Analyzing Jet Substructure via Energy Flow

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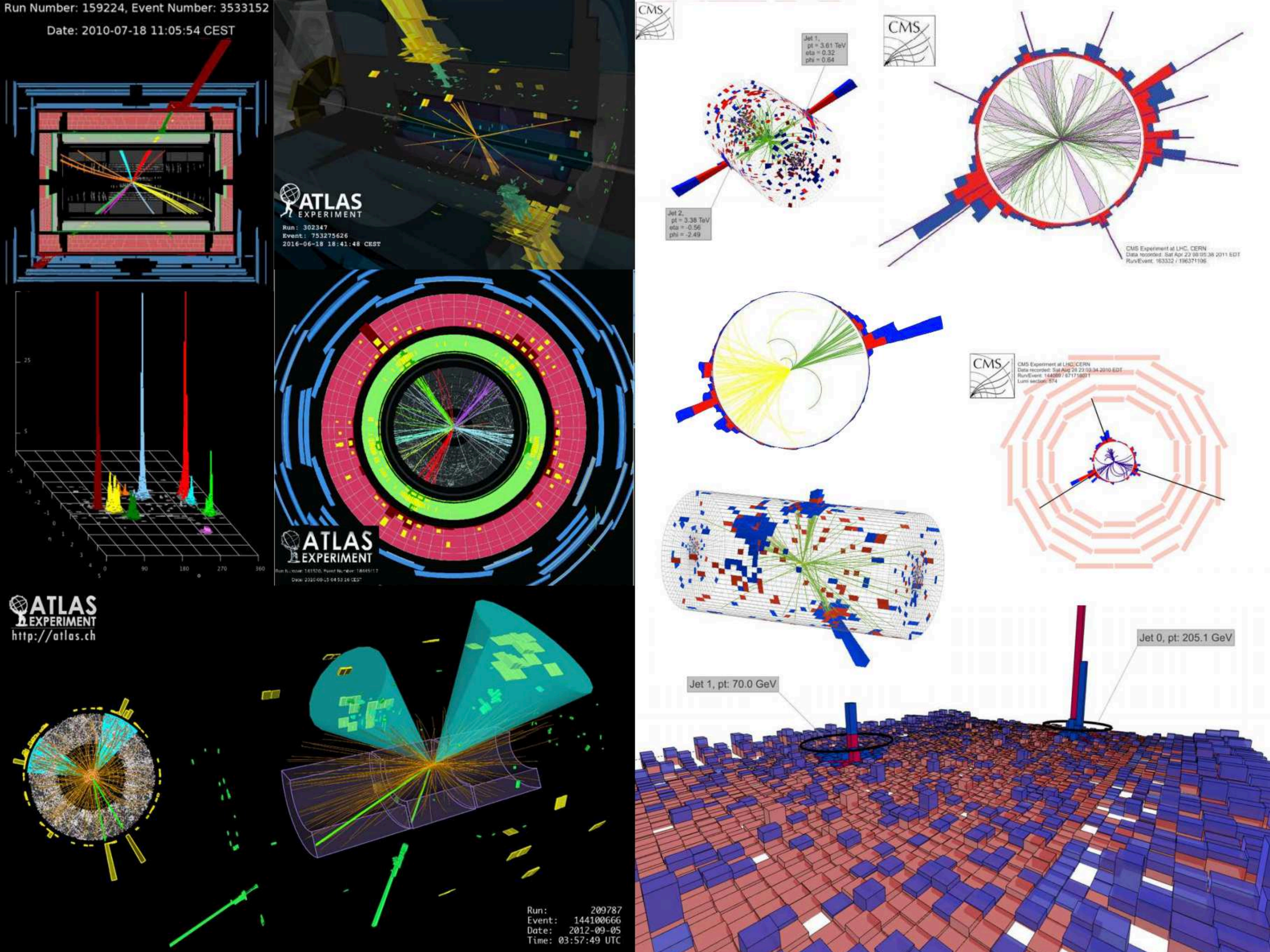
CTP QCD/LHC/DM/BSM/[ML?] Journal Club

Based on work with Eric Metodiev and Jesse Thaler

1712.07124 1810.0xxxx



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Date: 2010-07-18 11:05:54 CEST
ATLAS EXPERIMENT
Run: 302347
Event: 753275626
2016-06-18 18:41:48 CEST
CMS
Jet 1, pt = 3.61 TeV eta = 0.32 phi = 0.64
Jet 2, pt = 3.38 TeV eta = -0.56 phi = -2.49
CMS
CMS Experiment at LHC, CERN
ATLAS EXPERIMENT
CMS
ATLAS EXPERIMENT
http://atlas.ch
Jet 0, pt: 205.1 GeV
Jet 1, pt: 70.0 GeV
Run: 209787
Event: 144100666
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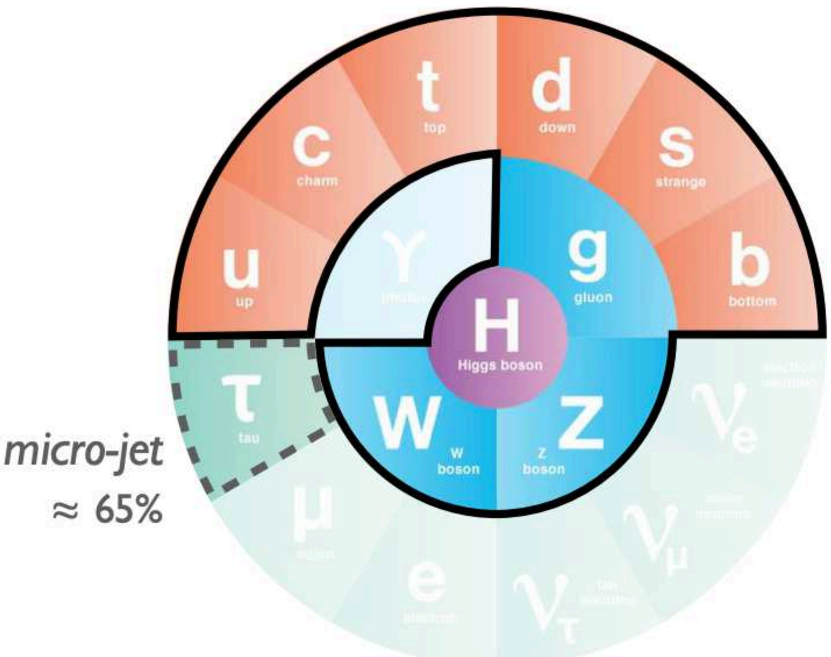
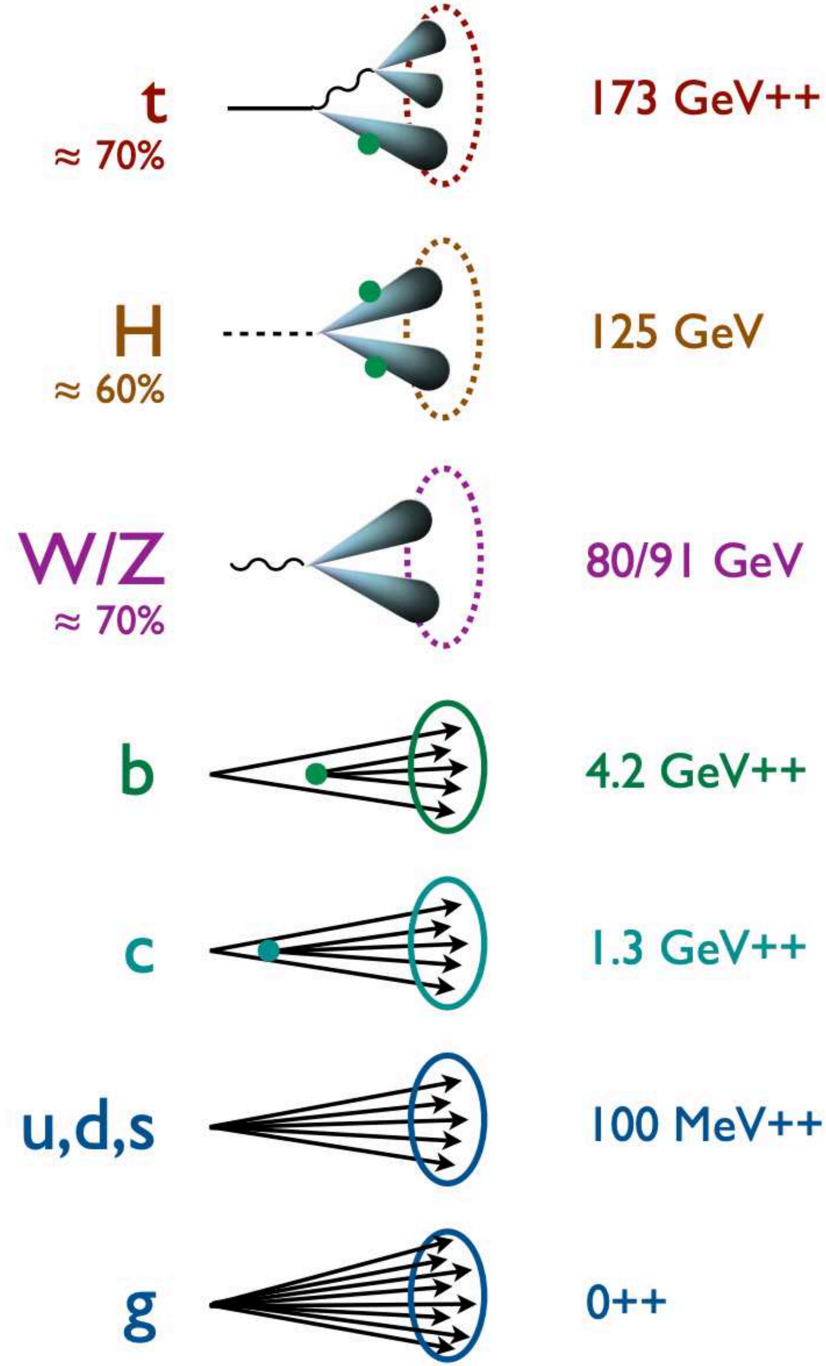
Time: 03:57:49 UTC





Jets from the Standard Model

++ = Mass from QCD Radiation

Slide by Jesse Thaler

Jet Representations ⟷ Analysis Tools

Two key choices when analyzing jets

How to represent the jet		How to analyze that representation
• Single expert observable		• Threshold cut
• A few expert observables		• Multidimensional likelihood
• Many expert observables		• Boosted decision tree (BDT), shallow neural network (NN)
• Jet images	→	• Convolutional NN (CNN)
• List of particles		• Recurrent/Recursive NN (RNN)
• Clustering tree		• Fancy RNN
• *N*-subjettiness basis		• Deep neural network (DNN)
• Energy flow polynomials	**Part I**	• Linear classification
• Set of particles	**Part II**	• Energy flow network

Outline

Part I 1712.07124

- IRC-Safe Jet Observables
- Energy Flow Polynomials (EFPs)

Part I.V 19??.xxxxx

- Energy Flow Moments (EFMs)

Part II 1810.0xxxx

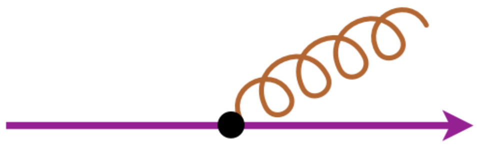
- Intrinsic Jet Symmetries
- Energy Flow Networks (EFNs)
- Opening the Box

Conclusions

Outline

Part I

- IRC-Safe Jet Observables
- Energy Flow Polynomials (EFPs)



Part I.V

- Energy Flow Moments (EFMs)

Part II

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Conclusions

What is IRC Safety?

Infrared (IR) safety – observable is unchanged under addition of a soft particle:

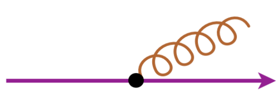
$$S(\{p_1^\mu, \ldots, p_M^\mu\}) = \lim_{\epsilon\to 0} S(\{p_1^\mu, \ldots, p_M^\mu, \epsilon p_{M+1}^\mu\}), \qquad \forall p_{M+1}^\mu$$

Collinear (C) safety – observable is unchanged under collinear splitting of a particle:

$$S(\{p_1^\mu, \ldots, p_M^\mu\}) = \lim_{\epsilon\to 0} S(\{p_1^\mu, \ldots, (1-\lambda)p_M^\mu, \lambda p_M^\mu\}), \qquad \forall\lambda \in [0,1]$$

A necessary and sufficient condition for soft/collinear divergences of a QFT to cancel at each order in perturbation theory (KLN theorem)

Divergences in QCD splitting function:



$$dP_{i\to ig} \simeq \frac{2\alpha_s}{\pi} C_i \frac{d\theta}{\theta} \frac{dz}{z}$$

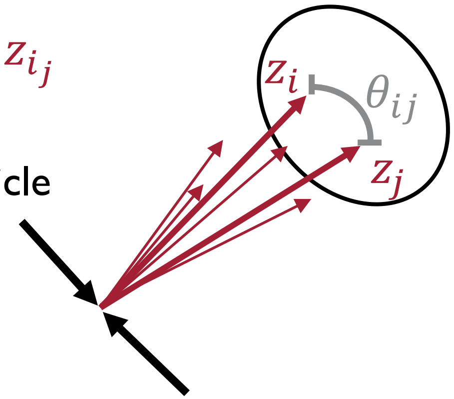
$$C_q = C_F = 4/3$$
$$C_g = C_A = 3$$

IRC-safe observables probe hard structure while being insensitive to low energy or small angle modifications

Expanding an Arbitrary IRC-safe Observable

Arbitrary IRC-safe observable: $S(p_1^\mu, \ldots, p_M^\mu)$

- Energy expansion*: Approximate S with polynomials of z_{i_j}
  - IR safety: S is unchanged under addition of soft particle
  - C safety: S is unchanged under collinear splitting of a particle
  - Relabeling symmetry: Particle index is arbitrary



Energy correlator parametrized by angular function f

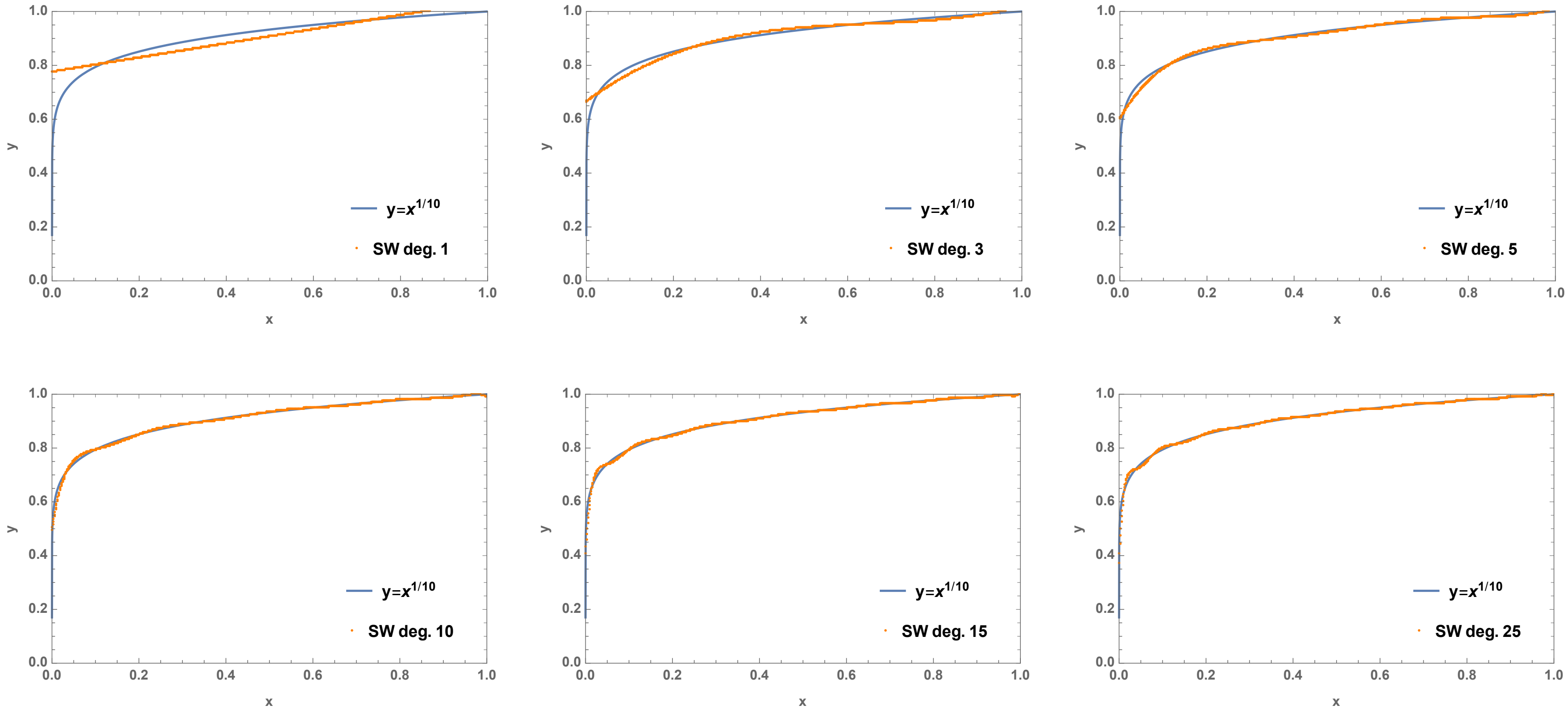
$$\sum_{i_1=1}^{M} \cdots \sum_{i_N=1}^{M} z_{i_1} \ldots z_{i_N} f(\hat{p}_{i_1}, \ldots, \hat{p}_{i_N})$$

