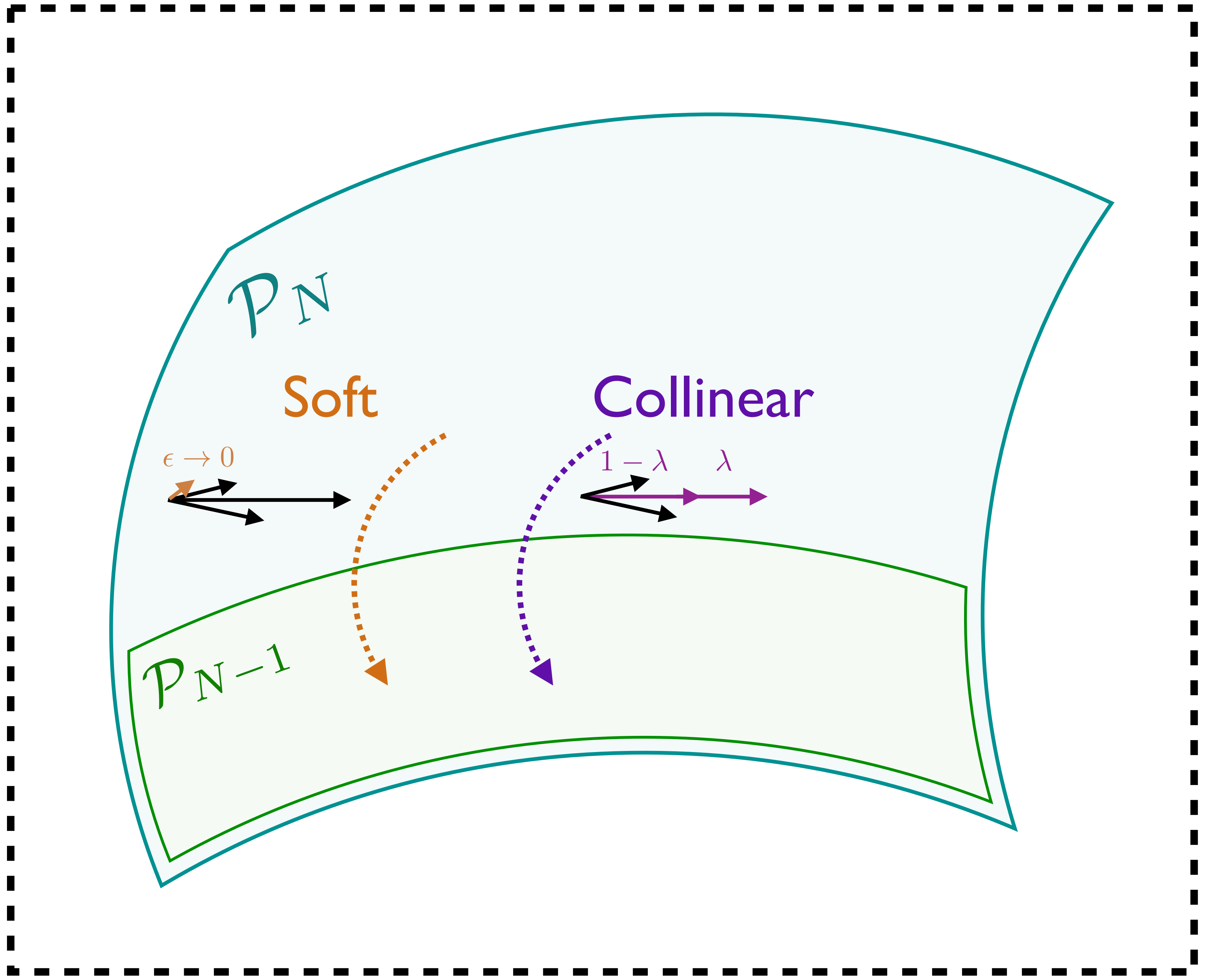
Some observables have discontinuities on P_N for some N

A resummed IRC-safe companion can mitigate the divergences

$$p(\mathcal{O}_{\text{Sudakov}}) = \int d\mathcal{O}_{\text{Comp.}}\, p(\mathcal{O}_{\text{Sudakov}}|\mathcal{O}_{\text{Comp.}})\, p(\mathcal{O}_{\text{Comp.}})$$

Event geometry suggests N-(sub)jettiness as universal companion

Infrared singularities of massless gauge theories appear on each P_N



Fixed-order calculability

[Sterman, PRD 1979; Banfi, Salam, Zanderighi, JHEP 2005]

Is a statement of integrability on each P_N

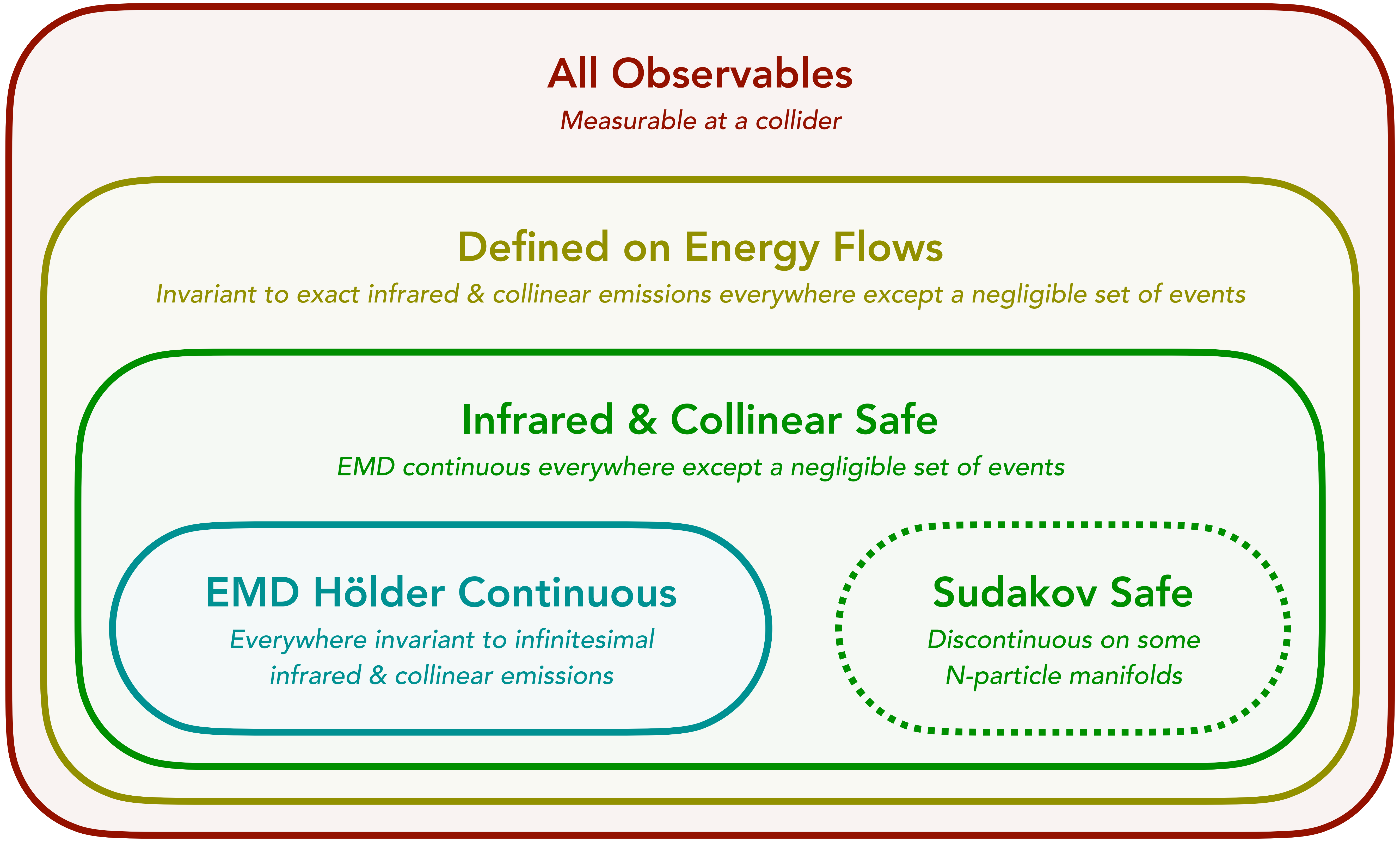
EMD continuity must be upgraded to EMD-Hölder continuity on each P_N

$$\lim_{\mathcal{E}\to\mathcal{E}'} \frac{\mathcal{O}(\mathcal{E}) - \mathcal{O}(\mathcal{E}')}{\text{EMD}(\mathcal{E},\mathcal{E}')^c} = 0, \quad c > 0$$

Example: $V(\mathcal{E}) = \mathcal{T}_2(\mathcal{E})\left(1 + \frac{1}{\ln E(\mathcal{E})/\mathcal{T}_3(\mathcal{E})}\right)$ is EMD continuous but not EMD Hölder continuous (it is Sudakov safe)

Hierarchy of IRC Safety Definitions

[PTK, Metodiev, Thaler, 2004.04159]



All Observables	Comments
Multiplicity ($\sum_i 1$)	IR unsafe and C unsafe
Momentum Dispersion [65] ($\sum_i E_i^2$)	IR safe but C unsafe
Sphericity Tensor [66] ($\sum_i p_i^\mu p_i^\nu$)	IR safe but C unsafe
Number of Non-Zero Calorimeter Deposits	C safe but IR unsafe
Defined on Energy Flows	
Pseudo-Multiplicity ($\min\{N \mid \mathcal{T}_N = 0\}$)	Robust to exact IR or C emissions
Infrared & Collinear Safe	
Jet Energy ($\sum_i E_i$)	Disc. at jet boundary
Heavy Jet Mass [67]	Disc. at hemisphere boundary
Soft-Dropped Jet Mass [38, 68]	Disc. at grooming threshold
Calorimeter Activity [69] (N_{95})	Disc. at cell boundary
Sudakov Safe	
Groomed Momentum Fraction [39] (z_g)	Disc. on 1-particle manifold
Jet Angularity Ratios [37]	Disc. on 1-particle manifold
N-subjettiness Ratios [47, 48] (τ_{N+1}/τ_N)	Disc. on N-particle manifold
V parameter [36] (Eq. (2.11))	Hölder disc. on 3-particle manifold
EMD Hölder Continuous Everywhere	
Thrust [40, 41]	
Spherocity [42]	
Angularities [70]	
N-jettiness [44] ($\mathcal{T}_N$)	
C parameter [71–74]	Resummation beneficial at $C = \frac{3}{4}$
Linear Sphericity [72] ($\sum_i E_i n_i^\mu n_i^\nu$)	
Energy Correlators [36, 75–77]	
Energy Flow Polynomials [15, 17]	

