

The Hidden Geometry of Particle Collisions

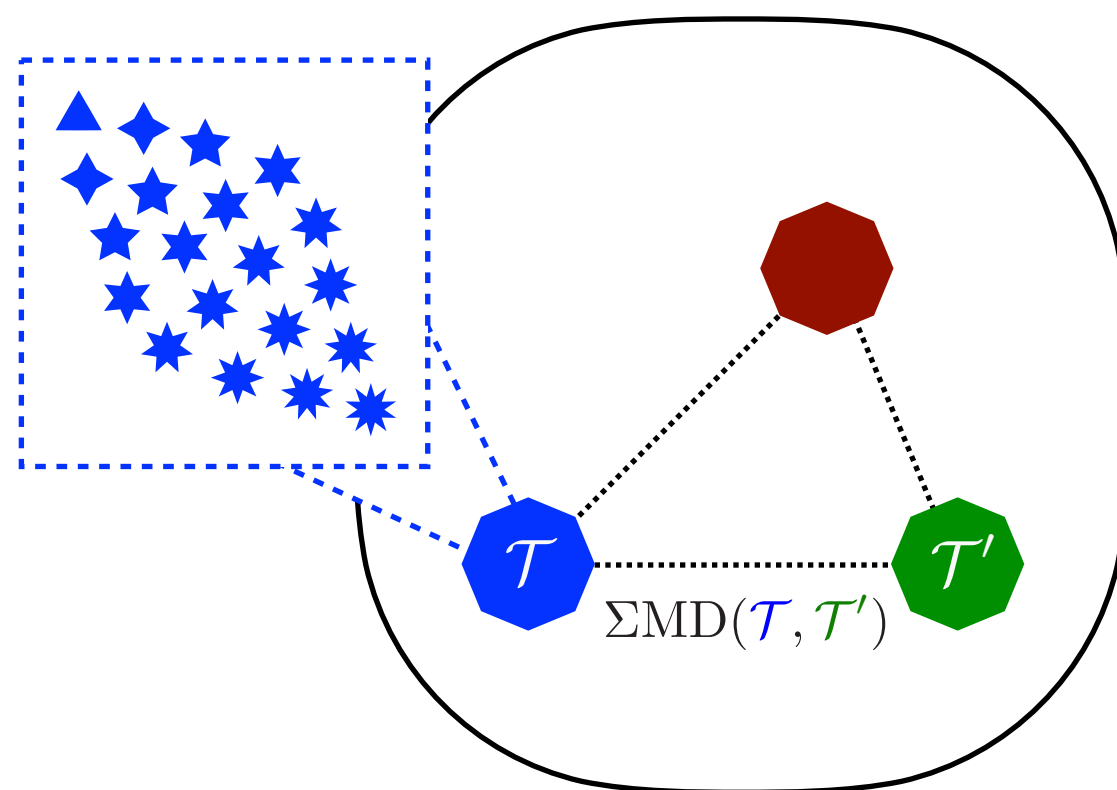
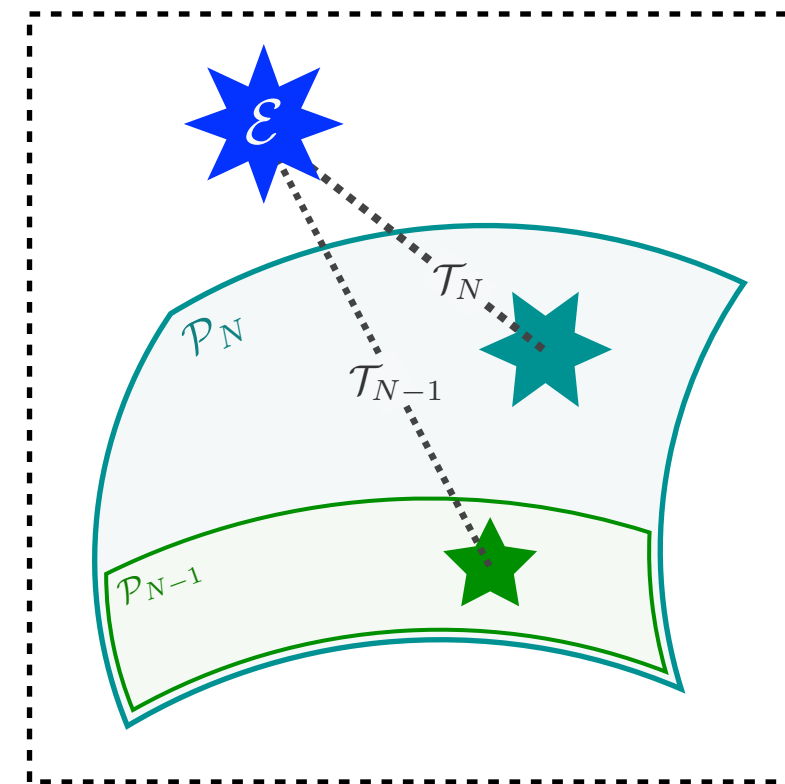
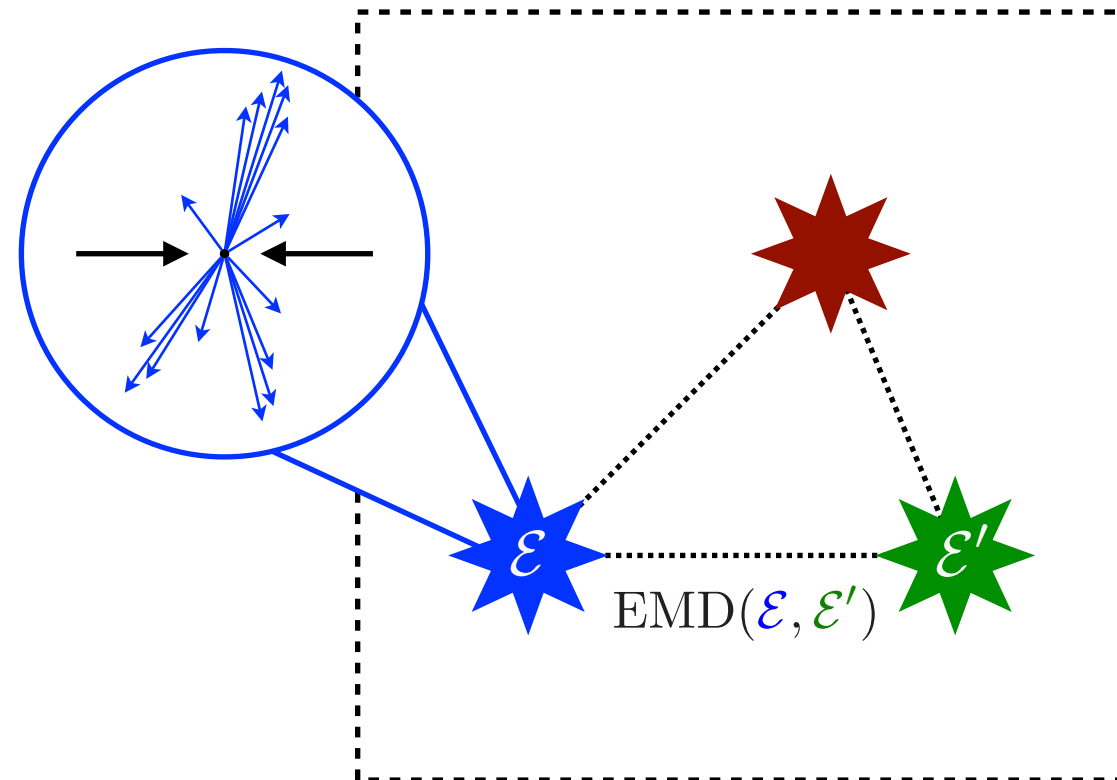
Patrick T. Komiske III

Massachusetts Institute of Technology
Center for Theoretical Physics

Based on work with Eric Metodiev and Jesse Thaler
[\[PRL 2019, 2004.04159\]](#)

Particle Physics Phenomenology Series
Università di Genova

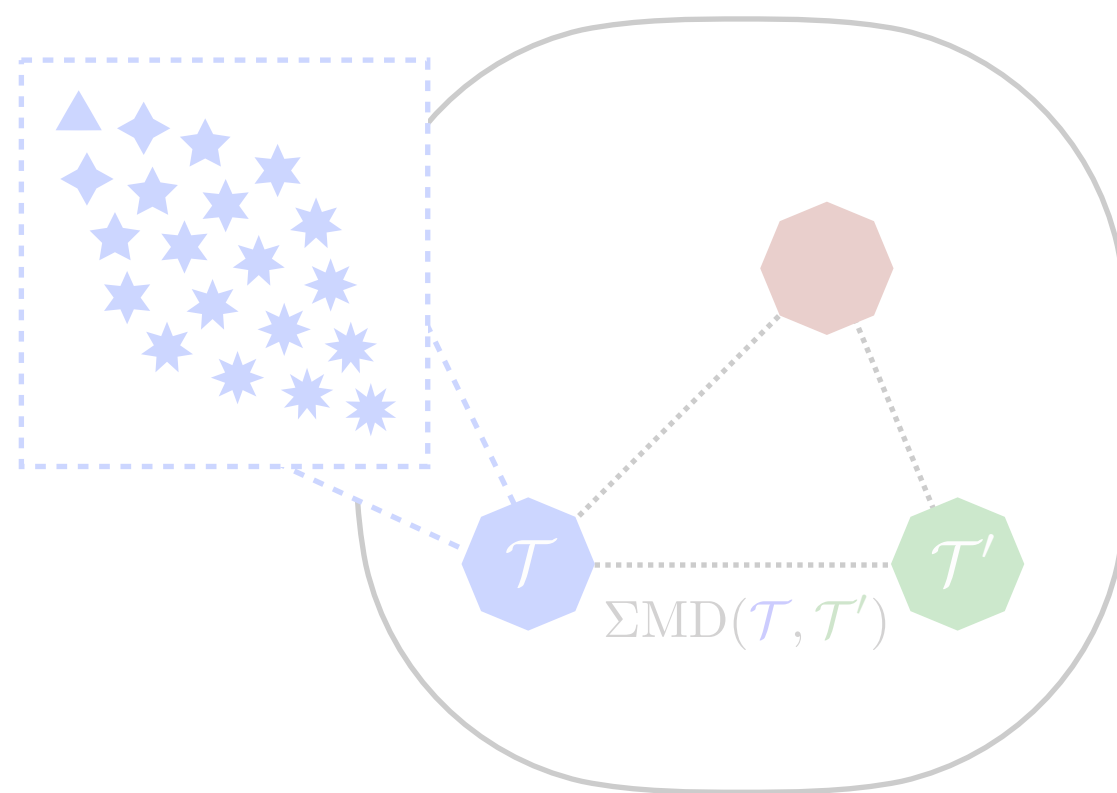
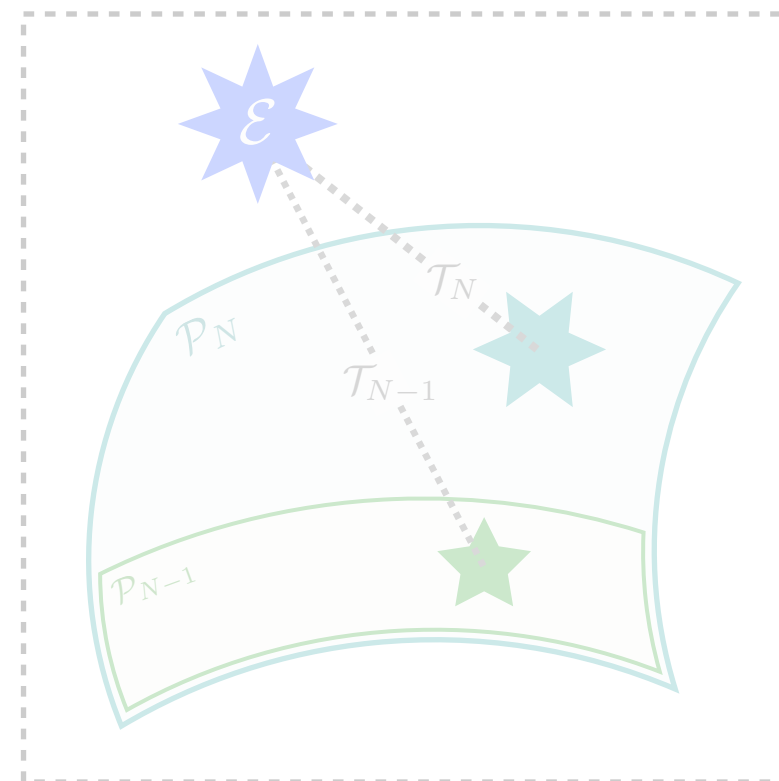
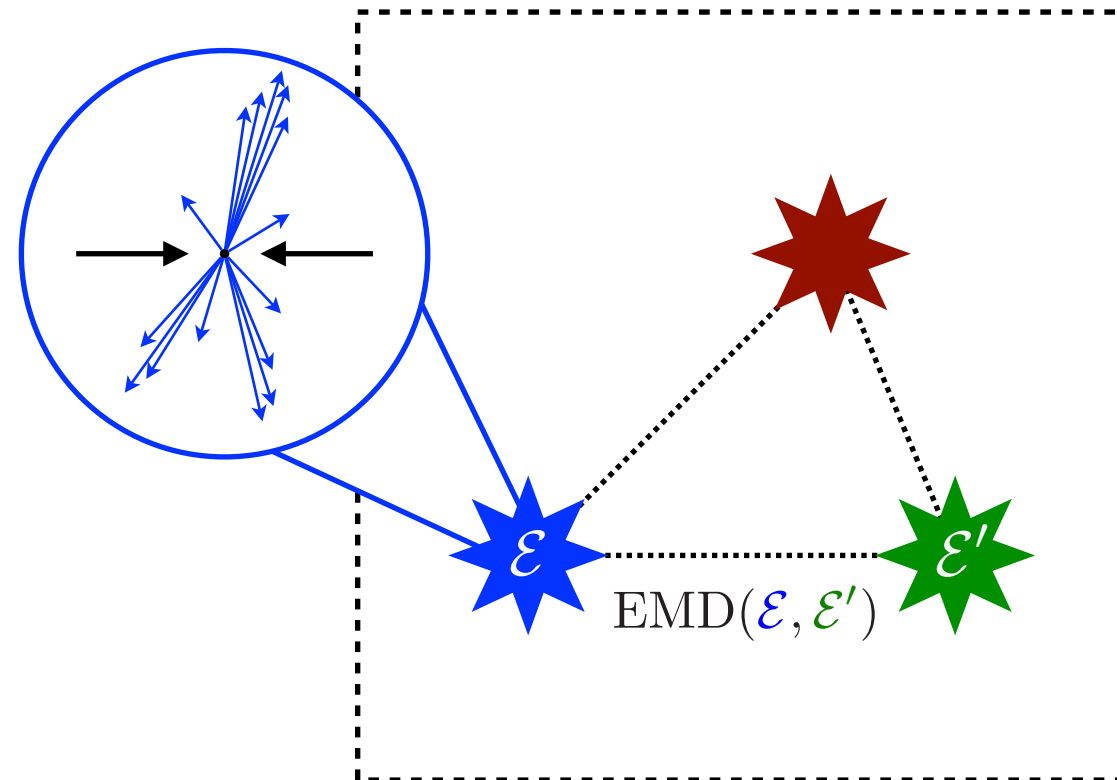
June 4, 2020



The (Metric) Space of Events

Revealing Hidden Geometry

Theory Space



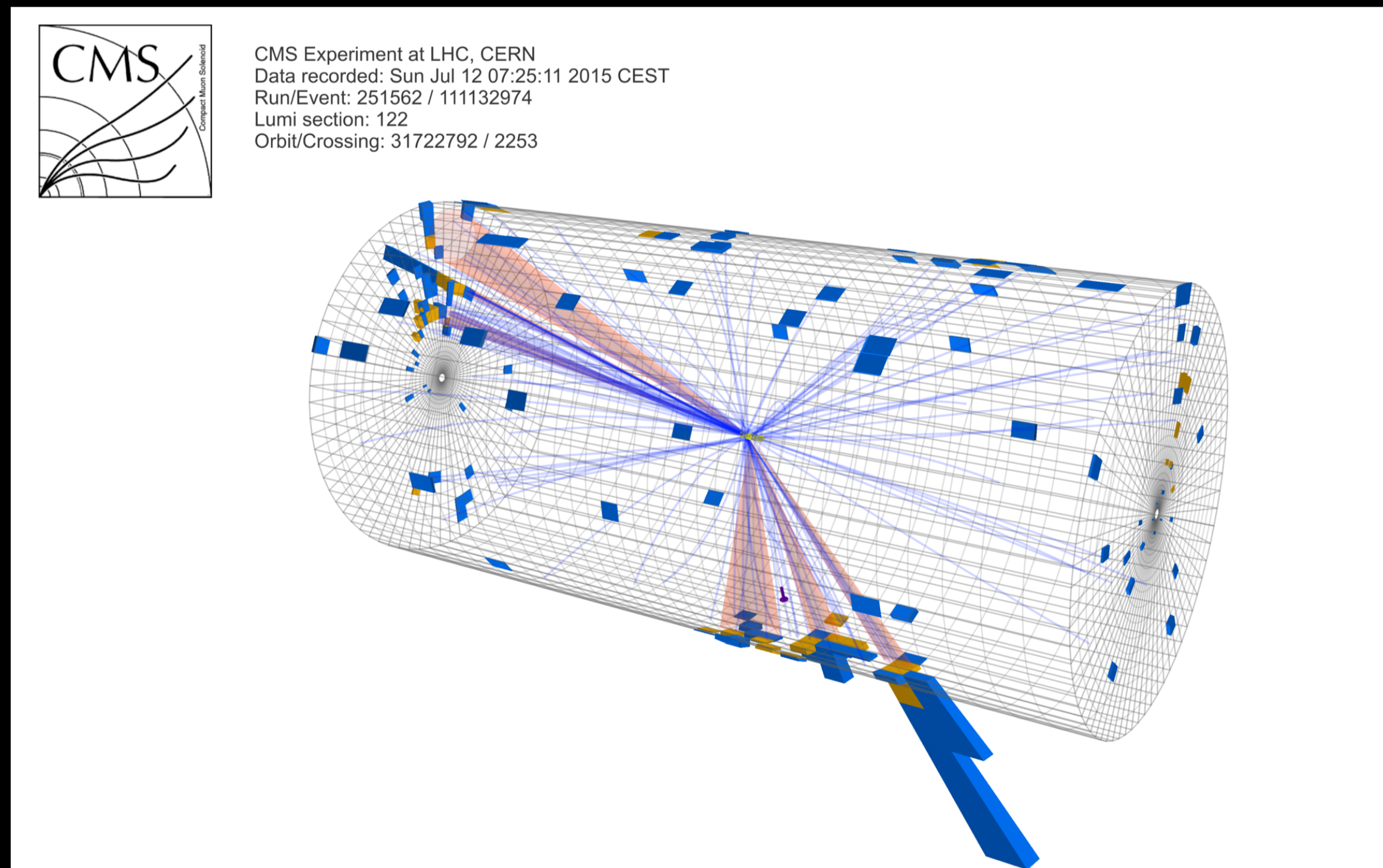
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Revealing Hidden Geometry

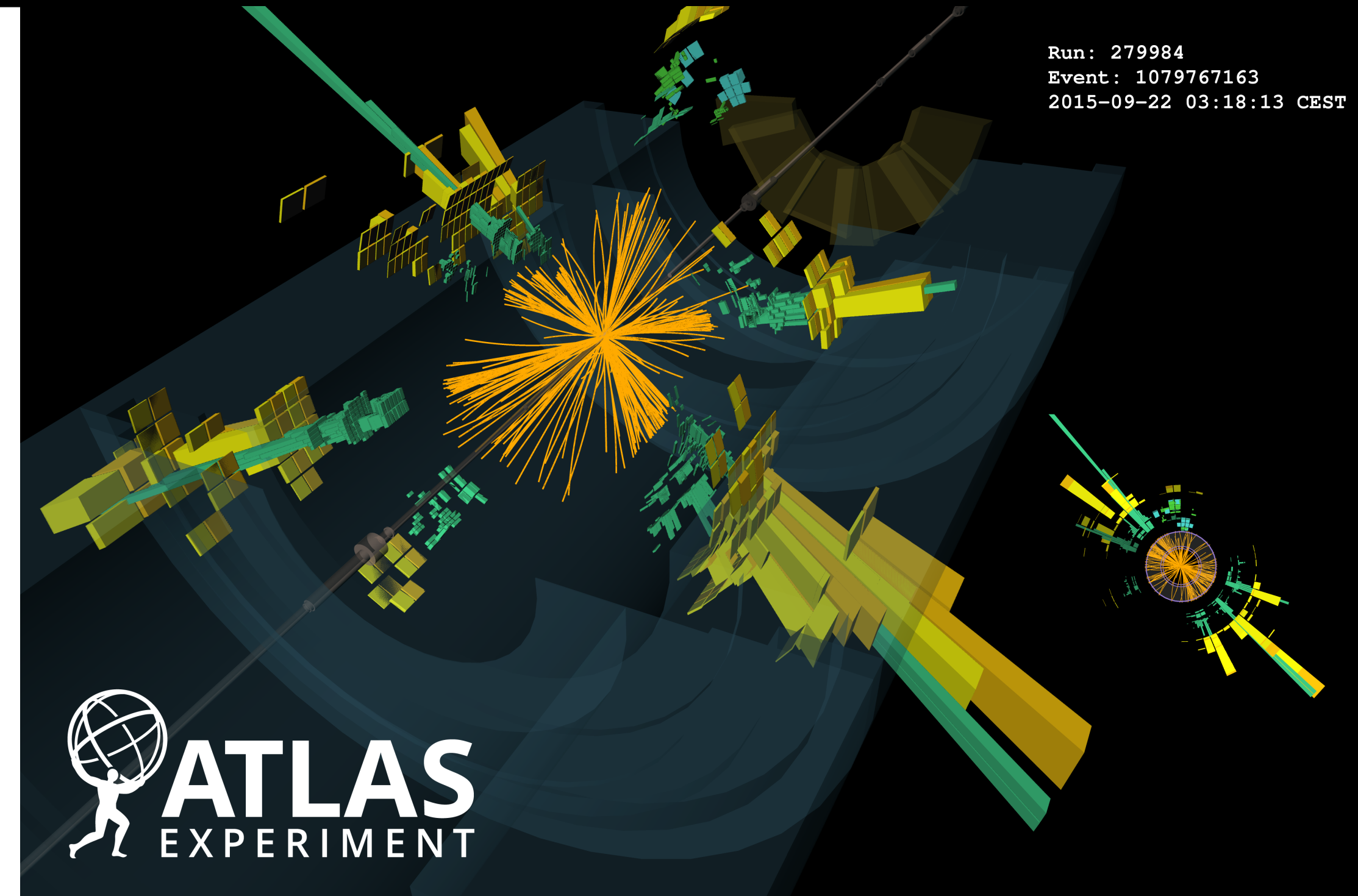
Theory Space

Explicit Geometry – Individual Events at the LHC

High-energy collisions produce final state particles with *energy*, *direction*, *charge*, *flavor*, and *other quantum numbers*