[F. Tkachov, hep-ph/9601308]

**Generically, approximations exist by the Stone-Weierstrass theorem

Fun with the Stone-Weierstrass Theorem

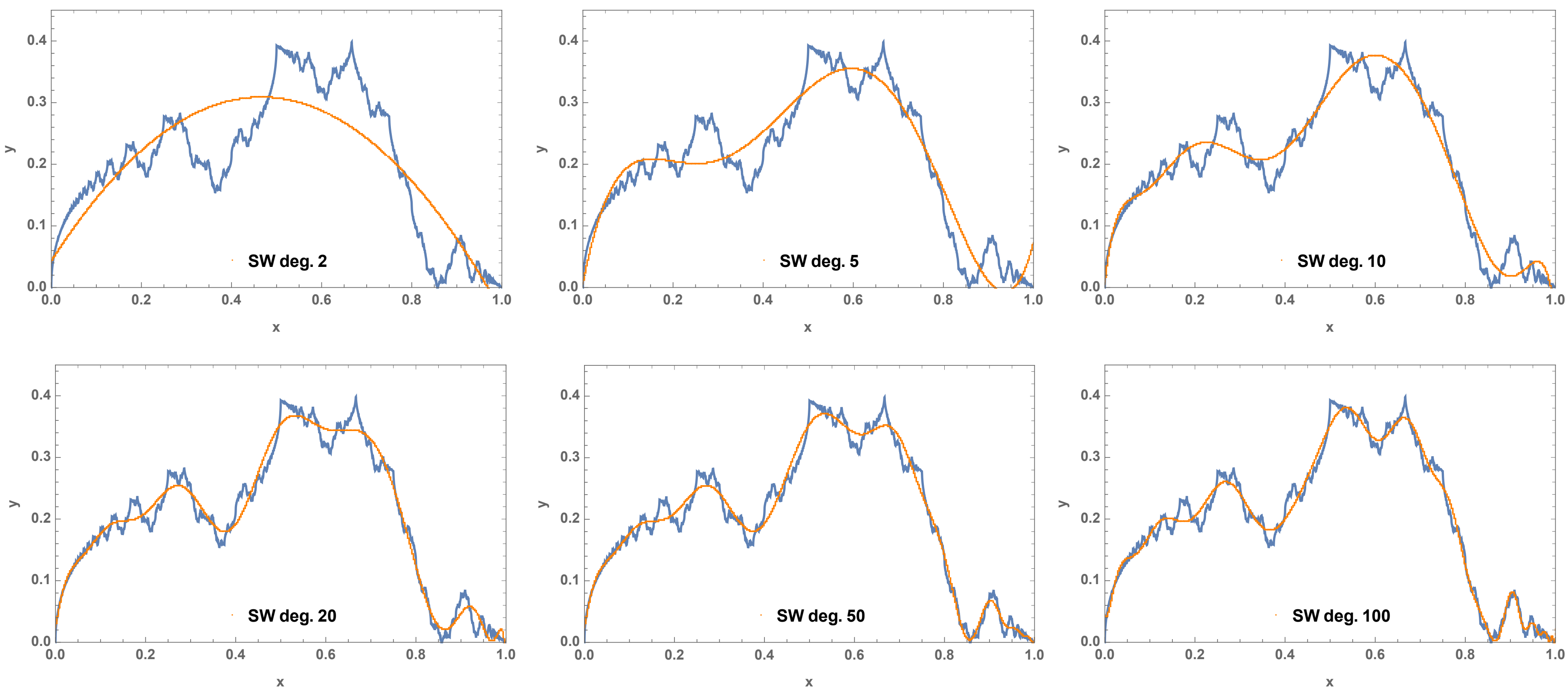
Try to approximate function that has no Taylor expansion around zero



Fun with the Stone-Weierstrass Theorem

That was too easy, try the Weierstrass function (continuous everywhere, differentiable on a measure zero set of points)

$$y = \sum_{k=1}^{\infty} \frac{\sin(\pi k^2 x)}{\pi k^2}$$



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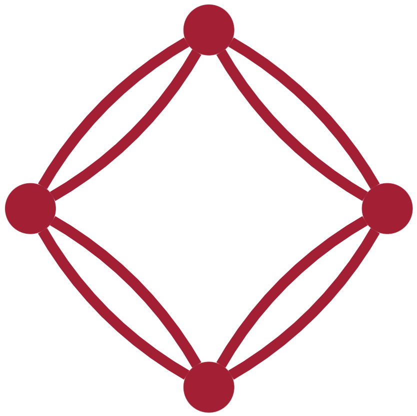
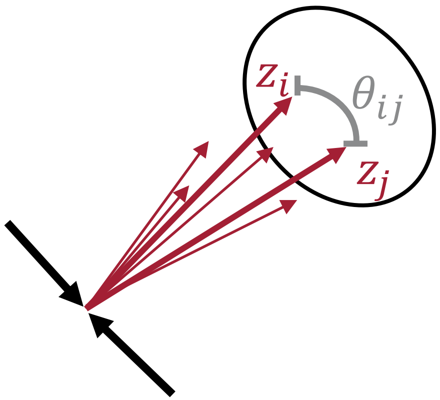


Energy Flow Polynomials (EFPs)

[**PTK**, E. Metodiev, J. Thaler, 1712.07124]



Energy Fraction | Pairwise Angular Distance

$$e^+e^-:\ z_i = \frac{E_j}{\sum_k E_k}, \quad \theta_{ij} = \left(\frac{2p_i^\mu p_{j\mu}}{E_i E_j}\right)^{\frac{\beta}{2}}$$

$$\text{Hadronic:}\ z_i = \frac{p_{Tj}}{\sum_k p_{Tk}}, \quad \theta_{ij} = \left(\Delta y_{ij}^2 + \Delta\phi_{ij}^2\right)^{\frac{\beta}{2}}$$

In equations:

$$\mathrm{EFP}_G = \underbrace{\sum_{i_1=1}^{M}\sum_{i_2=1}^{M}\cdots\sum_{i_N=1}^{M}}_{\text{Correlator}}\ \underbrace{z_{i_1} z_{i_2}\cdots z_{i_N}}_{\text{Energies}}\ \underbrace{\prod_{(k,l)\in G}\theta_{i_k i_l}}_{\text{Angles}}$$

multigraph

In words:

Correlator of Energies and Angles

Correlator: Sum over all N-tuples of particle in the event

Energies: Product of the N energy fractions

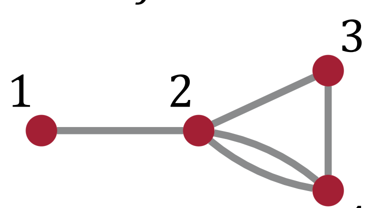
Angles: One $\theta_{i_k i_l}$ for each edge in $(k,l) \in G$

In pictures:

$\longleftrightarrow z_{i_j}$

$\longleftrightarrow \theta_{i_k i_l}$

(e.g. "fly swatter")



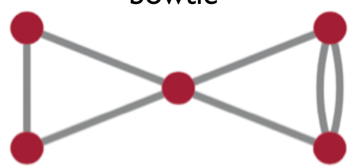
(any index labelling works)

$$= \sum_{i_1=1}^{M}\sum_{i_2=1}^{M}\sum_{i_3=1}^{M}\sum_{i_4=1}^{M} z_{i_1} z_{i_2} z_{i_3} z_{i_4}\, \theta_{i_1 i_2}\theta_{i_2 i_3}\theta_{i_3 i_4}\theta_{i_2 i_4}^2$$

Multigraph/EFP Correspondence

Multigraph $\longleftrightarrow$ EFP

"bowtie"



$$= \sum_{i_1=1}^{M}\sum_{i_2=1}^{M}\sum_{i_3=1}^{M}\sum_{i_4=1}^{M}\sum_{i_5=1}^{M} z_{i_1} z_{i_2} z_{i_3} z_{i_4} z_{i_5} \theta_{i_1 i_2} \theta_{i_2 i_3} \theta_{i_1 i_3} \theta_{i_1 i_4} \theta_{i_1 i_5} \theta_{i_4 i_5}^2$$

j $\longleftrightarrow$ z_{i_j}

k l $\longleftrightarrow$ $\theta_{i_k i_l}$

N Number of vertices $\longleftrightarrow$ *N*-particle correlator

d Number of edges $\longleftrightarrow$ Degree of angular monomial

χ Treewidth + 1 $\longleftrightarrow$ Optimal VE Complexity

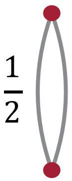
Connected $\longleftrightarrow$ Prime

Disconnected $\longleftrightarrow$ Composite

⋮

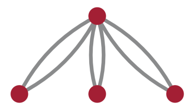
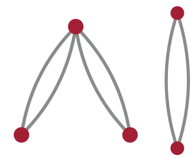
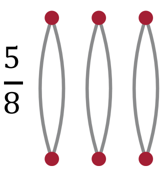
Familiar Jet Substructure Observables as EFPs

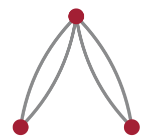
Scaled Jet Mass:

$$\frac{m_J^2}{p_{TJ}^2} = \sum_{i_1=1}^{M}\sum_{i_2=1}^{M} z_{i_1} z_{i_2} (\cosh \Delta y_{i_1 i_2} - \cos \Delta\phi_{i_1 i_2}) = \frac{1}{2}$$  $$+ \cdots$$

Jet Angularities:

$$\lambda^{(\alpha)} = \sum_{i}^{M} z_i \theta_i^{\alpha}$$

$$\lambda^{(6)} =$$  $$-\frac{3}{2}$$  $$+\frac{5}{8}$$ 

$$\lambda^{(4)} =$$  $$-\frac{3}{4}$$ 

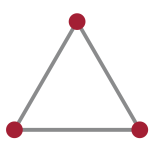
[C. Berger, T. Kucs, and G. Sterman, hep-ph/0303051]
[S. Ellis, *et al.*, 10010014]
[A. Larkoski, J. Thaler, and W. Waalewijn, 1408.3122]

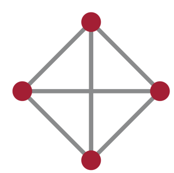
Energy Correlation Functions(ECFs):

$$e_N^{(\beta)} = \sum_{i_1=1}^{M}\sum_{i_2=1}^{M}\cdots\sum_{i_N=1}^{M} z_{i_1} z_{i_2} \cdots z_{i_N} \prod_{k<l\in\{1,\cdots,N\}} \theta_{i_k i_l}^{\beta}$$

[A. Larkoski, G. Salam, and J. Thaler, 1305.0007]

$$e_2^{(\beta)} =$$ 

$$e_3^{(\beta)} =$$ 

$$e_4^{(\beta)} =$$ 

and many more…

Organizing the Basis

EFPs are most naturally truncated by the degree *d*, the order of the angular expansion (other truncations possible)

Online Encyclopedia of Integer Sequences (OEIS)

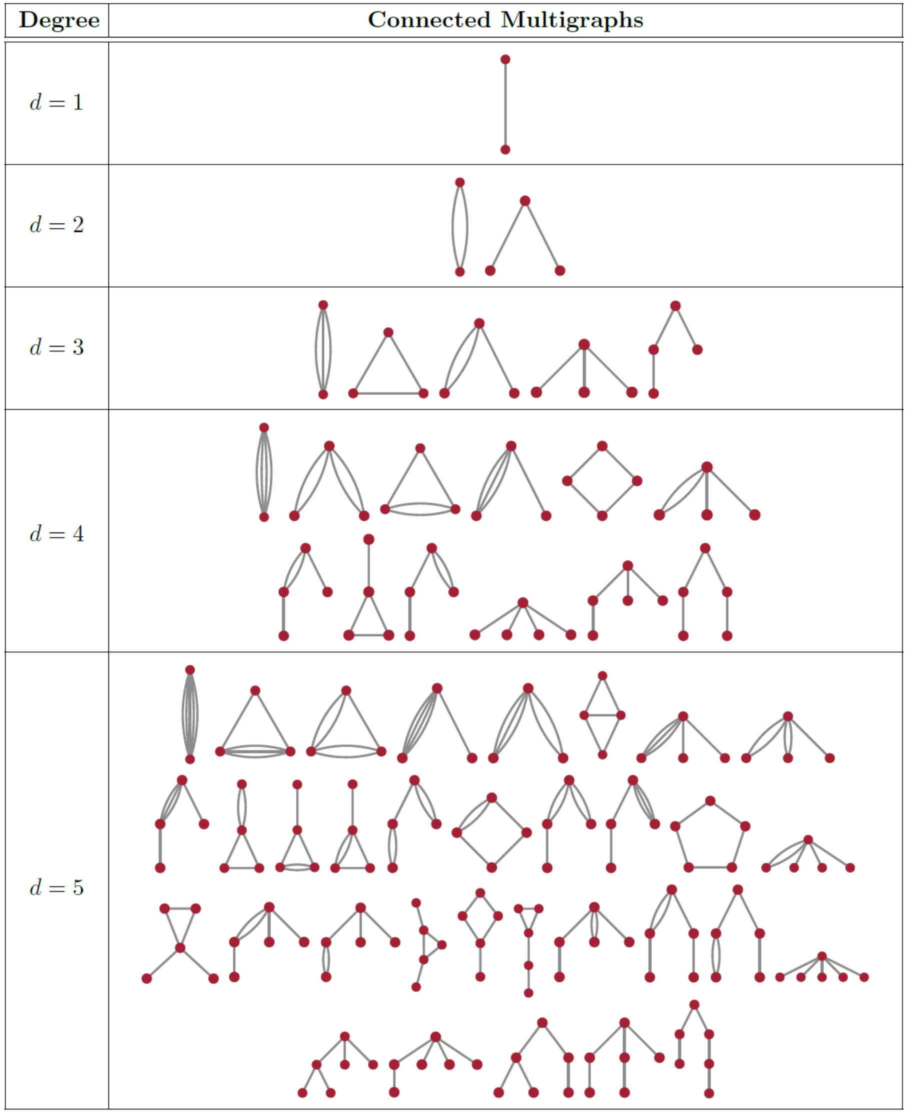
[A050535](A050535) # of multigraphs with *d* edges
# of EFPs of degree *d*

[A076864](A076864) # of connected multigraphs with *d* edges
# of prime EFPs of degree *d*

↓

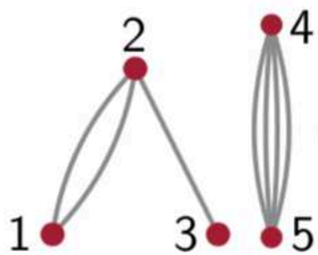
Exactly 1000 EFPs up to degree *d*=7*!*

There exist many linear redundancies of several types in the set of EFPs



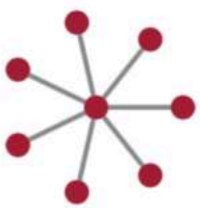
Computational Complexity of EFPs

- Like other energy correlators, EFPs are naively $\mathcal{O}(M^N)$
- *Factorability* of summand in EFP formula can speed up computation



$$= \left(\sum_{i_1=1}^{M}\sum_{i_1=1}^{M}\sum_{i_3=1}^{M} z_{i_1} z_{i_2} z_{i_3} \theta_{i_1 i_2}^2 \theta_{i_2 i_3}\right)\left(\sum_{i_4=1}^{M}\sum_{i_5=1}^{M} z_{i_4} z_{i_5} \theta_{i_4 i_5}^4\right)$$