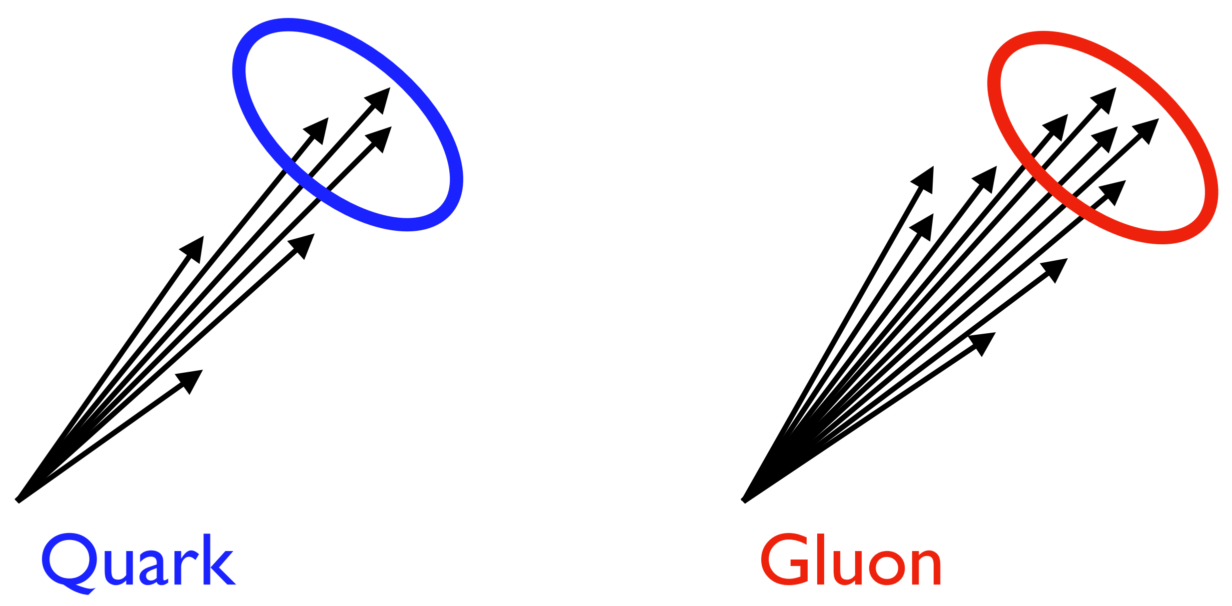
Quark and Gluon Correlation Dimensions

$$\dim(Q) = Q\frac{\partial}{\partial Q}\ln\sum_i\sum_j\Theta(\mathrm{EMD}(\mathcal{E}_i,\mathcal{E}'_j) < Q)$$

Leading log (single emission) calculation:

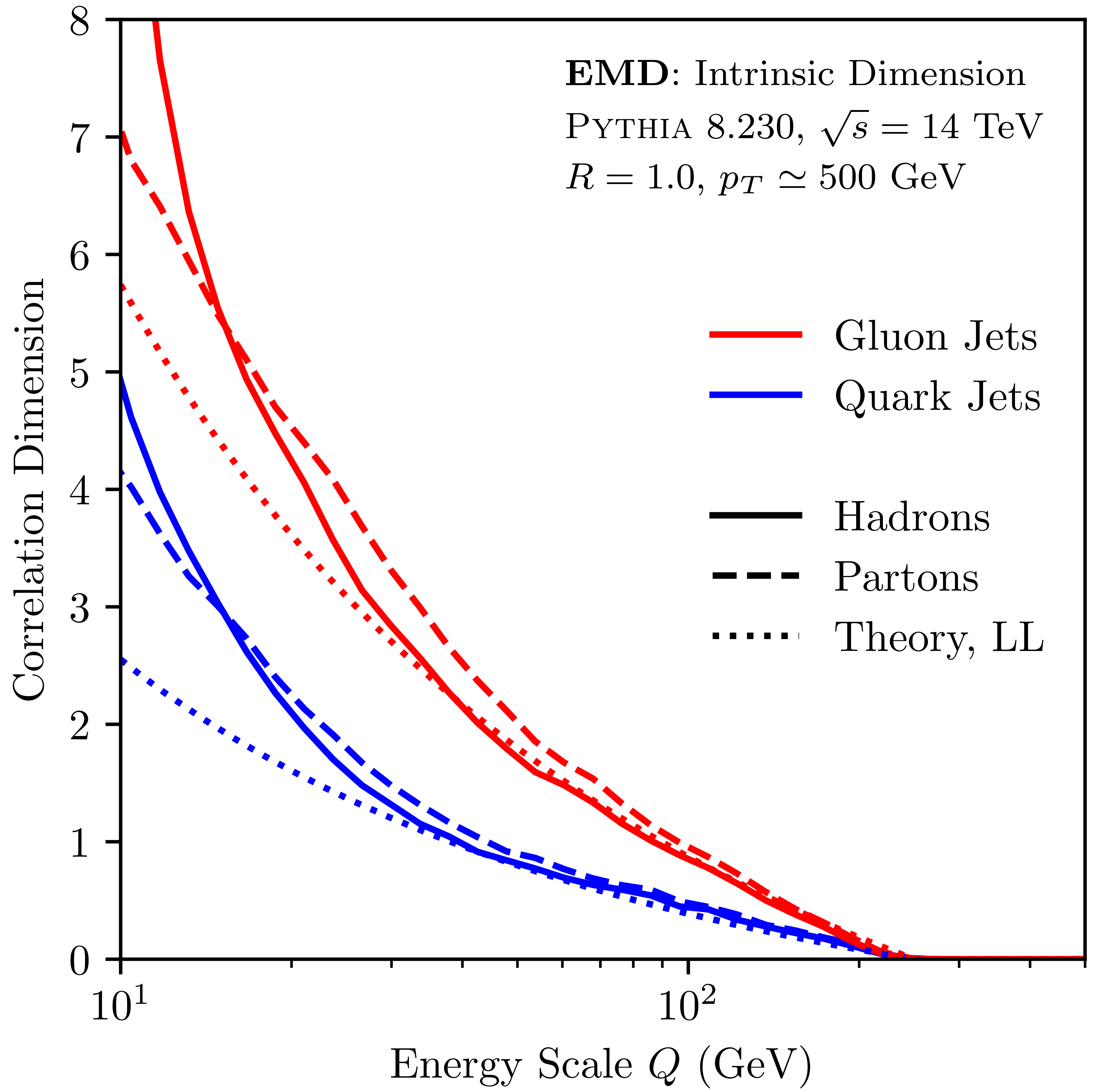
$$\dim_i(Q) \simeq -\frac{8\alpha_s}{\pi}C_i\ln\frac{Q}{p_T/2}$$

color factor



$C_F = 4/3$

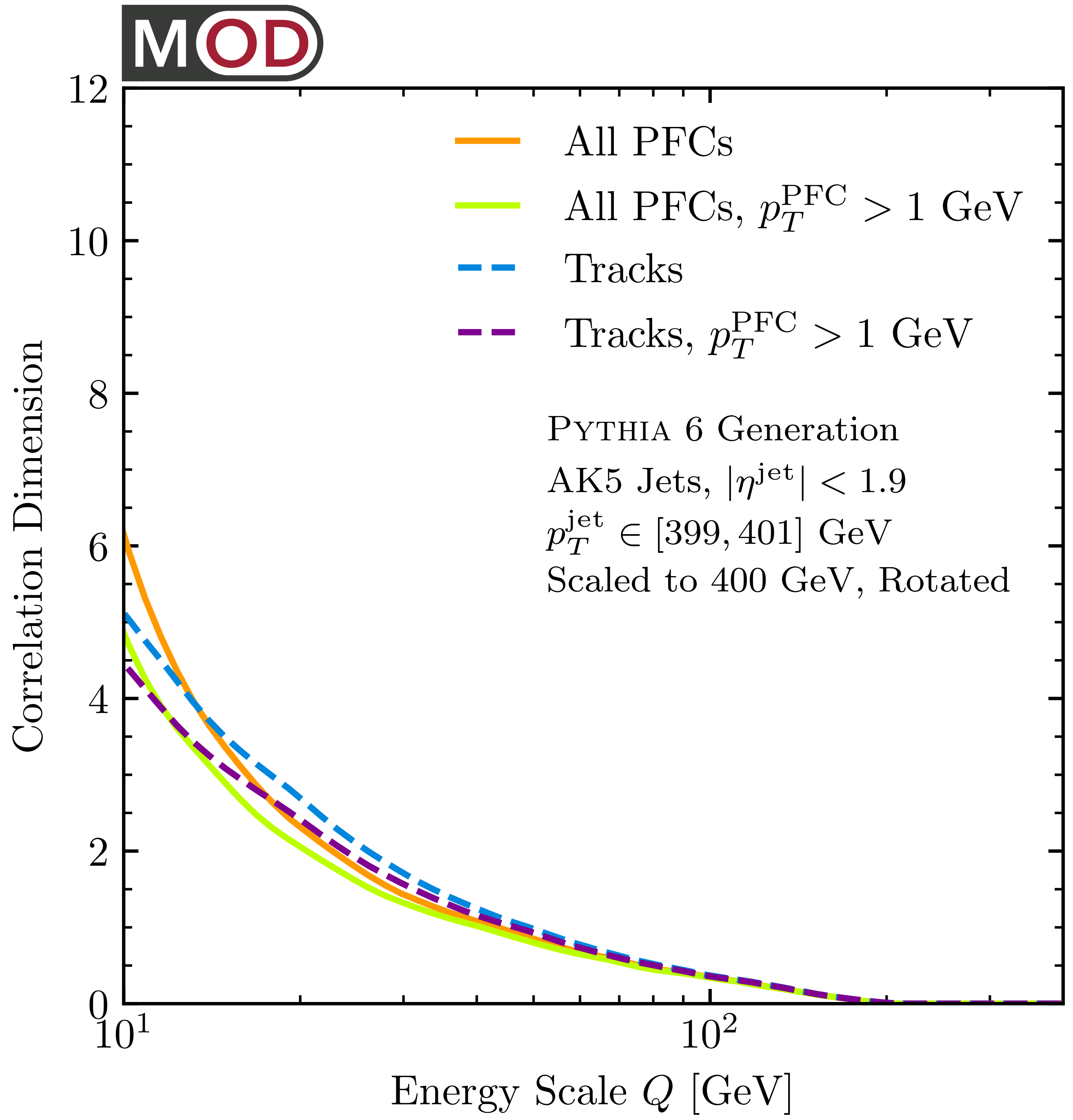
$C_A = 3$



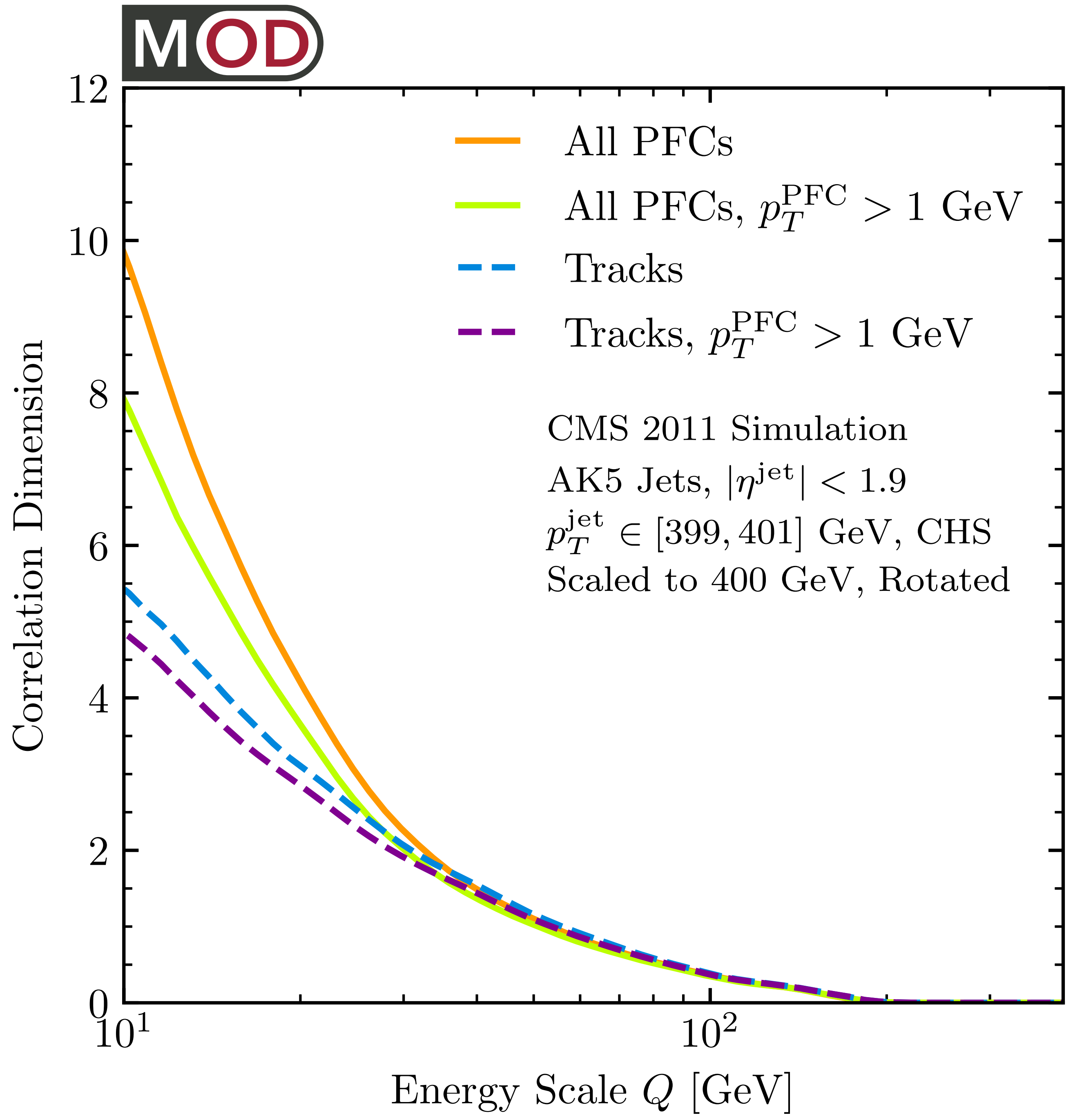
[PTK, Metodiev, Thaler, *to appear soon*]