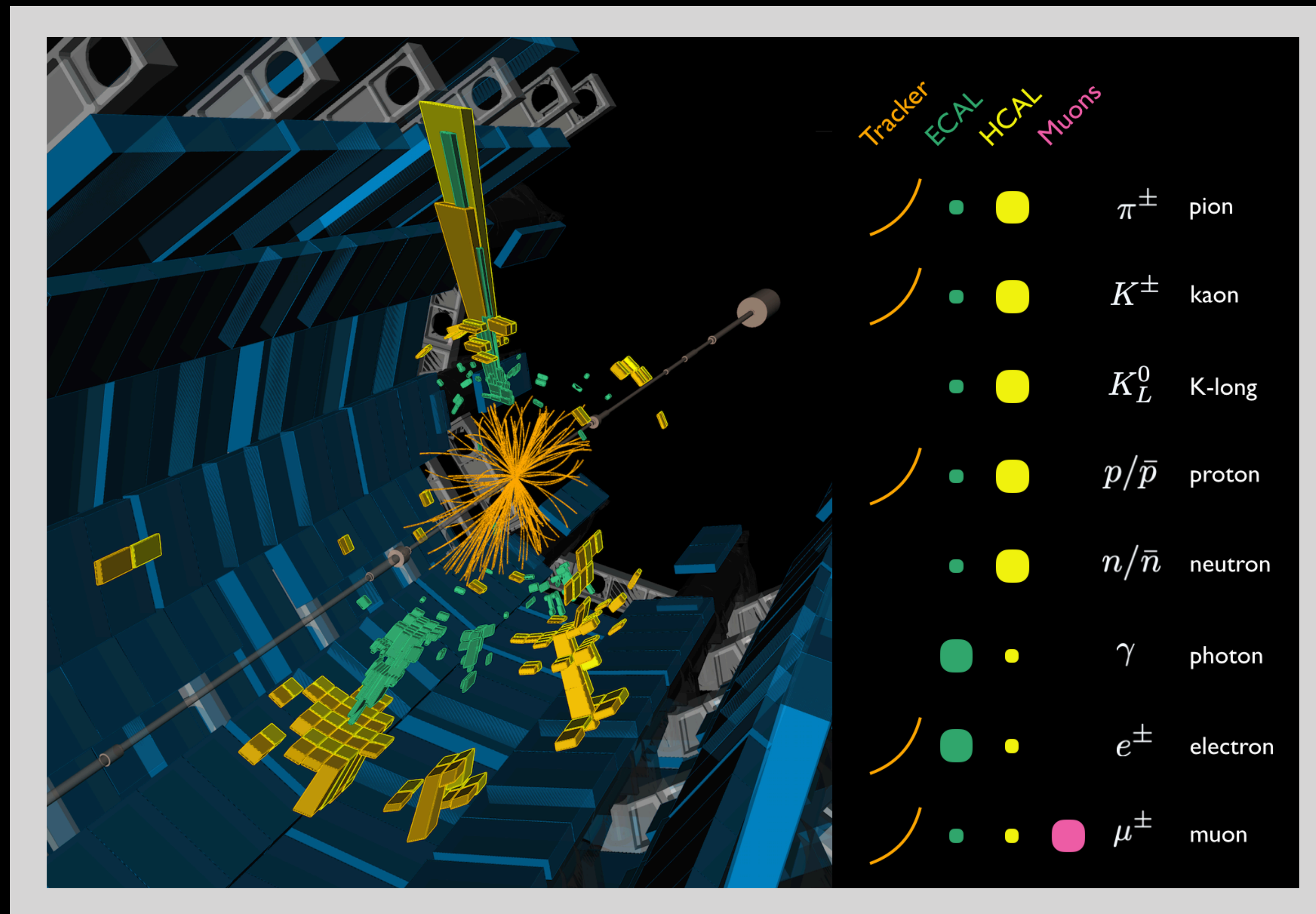
CMS hadronic $t\bar{t}$ event



ATLAS high jet-multiplicity event

Explicit Geometry – Individual Events at the LHC

High-energy collisions produce final state particles with *energy*, *direction*, *charge*, *flavor*, and *other quantum numbers*



Explicit Geometry – Individual Events in Theory

Hard collision

Good understanding via perturbation theory

Fragmentation

Semi-classical parton shower, effective field theory

Hadronization

Poorly understood (non-perturbative), modeled empirically

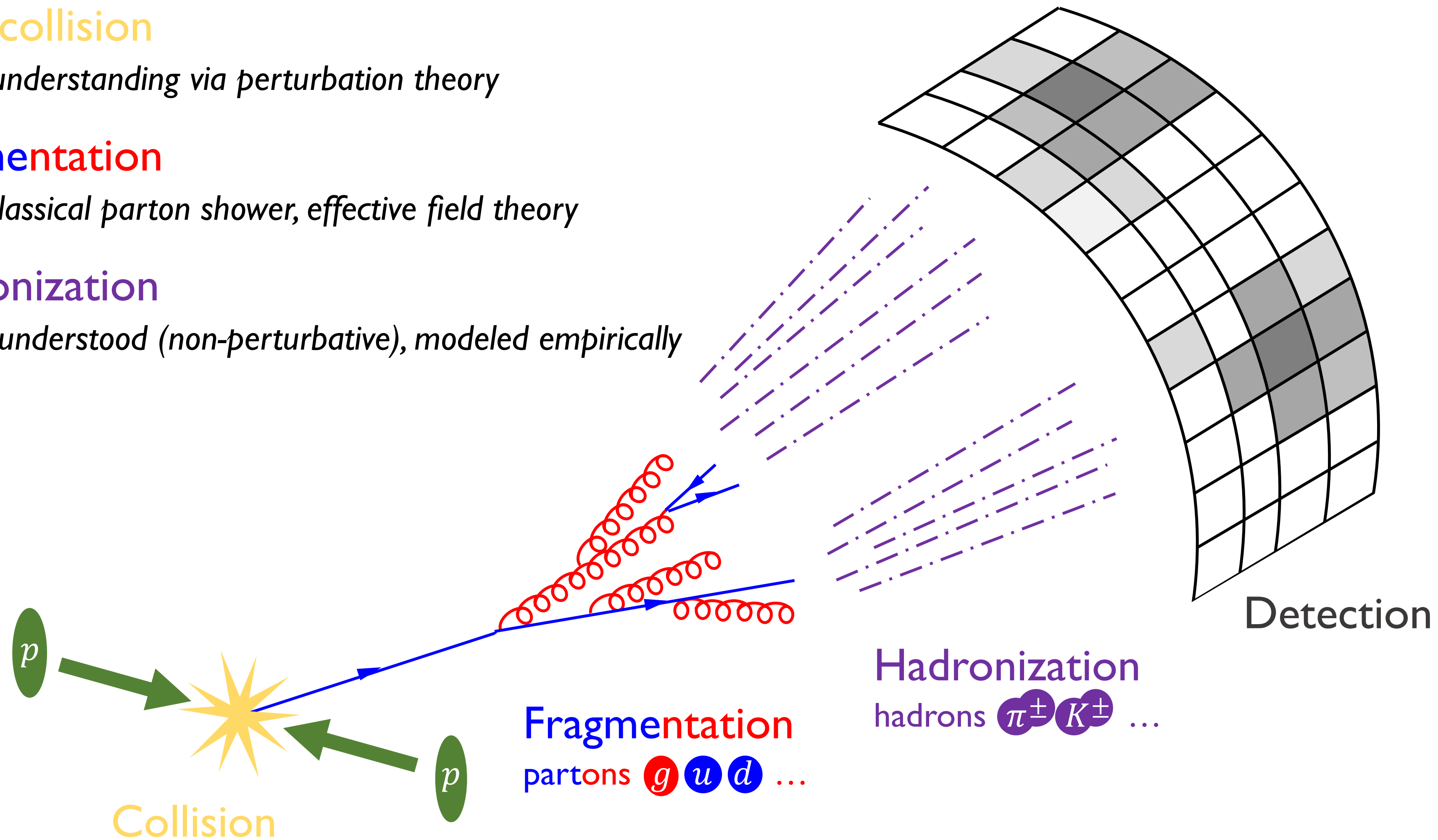


Diagram by Eric Metodiev

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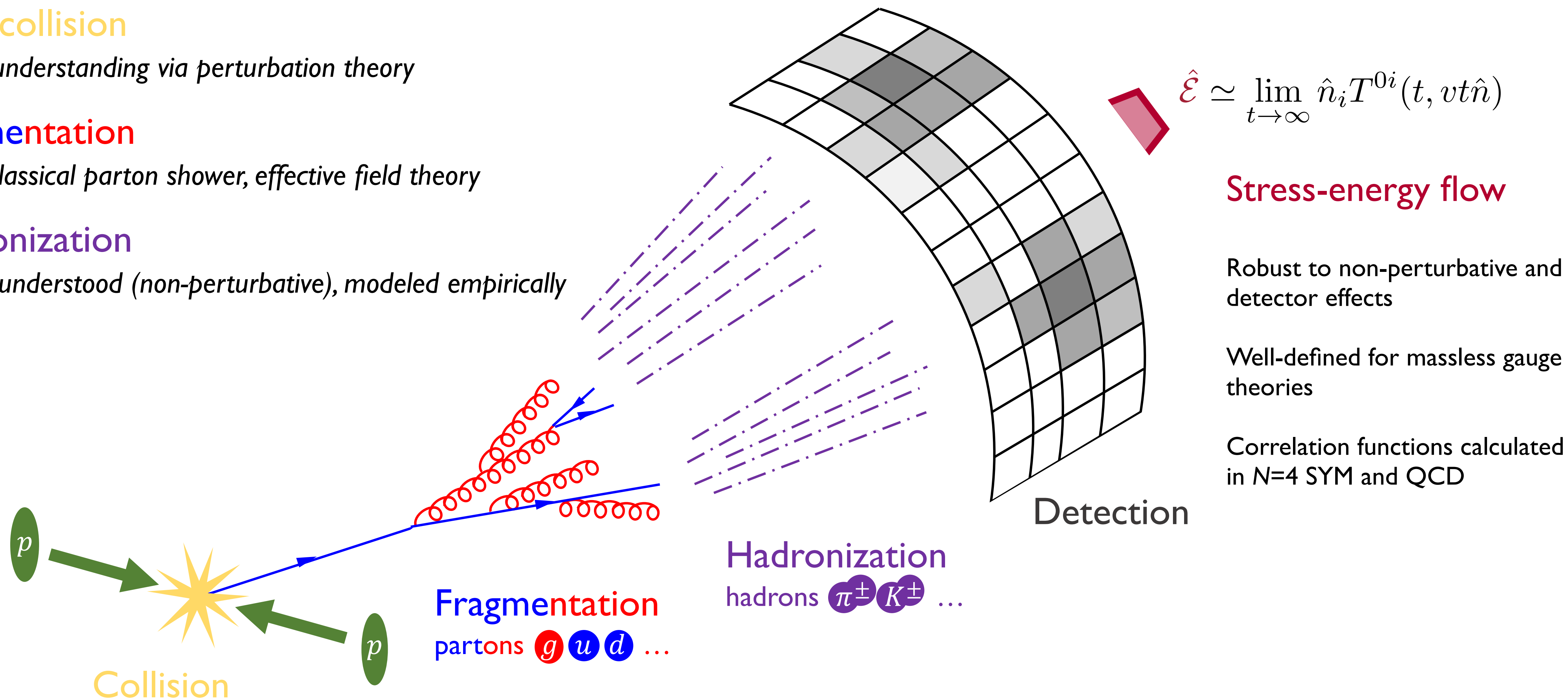
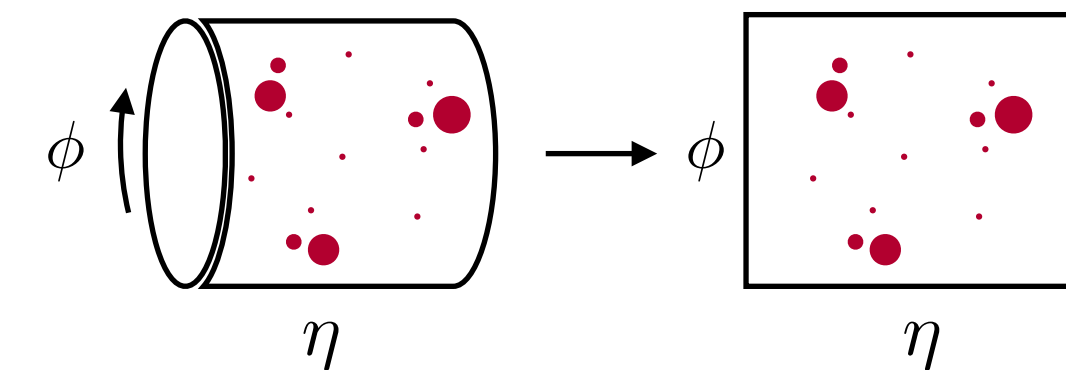


Diagram by Eric Metodiev

[Sveshnikov, Tkachov, [PLB 1996](#); Hofman, Maldacena, [JHEP 2008](#); Mateu, Stewart, Thaler, [PRD 2013](#); Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, [PRL 2014](#); Chen, Moul, Zhang, Zhu, [2004.11381](#); Dixon, PTK, Moul, Thaler, Zhu, *to appear soon*]

Explicit Geometry – Events as Distributions of Energy

[PTK, Metodiev, Thaler, [JHEP 2019](#); PTK, Metodiev, Thaler, [2004.04159](#)]



Energy flow distribution fully captures *IRC*-safe information

$$\mathcal{E}(\hat{n}) = \sum_{i=1}^M E_i \delta(\hat{n} - \hat{n}_i)$$

↑

Energy Flow
Distribution

↑

Energy
(p_T)

↑

Direction
(y, φ)

Explicit Geometry – Events as Distributions of Energy

[PTK, Metodiev, Thaler, [JHEP 2019](#); PTK, Metodiev, Thaler, [2004.04159](#)]

Energy flow distribution fully captures IRC-safe information

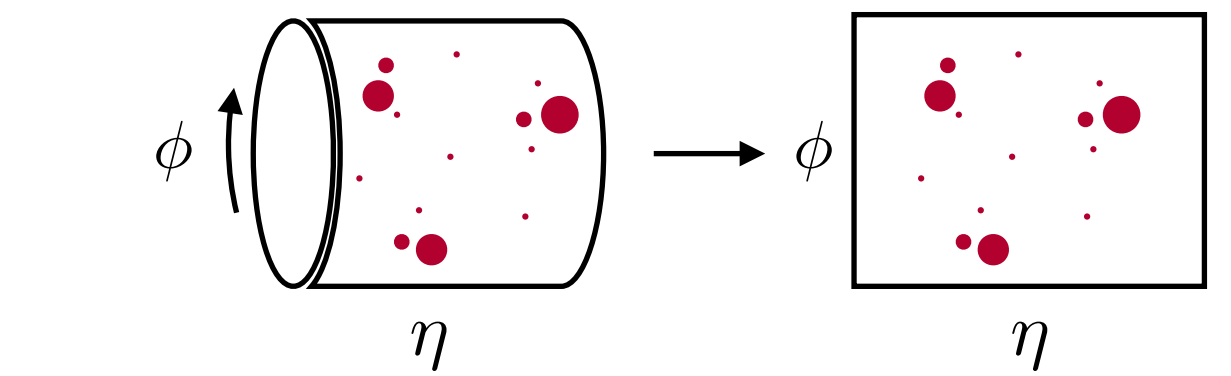
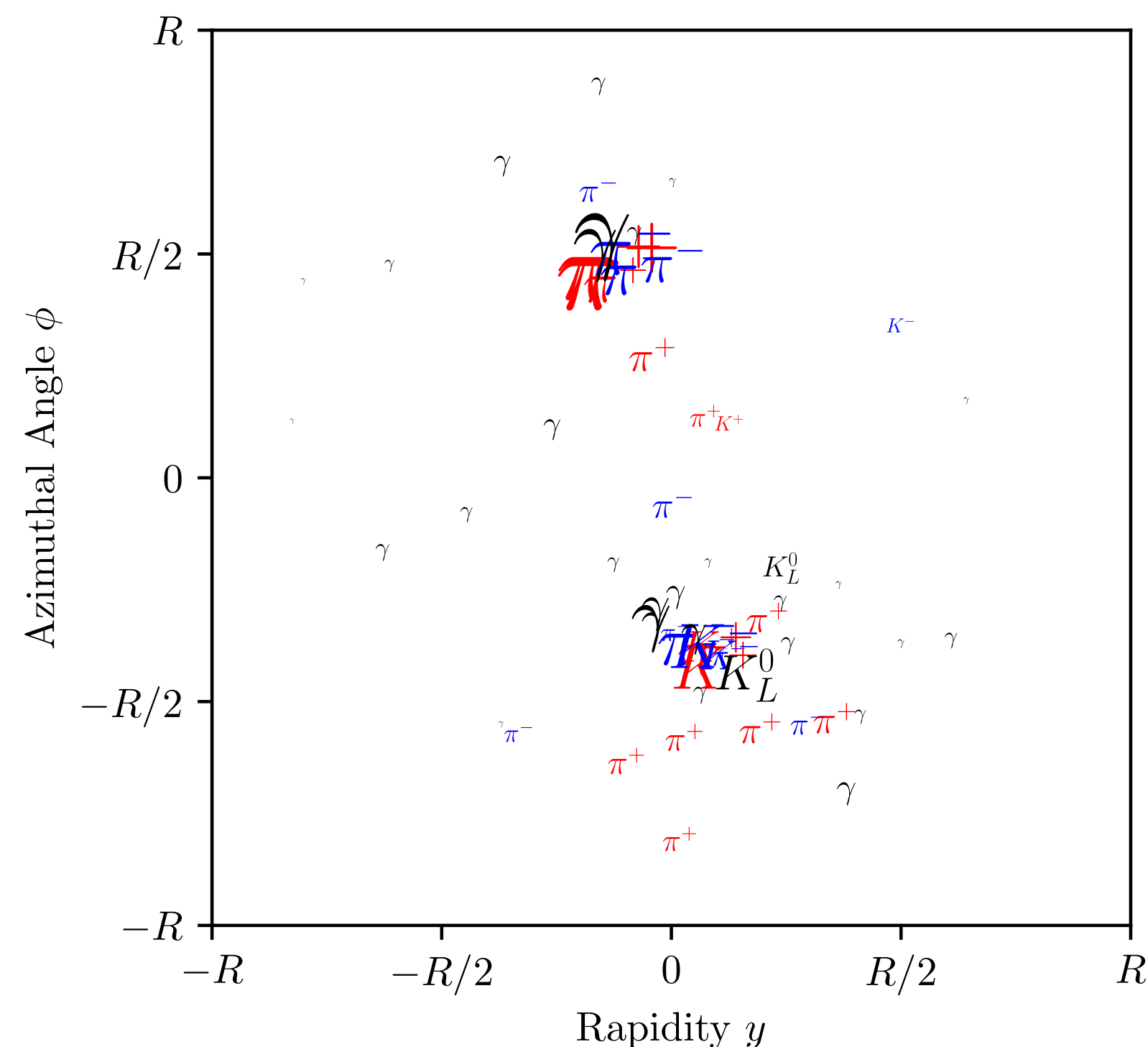
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\uparrow
 Energy Flow
Distribution