Composite EFPs are products of prime EFPs



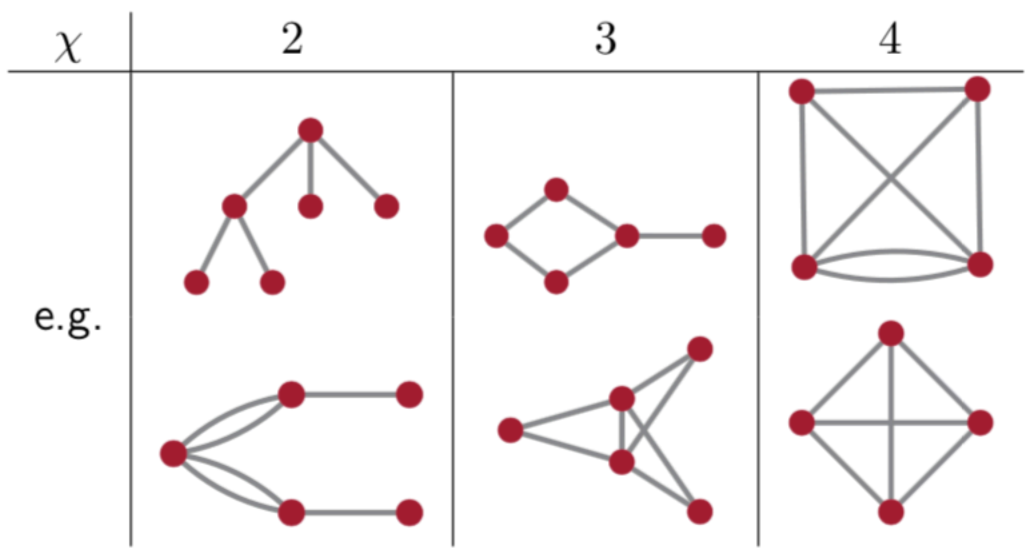
$$= \underbrace{\sum_{i_1=1}^{M}\sum_{i_2=1}^{M}\sum_{i_3=1}^{M}\sum_{i_4=1}^{M}\sum_{i_5=1}^{M}\sum_{i_6=1}^{M}\sum_{i_7=1}^{M}\sum_{i_8=1}^{M} z_{i_1} z_{i_2} z_{i_3} z_{i_4} z_{i_5} z_{i_6} z_{i_7} z_{i_8} \theta_{i_1 i_2}\theta_{i_1 i_3}\theta_{i_1 i_4}\theta_{i_1 i_5}\theta_{i_1 i_6}\theta_{i_1 i_7}\theta_{i_1 i_8}}_{\mathcal{O}(M^8)}$$

$$= \underbrace{\sum_{i_1=1}^{M} z_{i_1}\left(\sum_{i_2=1}^{M} z_{i_2}\theta_{i_1 i_2}\right)^7}_{\mathcal{O}(M^2)}$$

Other algebraic simplifications are also possible by choosing parentheses wisely

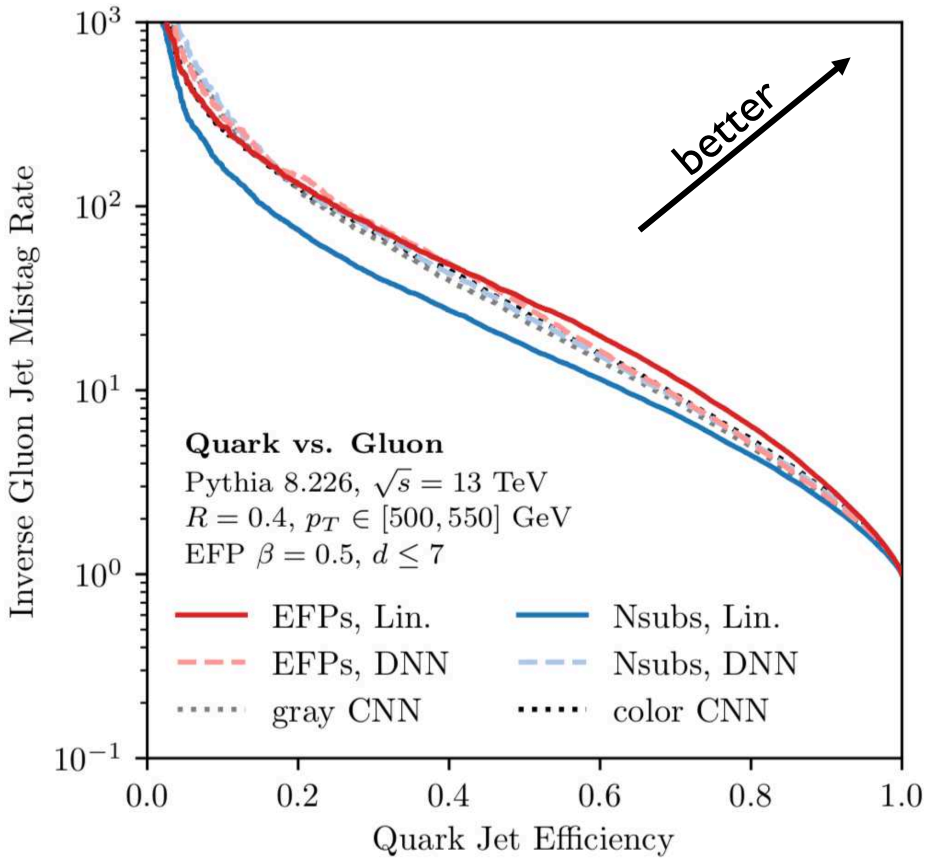
Variable Elimination (VE)

- Algorithm for finding optimal parentheses placement in EFP formula
- Reduces EFP computational complexity to $\mathcal{O}(M^\chi)$:
  - Best case (NP-hard): $\chi = treewidth + 1$
  - Heuristics can be used that work with our small graphs
  - $\chi = 2$ for all tree graphs, $\chi = 3$ for single cycle graphs, $\chi = N$ for K_N



Jet Tagging Performance – Quark vs. Gluon Jets

ROC curves for quark vs. gluon jet tagging



W vs. QCD and top vs. QCD in backup

N-subjettiness:
[J. Thaler, K. Van Tilburg, 1011.2268, 1108.2701]

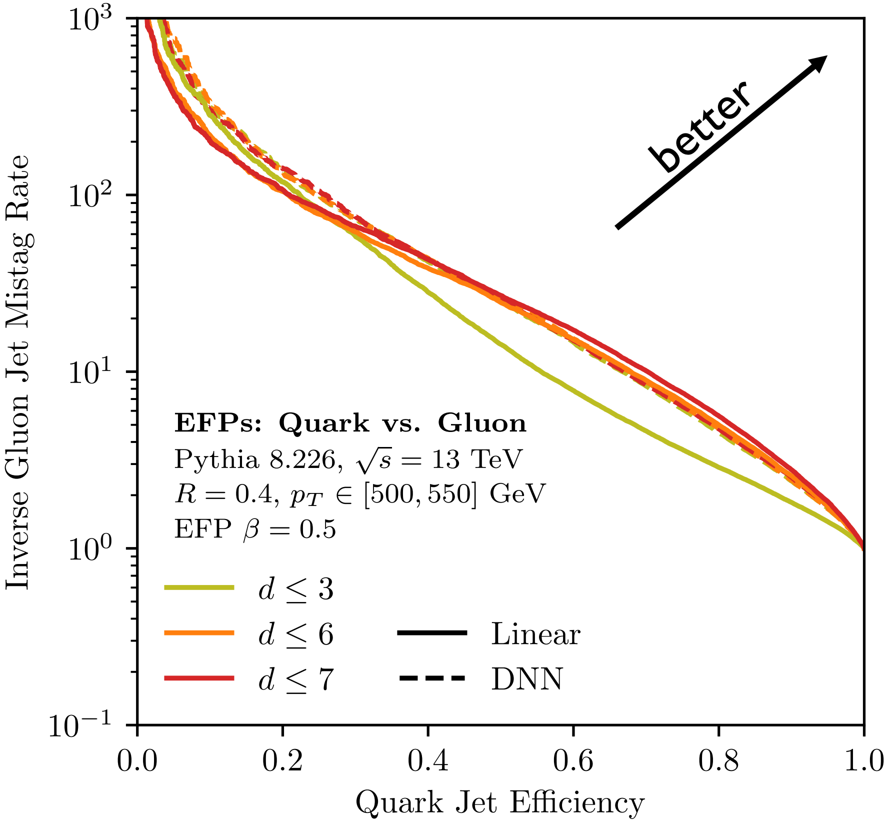
N-subjettiness basis:
[K. Datta, A. Larkoski, 1704.08249]

QG CNNs:
[**PTK**, E. Metodiev, M. Schwartz, 1612.01551]

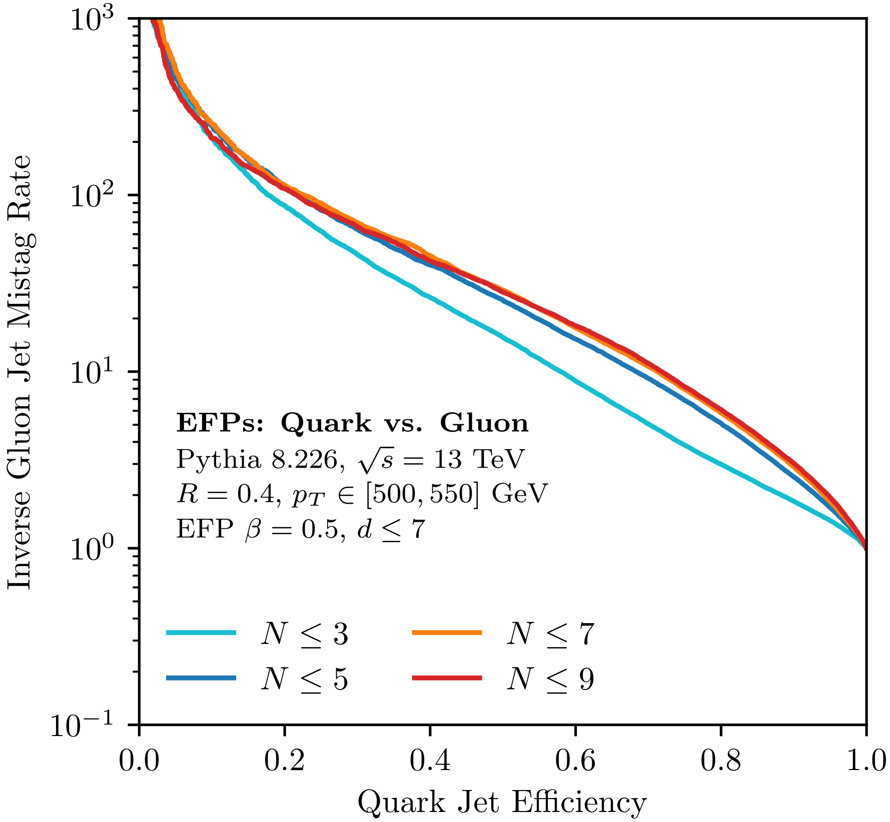
ML/NN review:
[A. Larkoski, I. Moult, B. Nachman, 1709.04464]

(Linear classification with EFPs) ~ (MML) for efficiency > 0.25!

Additional EFP Tagging Plots – Quark vs. Gluon Jets

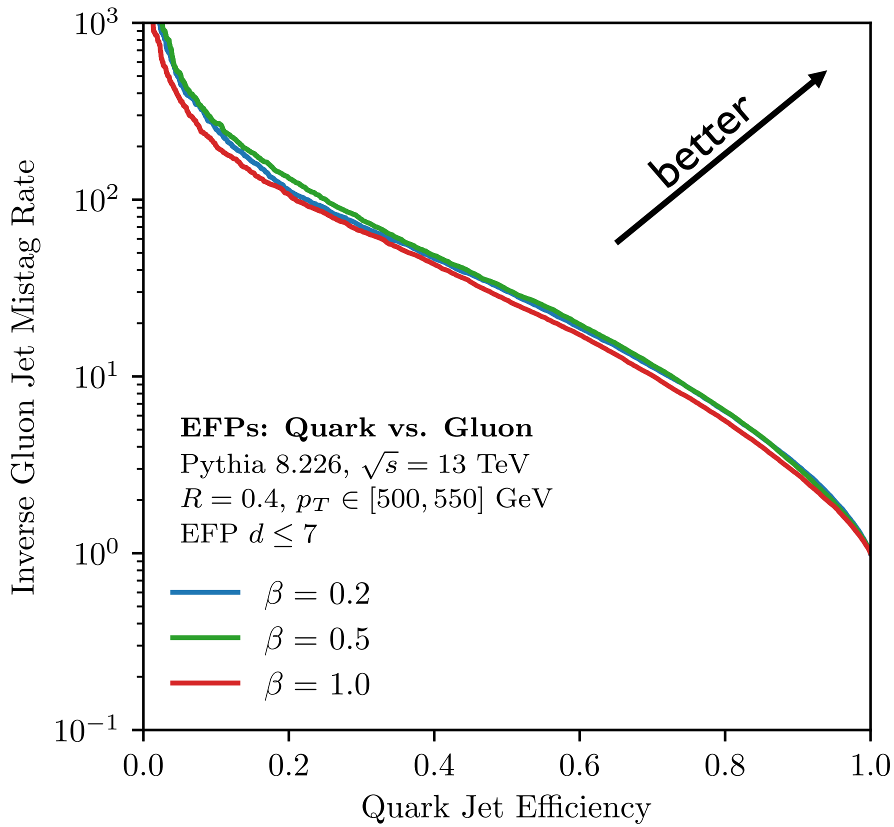


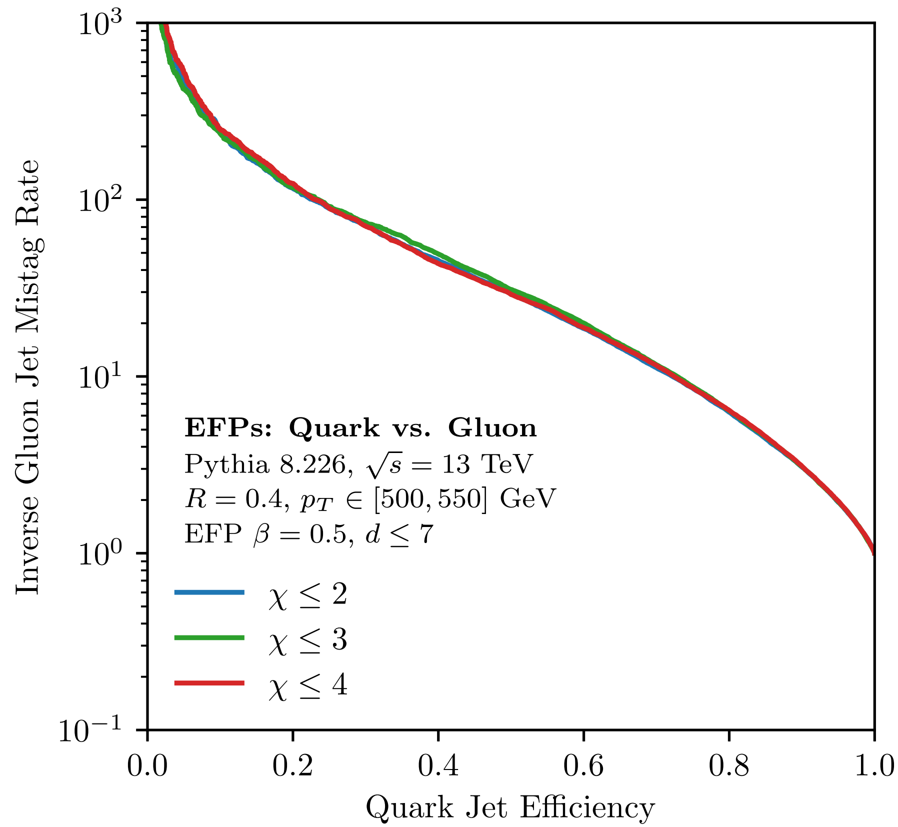
High d EFPs are important
Convergence by $d \leq 7$



High N EFPs are important

Additional EFP Tagging Plots – Quark vs. Gluon Jets





Smaller β slightly better, use ½ as default

Performance captured even with just $\chi \leq 2$!