Correlation Dimension at Particle and Detector Levels

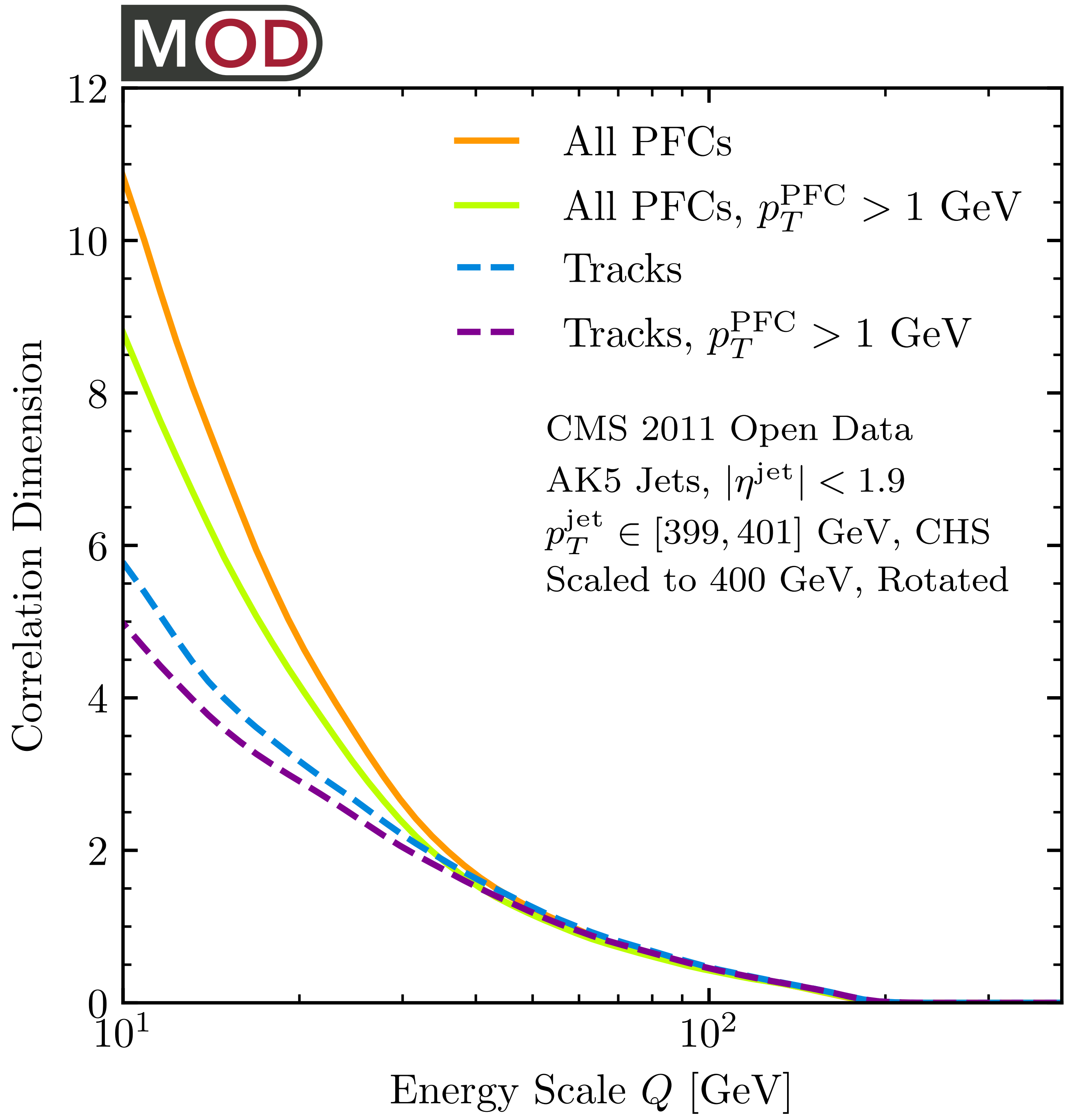
Particle-level (PYTHIA)



Detector-level (PYTHIA + GEANT 4)

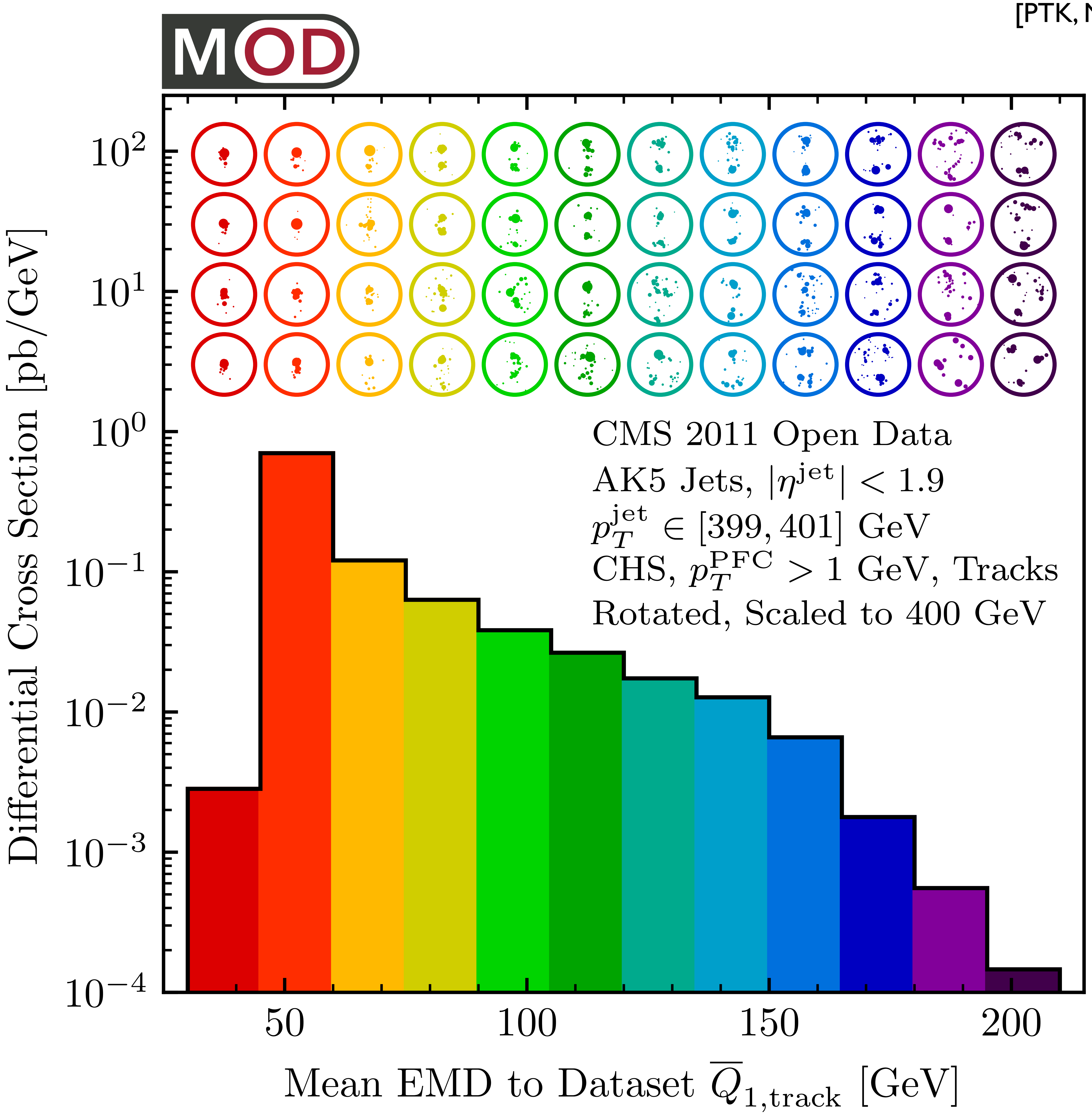


CMS Open Data



Visualizing Geometry in CMS Open Data

[PTK, Mastandrea, Metodiev, Naik, Thaler, PRD 2019; code and datasets at energyflow.network]



EMD for anomaly detection

4 medoids in each bin of anomaliness $\bar{Q}_1$

n^th moment of EMD distribution for a dataset

$$\bar{Q}_n(\mathcal{I}) = \sqrt[n]{\frac{1}{N}\sum_{k=1}^{N}(\text{EMD}(\mathcal{I},\mathcal{J}_k))^n}$$

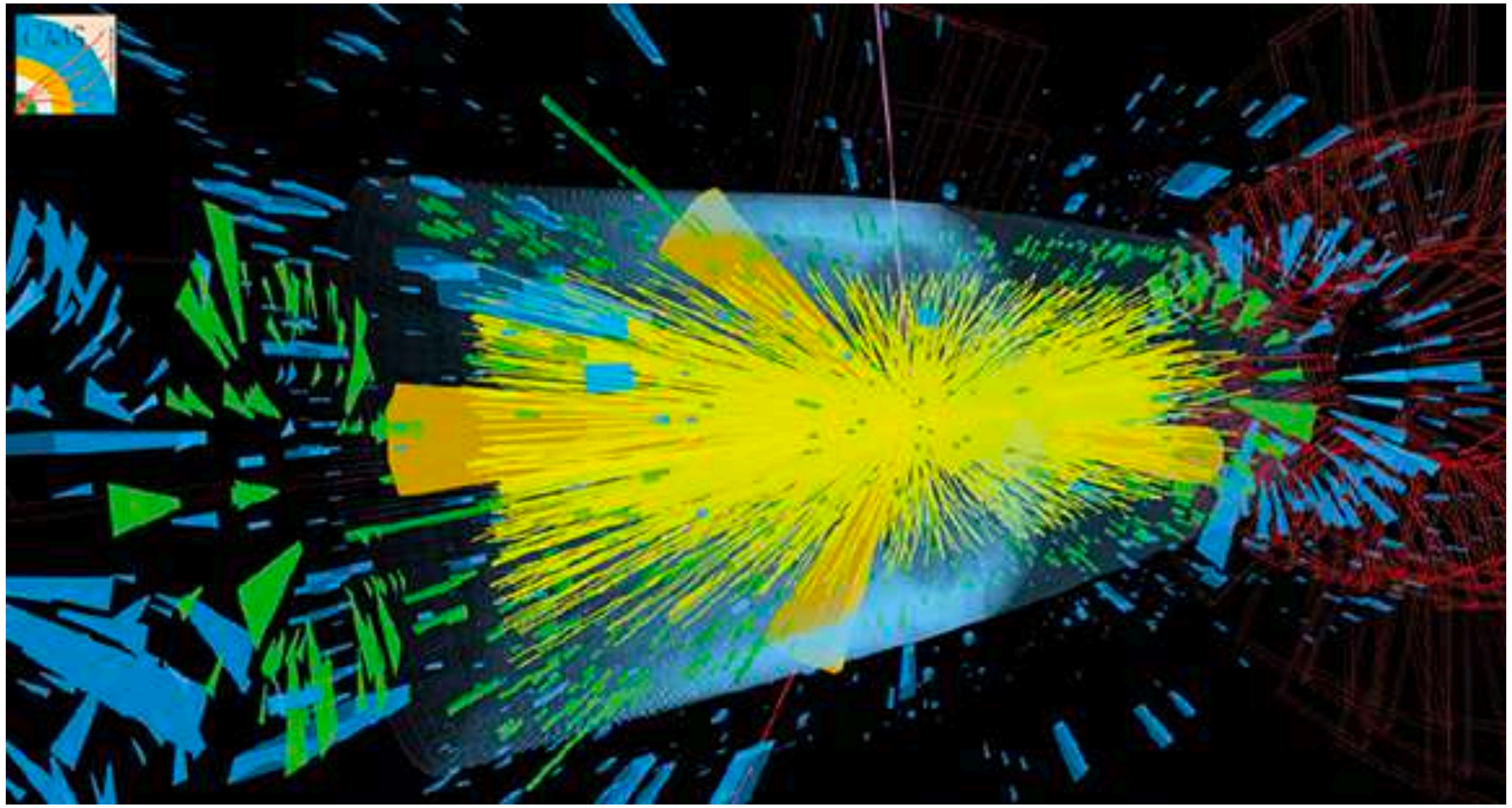
How far does this go?