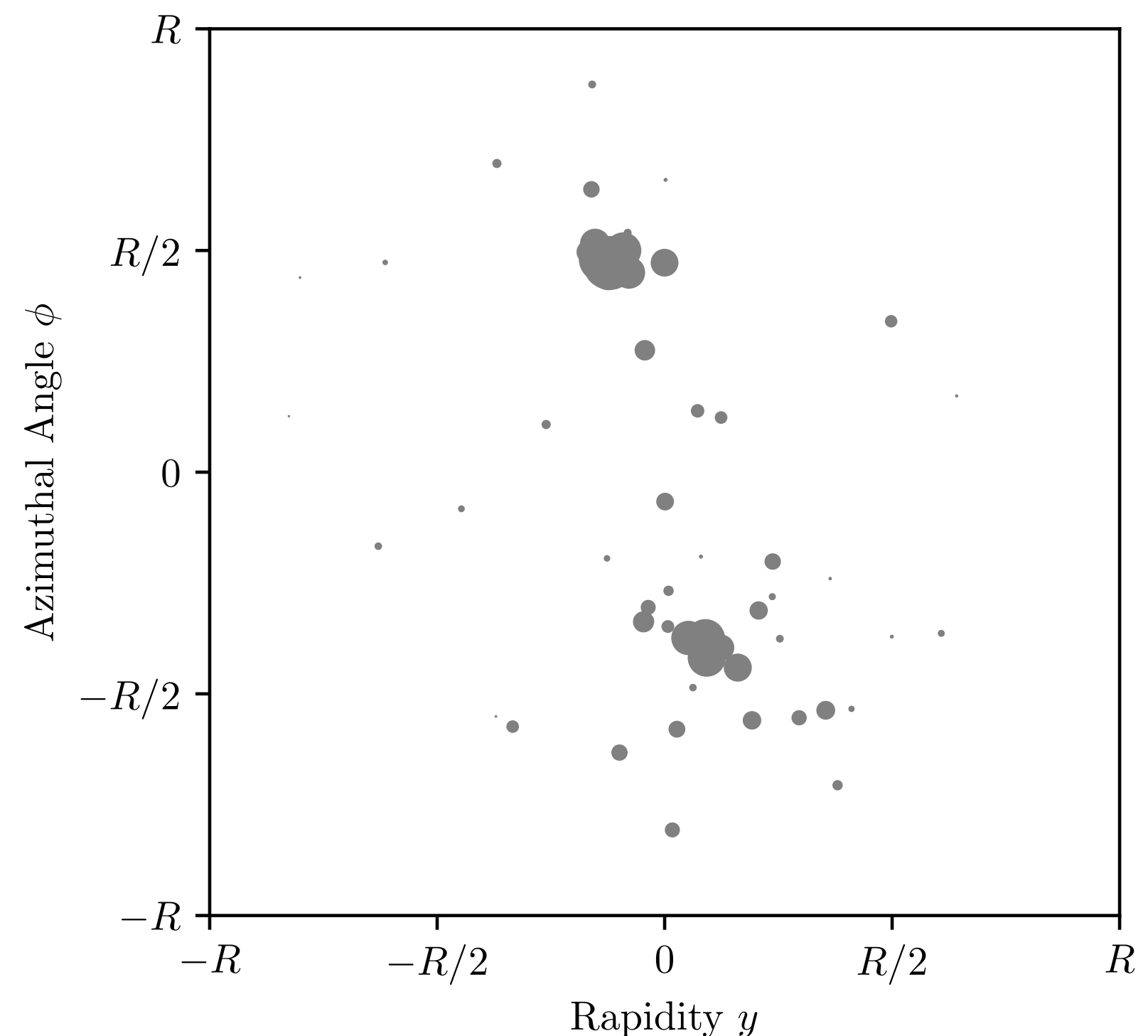
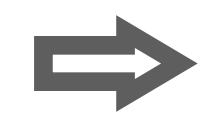
\uparrow
 Energy
(pT)

\uparrow
 Direction
(y, ϕ)

Full event is a set of particles having momentum and charge/flavor



The **energy** flow is unpixelized and ignores charge/flavor information



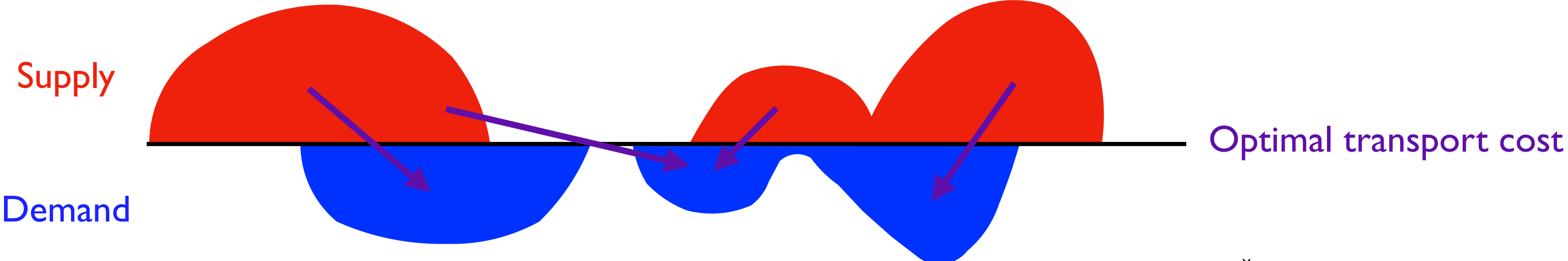
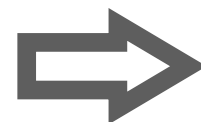
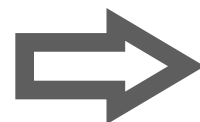
When are Two Distributions Similar?

Optimal transport minimizes the “work” (stuff x distance) required to transport supply to demand

[Monge, 1781; Vaseršteĭn, 1969; Peleg, Werman, Rom, IEEE 1989;
Rubner, Tomasi, Guibas, ICCV 1998, ICJV 2000; Pele, Werman, ECCV 2008; Pele, Taskar, GSI 2013]

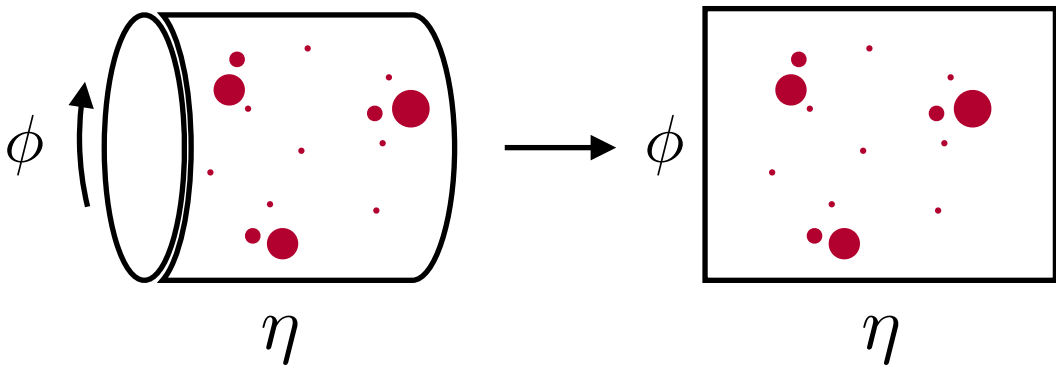
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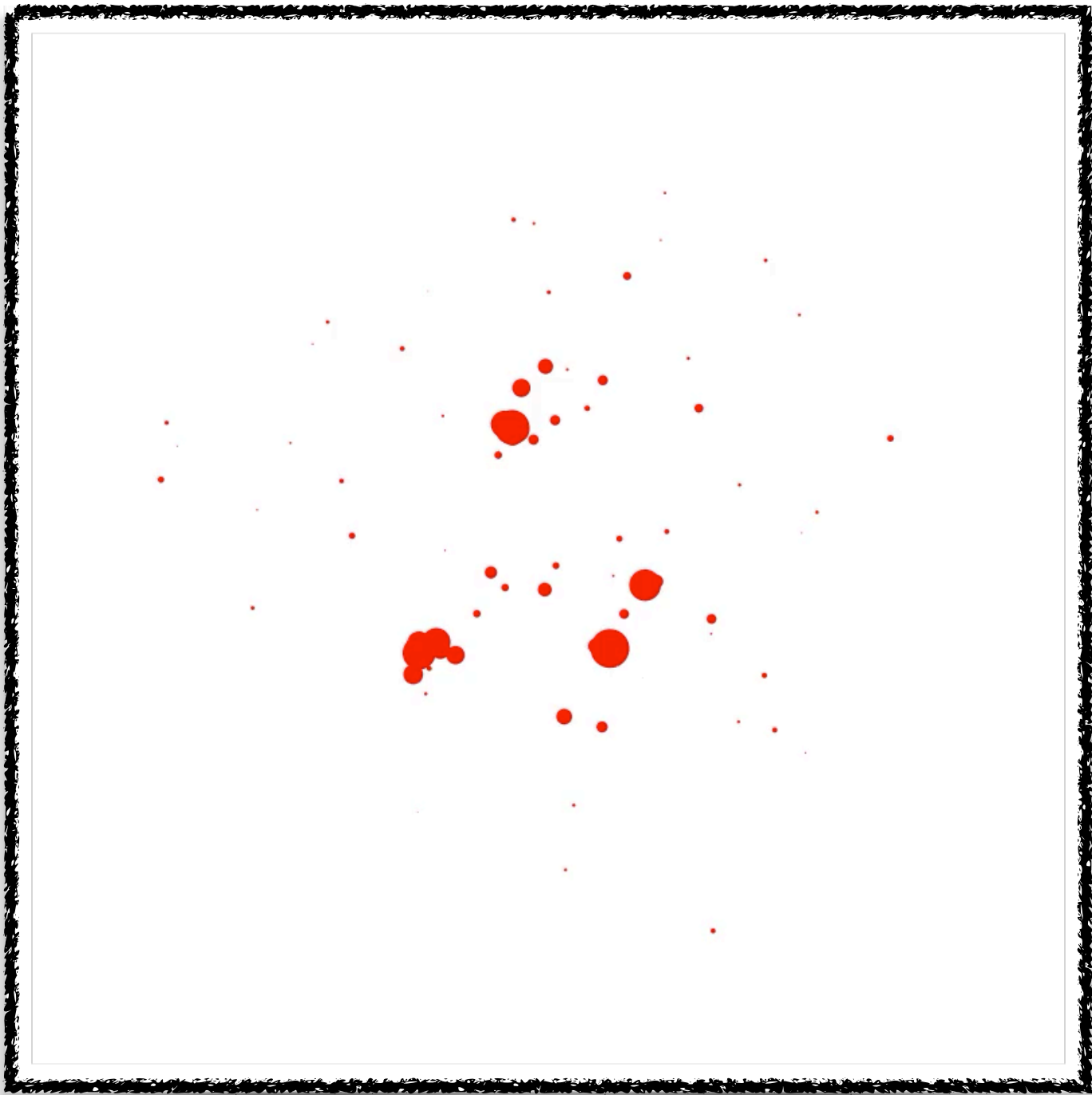
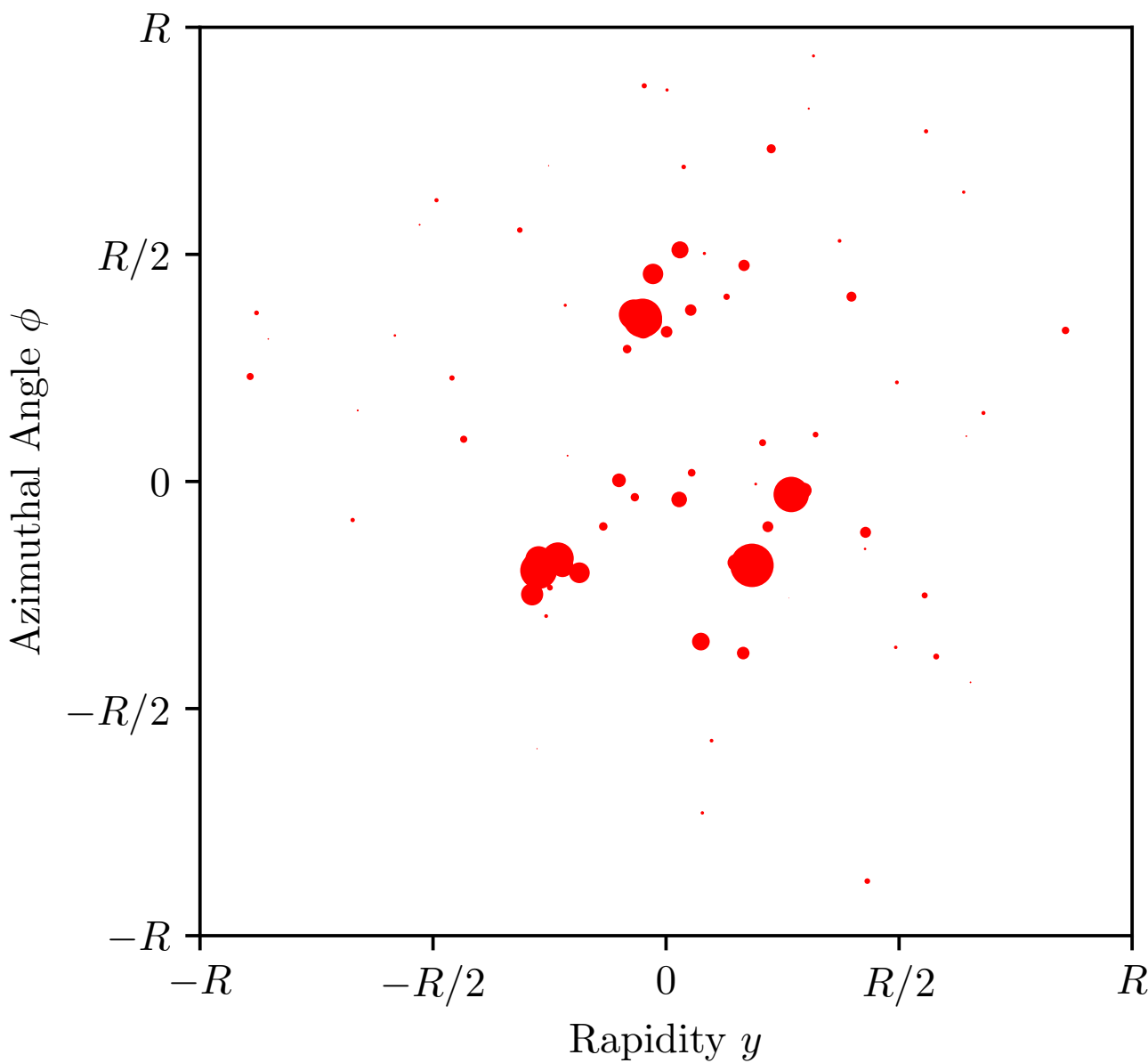
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Towards a Hidden Geometry – When are two events similar?

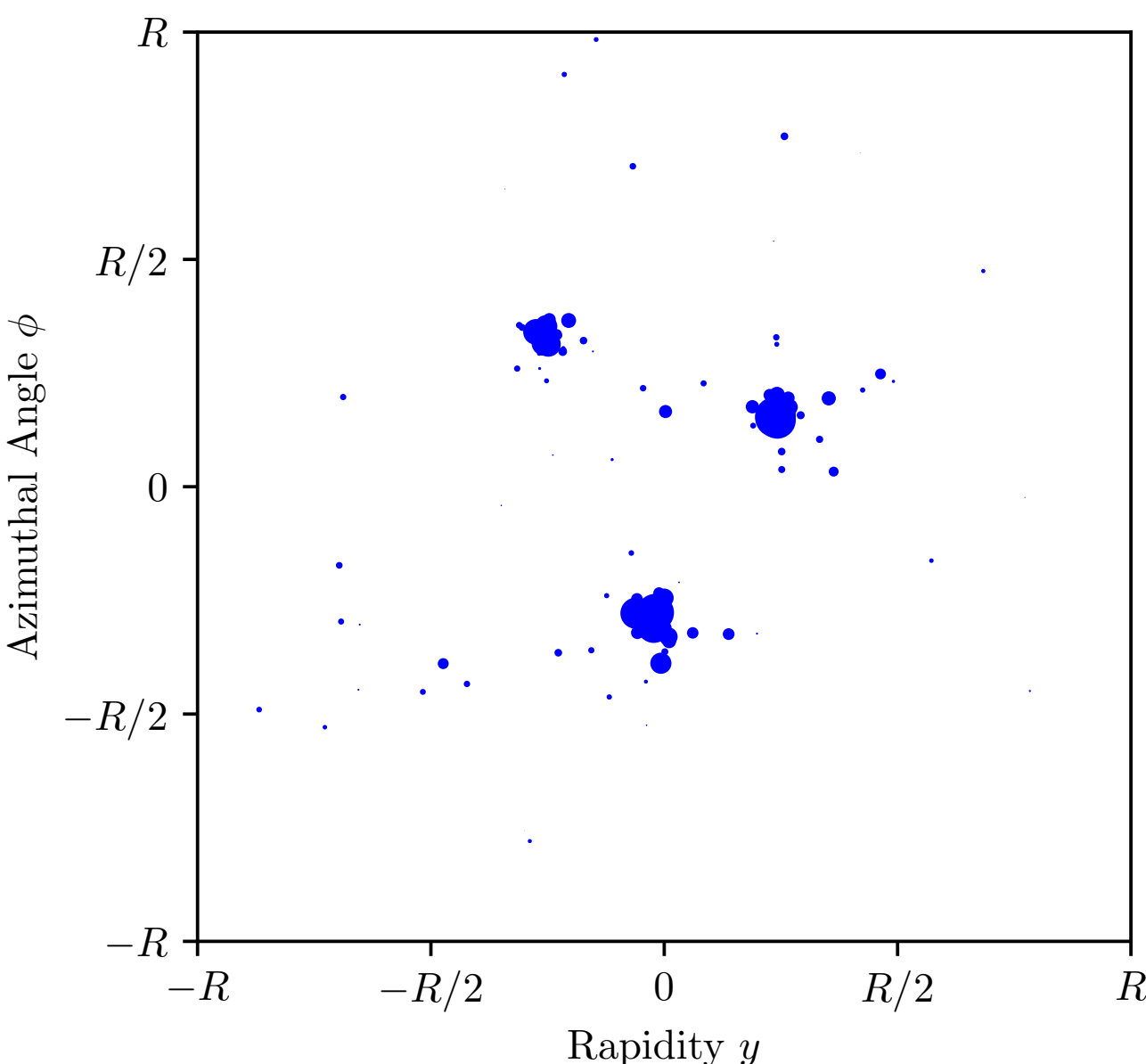


Optimal transport minimizes the “work” (stuff x distance) required to transport supply to demand

Top Jet 1



Top Jet 2



$$\mathcal{E}(\hat{n}) = \sum_{i=1}^M E_i \delta(\hat{n} - \hat{n}_i)$$

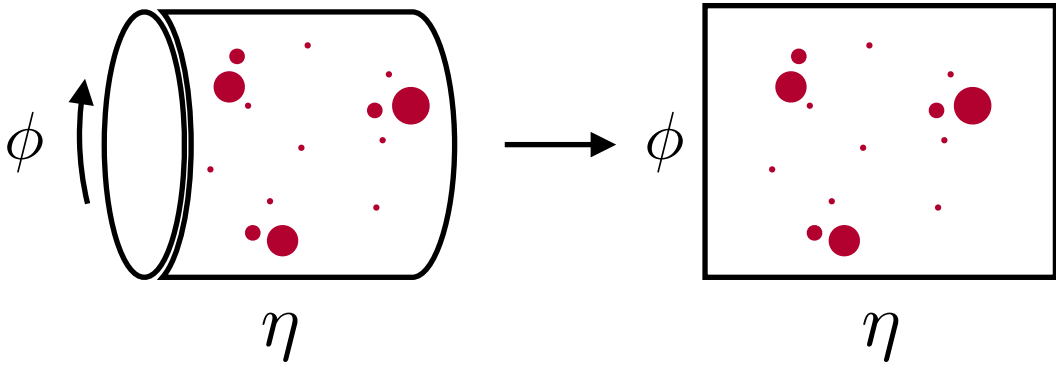
Provides a metric on normalized distributions in a space with a ground distance measure