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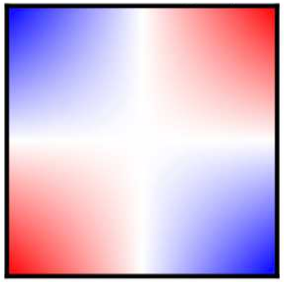

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Conclusions

$\beta = 2$ and Energy Flow Moments

[**PTK**, E. Metodiev, J. Thaler, work in progress]

Consider a slightly different hadronic angular measure, $\theta_{ij} = (2\hat{p}_i^\mu \hat{p}_{j\mu})^{\frac{\beta}{2}}, \quad \hat{p}_i^\mu = \dfrac{p_i^\mu}{p_{Ti}}$

Agrees with previous hadronic measure in the limit of narrow, central jets

When $\beta = 2$, angular measure can be factored, which motivates defining:

Energy Flow Moment (EFM) of valency v: $$\mathcal{I}^{\mu_1\cdots\mu_v} = \sum_{i=1}^{M} z_i \hat{p}_i^{\mu_1} \cdots \hat{p}_i^{\mu_v}$$

$\beta = 2$ EFPs can be rewritten in terms of EFMs, which are $\boxed{\text{linear in M to compute!}}$

$$= \sum_{i_1=1}^{M} \sum_{i_2=1}^{M} \sum_{i_3=1}^{M} z_{i_1} z_{i_2} z_{i_3} \theta_{i_1 i_2}^2 \theta_{i_1 i_3}^2 \theta_{i_2 i_3}$$

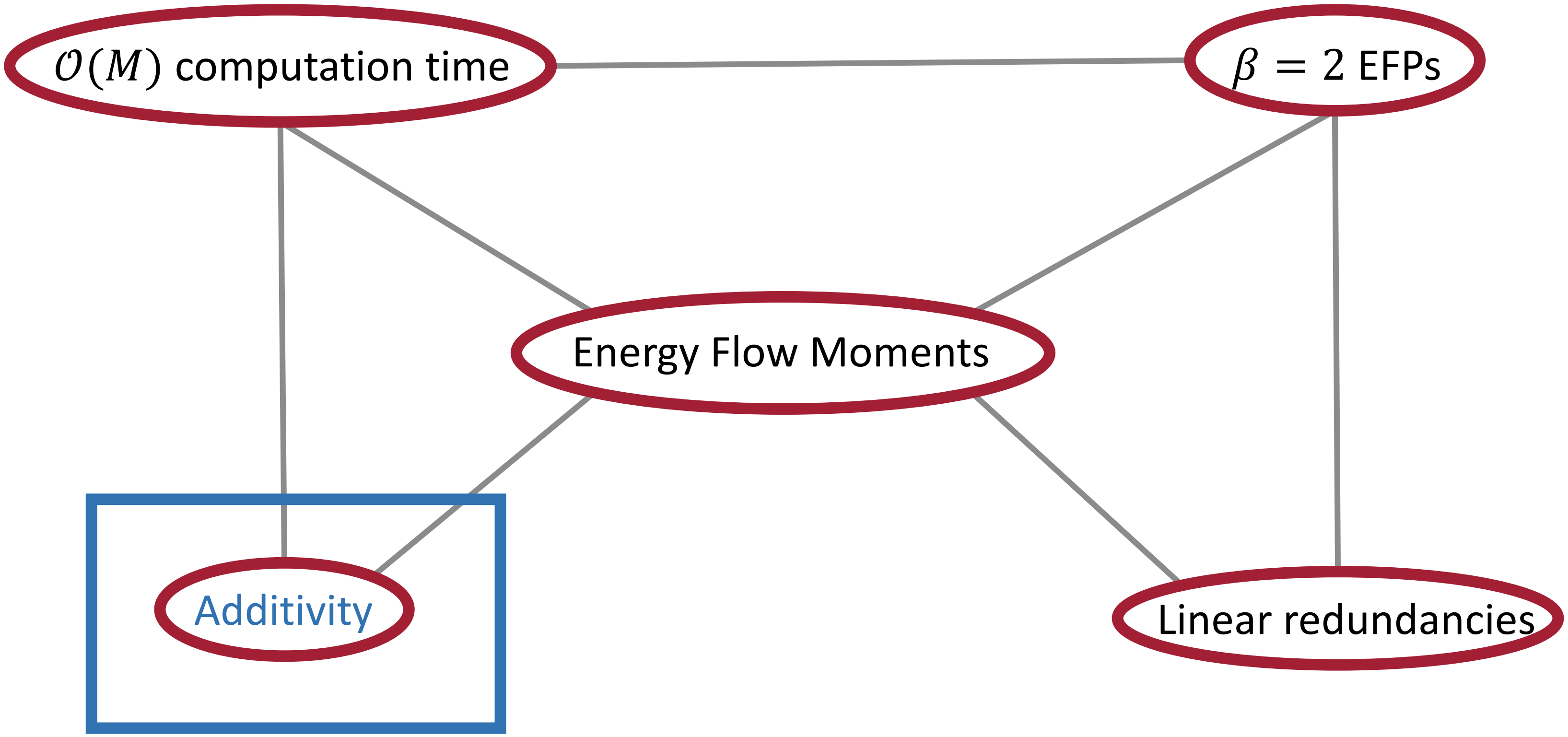
$$= 2^5 \underbrace{\left(\sum_{i_1=1}^{M} z_{i_1} \hat{p}_{i_1}^\alpha \hat{p}_{i_1}^\beta \hat{p}_{i_1}^\gamma \hat{p}_{i_1}^\delta\right)}_{\mathcal{I}^{\alpha\beta\gamma\delta}} \underbrace{\left(\sum_{i_2=1}^{M} z_{i_2} \hat{p}_{i_2\alpha} \hat{p}_{i_2\beta} \hat{p}_{i_2}^\epsilon\right)}_{\mathcal{I}_{\alpha\beta}{}^{\epsilon}} \underbrace{\left(\sum_{i_3=1}^{M} z_{i_3} \hat{p}_{i_3\gamma} \hat{p}_{i_3\delta} \hat{p}_{i_3\epsilon}\right)}_{\mathcal{I}_{\gamma\delta\epsilon}}$$

A multigraph correspondence also exists for EFMs:

$\Longleftrightarrow \mathcal{I}^{\mu_k\cdots\mu_\ell}$

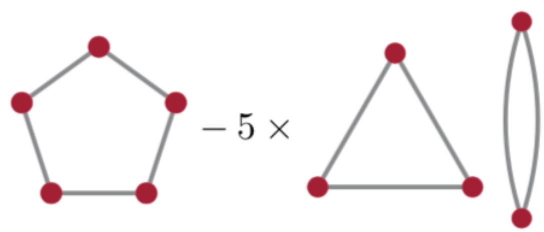
$\Longleftrightarrow \eta_{\mu_i\mu_j}$

EFM Concept Map



Can the additive structure that EFMs have be generalized?

$$5! \times \mathcal{I}^{\mu_2}_{[\mu_1}\mathcal{I}^{\mu_3}_{\mu_2}\mathcal{I}^{\mu_4}_{\mu_3}\mathcal{I}^{\mu_5}_{\mu_4}\mathcal{I}^{\mu_1}_{\mu_5]} = 6 \times \quad - 5 \times \quad = 0$$



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Conclusions

What are Jets?

Jets are variable length, unordered collections of particles

Due to quantum-mechanical indistinguishability

$$J(\{p_1^\mu, \dots, p_M^\mu\}) = J\left(\{p_{\pi(1)}^\mu, \dots, p_{\pi(M)}^\mu\}\right), \qquad \forall\, \pi \in S_M$$

M is multiplicity of the jet

Permutation group on M elements

Particle features:

- Four-momenta p_i^μ
- Other quantum numbers (e.g. particle charge or flavor)
- Experimental information (e.g. vertex info)

Variable jet length requires at least one of:

- Preprocessing into another representation (jet images, EFPs, N-subs, etc.)
- Truncation to an (arbitrary) fixed size
- Recurrent NN structure – induces a dependence on the particle order!

Additivity makes EFMs well behaved for variable length, unordered particles

Desire an architecture that can handle variable length, permutation symmetric inputs

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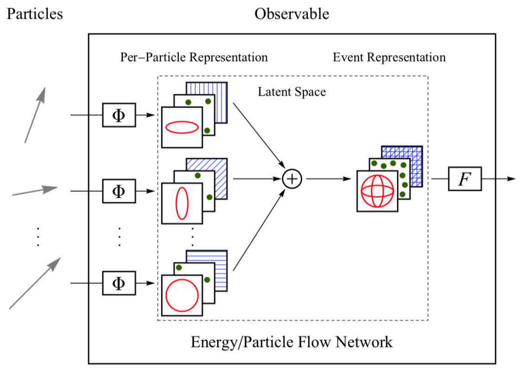


Conclusions

Deep Sets

It turns out that all permutation symmetric functions have an additivity similar to EFMs

Deep Sets

1703.06114

Manzil Zaheer[1,2], **Satwik Kottur**[1], **Siamak Ravanbhakhsh**[1],
Barnabás Póczos[1], **Ruslan Salakhutdinov**[1], **Alexander J Smola**[1,2]
[1] Carnegie Mellon University [2] Amazon Web Services

Deep Sets Theorem [60]. *Let $\mathfrak{X} \subset \mathbb{R}^d$ be compact, $X \subset 2^{\mathfrak{X}}$ be the space of sets with bounded cardinality of elements in $\mathfrak{X}$, and $Y \subset \mathbb{R}$ be a bounded interval. Consider a continuous function $f : X \to Y$ that is invariant under permutations of its inputs, i.e. $f(x_1, \ldots, x_M) = f(x_{\pi(1)}, \ldots, x_{\pi(M)})$ for all $x_i \in \mathfrak{X}$ and $\pi \in S_M$. Then there exists a sufficiently large integer ℓ and continuous functions $\Phi : \mathfrak{X} \to \mathbb{R}^\ell$, $F : \mathbb{R}^\ell \to Y$ such that the following holds to an arbitrarily good approximation:*[1]

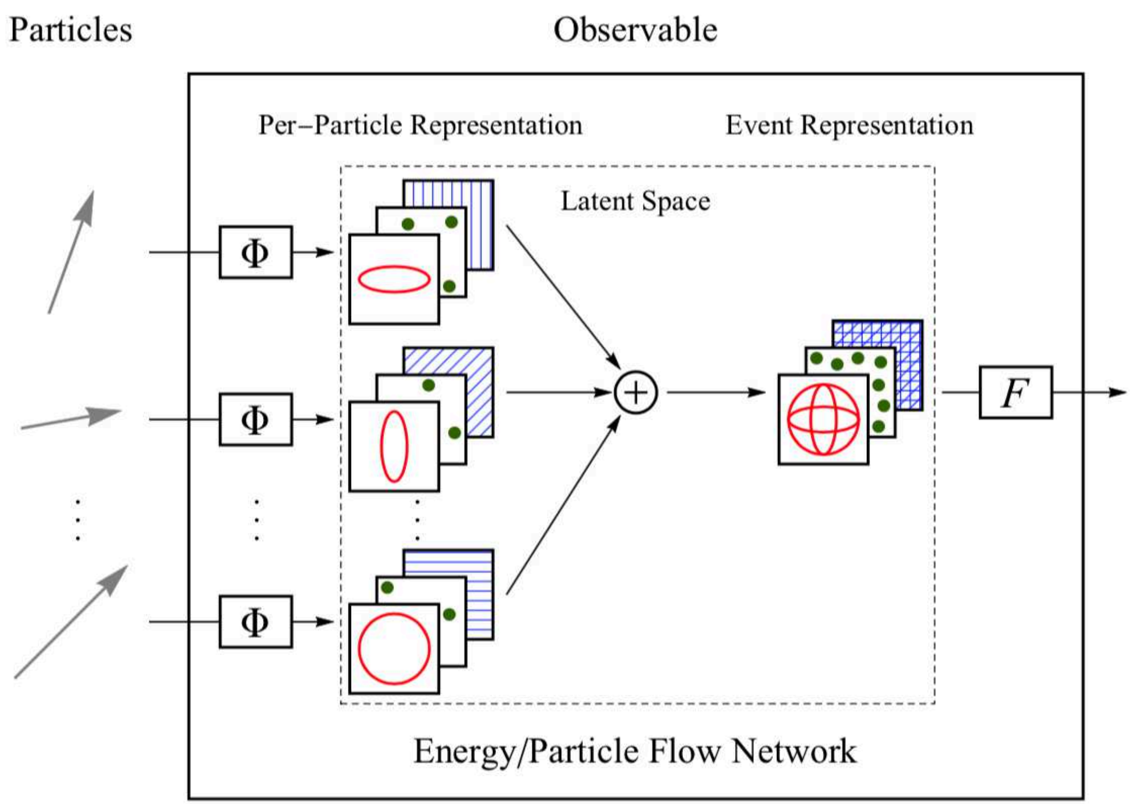
$$f(\{x_1, \ldots, x_M\}) = F\left(\sum_{i=1}^{M} \Phi(x_i)\right).$$

Proof sketch is simple: Stone-Weierstrass theorem followed by the fundamental theorem of symmetric polynomials

Deep Sets for Particle Jets

[**PTK**, E. Metodiev, J. Thaler, 1810.0xxxx]

$$f(\{x_1,\ldots,x_M\}) = F\left(\sum_{i=1}^{M}\Phi(x_i)\right)$$



$$\mathrm{EFN}\left(\{p_1^\mu,\ldots,p_M^\mu\}\right) = F\left(\sum_{i=1}^{M} z_i\Phi(\hat{p}_i)\right)$$

Manifestly IRC-safe latent space

$$\mathrm{PFN}\left(\{p_1^\mu,\ldots,p_M^\mu\}\right) = F\left(\sum_{i=1}^{M}\Phi(p_i^\mu)\right)$$

Fully general latent space

Familiar Jet Substructure Observables as EFNs or PFNs

Observable $\mathcal{O}$		Map Φ	Function F
Mass	m	p^μ	$F(x^\mu) = \sqrt{x^\mu x_\mu}$
Multiplicity	M	1	$F(x) = x$
Track Mass	m_{track}	$p^\mu \mathbb{I}_{\text{track}}$	$F(x^\mu) = \sqrt{x^\mu x_\mu}$
Track Multiplicity	M_{track}	$\mathbb{I}_{\text{track}}$	$F(x) = x$
Jet Charge [69]	$\mathcal{Q}_\kappa$	$(p_T, Q\, p_T^\kappa)$	$F(x, y) = y/x^\kappa$
Eventropy [71]	$z \ln z$	$(p_T, p_T \ln p_T)$	$F(x, y) = y/x - \ln x$
Momentum Dispersion [90]	p_T^D	(p_T, p_T^2)	$F(x, y) = \sqrt{y/x^2}$
C parameter [91]	C	$(\lvert\vec{p}\rvert, \vec{p} \otimes \vec{p}/\lvert\vec{p}\rvert)$	$F(x, Y) = \frac{3}{2x^2}[(\operatorname{Tr} Y)^2 - \operatorname{Tr} Y^2]$

Many observables are easily interpreted in EFN language

Some observables not as easily handled (e.g. *N*-subjettiness) Iterated EFN structure could address this

$\beta = 2$ EFPs are also included via EFMs

$$\text{EFN}\left(\{p_1^\mu, \ldots, p_M^\mu\}\right) = F\left(\sum_{i=1}^{M} z_i \Phi(\hat{p}_i)\right)$$