$$\mathcal{V}_k = \frac{1}{N}\sum_{i=1}^{N}\min\{\text{EMD}(\mathcal{J}_i,\mathcal{K}_1),\ldots,\text{EMD}(\mathcal{J}_i,\mathcal{K}_k)\}$$

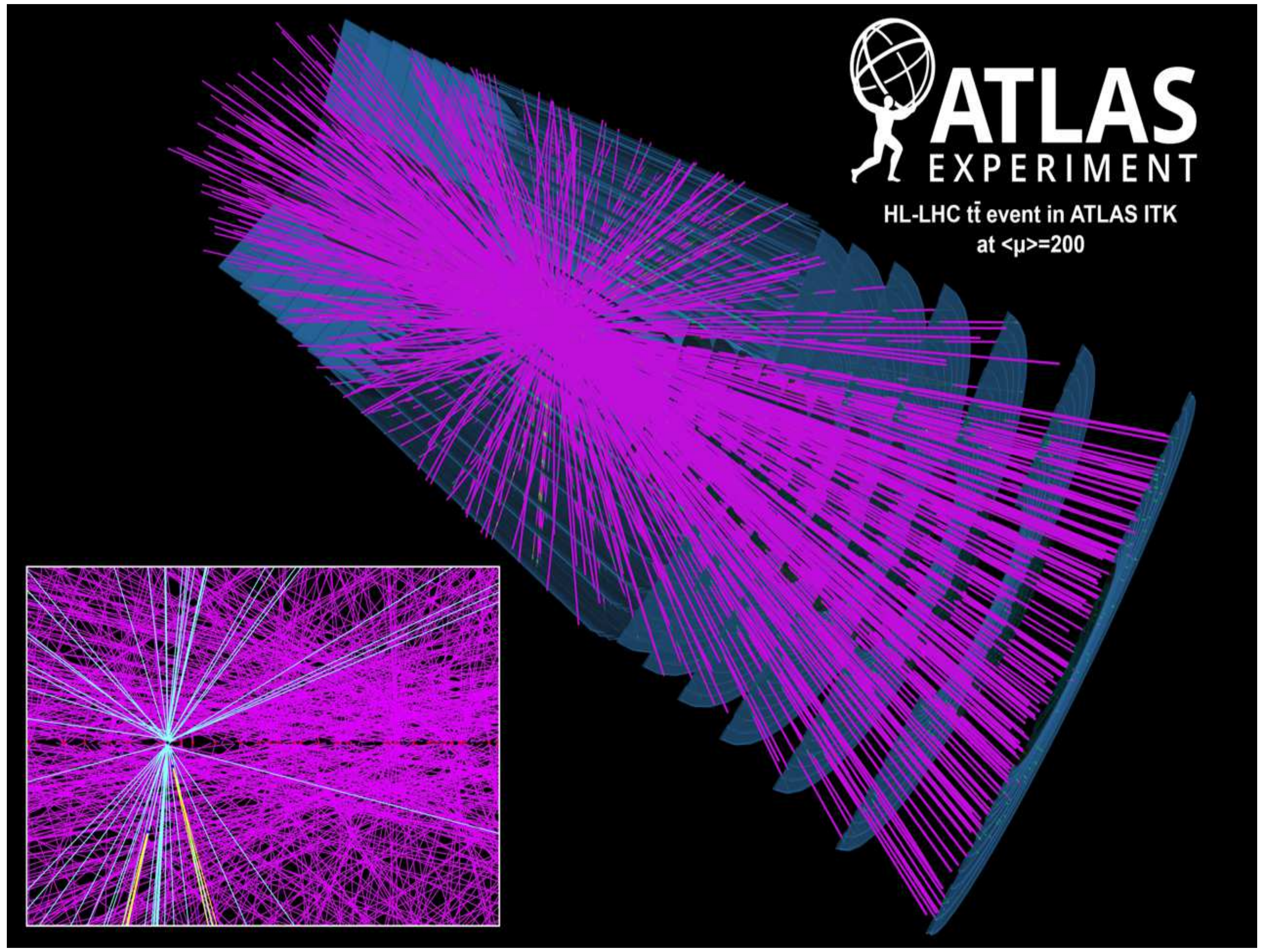
k-eventiness? jet from dataset medoids

Pileup at the (HL-)LHC

Pileup is uniform (on average) radiation from additional proton-proton collisions



VBF Higgs + 200 pileup vertices



$t\bar{t}$ + 200 pileup vertices

Pileup Mitigation in Event Space

Pileup: uniform (on average) radiation from additional proton-proton collisions

Pileup mitigation: "moving away" from the uniform event

Uniform radiation with unit energy density

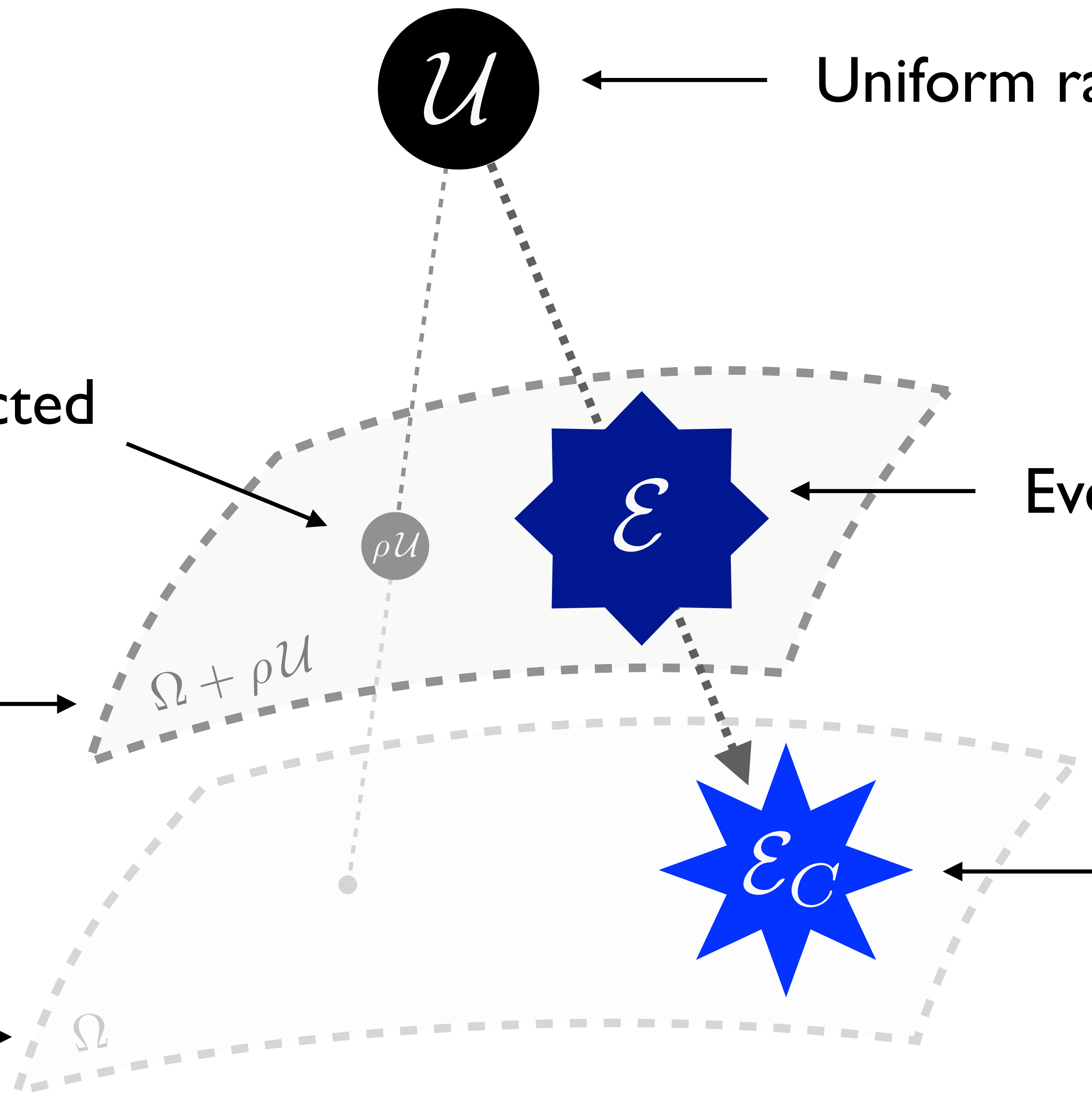
Pileup radiation to be subtracted
ρ: pileup energy density

Event with pileup contamination

$\Omega + \rho\mathcal{U}$: all events with a uniform component

"Cleaned" event, free of pileup

Ω: the space of all energy flows



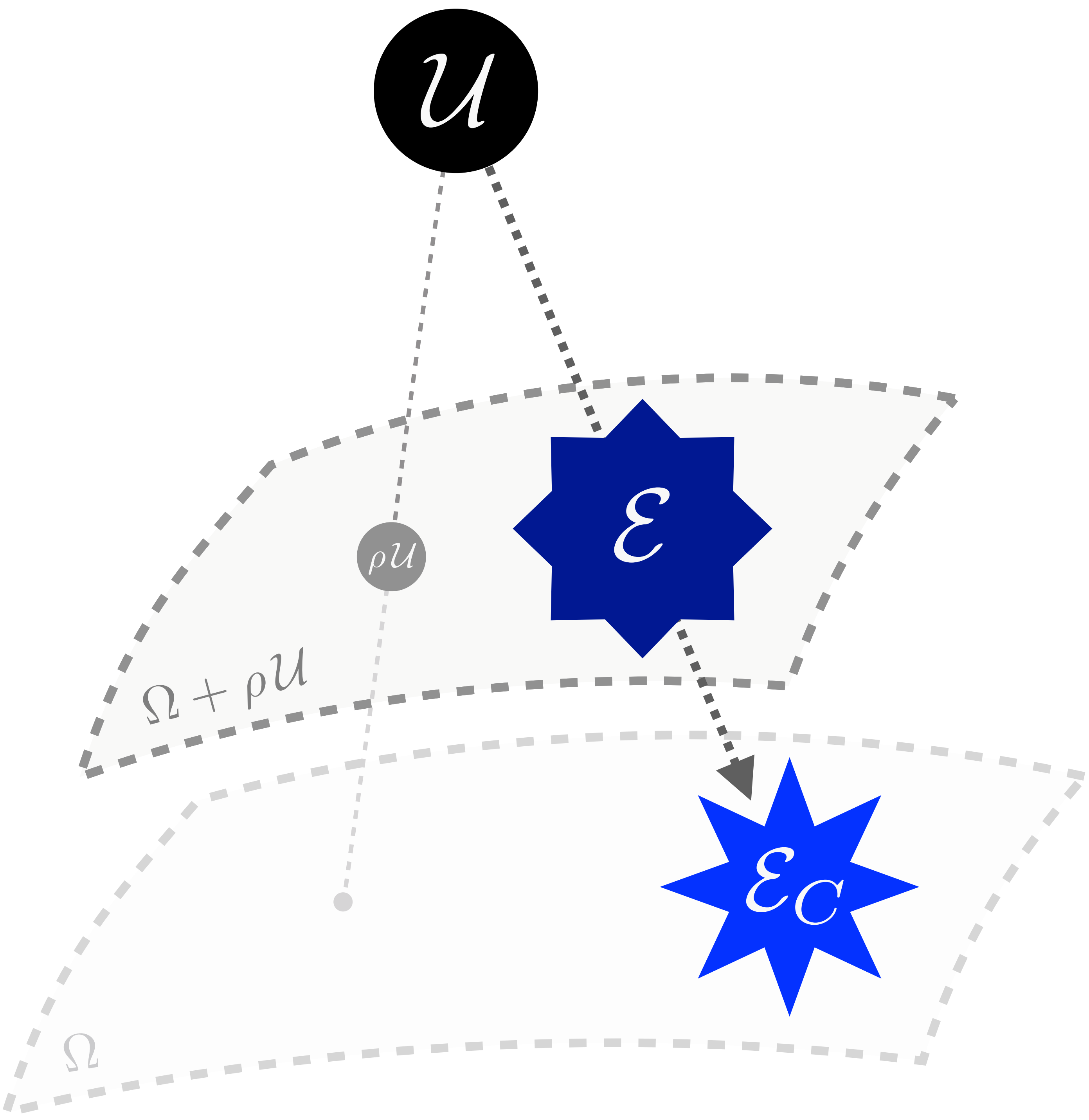
$$\mathcal{E}_C(\mathcal{E}, \rho) = \underset{\mathcal{E}' \in \Omega}{\arg\min}\ \mathrm{EMD}_\beta(\mathcal{E}, \mathcal{E}' + \rho\mathcal{U})$$