↳ symmetric, non-negative, triangle inequality, zero iff identical

$$\theta_{ij} = \sqrt{\Delta y_{ij}^2 + \Delta \phi_{ij}^2}$$

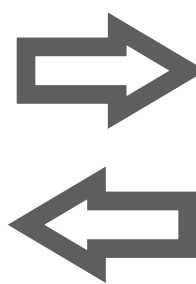
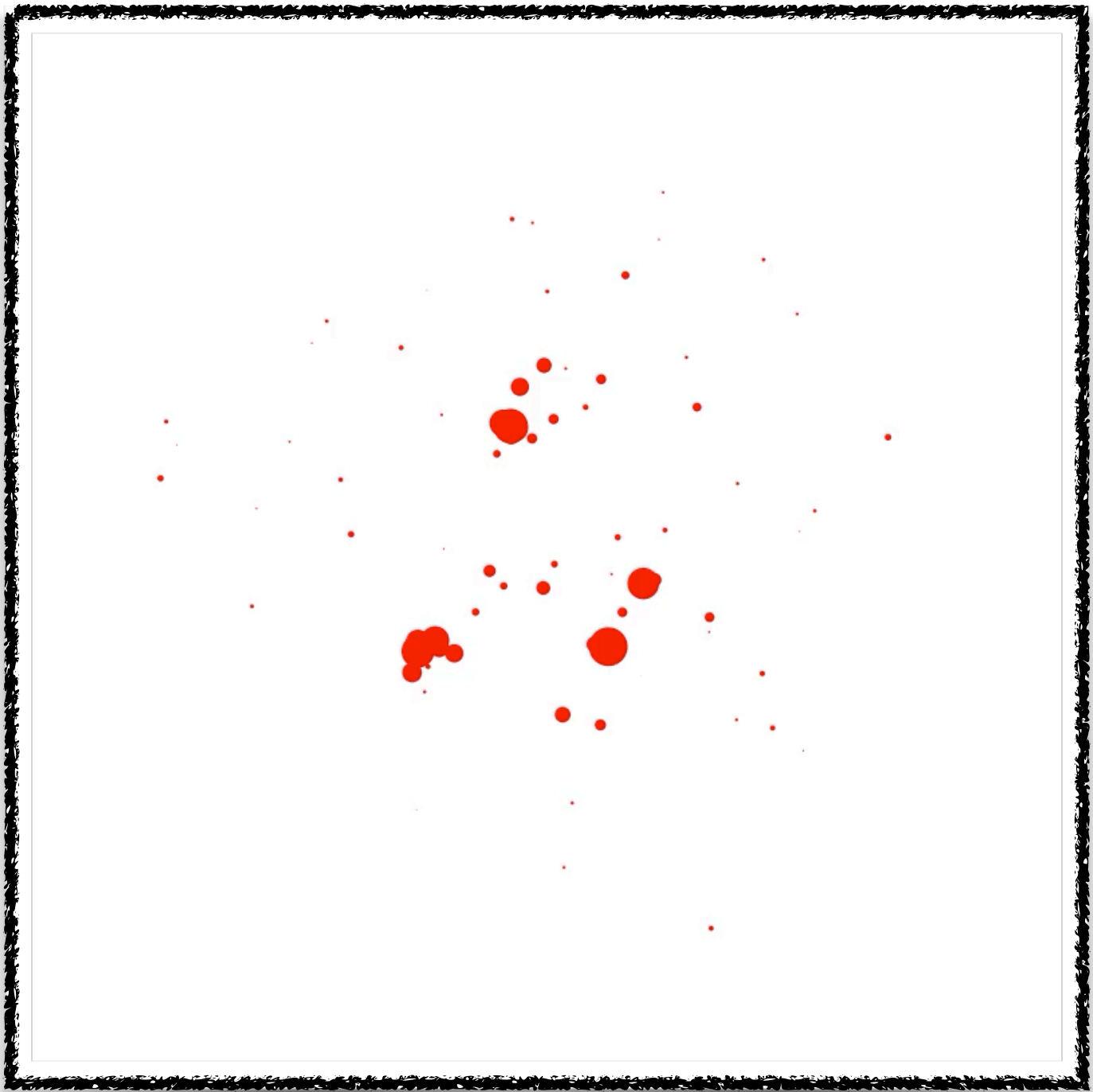
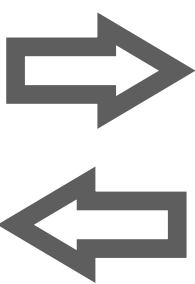
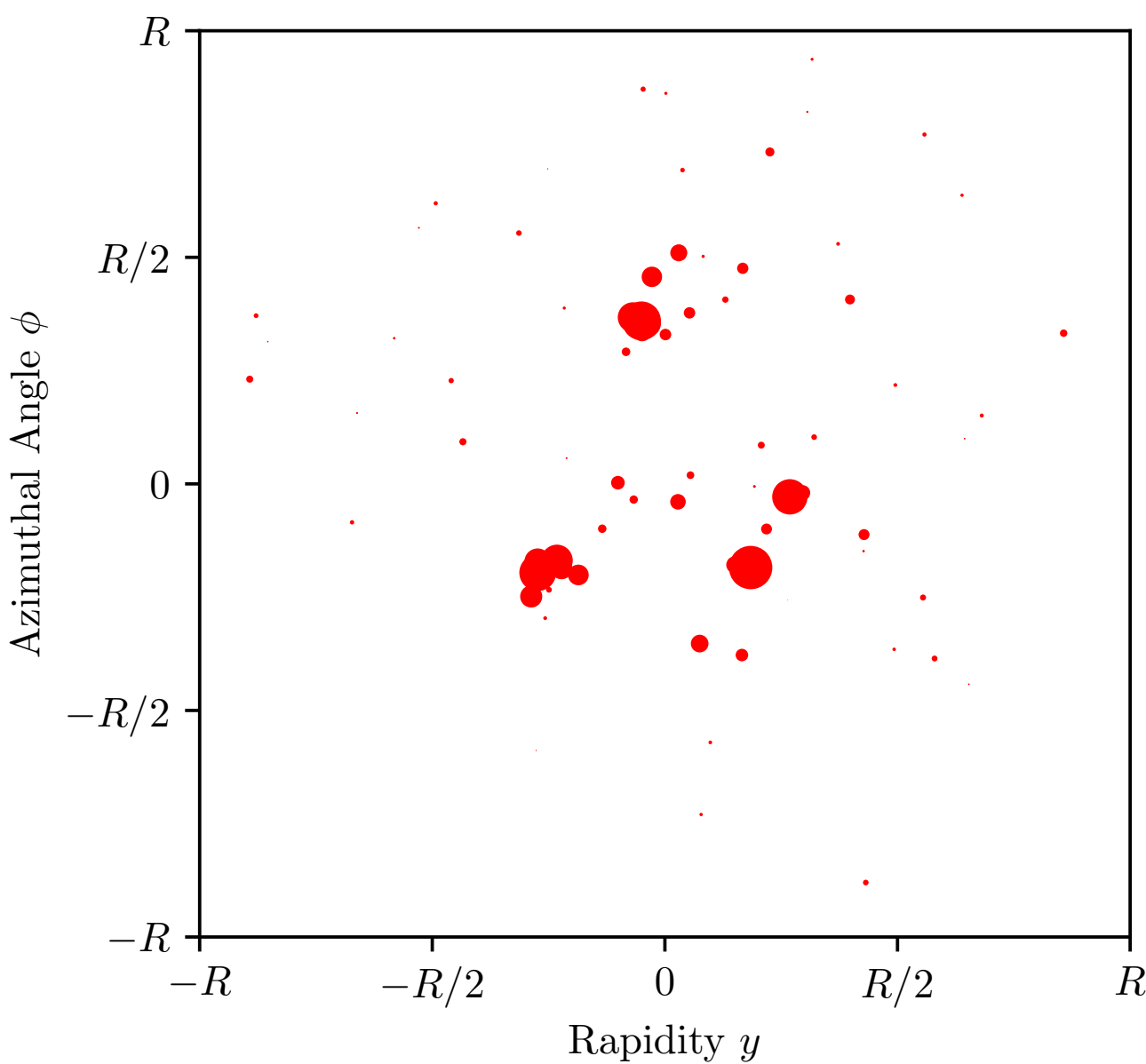
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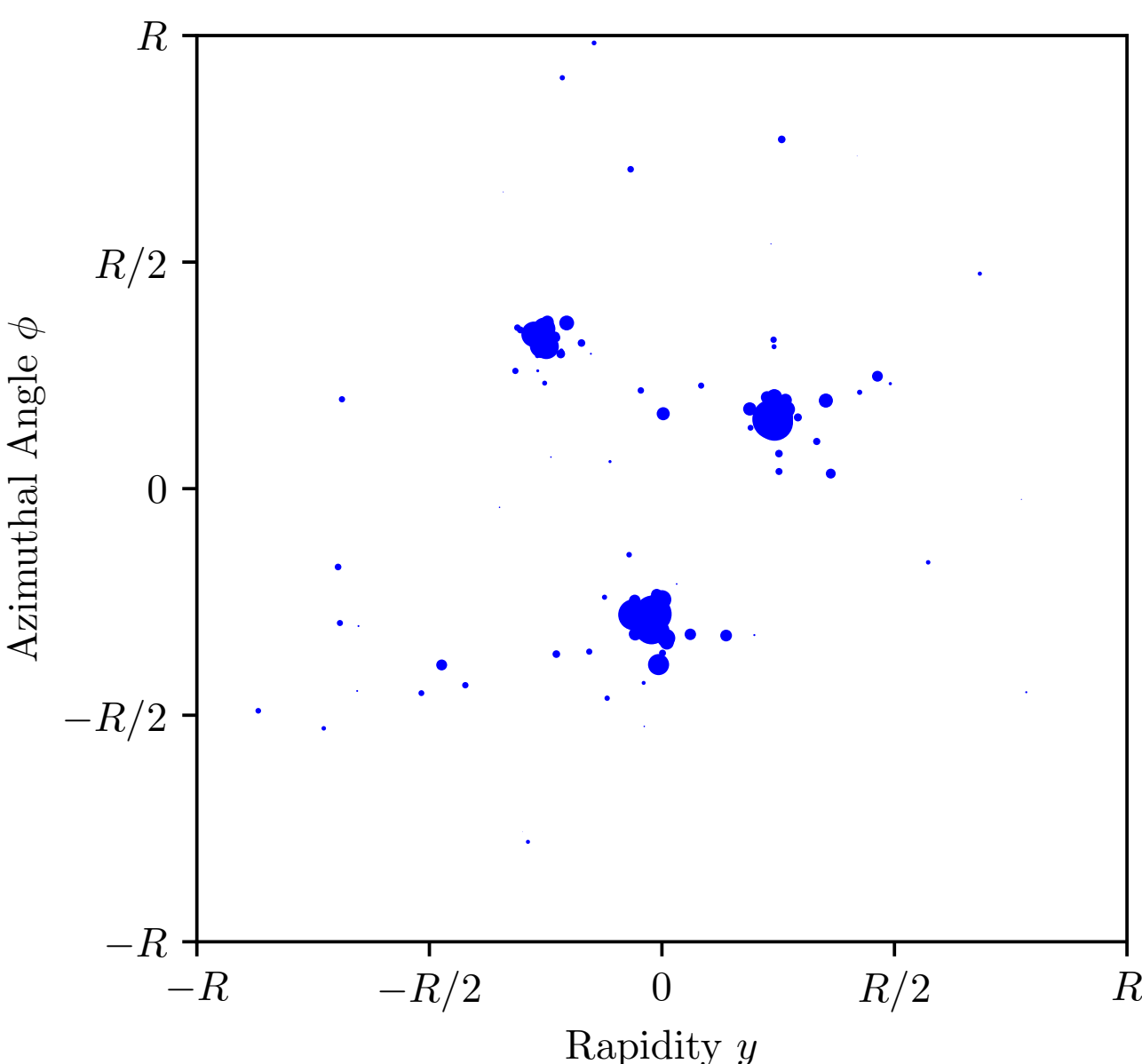


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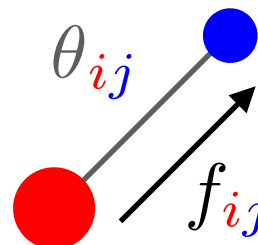
[Peleg, Werman, Rom, IEEE 1989; Rubner, Tomasi, Guibas, ICCV 1998, ICJV 2000; Pele, Werman, ECCV 2008; Pele, Taskar, GSI 2013]

The Energy Mover's Distance (EMD)

[PTK, Metodiev, Thaler, [PRL 2019](#)]

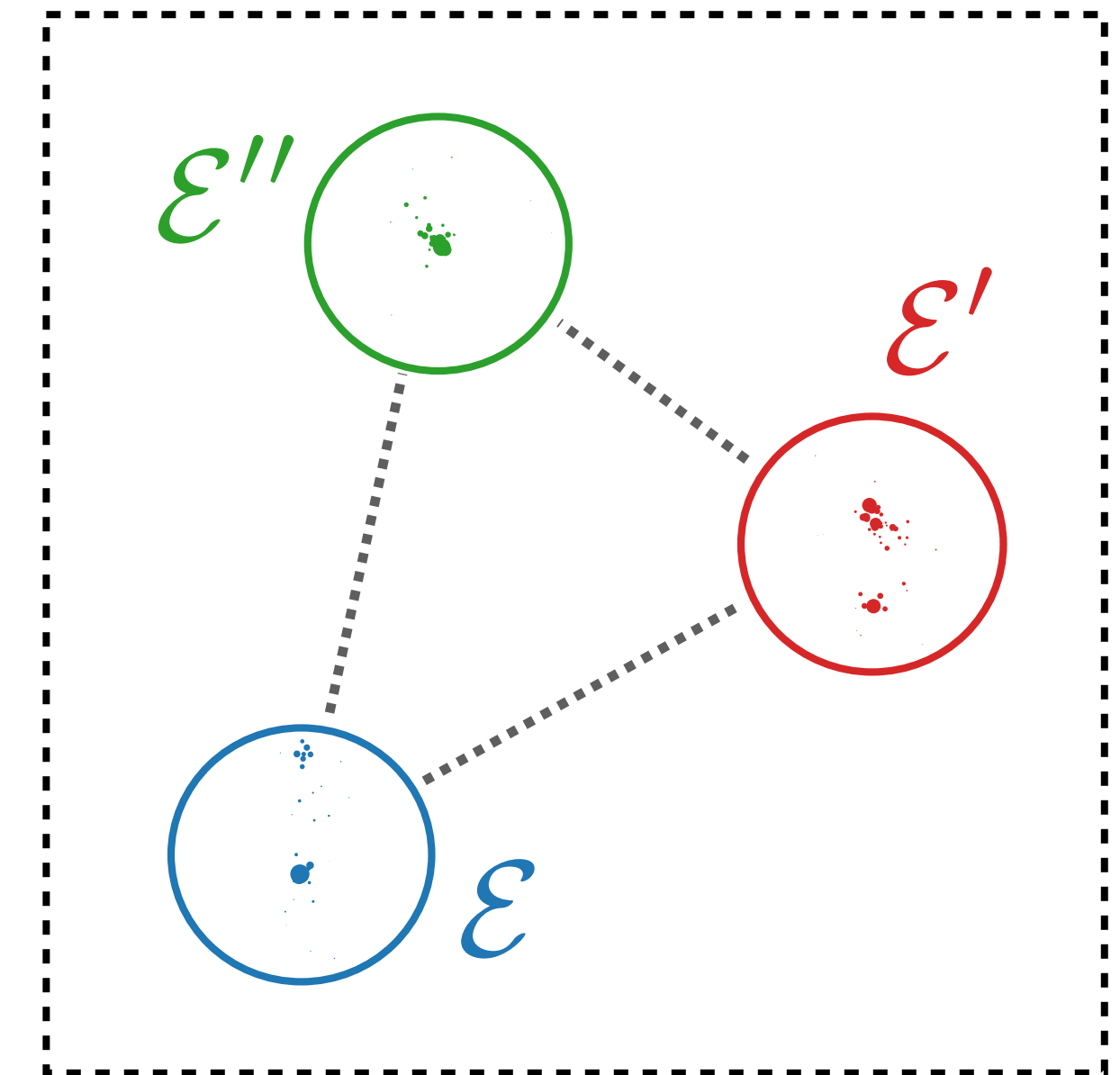
EMD between *energy* flows defines a *metric* on the space of events

$$\text{EMD}_{\beta,R}(\mathcal{E}, \mathcal{E}') = \underbrace{\min_{\{f_{ij} \geq 0\}} \sum_i \sum_j f_{ij} \left(\frac{\theta_{ij}}{R} \right)^\beta}_{\text{Cost of optimal transport}} + \underbrace{\left| \sum_i E_i - \sum_j E'_j \right|}_{\text{Cost of energy creation}}$$

$$\underbrace{\sum_j f_{ij} \leq E_i, \quad \sum_i f_{ij} \leq E'_j, \quad \sum_{ij} f_{ij} = \min \left(\sum_i E_i, \sum_j E'_j \right)}_{\text{Capacity constraints to ensure proper transport}}$$


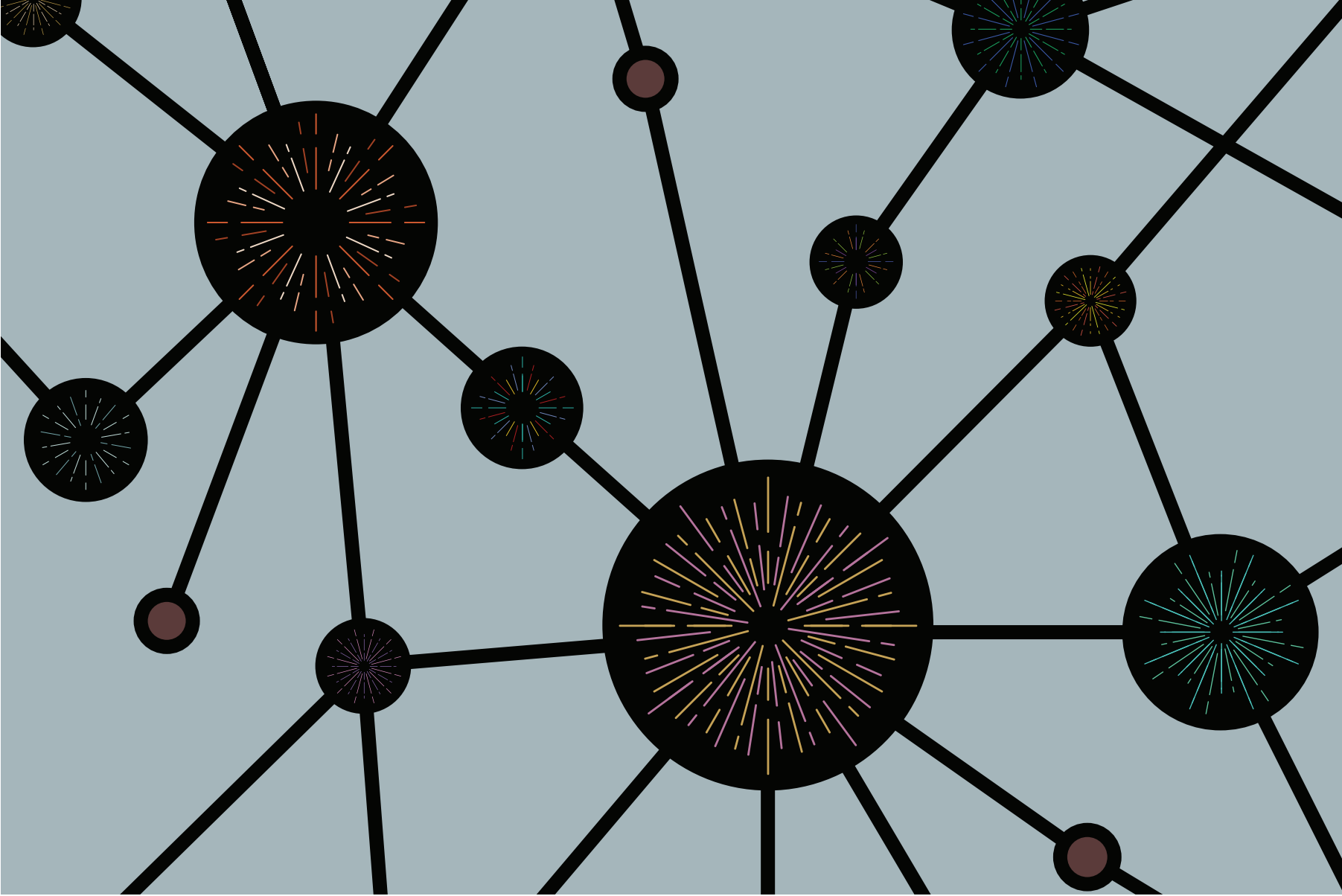
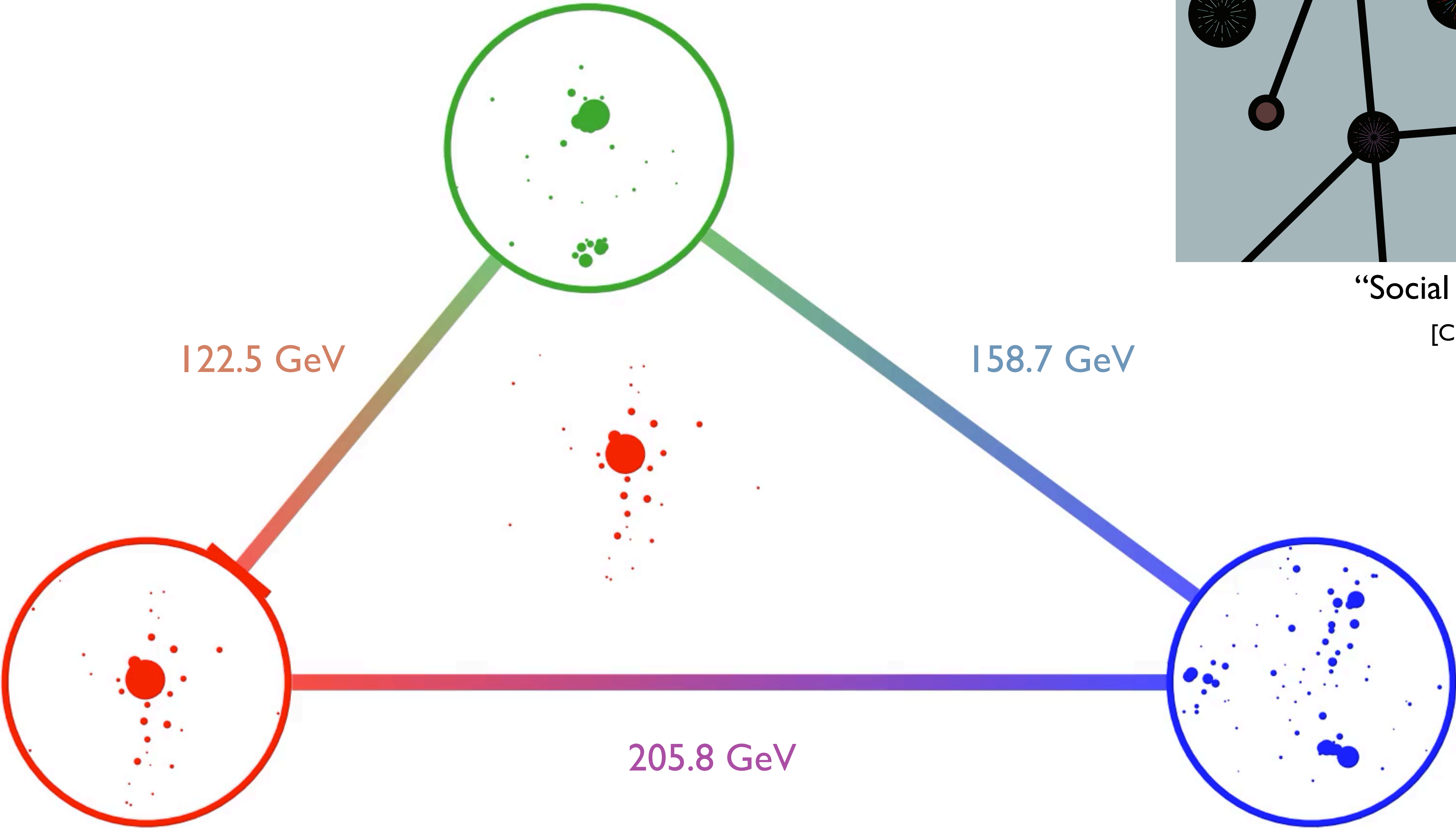
R : controls cost of transporting energy vs. destroying/creating it

β : angular weighting exponent



Triangle inequality satisfied for $R \geq d_{\max}/2$
 $0 \leq \text{EMD}(\mathcal{E}, \mathcal{E}') \leq \text{EMD}(\mathcal{E}, \mathcal{E}'') + \text{EMD}(\mathcal{E}'', \mathcal{E}')$
 i.e. $R \geq$ jet radius for conical jets