Manifestly IRC-safe latent space

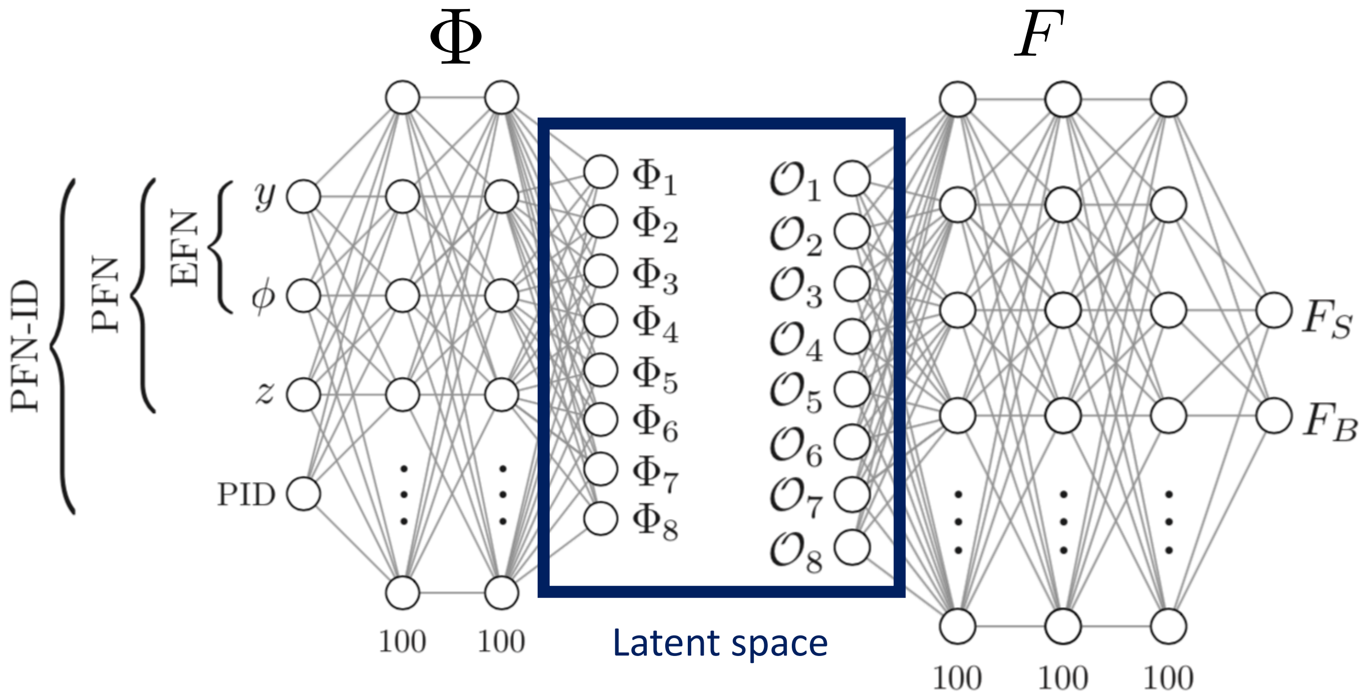
$$\text{PFN}\left(\{p_1^\mu, \ldots, p_M^\mu\}\right) = F\left(\sum_{i=1}^{M} \Phi(p_i^\mu)\right)$$

Fully general latent space

Approximating Φ and F with Neural Networks

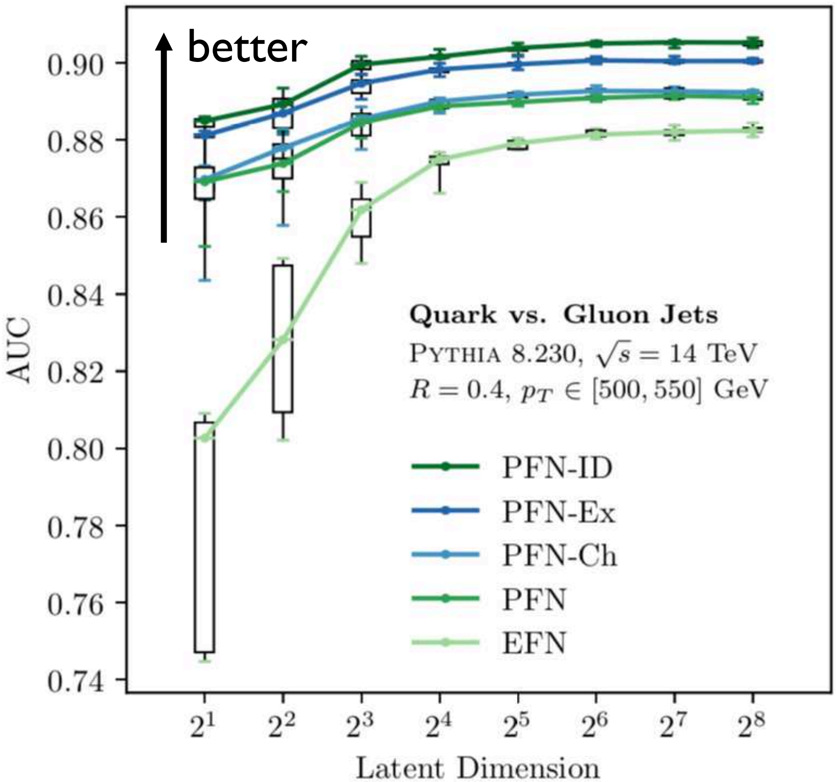
Employ neural networks as arbitrary function approximators

Use basic fully-connected networks for simplicity



$$\text{EFN:}\ \ \mathcal{O}_a = \sum_i z_i \Phi_a(y_i, \phi_i) \qquad\qquad \text{PFN:}\ \ \mathcal{O}_a = \sum_i \Phi_a(z_i, y_i, \phi_i, [\text{PID}_i])$$

EFN Latent Dimension Sweep – Quark vs. Gluon Jets



PFN-ID: Full particle flavor info
$(\pi^\pm, K^\pm, p, \bar{p}, n, \bar{n}, \gamma, K_L, e^\pm, \mu^\pm)$

PFN-Ex: Experimentally accessible info
$(h^{\pm,0}, \gamma, e^\pm, \mu^\pm)$

PFN-Ch: Particle charge info

PFN: No particle type info, arbitrary energy dependence

EFN: IRC-safe latent space

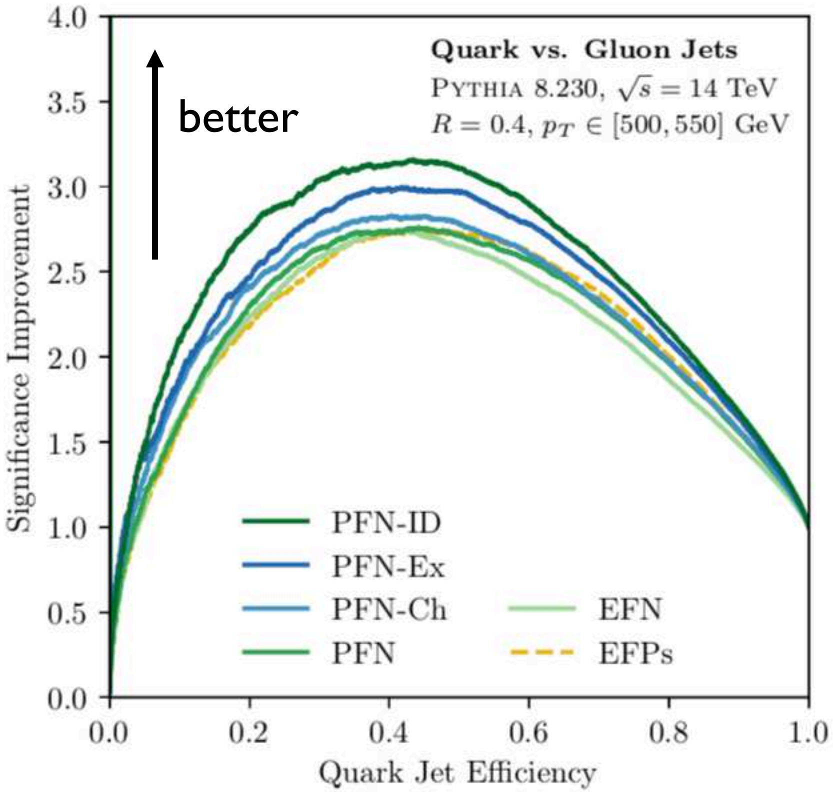
Performance saturates as latent dimension increases

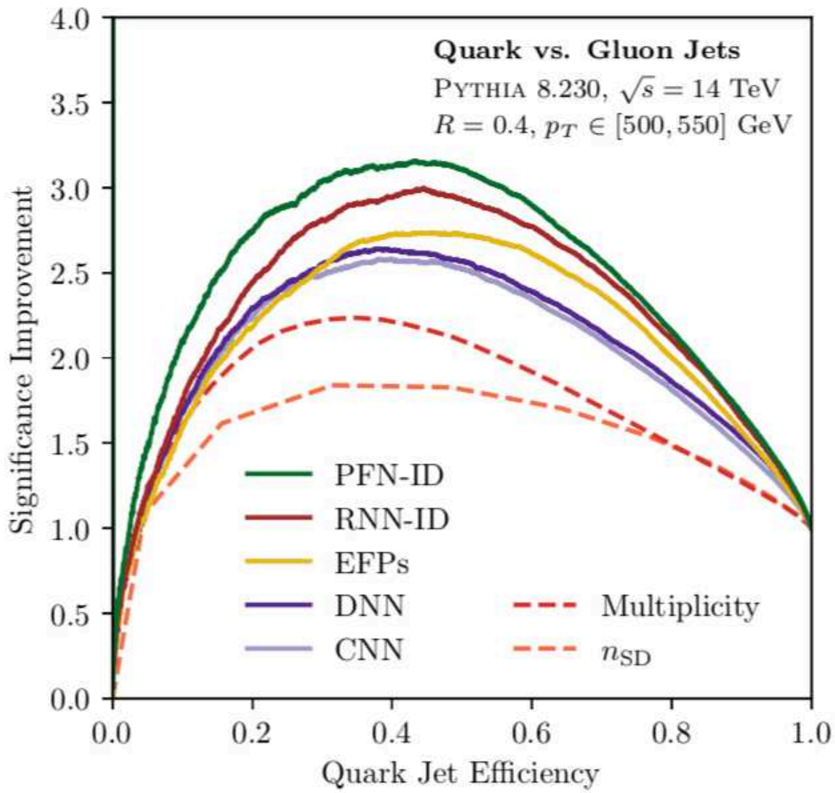
IRC-unsafe information clearly helpful

Adding particle type information helpful

Classification Performance – Quark vs. Gluon Jets

Top vs. QCD Jets in backup





Latent space dimension $\ell = 256$

EFPs are comparable to EFN

PFN-ID compares favorably to other architectures and observables

Outline

Part I

- IRC-Safe Jet Observables
- Energy Flow Polynomials (EFPs)

Part I.V

- Energy Flow Moments (EFMs)

Part II

- Intrinsic Jet Symmetries
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- Opening the Box

Conclusions

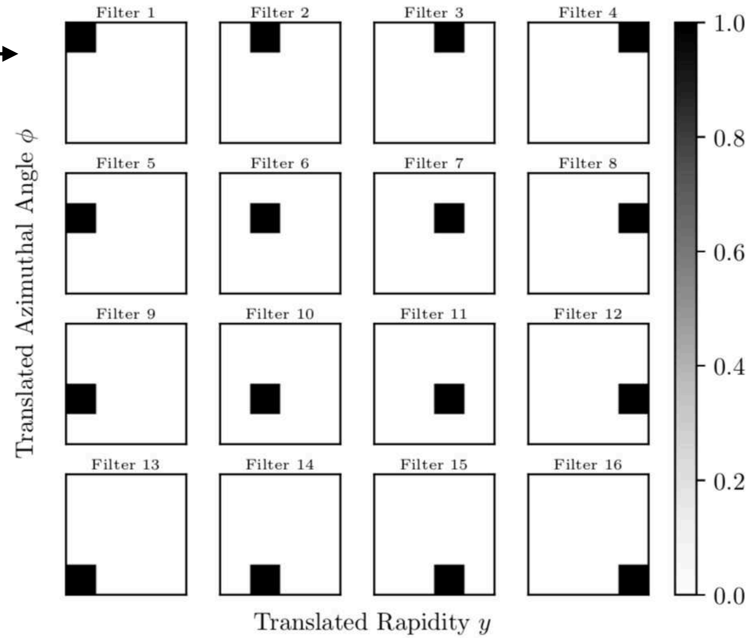
Visualizing the Filters

Given trained model, examine values of latent observables, $\Phi(\hat{p}) = (\Phi_1(\hat{p}), \ldots, \Phi_\ell(\hat{p}))$

EFN observables are purely geometric functions of (y, ϕ) and can be shown as two-dimensional images (similar to jet images)

Example: Jet images as EFN filters

EFN structure encompasses many representations, e.g. jet images



What will the EFN learn?

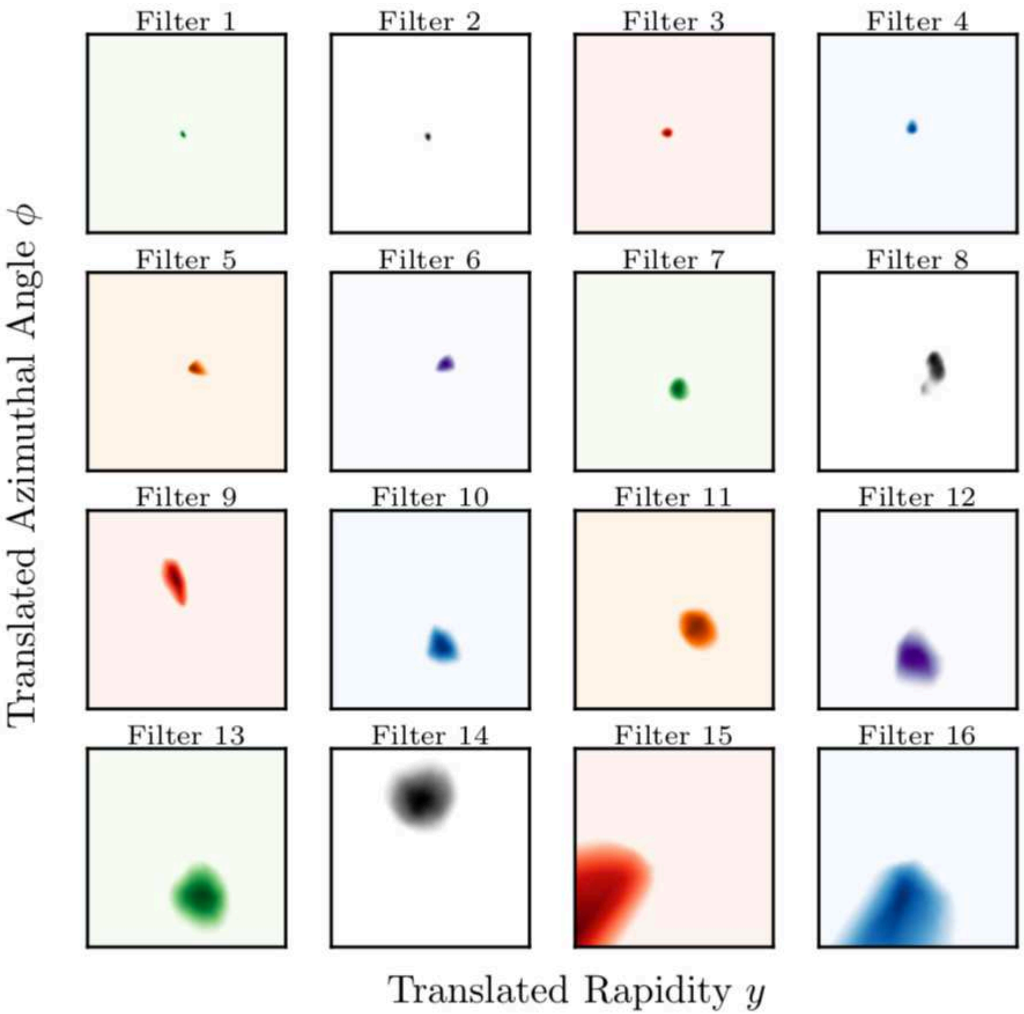
- EFPs (via EFMs)?
- Jet images?
- Something uninterpretable?
- Something interpretable but completely new?