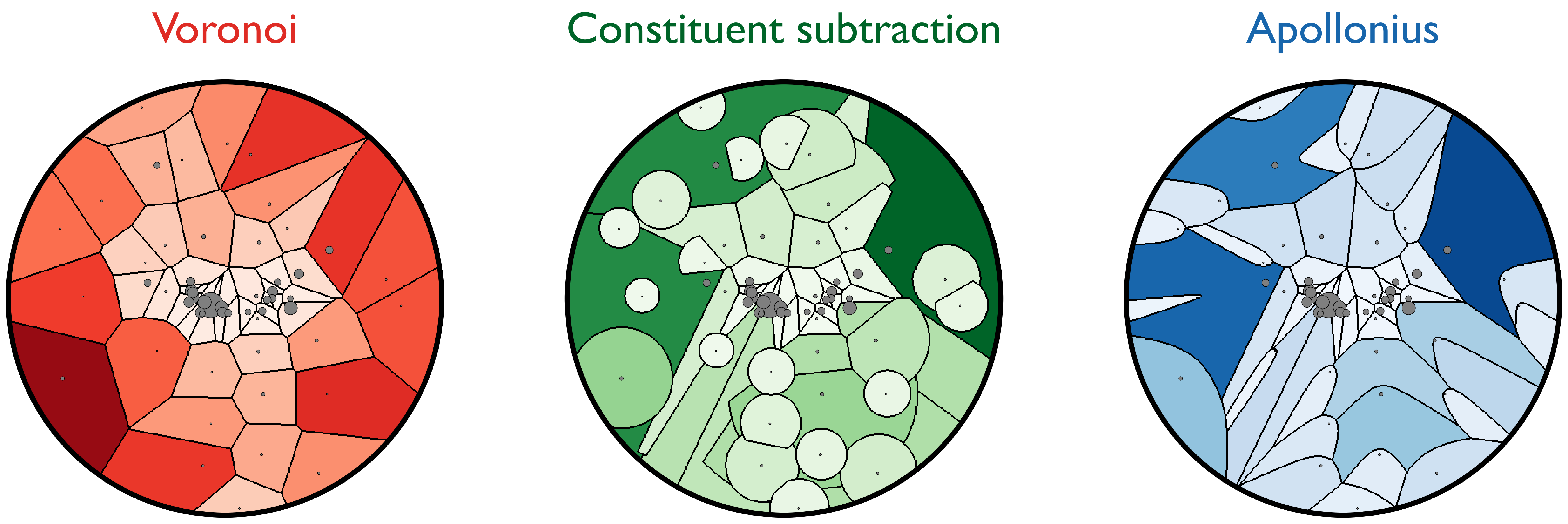
Pileup Mitigation in Event Space

Area subtraction

Particle energies corrected proportional to area of associated region

$$\mathcal{E}_C(\mathcal{E},\rho) = \underset{\mathcal{E}'\in\Omega}{\arg\min}\ \mathrm{EMD}_\beta(\mathcal{E},\mathcal{E}'+\rho\mathcal{U})$$





Voronoi

[Cacciari, Salam, Soyez, JHEP 2008]

Voronoi regions IRC unsafe

Sensitive to small modifications

Constituent subtraction

[Berta, Spousta, Miller, Leitner, JHEP 2008]

Lays down grid of "ghost" particles

Ghosts associate to nearest particle

Vanished particles don't attract ghosts

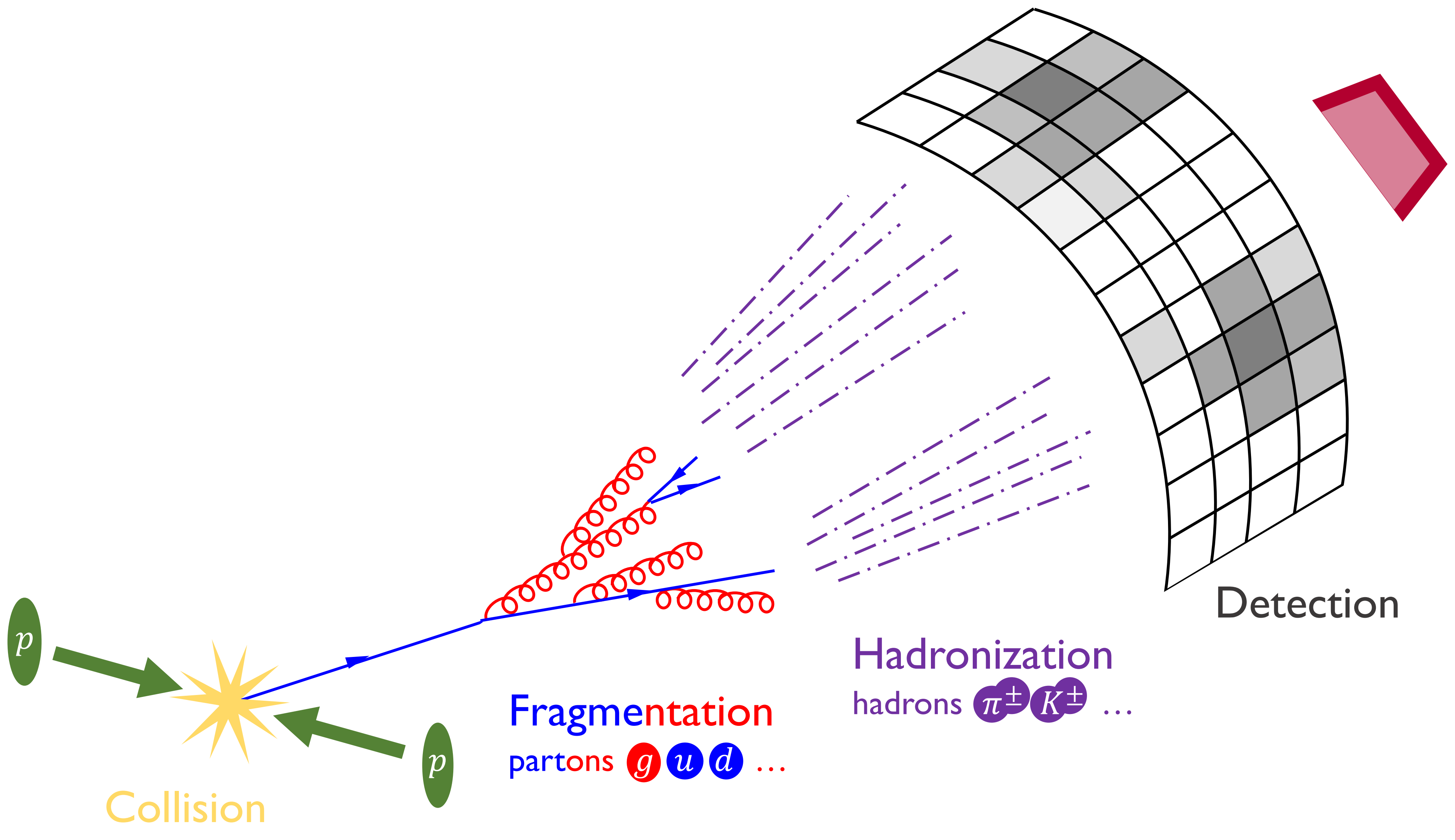
Apollonius

[PTK, Metodiev, Thaler, 2004.04159]

Ghosts are optimally assigned to particles by minimizing EMD

Apollonius regions have an understood continuum limit

Beyond Observables via Weighted Cross Sections



$$\mathcal{E}(\hat{n}) = \int_0^\infty dt \lim_{r\to\infty} r^2 n^i T_{0i}(t, r\hat{n})$$

Stress-energy flow

Standard observable (e.g. EFPs)

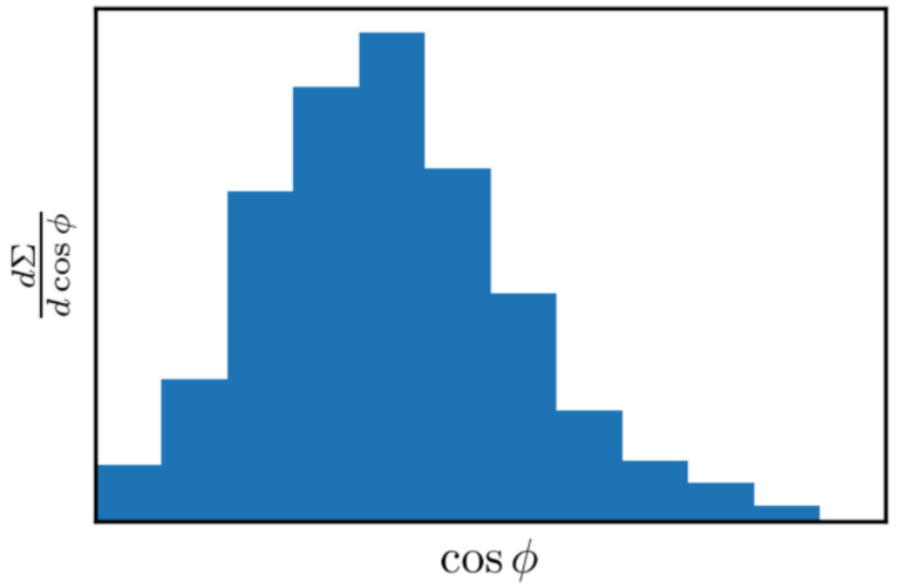
Calculate a single number for each jet/event and study distribution of values