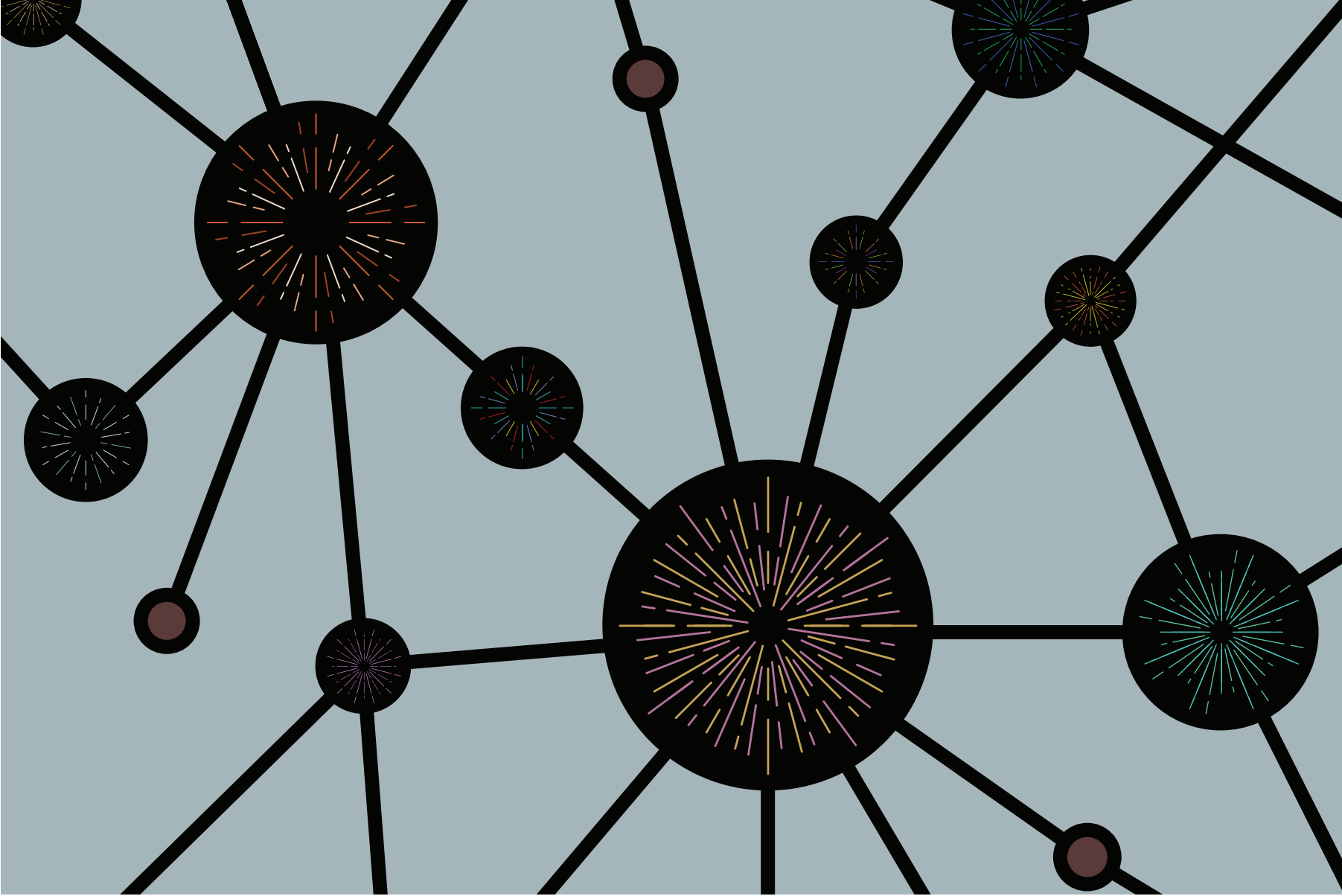
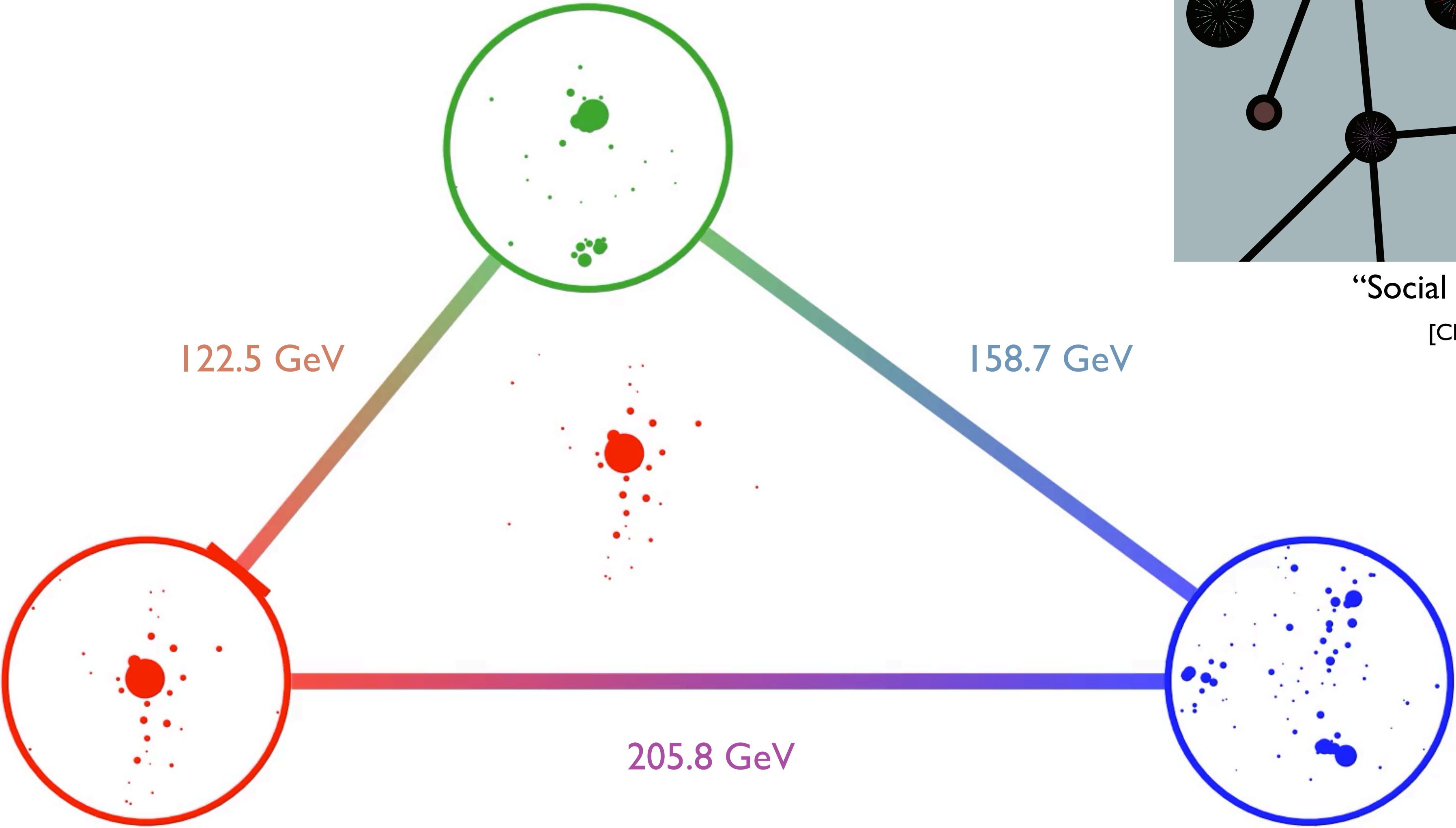
Geodesics in the Space of Events



“Social networking of jets”

[Chu, [MIT News](#) 2019]

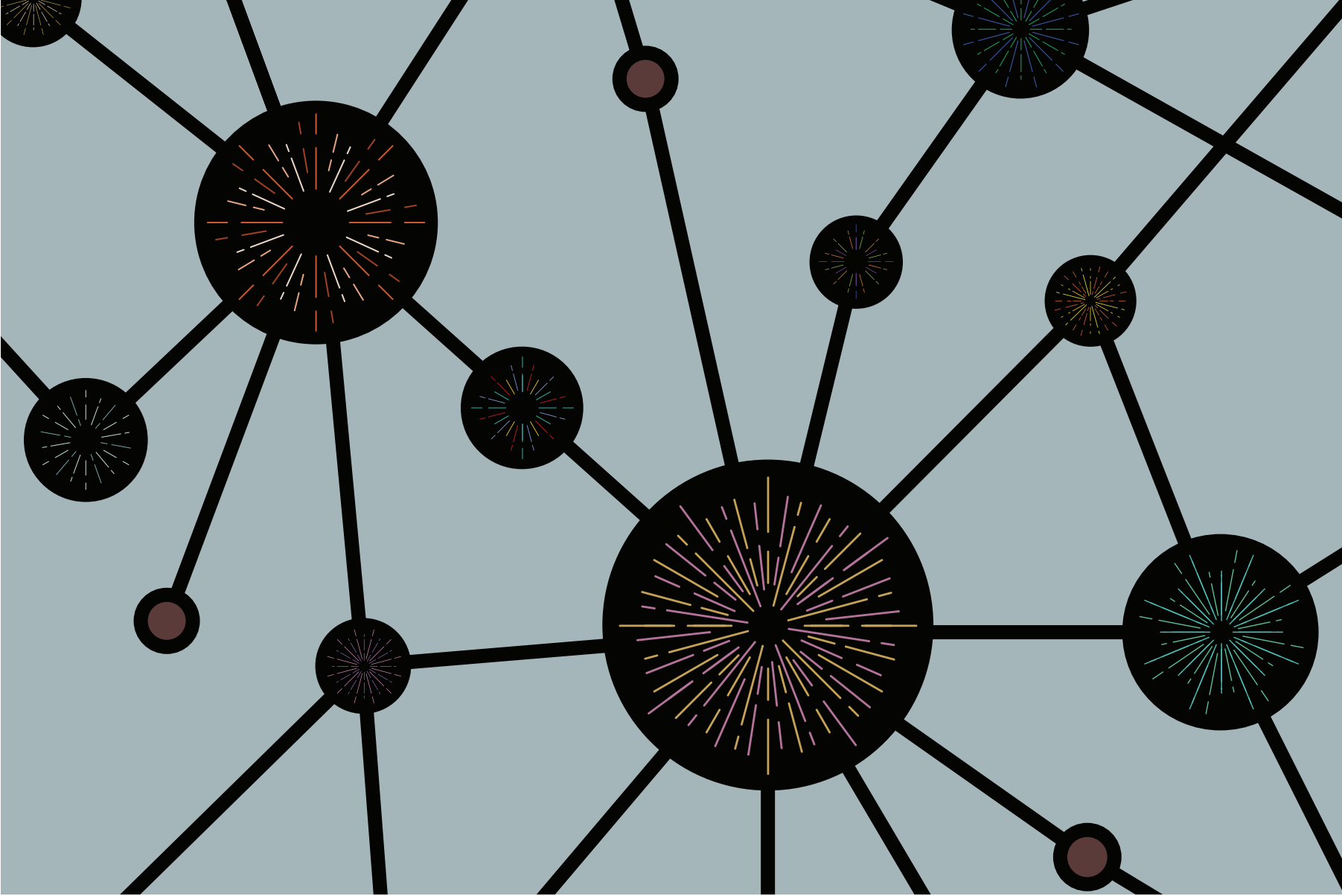
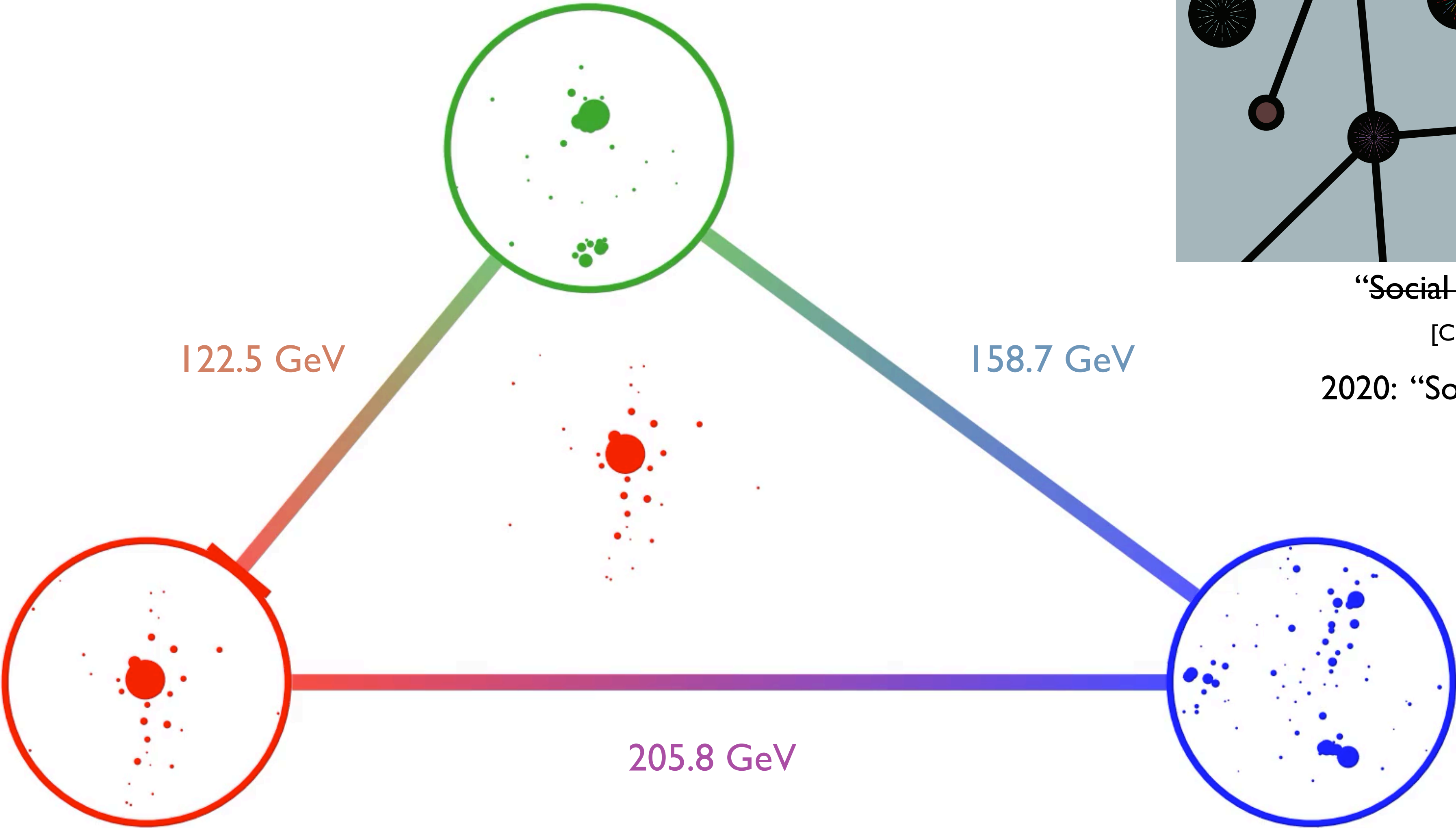
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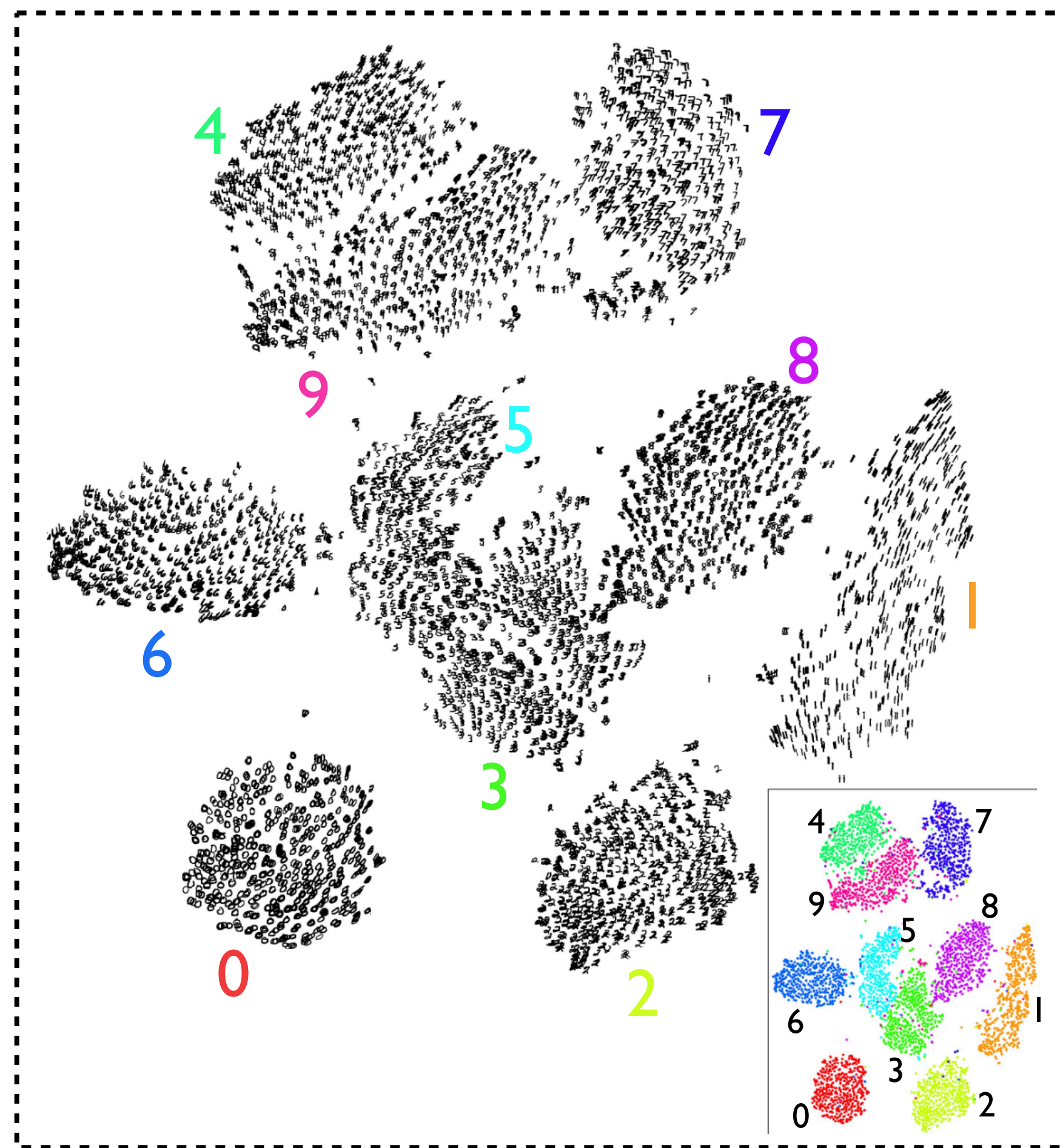
[Chu, MIT News 2019]

2020: “Social distancing of jets”

Visualizing Geometry in the Space of Events

t-Distributed Stochastic Neighbor Embedding (t-SNE)

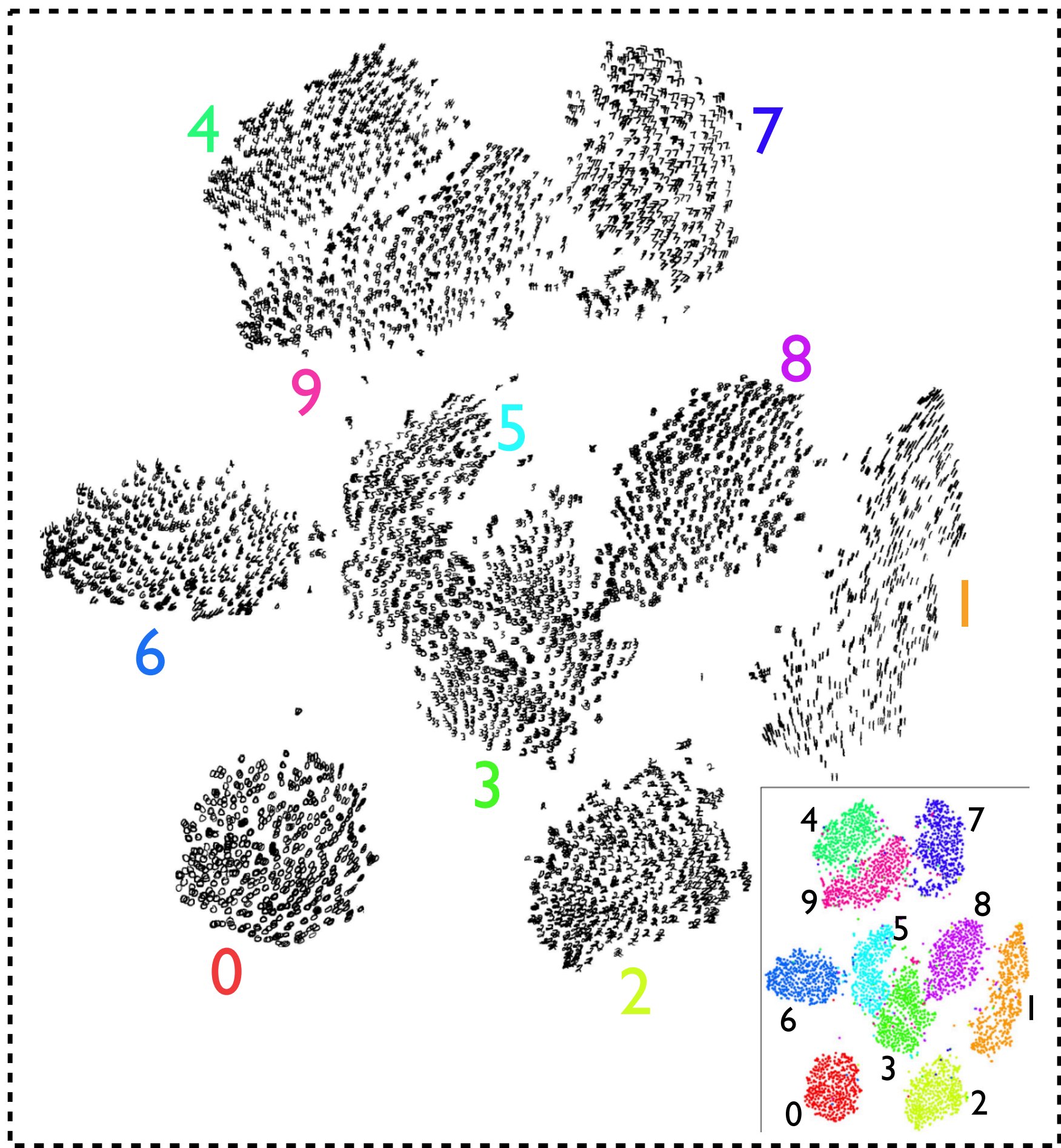
MNIST handwritten digits



[L. van der Maaten, G. Hinton, JMLR 2008]