Jet images:

[J. Cogan, M. Kagan, E. Strauss, A. Schwartzman, 1407.5675]

[L. de Oliviera, M. Kagan, L. Mackey, B. Nachman, A. Schwartzman, 1511.05190]

Visualizing the Filters – Quark vs. Gluon Jets



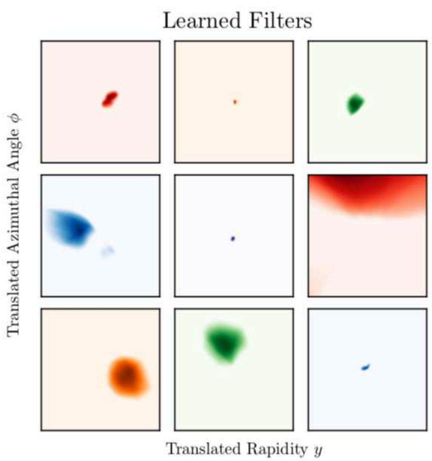
Generally see "peanuts" and "lobes"

EFN_{256} randomly selected filters, sorted by active filter size

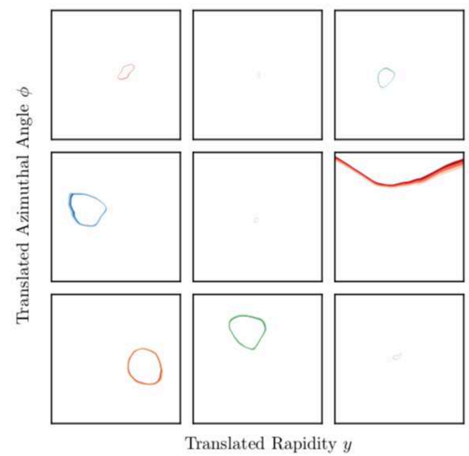
Local nature of activated pixel regions is fascinating!

Visualizing the Filters – Quark vs. Gluon Jets

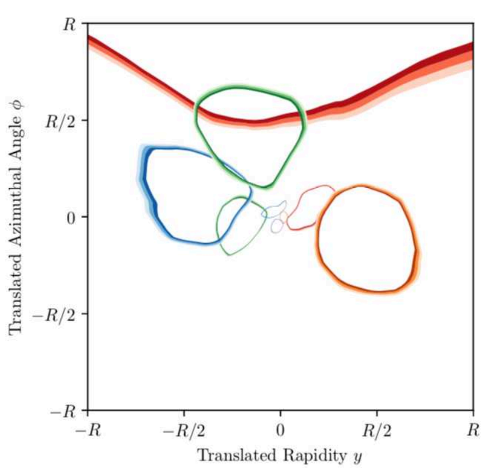
Simultaneous visualization strategy



Contour

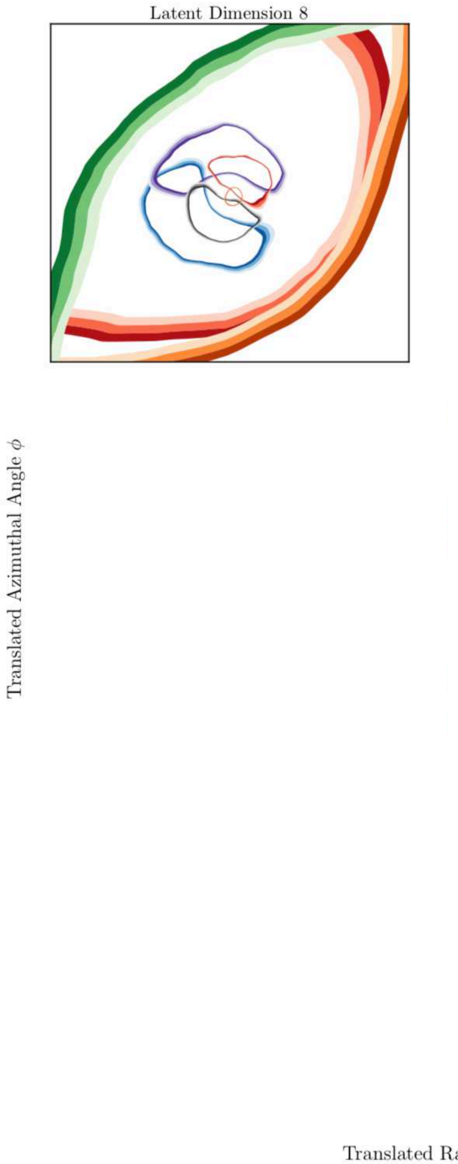


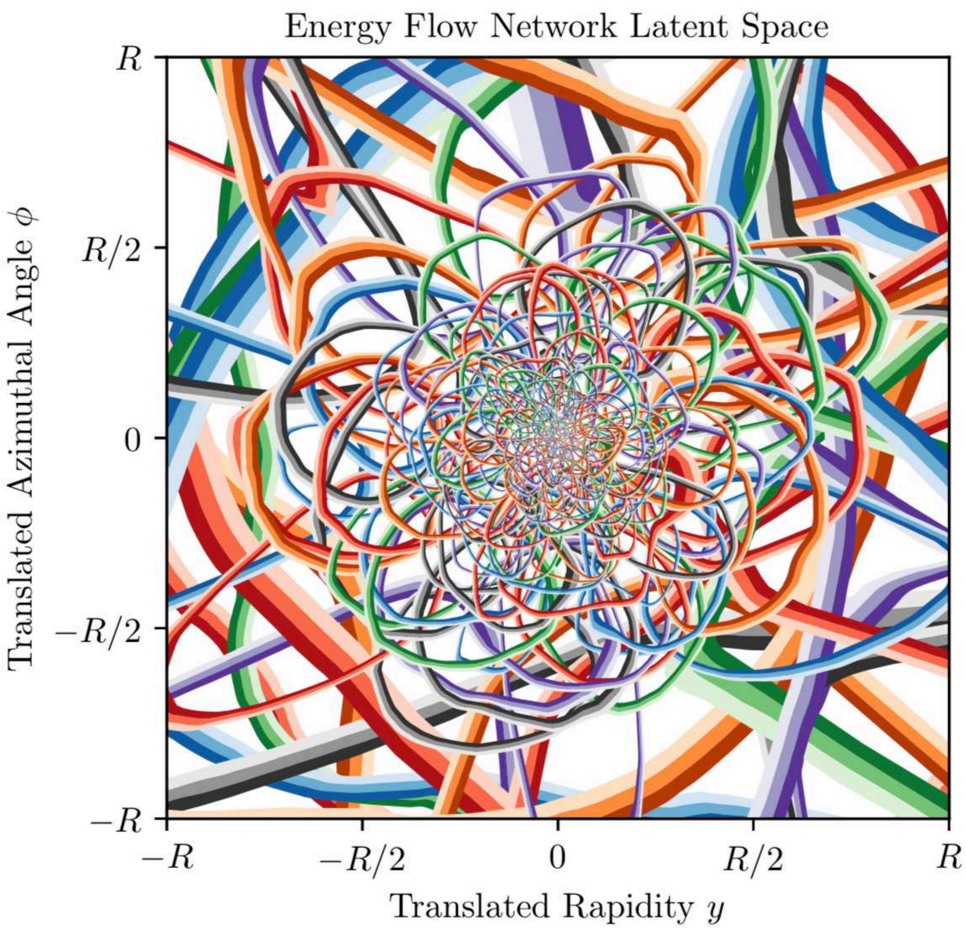
Overlay



Colored region is 10% around median

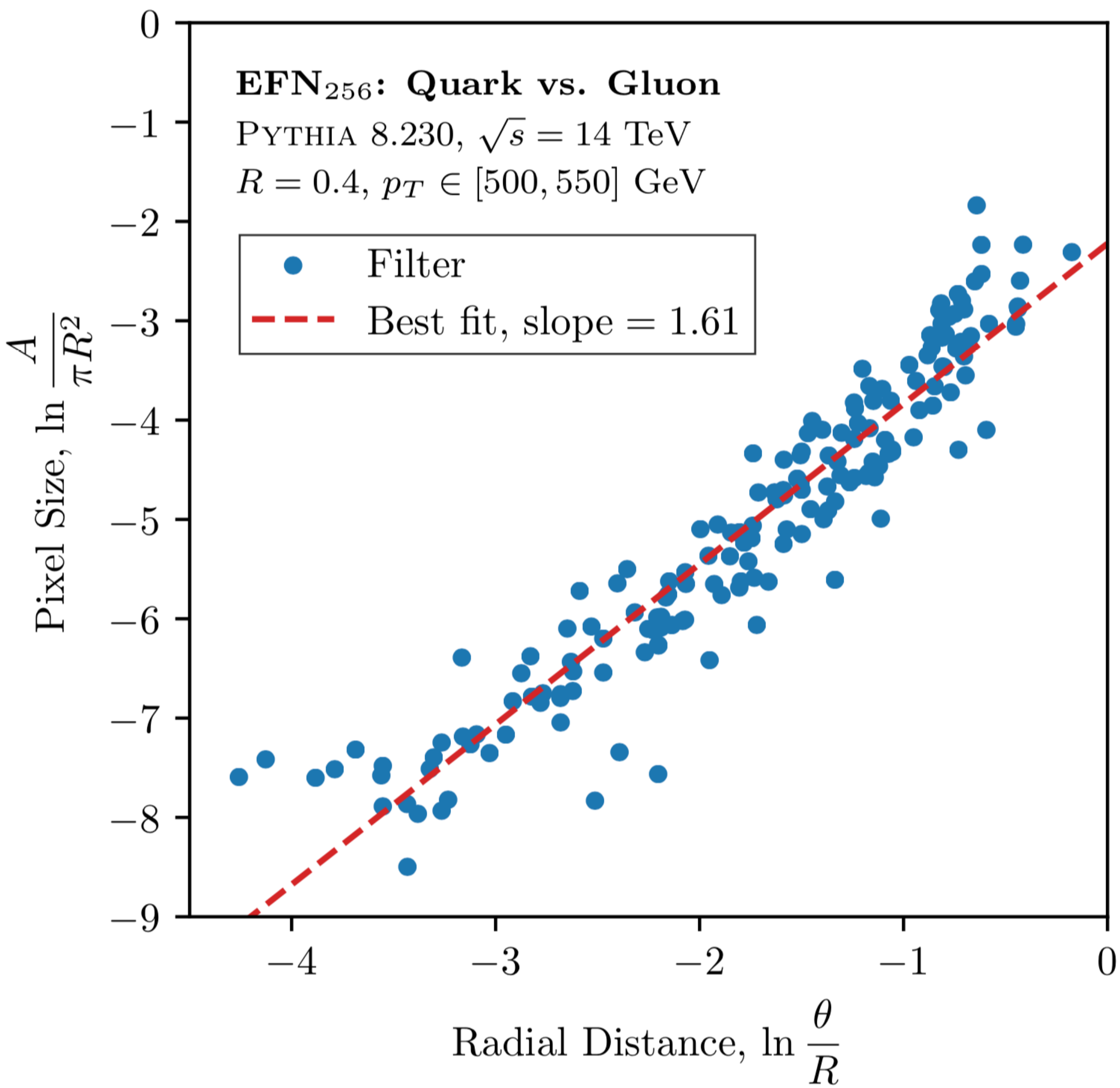
Visualizing the Filters – Quark vs. Gluon Jets





Latent space dimension $\ell = 256$

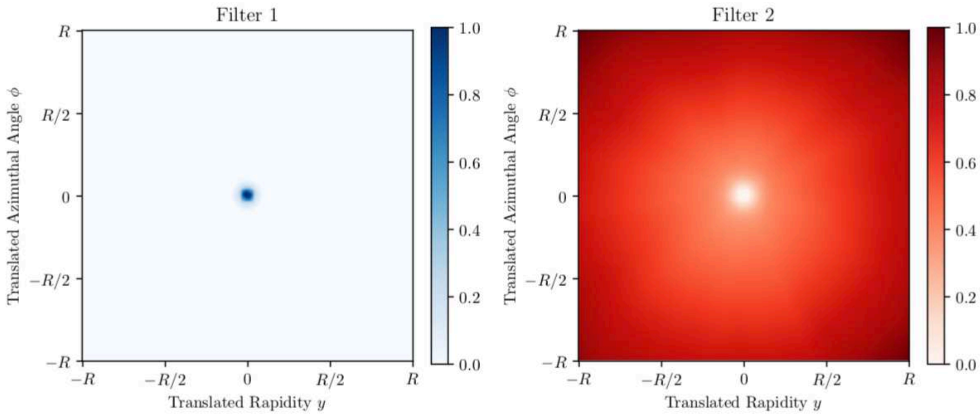
Measuring the Filters – Quark vs. Gluon Jets



Power-law dependence between filter size and distance from center

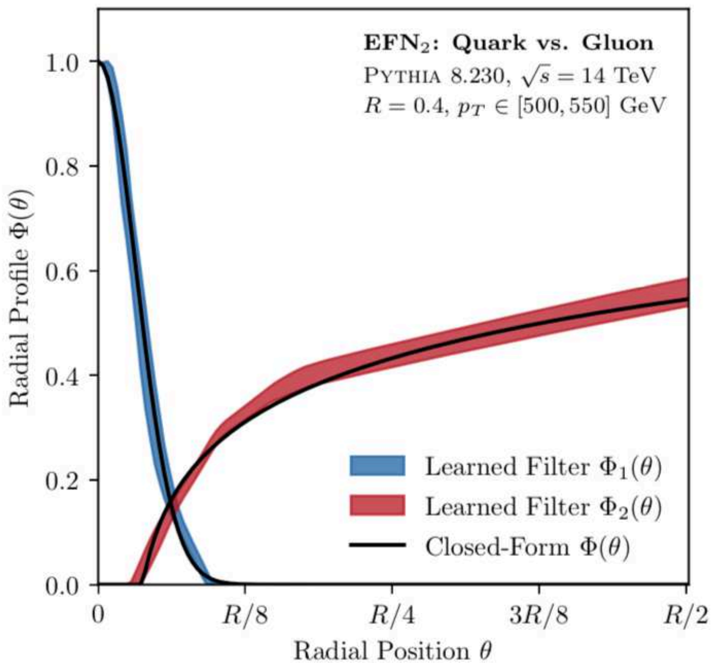
Slope of 2 is expected from scale-invariance

Extracting New Analytic Observables



Latent space dimension $\ell = 2$ has approximately radially symmetric filters:

$$\mathcal{O}_1 = \sum_{i=1}^{M} z_i \, \Phi_1(\theta_i) \qquad \mathcal{O}_2 = \sum_{i=1}^{M} z_i \, \Phi_2(\theta_i)$$



Take radial slices to obtain envelope

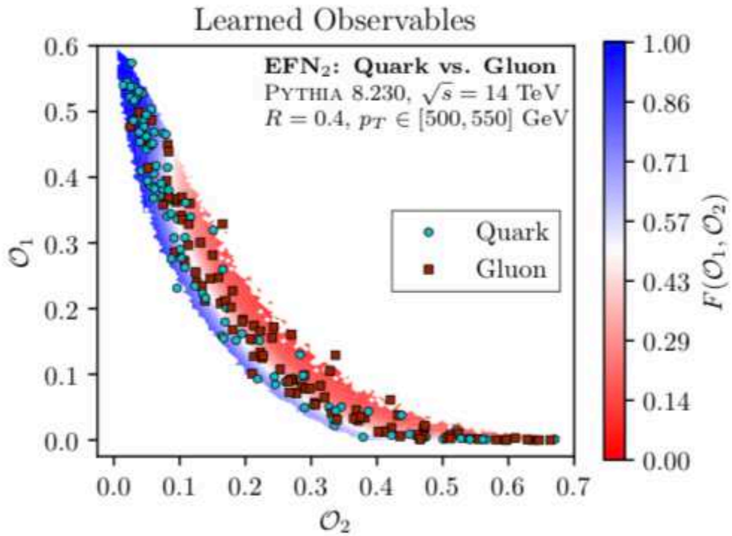
Fit functions of the following forms:

$$A_{r_0} = \sum_i z_i \, e^{-\theta_i^2/r_0^2}, \qquad B_{r_1,\beta} = \sum_i z_i \ln(1 + \beta(\theta_i - r_1)) \, \Theta(\theta_i - r_1)$$

A and B separate collinear and wide-angle regions of phase space, unlike traditional angularities which mix them