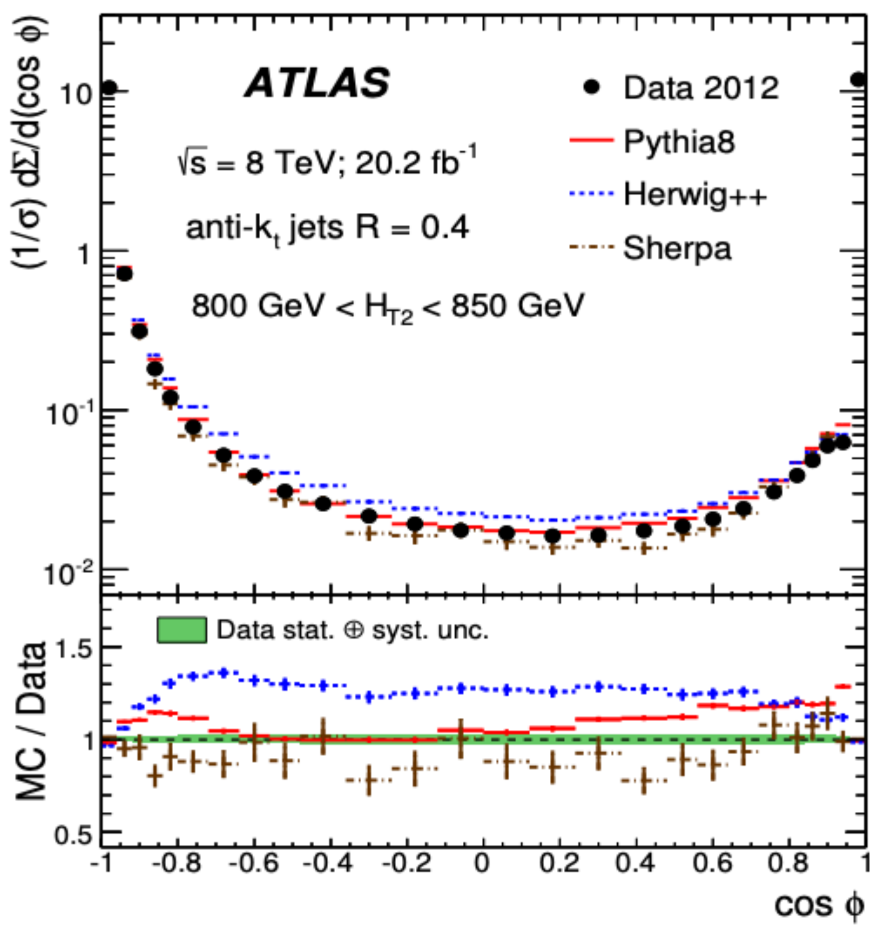
Weighted cross section

Calculate a distributional quantity per event and study the mean distribution

e.g. energy-energy correlator (EEC)

$$\frac{d\Sigma}{d\cos\phi} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta(\cos\theta_{ij} - \cos\phi)$$





$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma}{d\hat{n}_1 \cdots d\hat{n}_N} = \frac{\langle \mathcal{O}\mathcal{E}(\hat{n}_1)\cdots\mathcal{E}(\hat{n}_N)\mathcal{O}^\dagger \rangle}{\langle \mathcal{O}\mathcal{O}^\dagger \rangle}$$

Correlations of energy flow operators can be directly studied!

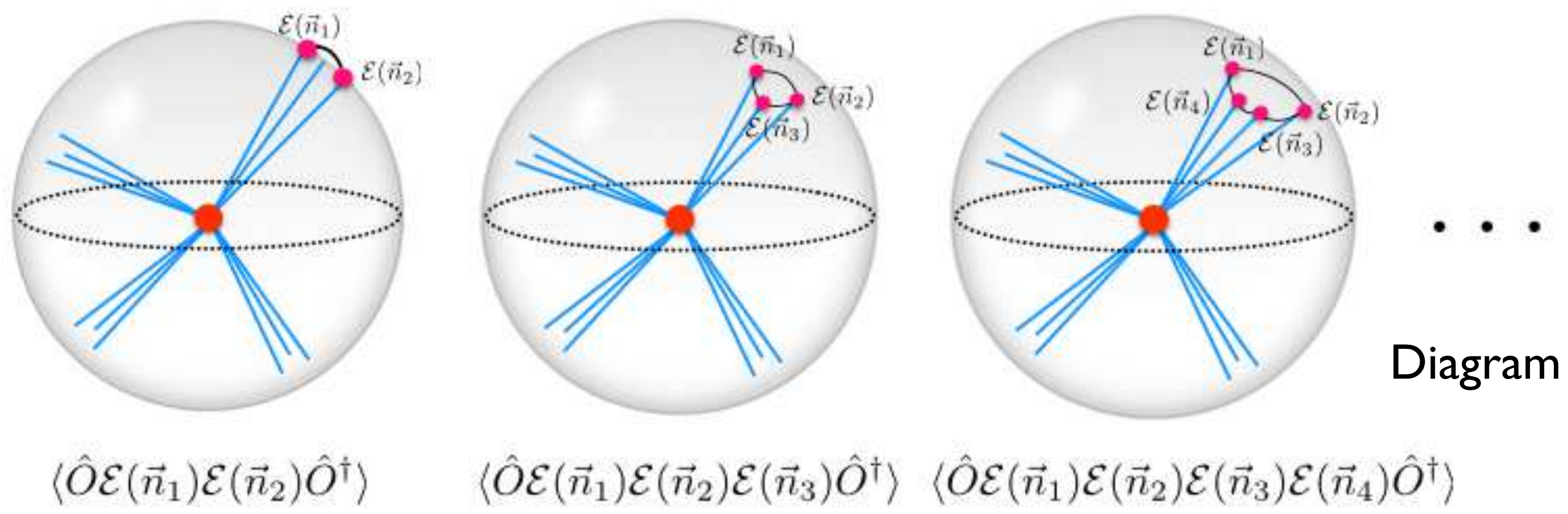


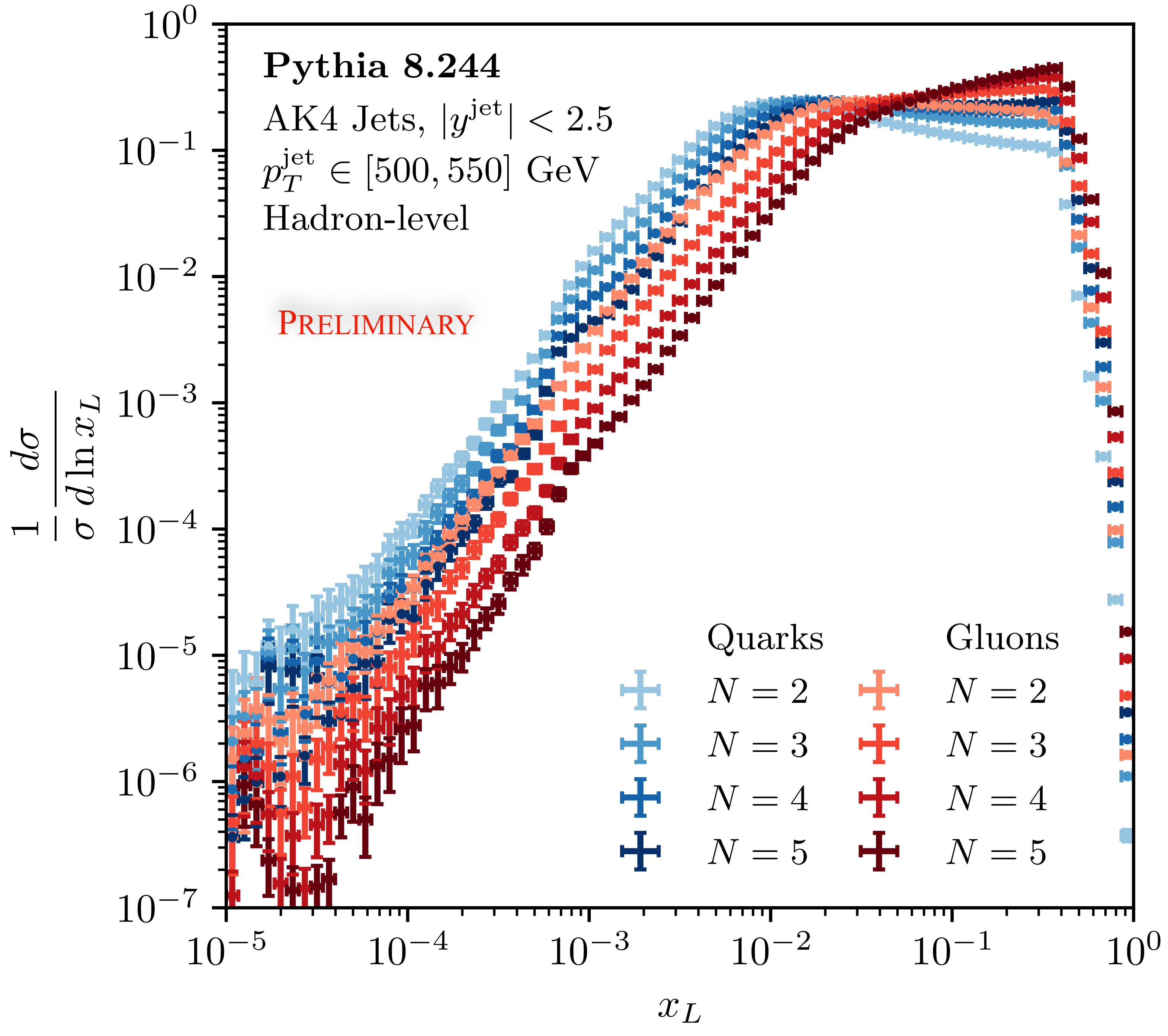
Diagram by Ian Moult

Energy-Energy Correlators – Projection to Longest Side

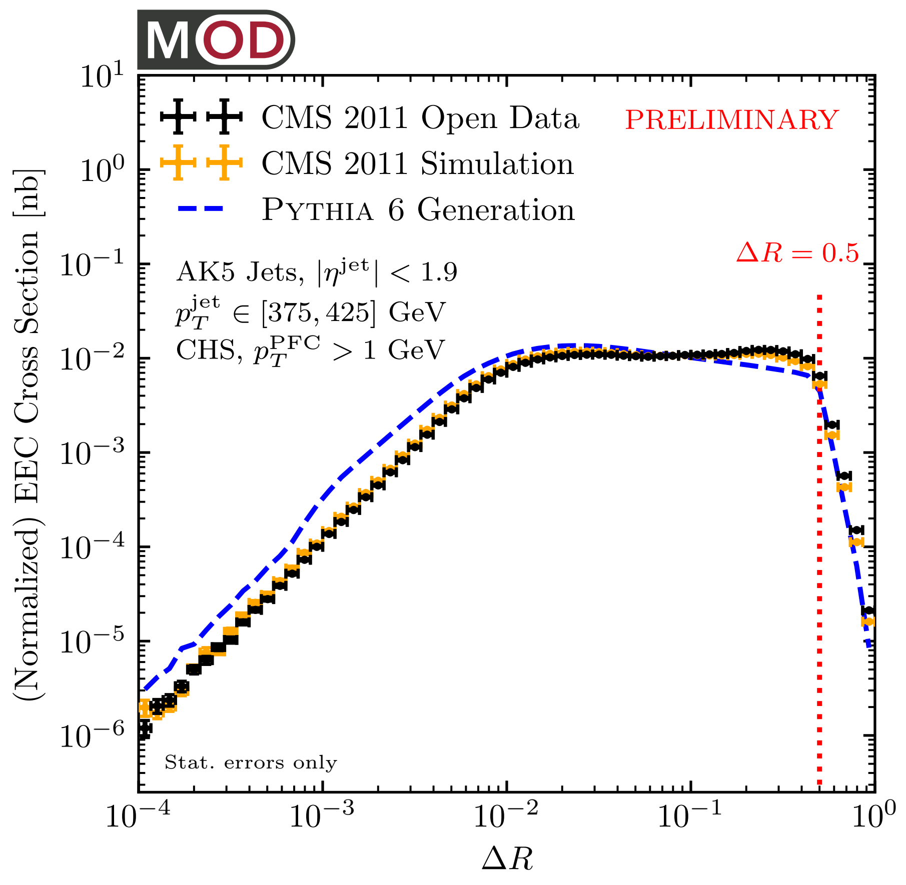
[PTK, Moult, Thaler, Zhu, *to appear soon*]