Visualizing Geometry in the Space of Events

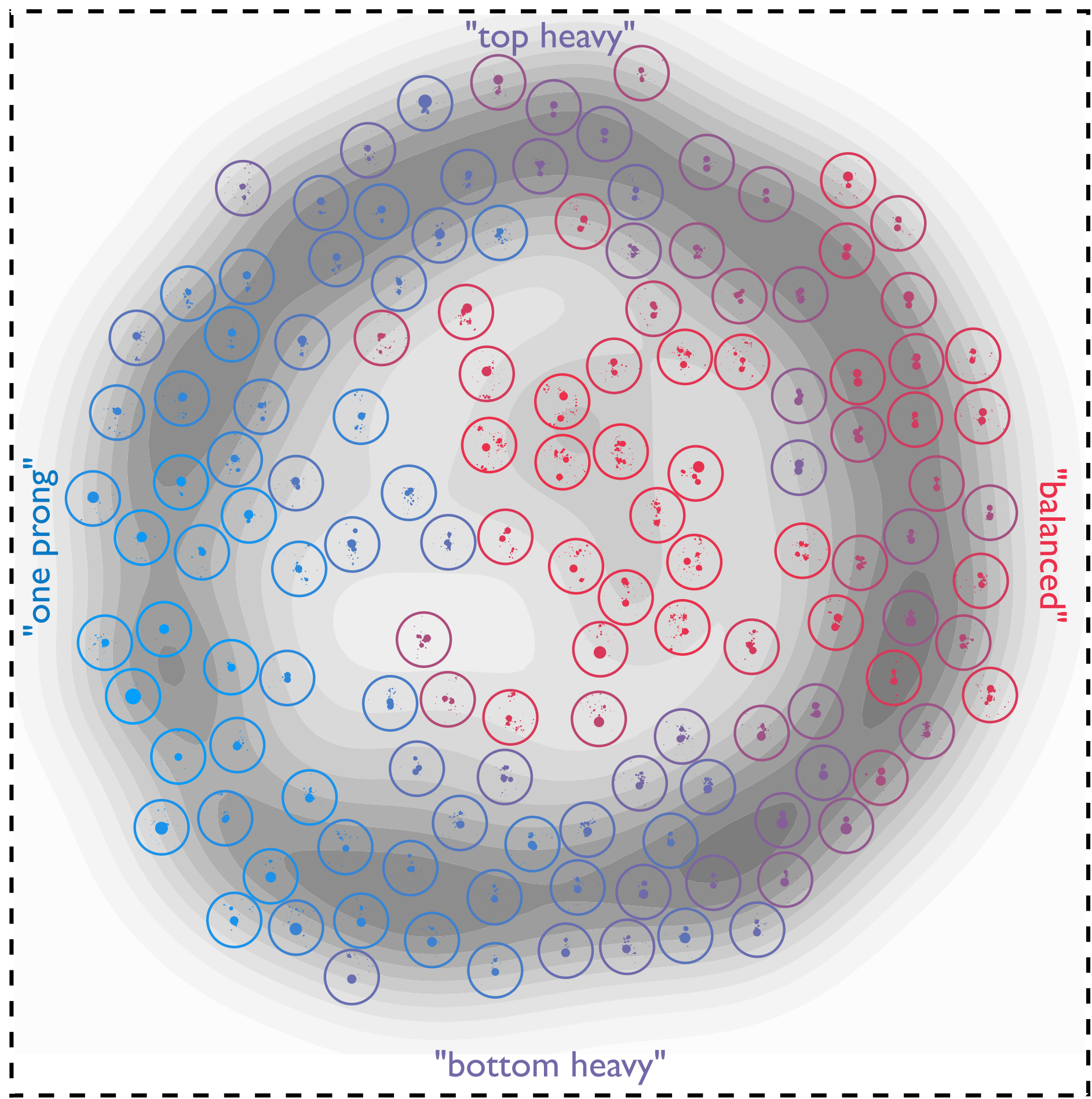
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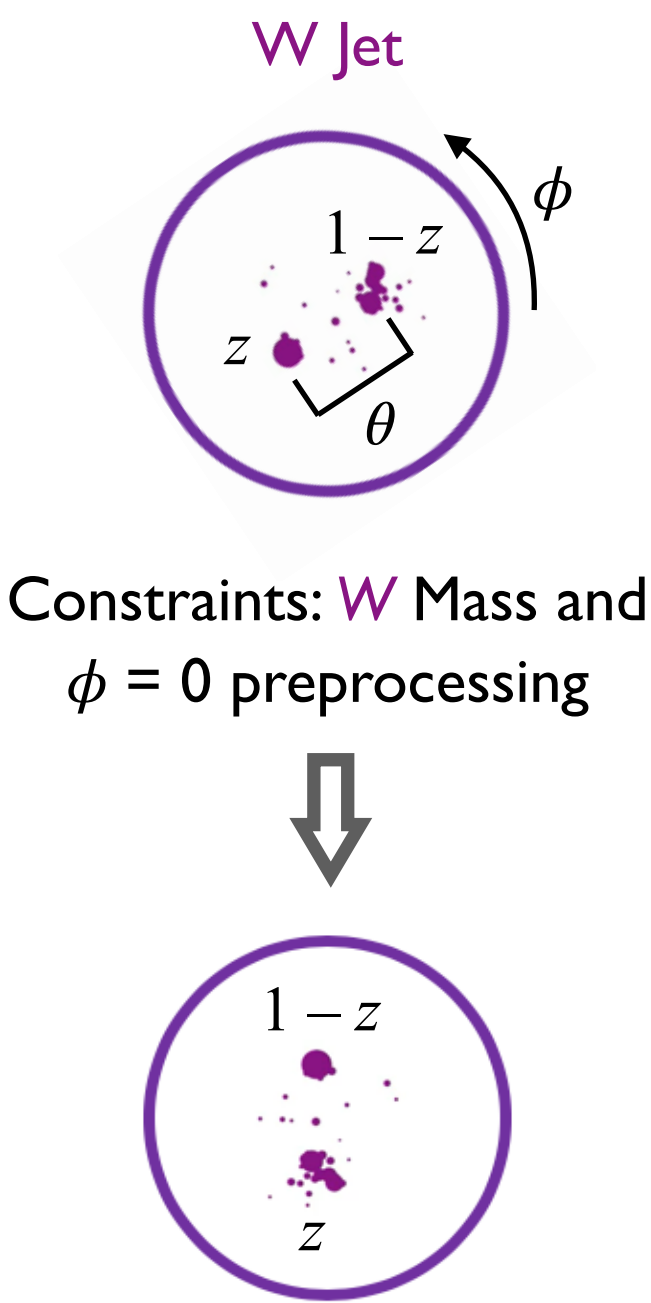
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[PTK, Metodiev, Thaler, PRL 2019]

Geometric space of W jets

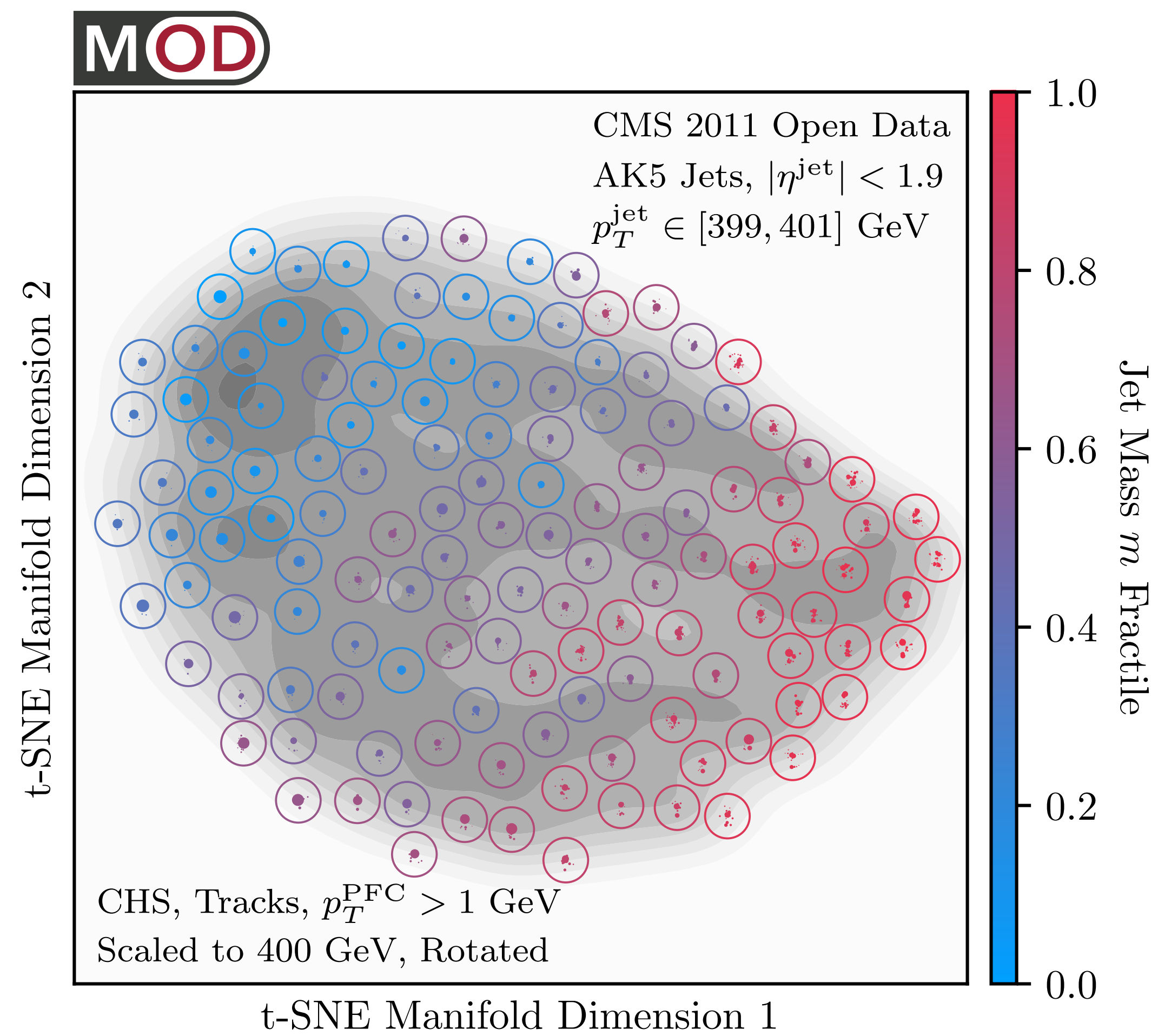


Gray contours represent the density of jets
Each circle is a particular W jet



Visualizing Geometry in CMS Open Data

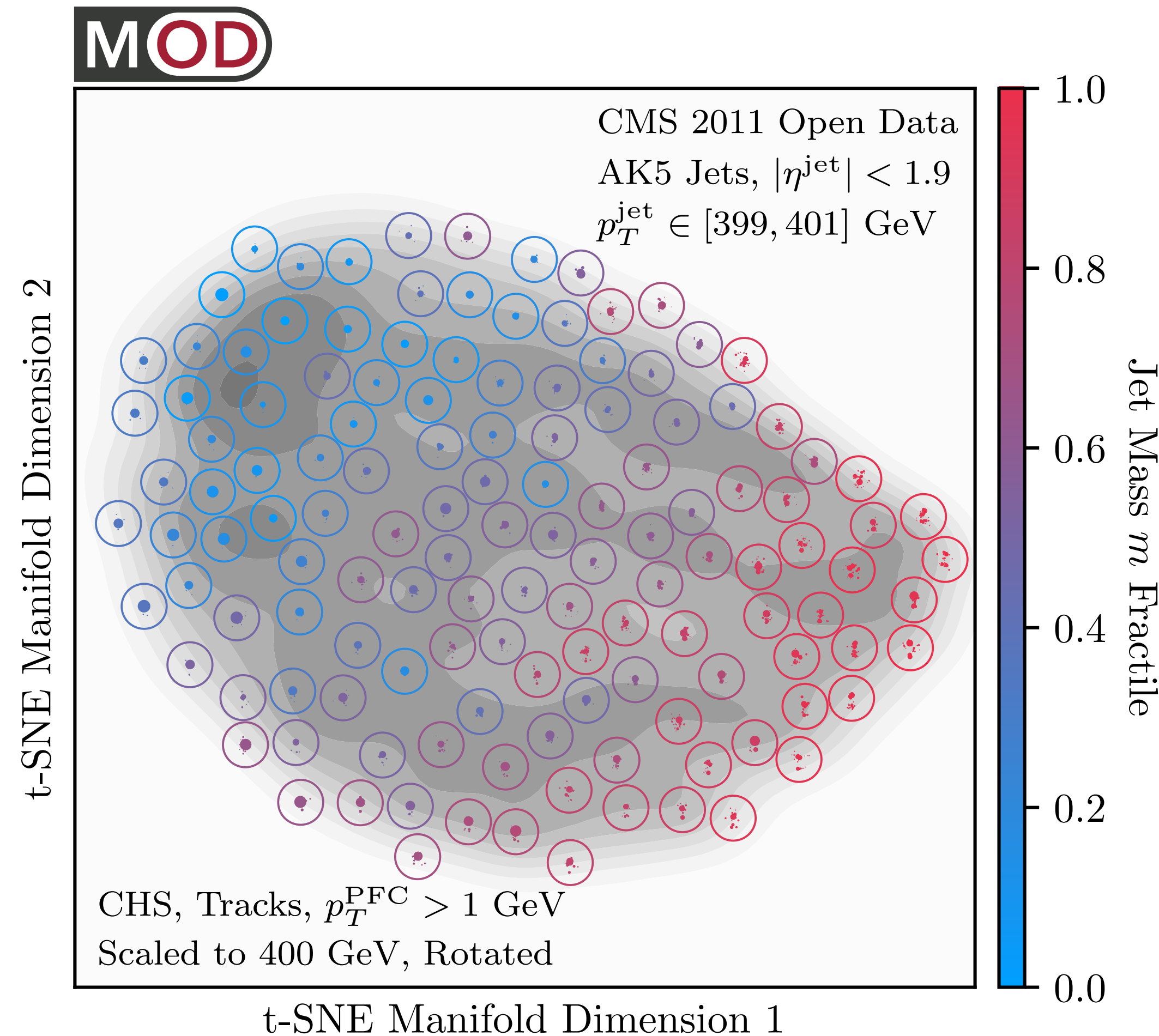
[PTK, Mastandrea, Metodiev, Naik, Thaler, [PRD 2019](#); code and datasets at [energyflow.network](#)]



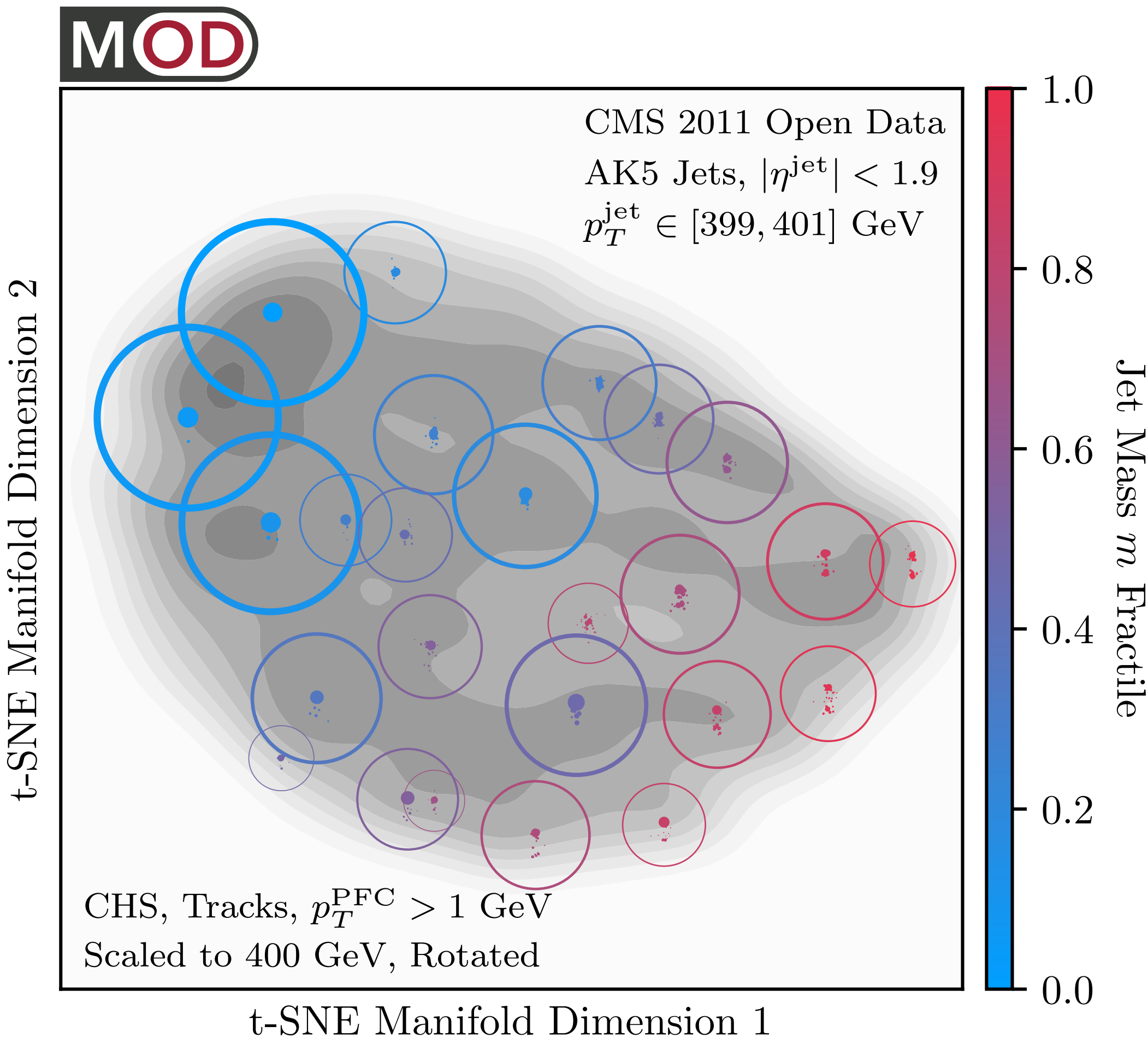
Example jets sprinkled throughout

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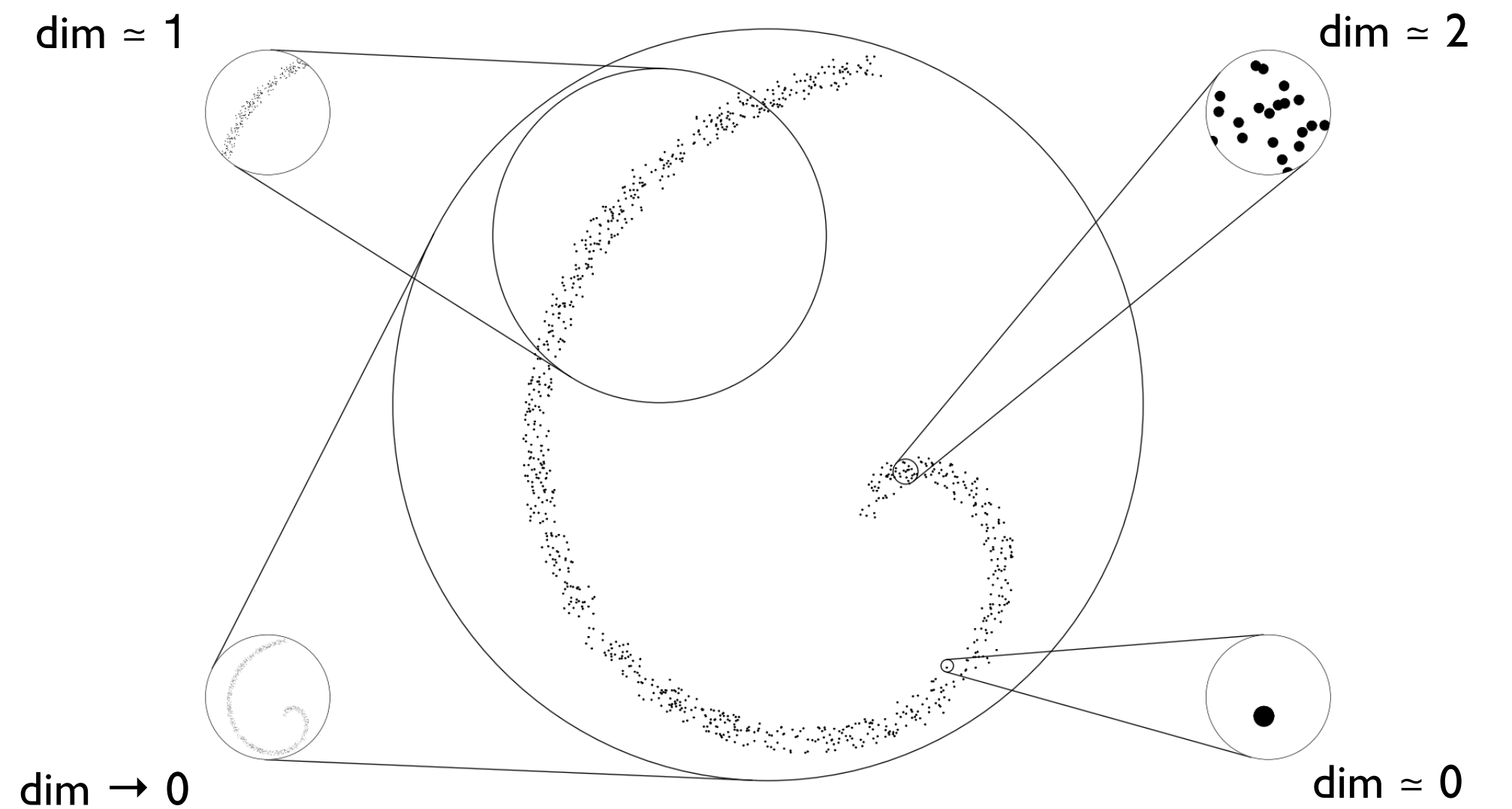
Example jets sprinkled throughout