Extracting New Analytic Observables

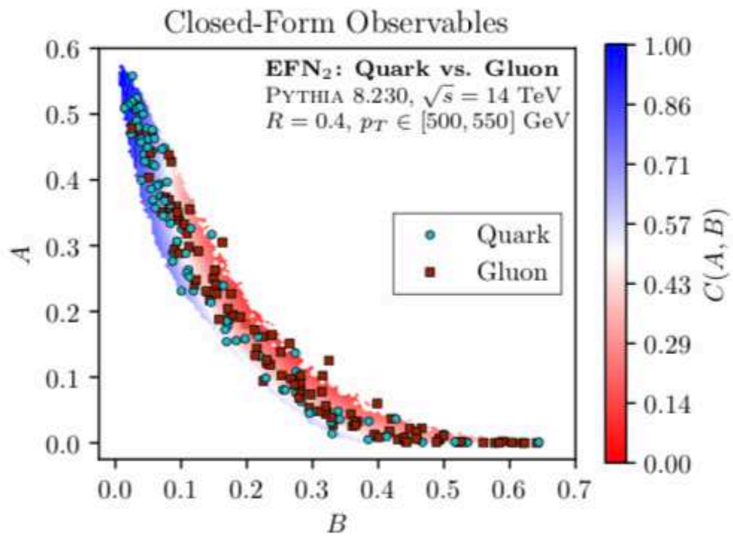
Can also visualize F in the two-dimensional $(\mathcal{O}_1, \mathcal{O}_2)$ phase space



Learned

Extract analytic form for F as distance from a point:

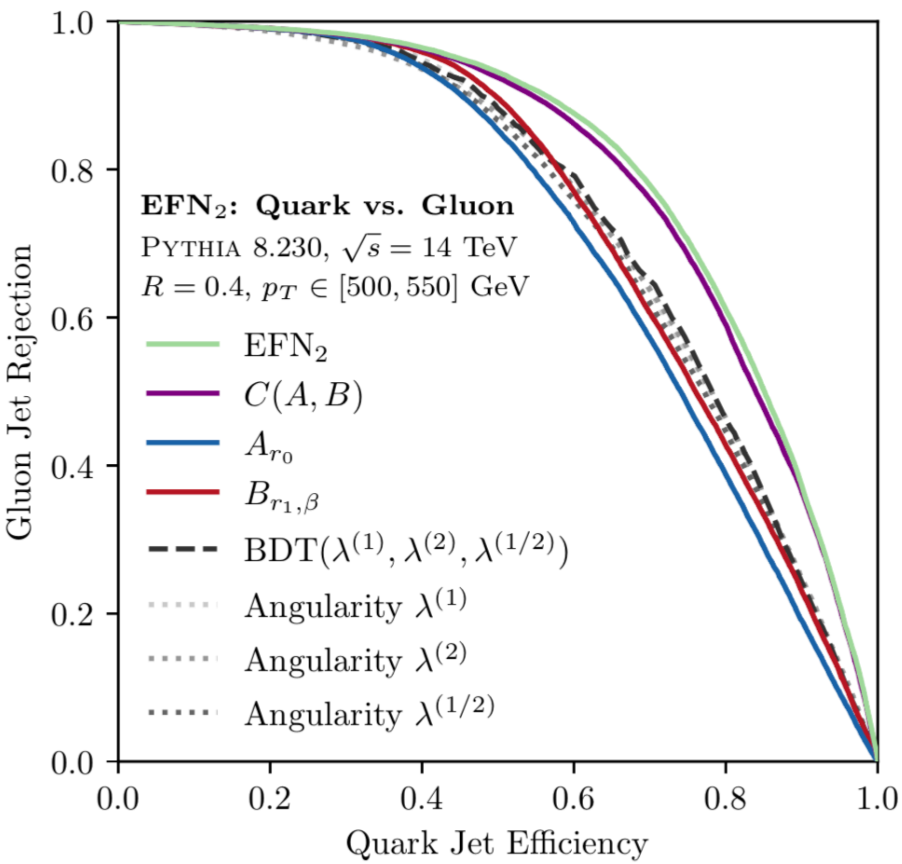
$$C(A, B) = (A - a_0)^2 + (B - b_0)^2$$



Extracted

Extracted C, A, B do a good job of reproducing learned $\mathcal{O}_1, \mathcal{O}_2, F$

Benchmarking New Analytic Observables



Extracted $C(A, B)$ performs nearly as well as EFN_2

Multivariate combination (BDT) of three other angularities does not do as well

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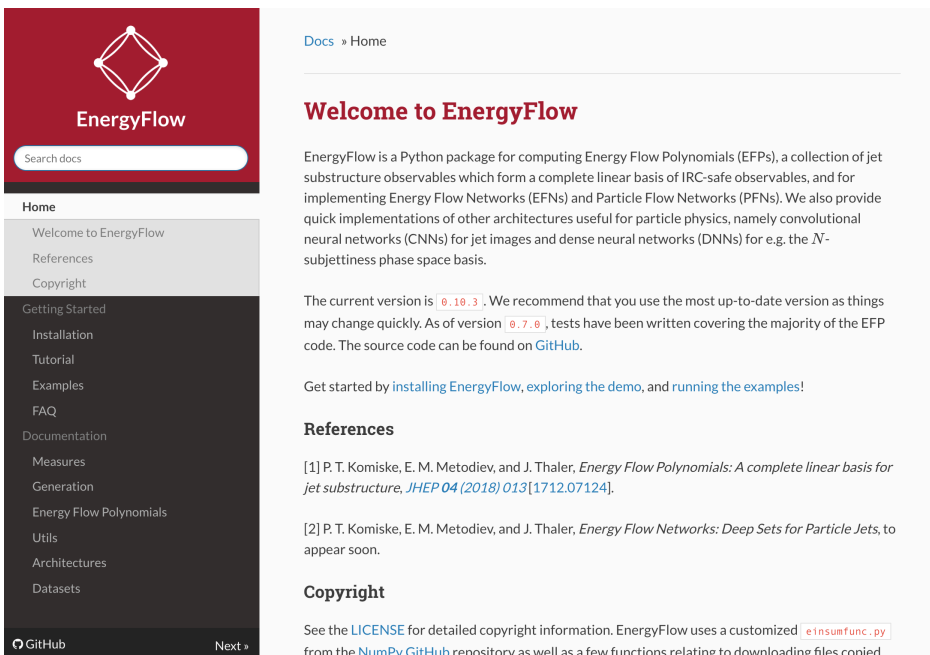
EnergyFlow Python Package

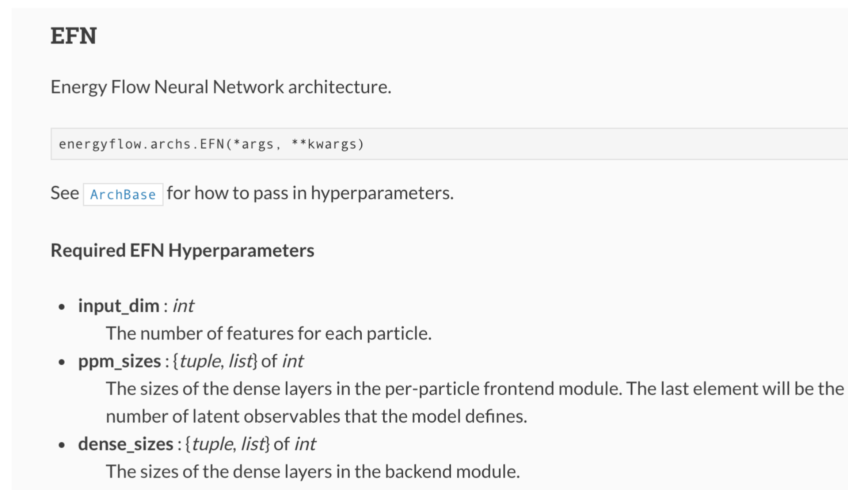
EnergyFlow package is available for Python 2 and Python 3

Computes EFPs using variable elimination, easy to select combinations of EFPs

EFN/PFN/CNN/DNN architectures included for easy model implementation

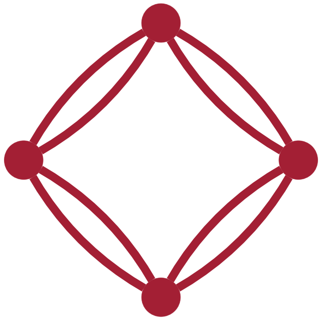
Includes comprehensive examples demonstrating how to make plots similar to those shown here



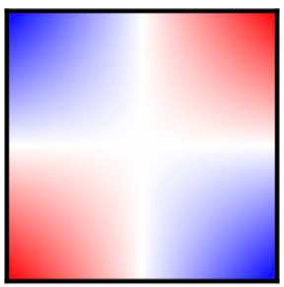


https://energyflow.network

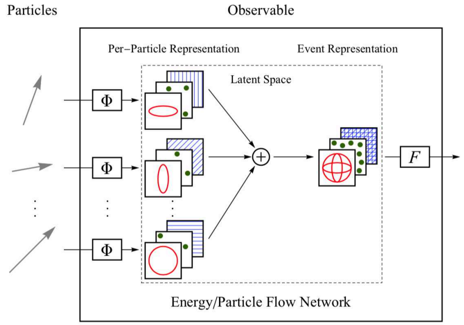
Conclusions



Energy Flow Polynomials are an interesting set of jet substructure observables that form a linear spanning basis of IRC-safe observables



Energy Flow Moments allow for linear in M computation of $\beta = 2$ EFPs, have other interesting properties (additivity, algebraic identities) to be explored

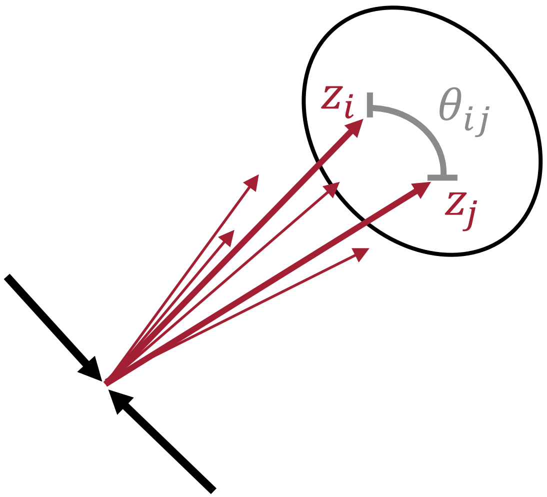


Energy Flow Networks are a natural and general way to process events, offer good performance, appropriate symmetries, and model interpretability

Azure

Backup Slides

Connection with the Stress-Energy Operator



At the heart is the Energy Flow Operator:

Energy Flow to infinity

$$\hat{\mathcal{E}}(\hat{n}, v) = \lim_{t\to\infty} \hat{n}_i T^{0i}(t, vt\hat{n})$$

in the $\hat{n}$ direction
at velocity v

[N. Sveshnikov and F. Tkachov, hep-ph/9512370]
[V. Mateu, I.W. Stewart, and J. Thaler, arXiv:1209.3781]

Progress has been made in computing correlations of $\hat{\mathcal{E}}(\hat{n}, v)$ in conformal field theory

[D. Hofman and J. Maldecena, 0803.1467]

IRC-safe observables are built out of energy correlators:

[F. Tkachov, hep-ph/9601308]

Rigid energy structure

$$C_f = \sum_{i_1=1}^{M} \cdots \sum_{i_N=1}^{M} z_{i_1} \cdots z_{i_N} f(\hat{p}_{i_1}, \cdots, \hat{p}_{i_N})$$

Arbitrary angular function f

Linear Regression and IRC Safety

$\frac{m_J}{p_{TJ}}$: IRC safe. No Taylor expansion due to square root.

$\lambda^{(\alpha=1/2)}$: IRC safe. No simple analytic relationship.

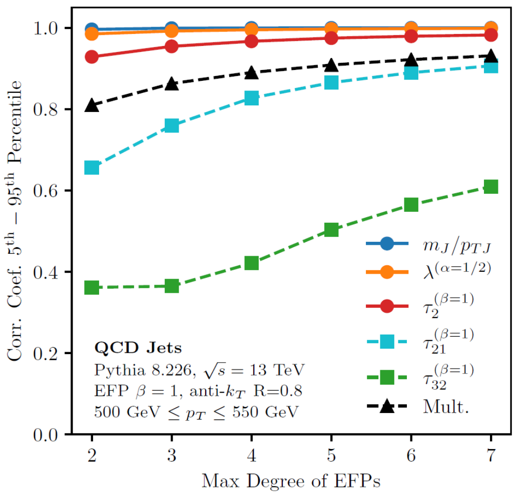
τ_2: IRC safe. Algorithmically defined.

τ_{21}: Sudakov safe. Safe for 2-prong jets and higher. [A. Larkoski, S. Marzani, and J. Thaler, 1502.01719]

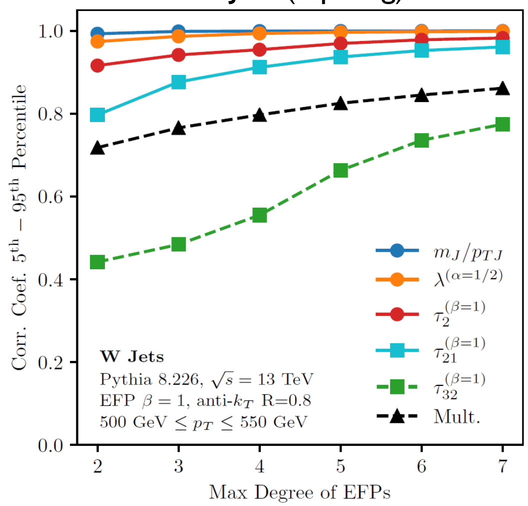
τ_{32}: Sudakov safe. Safe for 3-prong jets and higher.

Multiplicity: IRC unsafe.

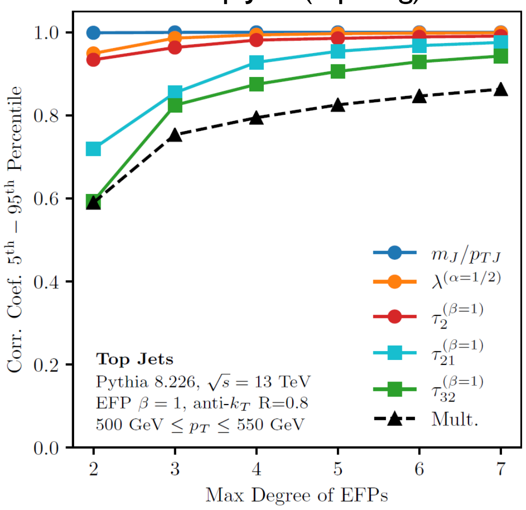
QCD Jets (1 prong)



W Jets (2 prong)



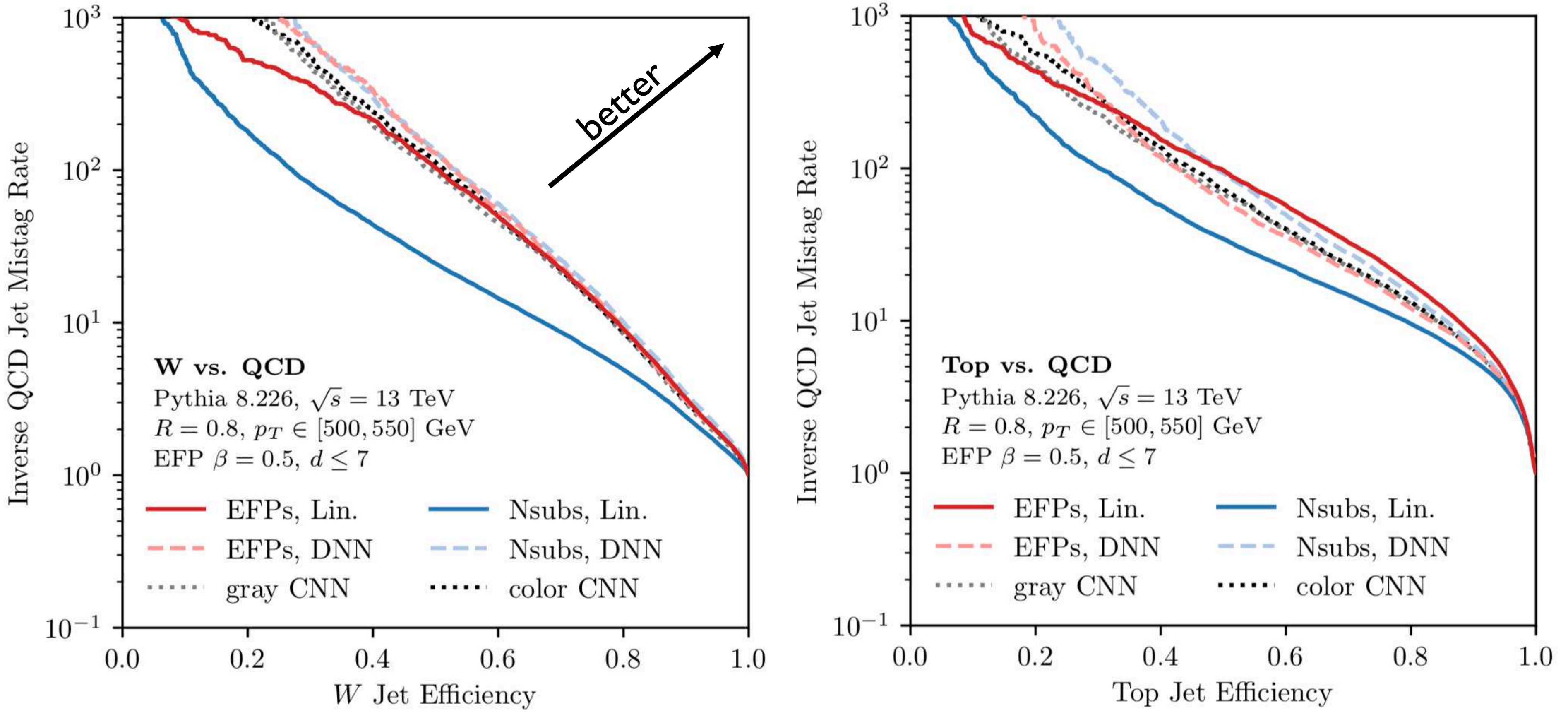
Top Jets (3 prong)



Expected to be IRC safe = Solid.
Expected to be IRC unsafe = Dashed.