Integrate out shape dependence but keep overall size dependence

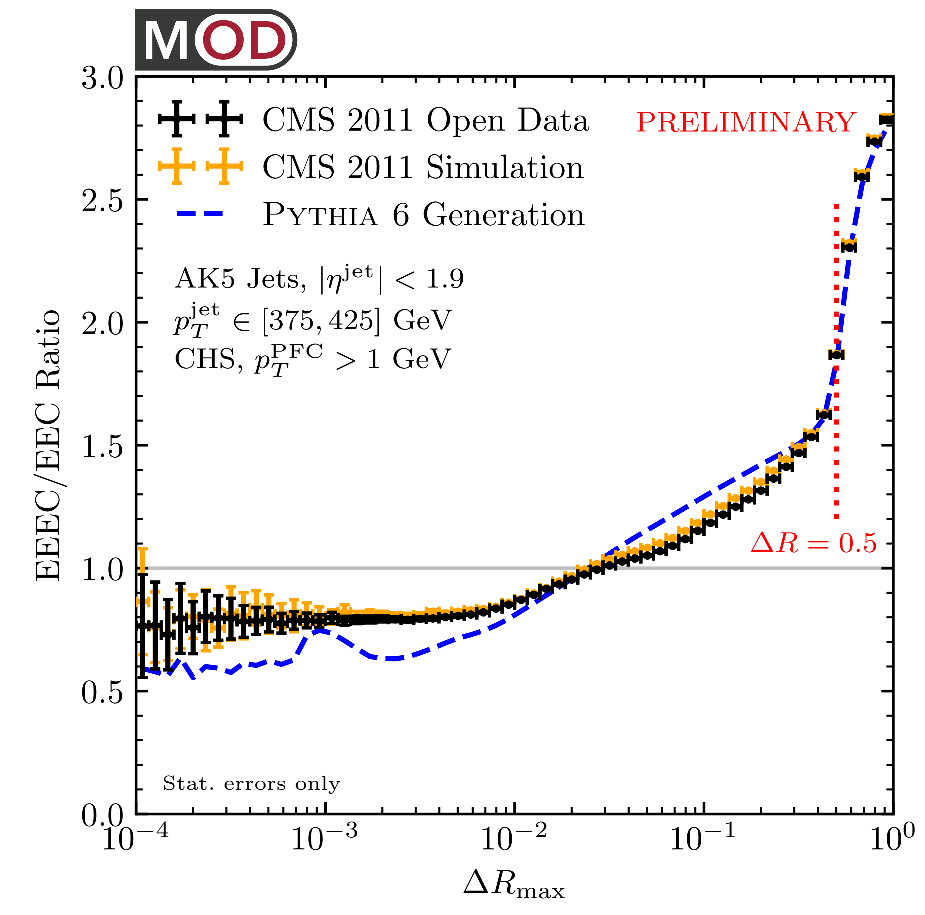
$$\frac{d\Sigma^{[N]}}{dx_L} = \sum_n \sum_{1\leq i_1 \leq \ldots \leq i_N \leq n} \int d\sigma_n \frac{E_{i_1}\cdots E_{i_N}}{Q^N}\delta(x_L - \max_{1\leq j<k\leq N}\{\theta_{i_j i_k}\})$$



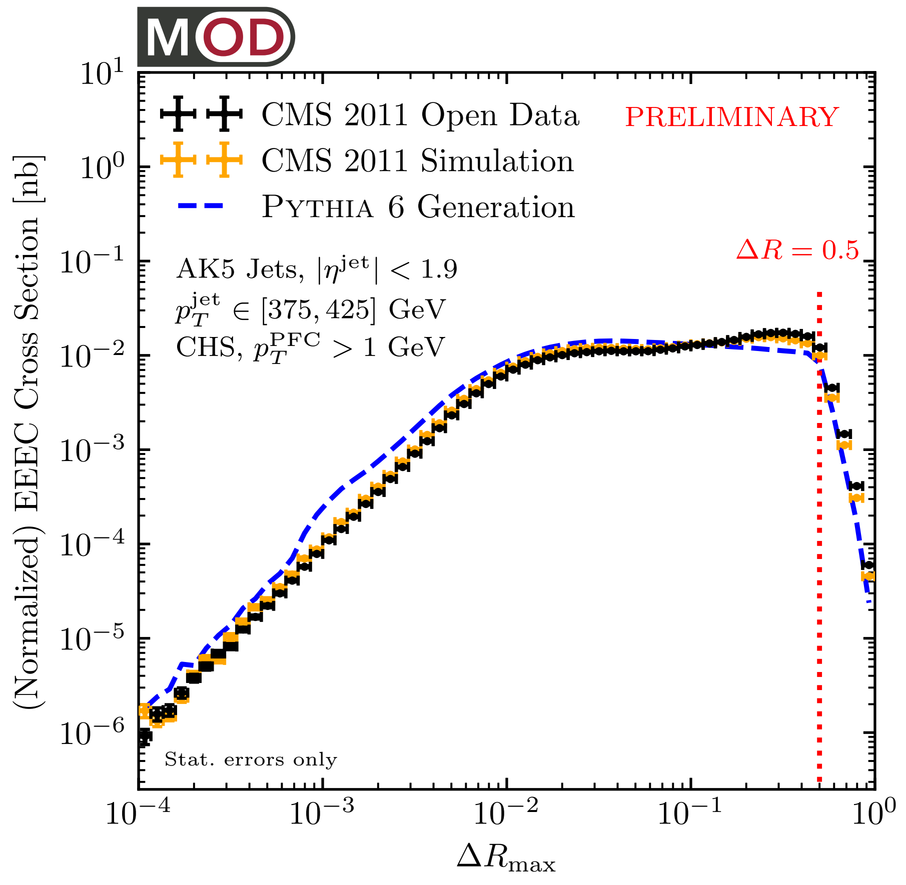
$N = 2$



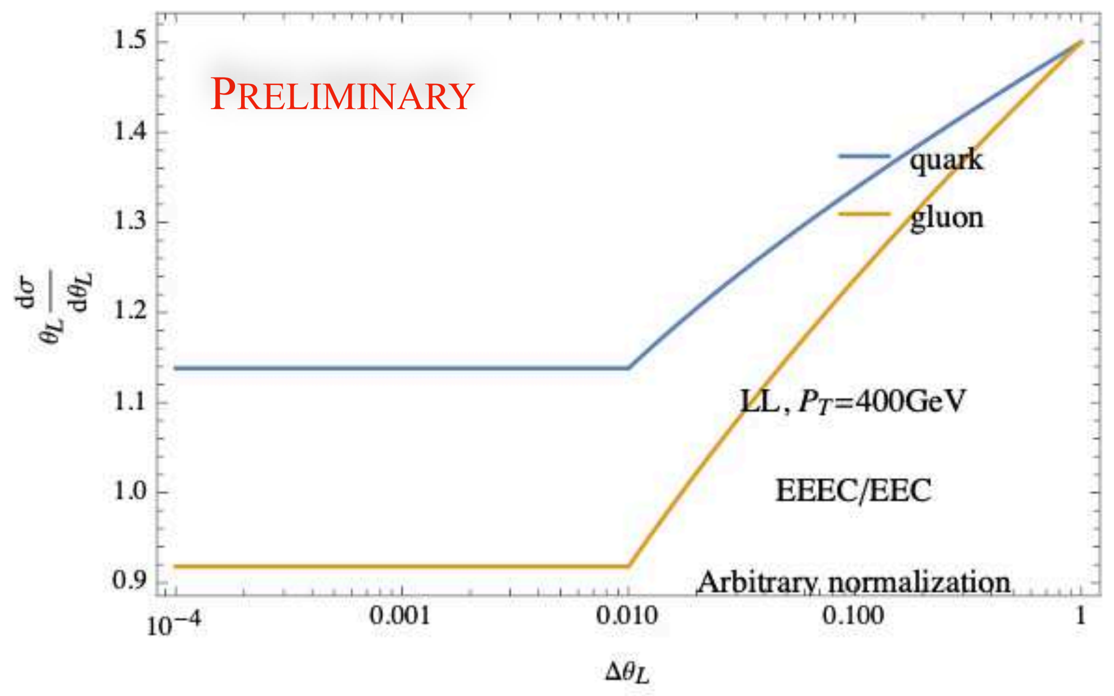
EEEC/EEC Ratio



$N = 3$



LL prediction of ratio



EEEC – Full Shape Dependence

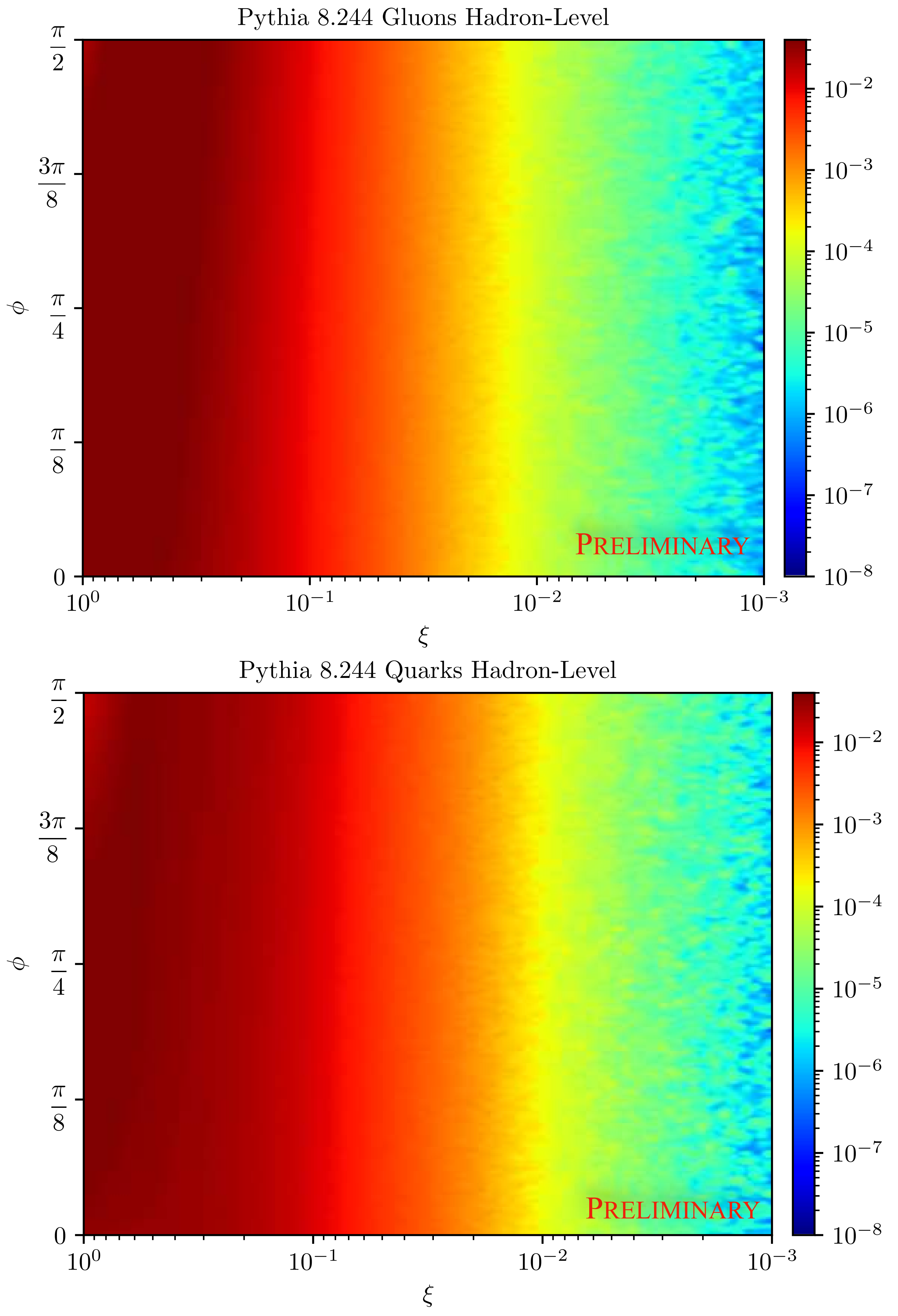
[PTK, Moult, Thaler, Zhu, *to appear soon*]