25 most representative jets (“medoids”)
Size is proportional to number of jets associated to that medoid

Quantifying Event-Space Manifolds

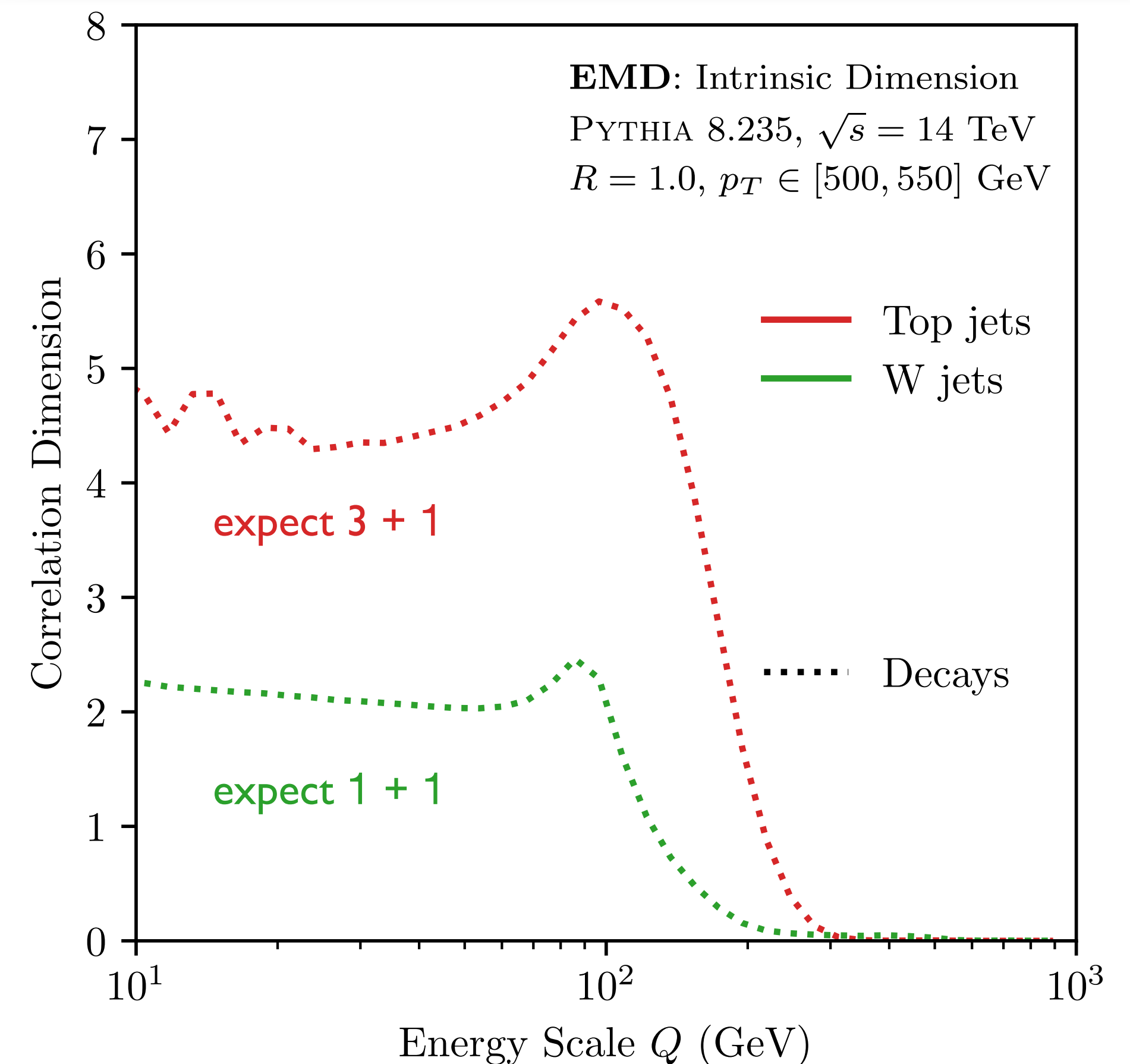
Correlation dimension: *how does the # of elements within a ball of size Q change?*



$$N_{\text{neigh.}}(Q) \propto Q^{\text{dim}} \implies \text{dim}(Q) = Q \frac{d}{dQ} \ln N_{\text{neigh.}}(Q)$$

Correlation dimension lessons:
Decays are "constant" dim. at low Q

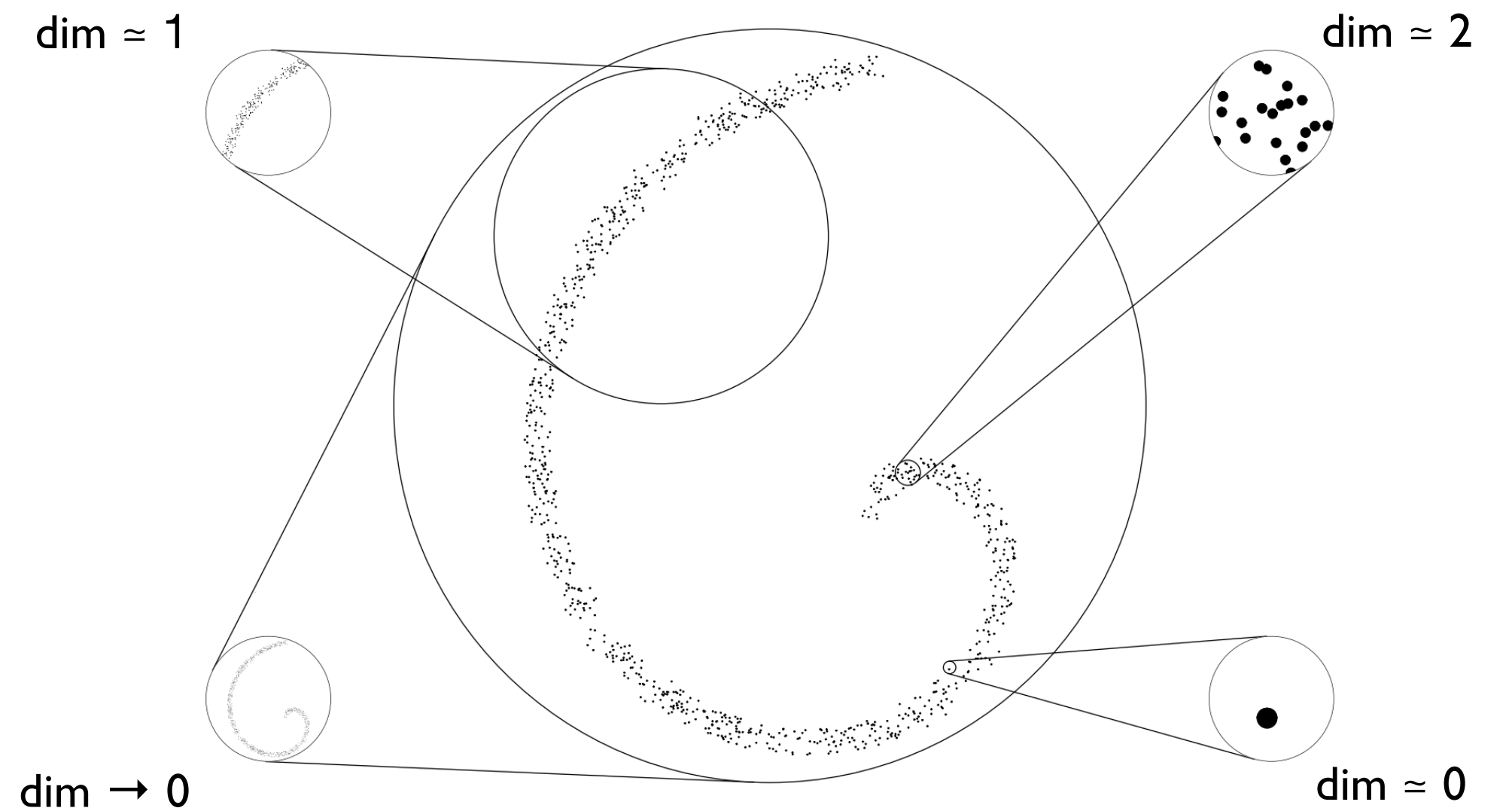
$$\text{dim}(Q) = Q \frac{\partial}{\partial Q} \ln \sum_i \sum_j \Theta(\text{EMD}(\mathcal{E}_i, \mathcal{E}'_j) < Q)$$



[Grassberger, Procaccia, [PRL 1983](#); PTK, Metodiev, Thaler, [PRL 2019](#)]

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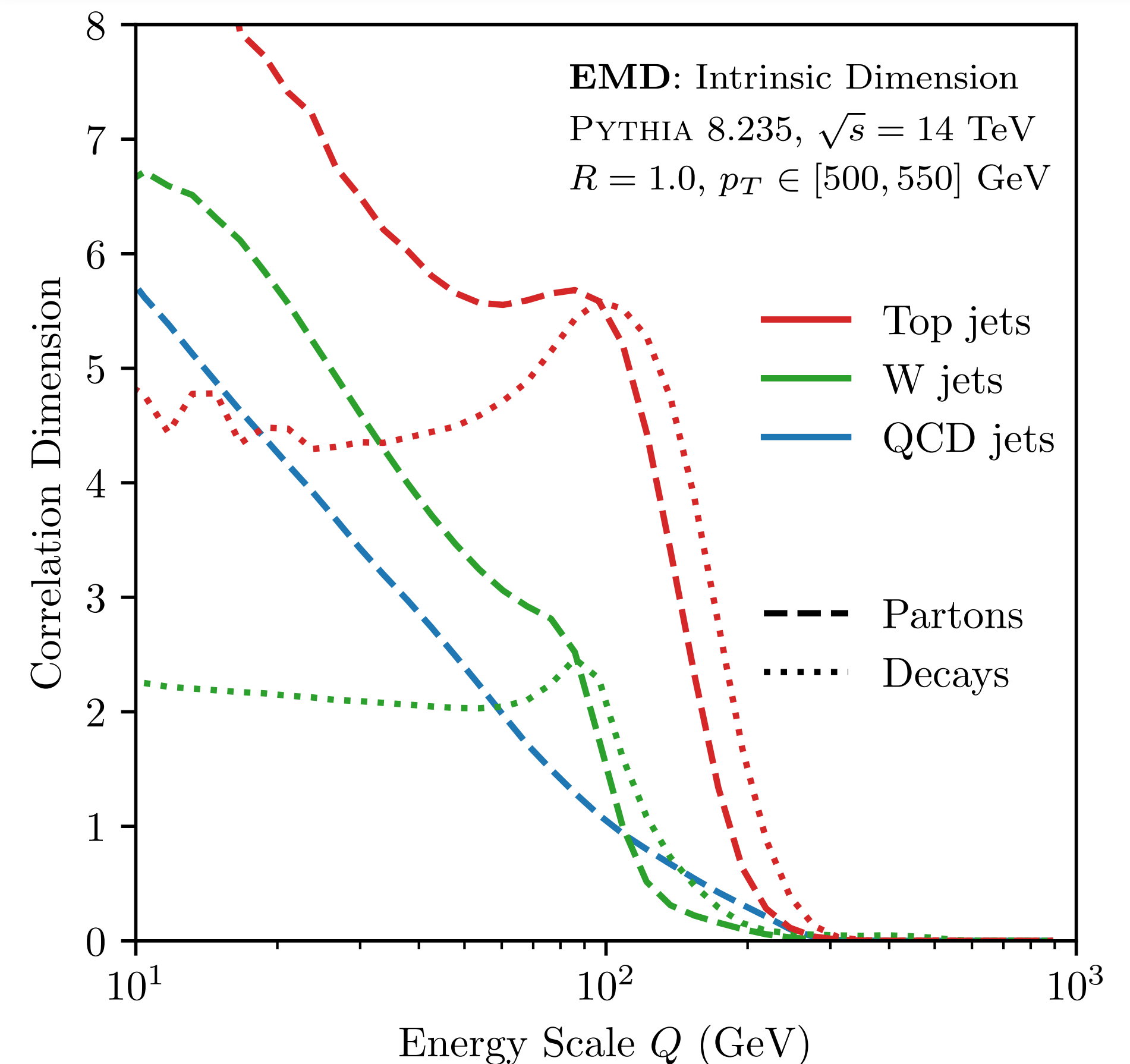
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Complexity hierarchy: QCD < W < Top

Fragmentation increases dim. at smaller scales

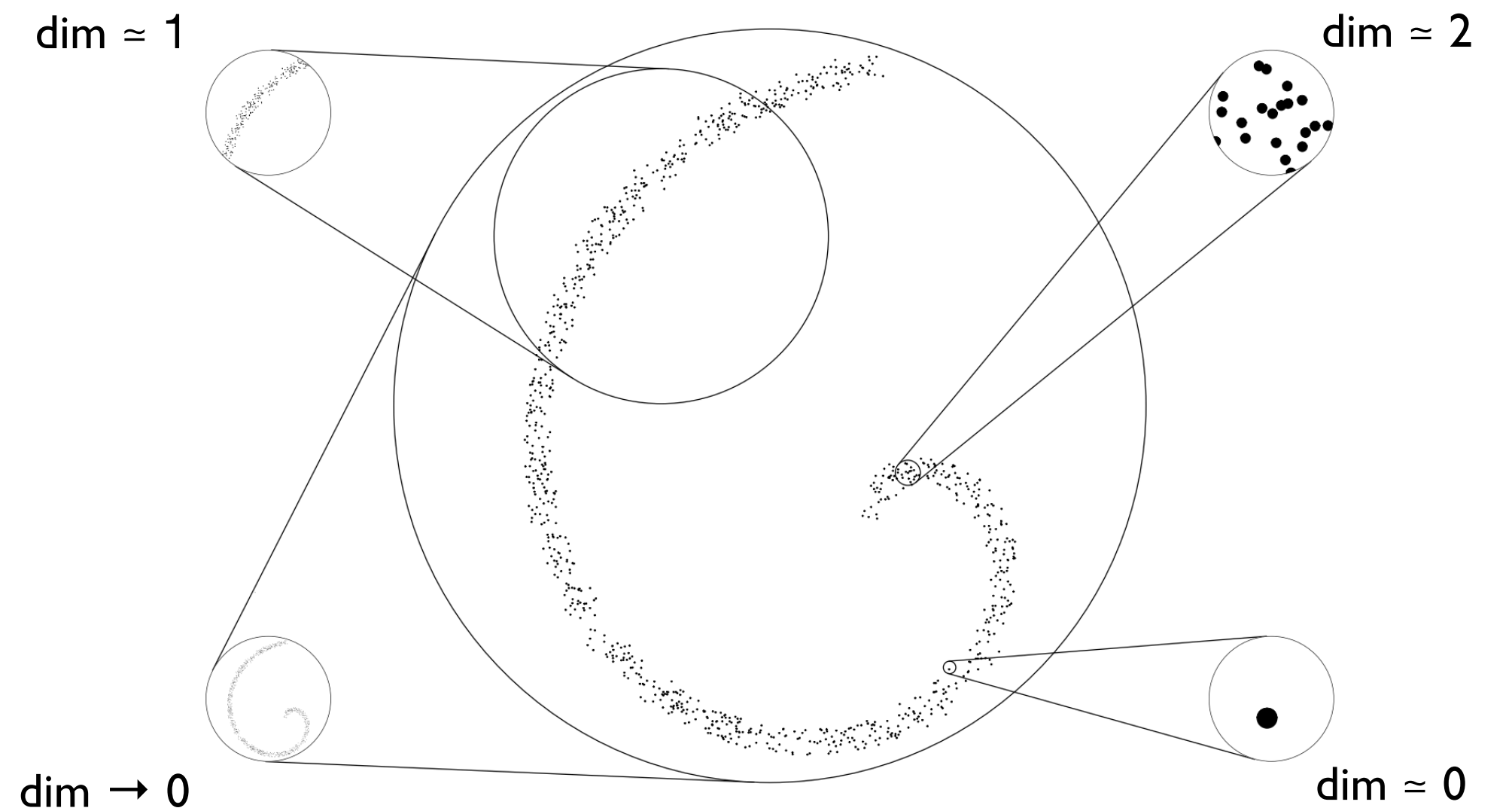
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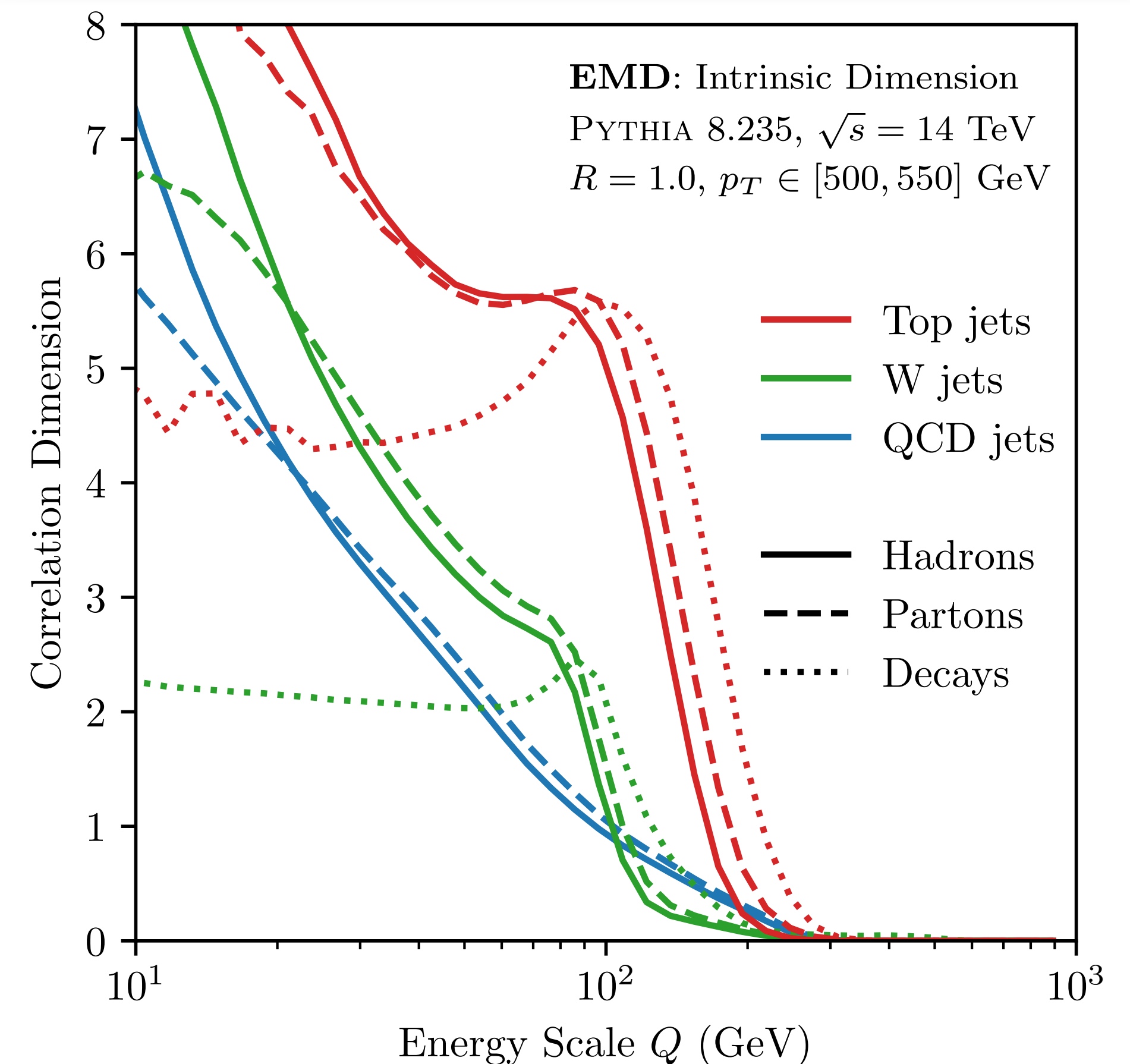
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Hadronization important around 20-30 GeV

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[Grassberger, Procaccia, [PRL 1983](#); PTK, Metodiev, Thaler, [PRL 2019](#)]