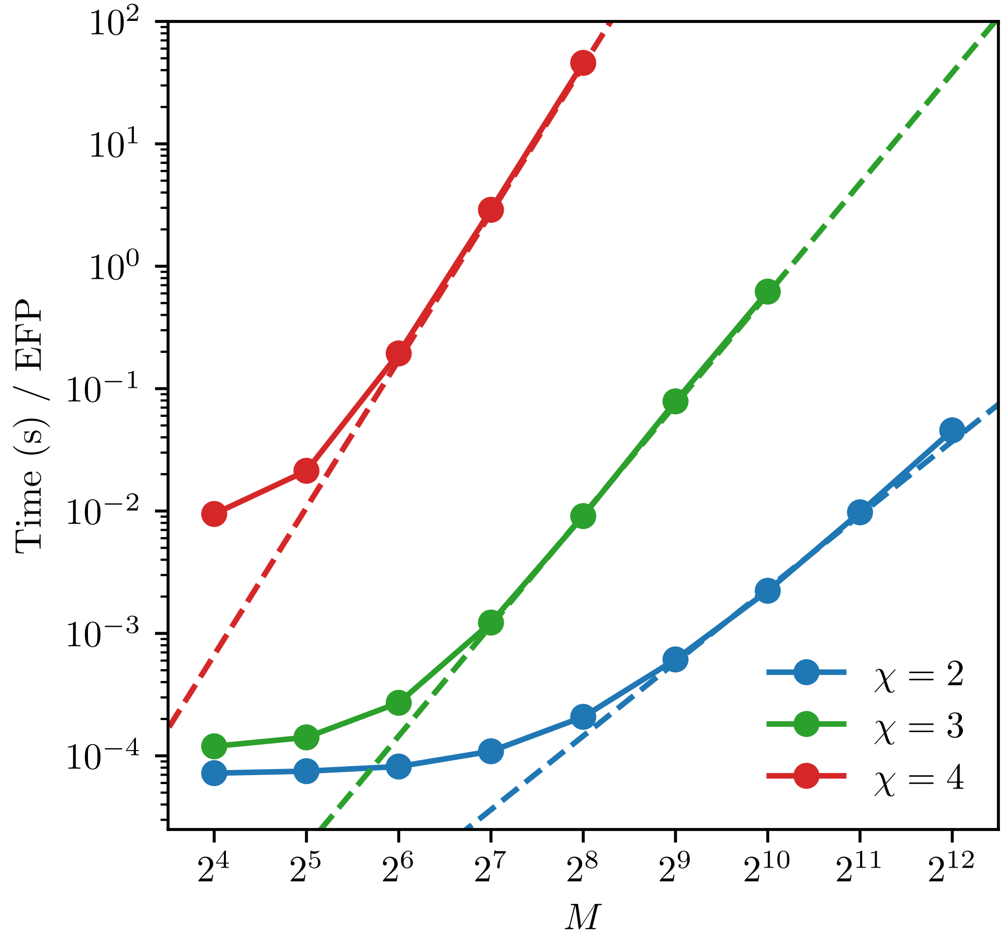
Jet Tagging Performance – 2-prong and 3-prong tagging

ROC curves for W vs. QCD and top vs. QCD jet tagging

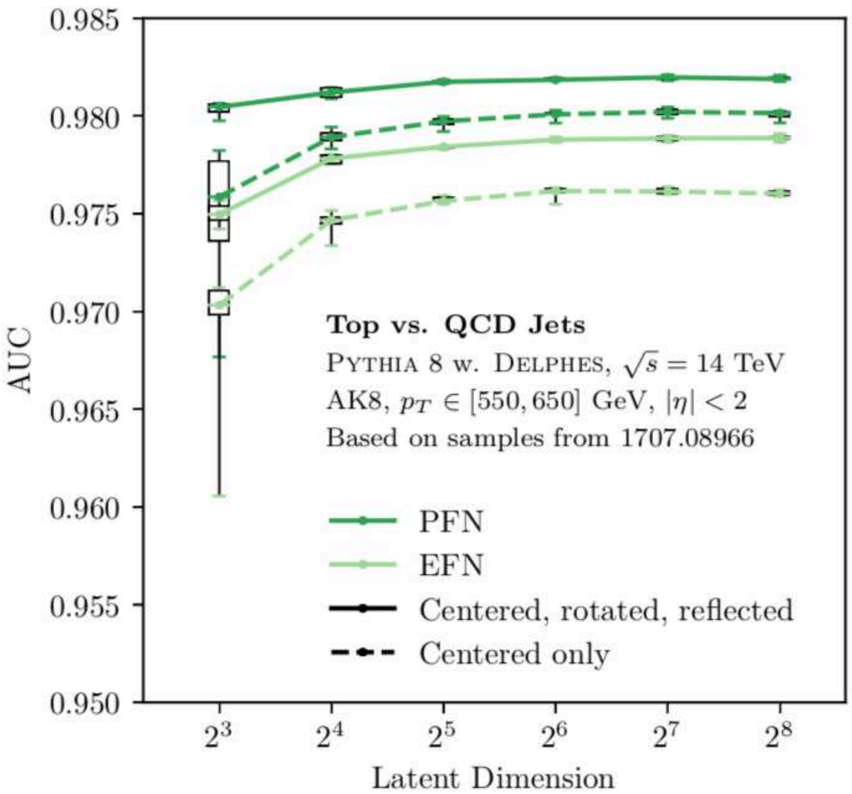


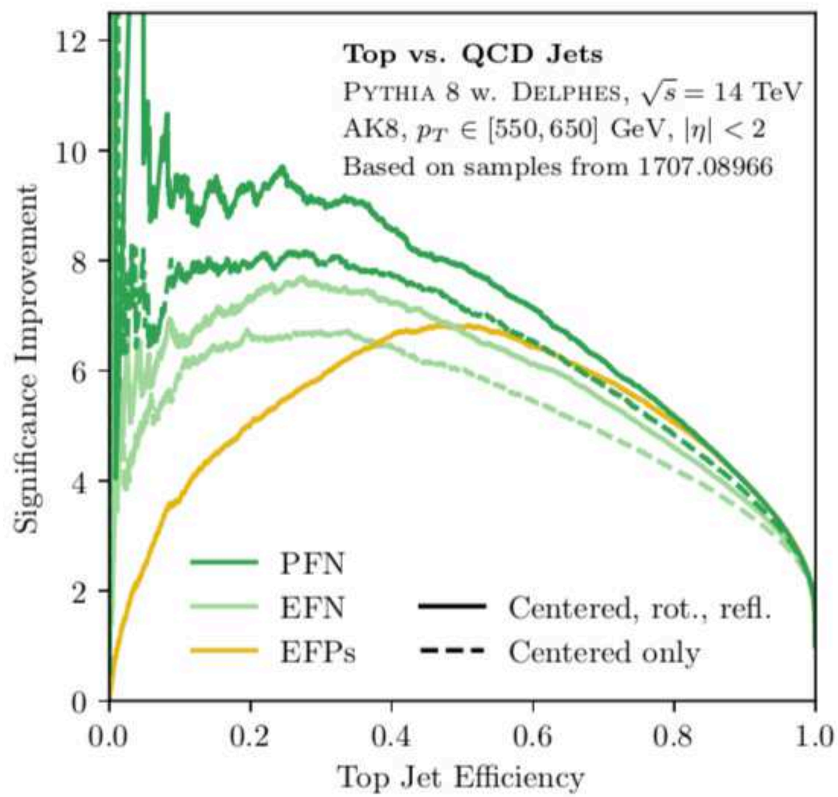
(Linear classification with EFPs) ~ (MML) for signal efficiency > 0.5!

EFP Computation Timing with Variable Elimination

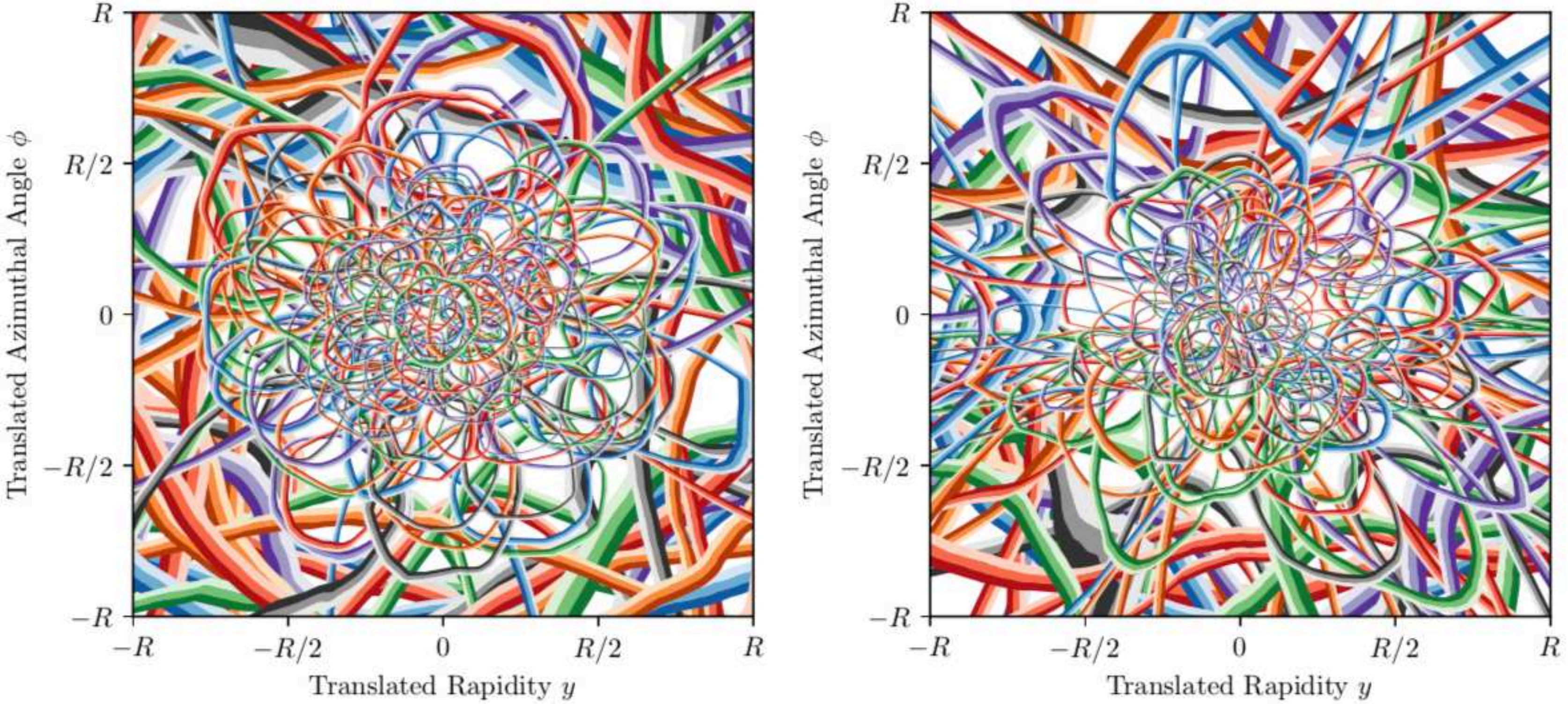


Linear Classification Performance – Top vs. QCD Jets



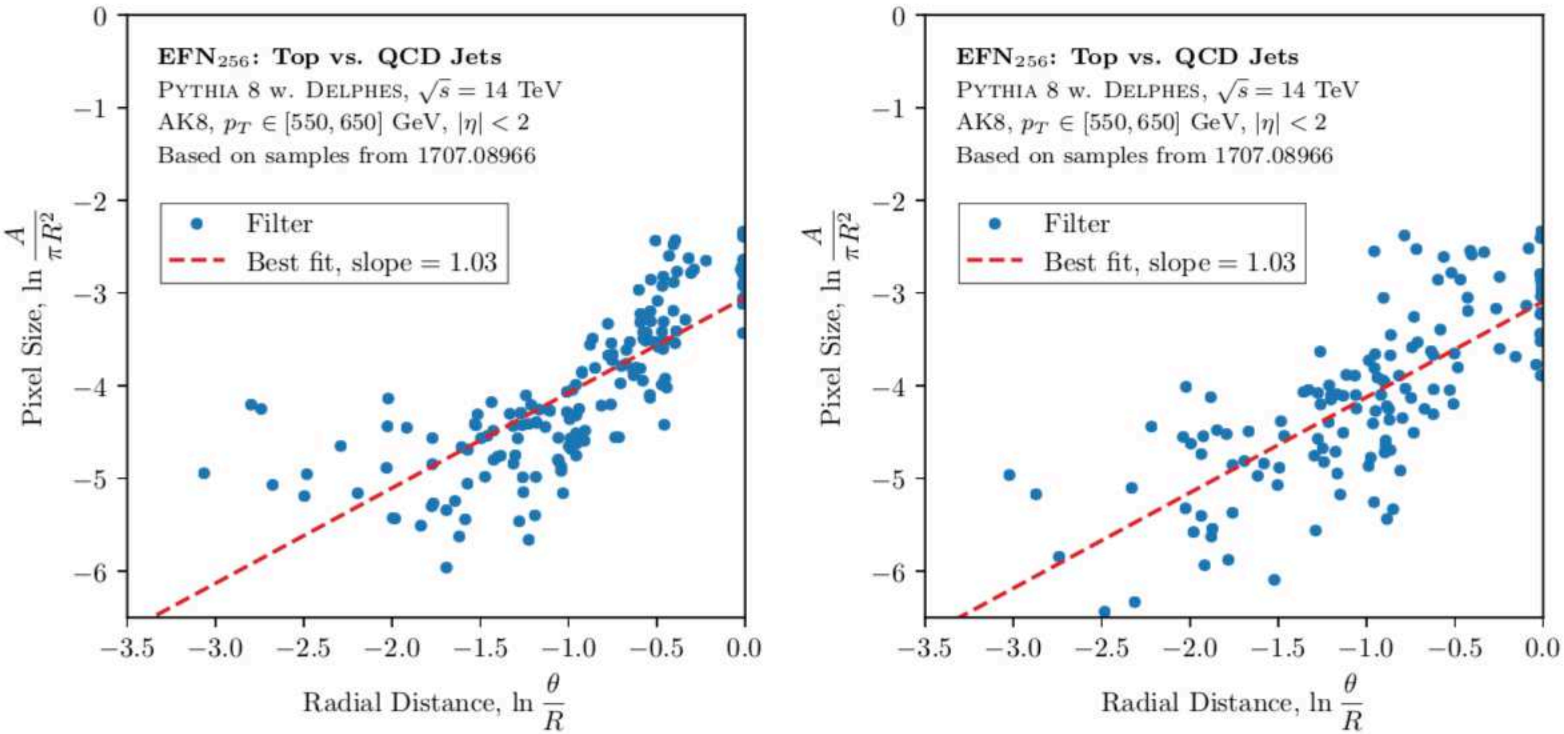


Linear Classification Performance – Top vs. QCD Jets



Preprocessed by rotating and reflecting

Linear Classification Performance – Top vs. QCD Jets



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