For $x_L \sim 0.01$

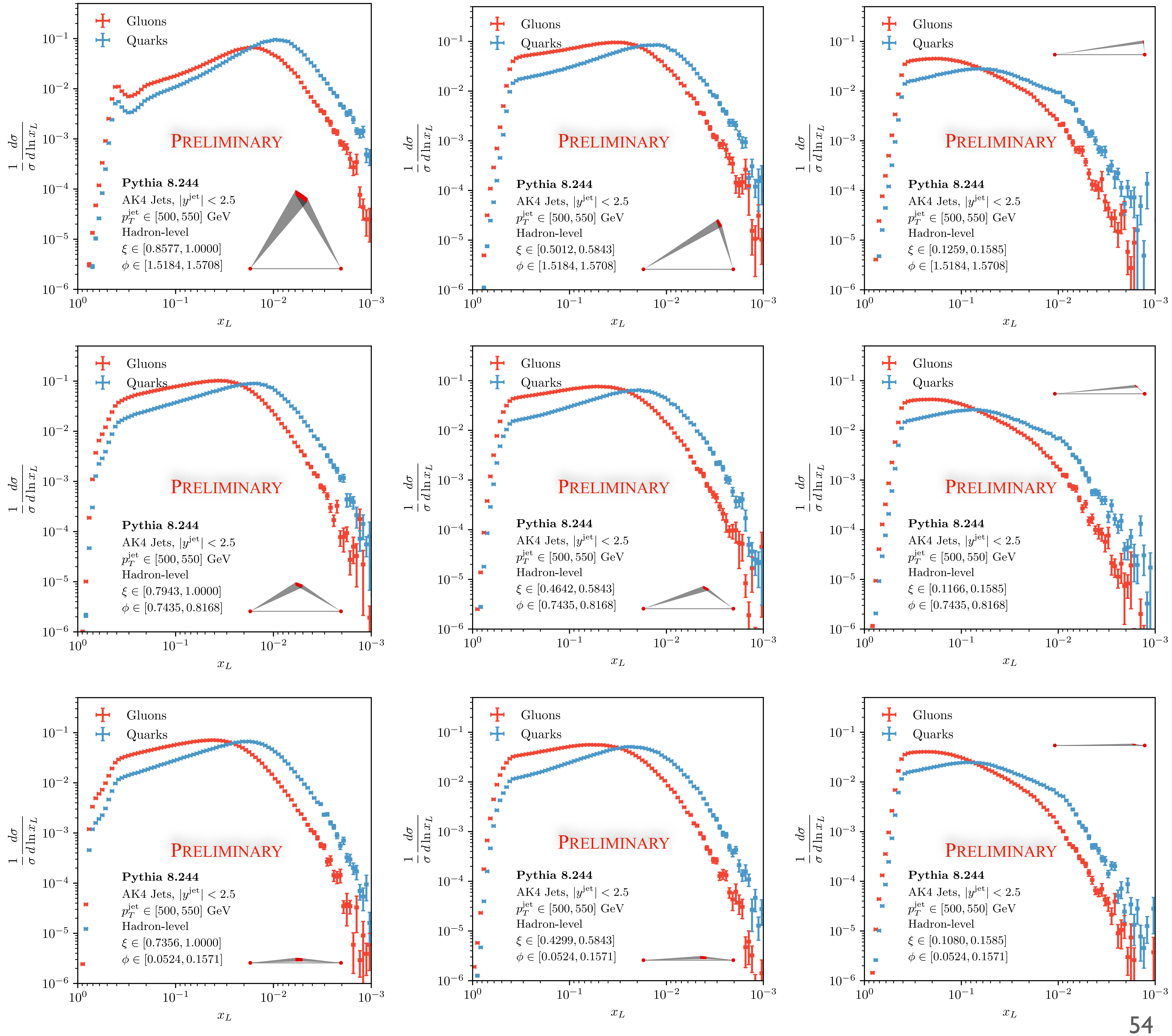
less aligned

ϕ

more aligned



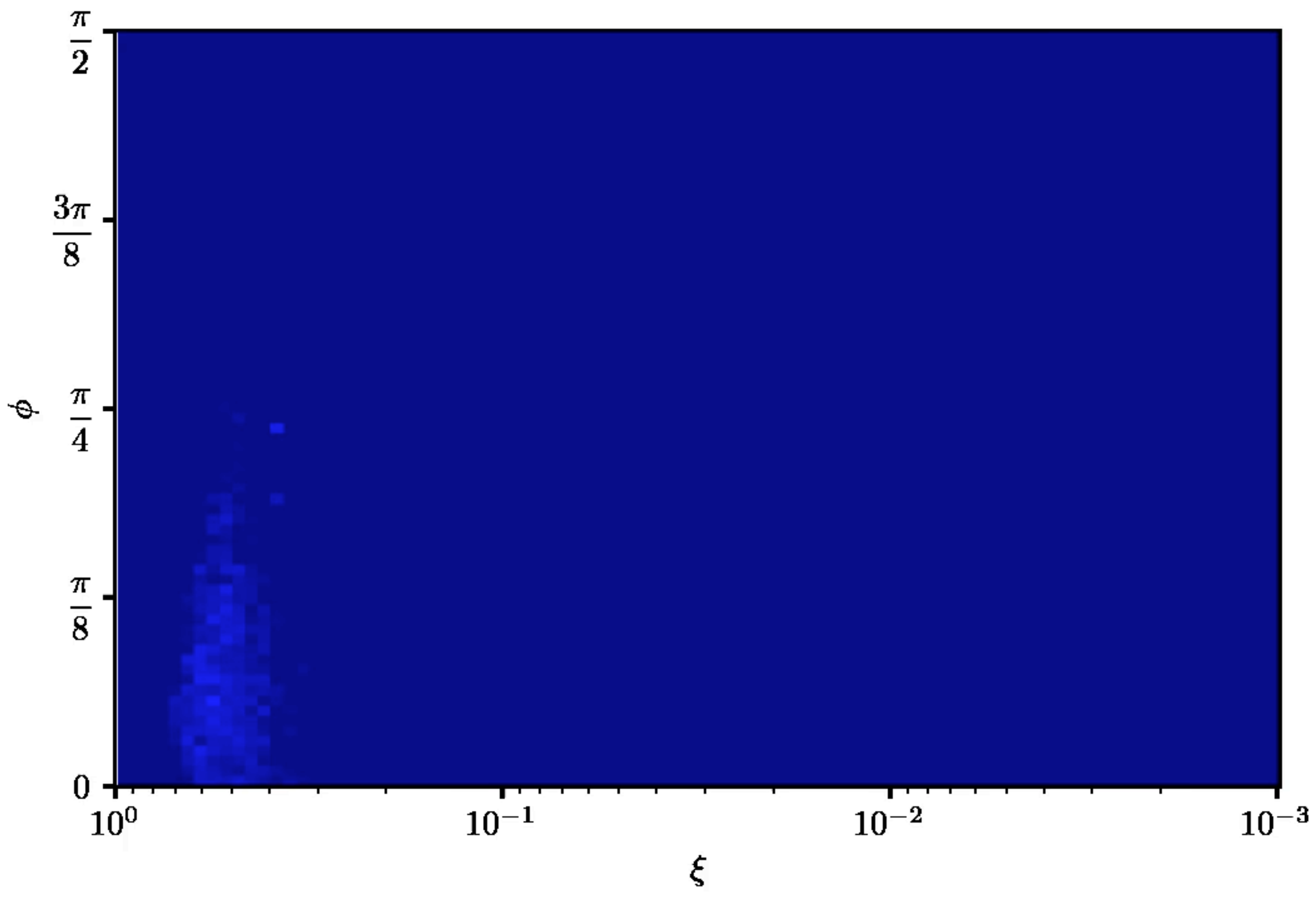
less collinear ⟷ more collinear



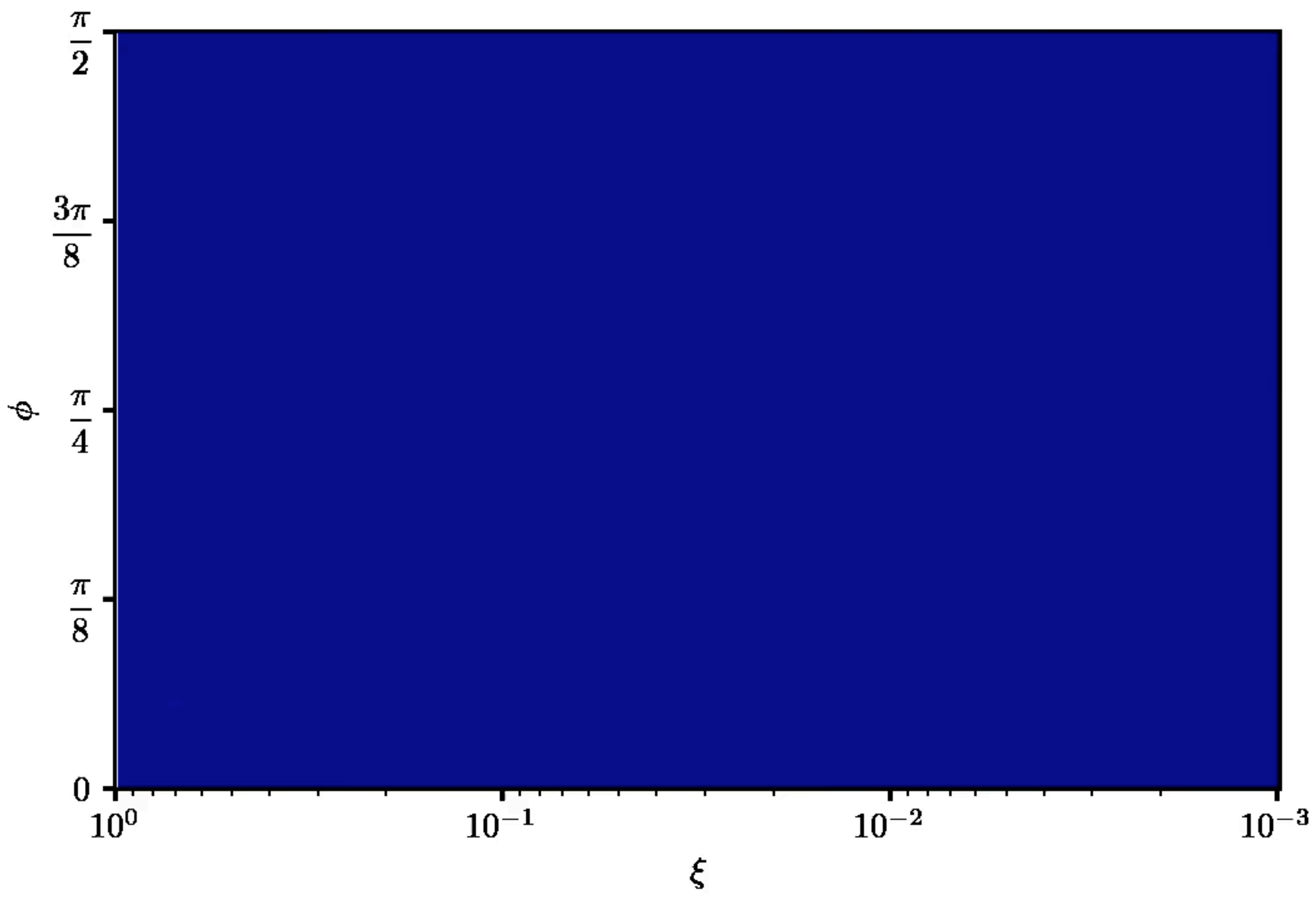
Visualizing the 3D EEEC

[PTK, Moult, Thaler, Zhu, *to appear soon*]

Pythia Gluon Jets
$p_T \in [500{,}550]$ GeV



Pythia Quark Jets
$p_T \in [500{,}550]$ GeV



Time in the videos corresponds to $\ln x_L$ going from 0 to $-\infty$

Color corresponds to log of EEC (red is large, blue is small)

Uniformly persistent red is roughly the perturbatively accessible region