Quantifying Event-Space Manifolds

Correlation dimension
elements within a ball

dim ≈ 1

dim → 0

$N_{\text{neigh.}}(Q) \propto Q^{\text{dim}} \implies \text{dim}$

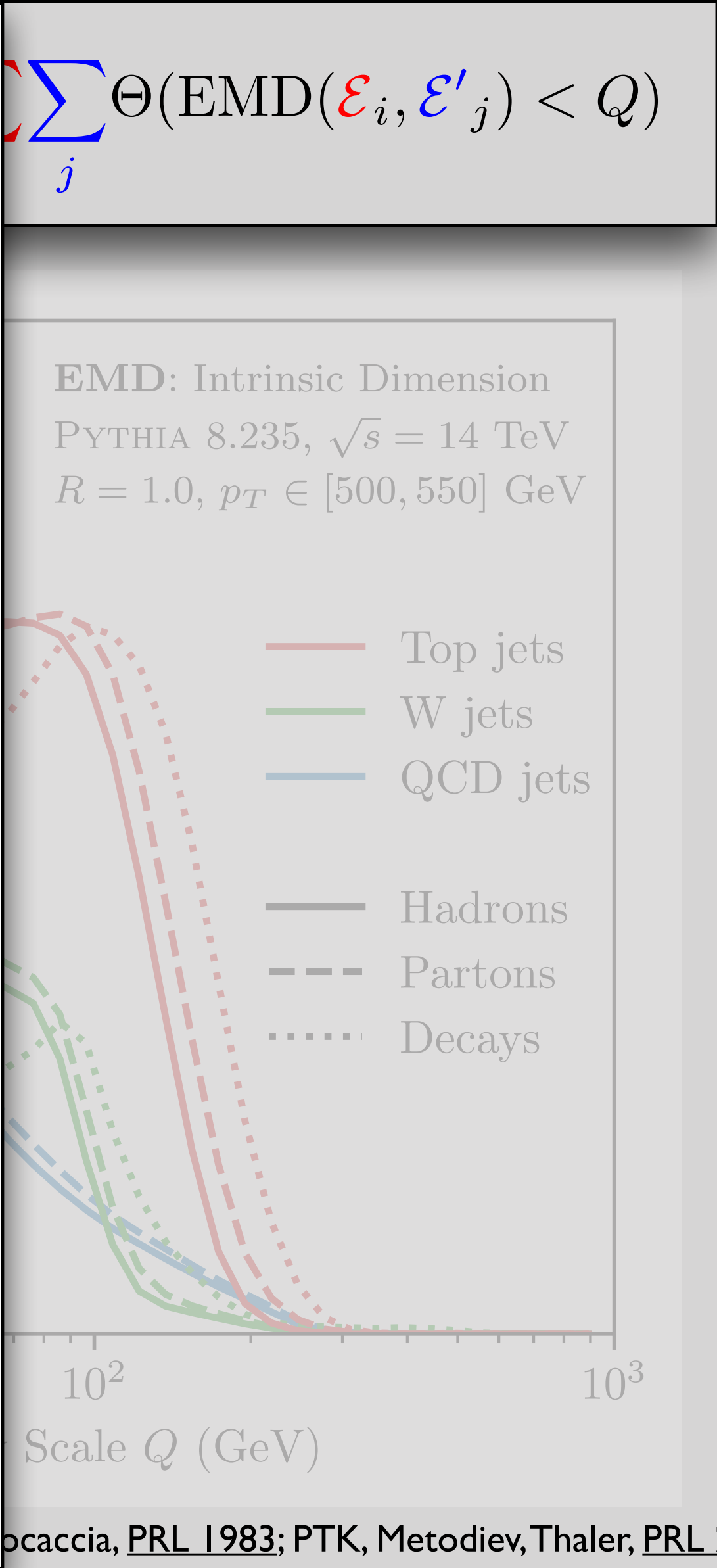
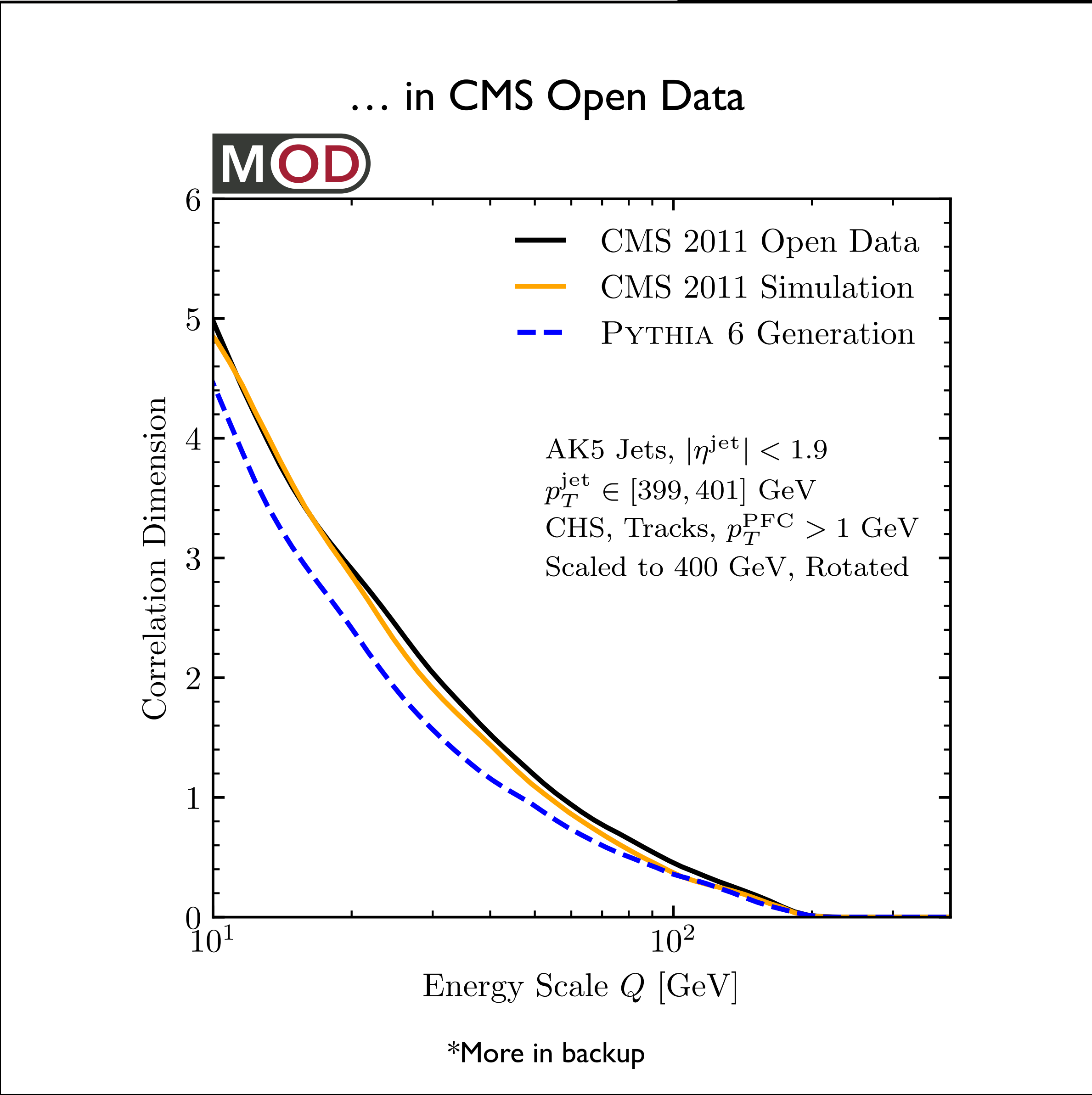
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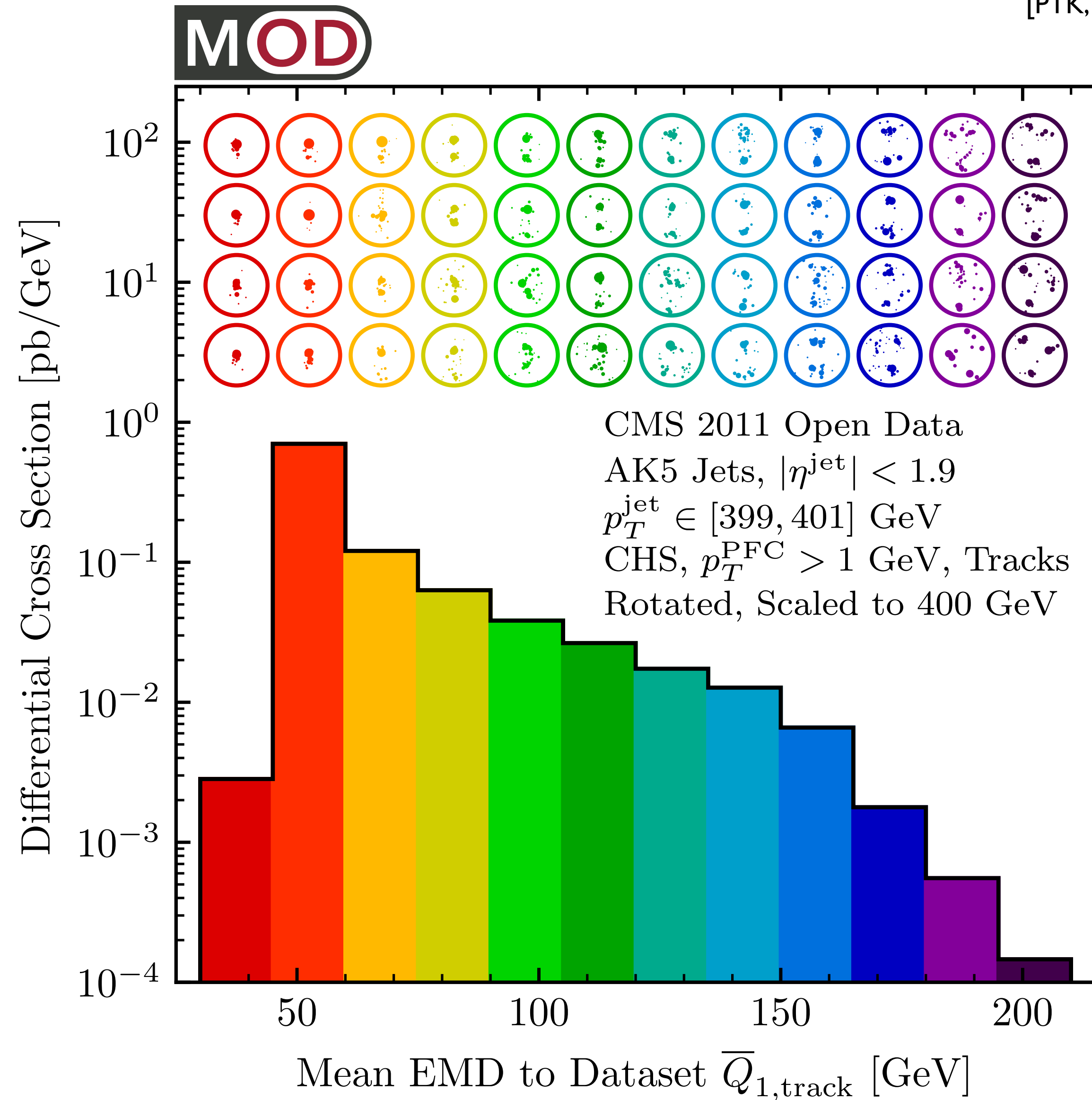
Fragmentation increases

Hadronization impacts



Visualizing Geometry in CMS Open Data

[PTK, Mastandrea, Metodiev, Naik, Thaler, [PRD 2019](#); code and datasets at [energyflow.network](#)]



EMD for anomaly detection

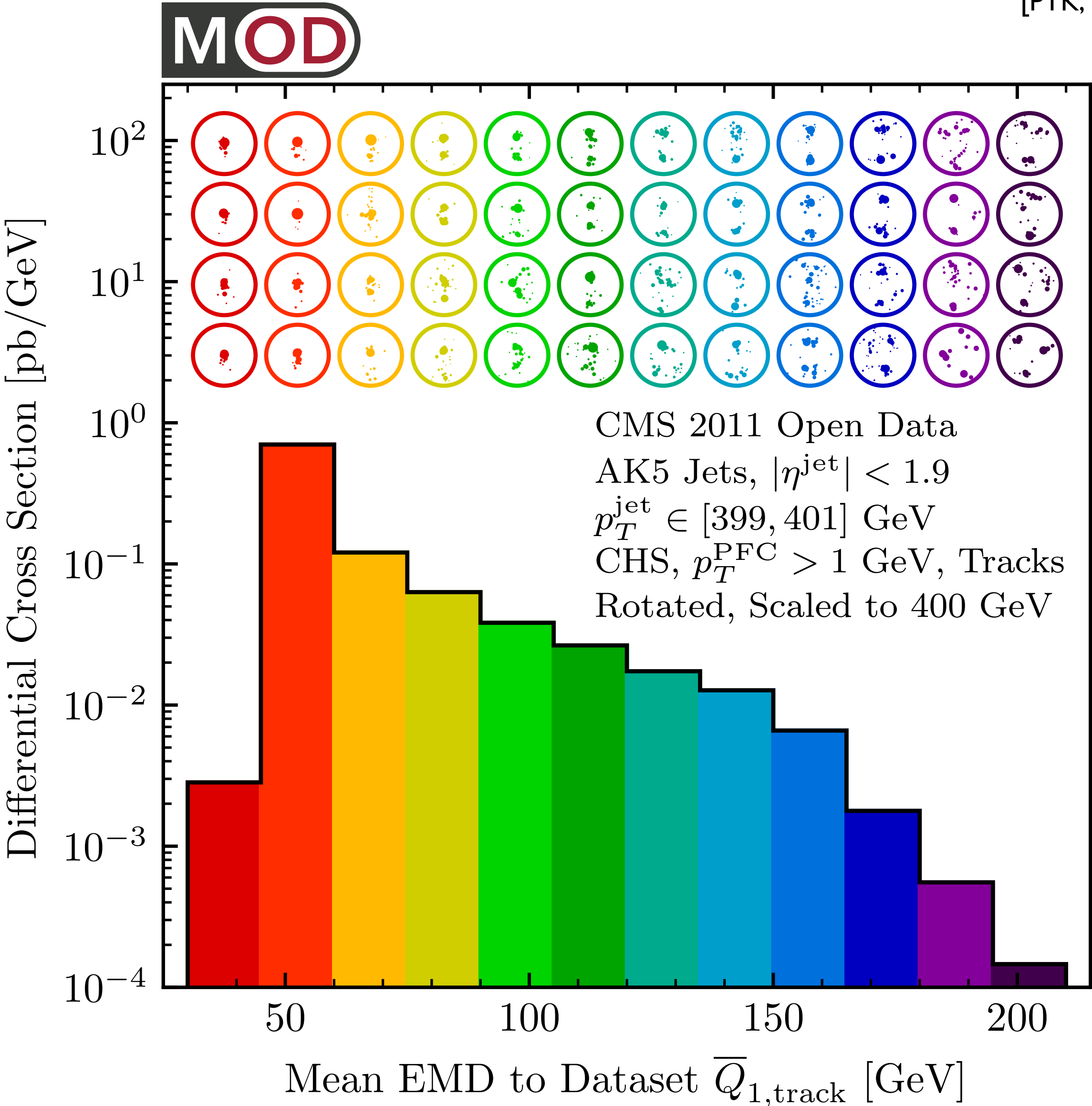
← 4 medoids in each bin of anomaliness \bar{Q}_1

n^{th} moment of EMD distribution for a dataset

$$\bar{Q}_n(\mathcal{I}) = \sqrt[n]{\frac{1}{N} \sum_{k=1}^N (\text{EMD}(\mathcal{I}, \mathcal{J}_k))^n}$$

Visualizing Geometry in CMS Open Data

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How far does this go?

$$\nu_k = \frac{1}{N} \sum_{i=1}^N \min \{ \text{EMD}(\mathcal{J}_i, \mathcal{K}_1), \dots, \text{EMD}(\mathcal{J}_i, \mathcal{K}_k) \}$$

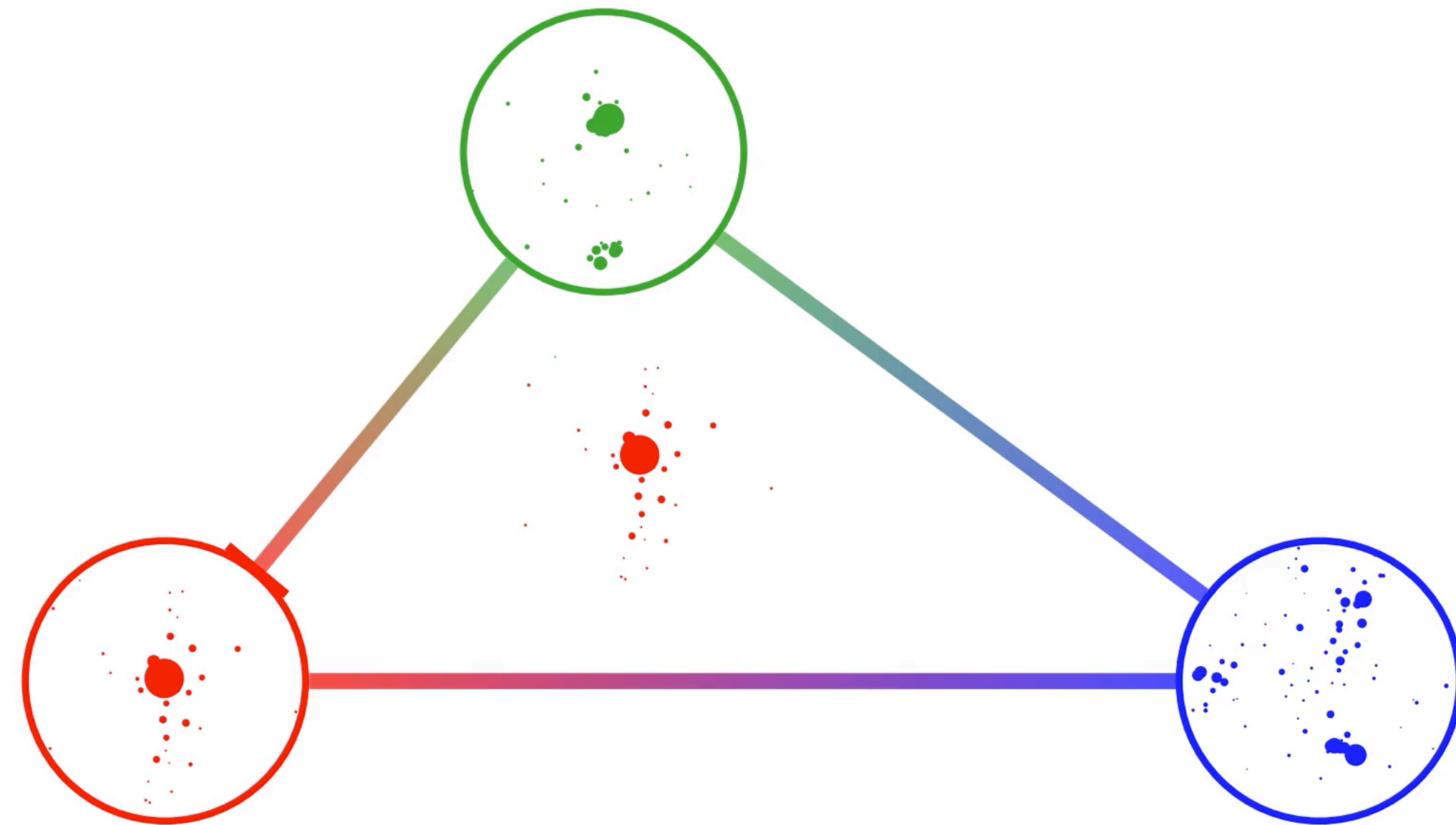
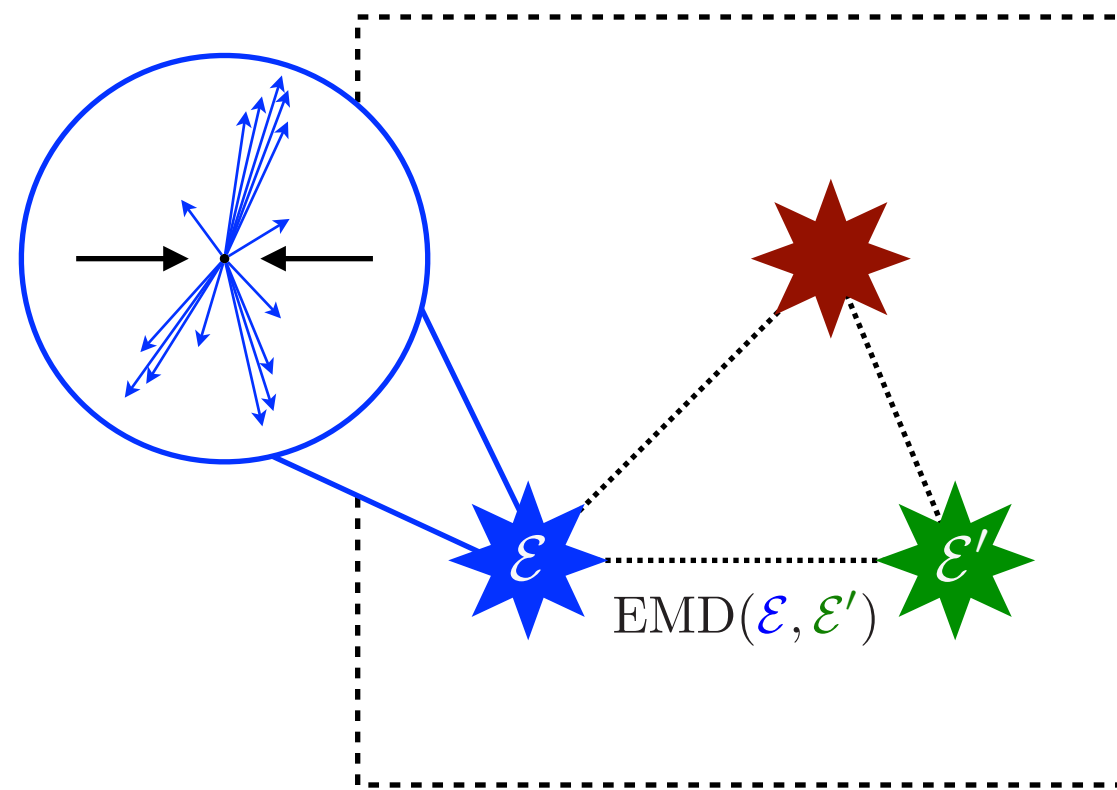
\uparrow
k-eventiness?

\uparrow
jet from dataset

\uparrow
medoids

The (Metric) Space of Events

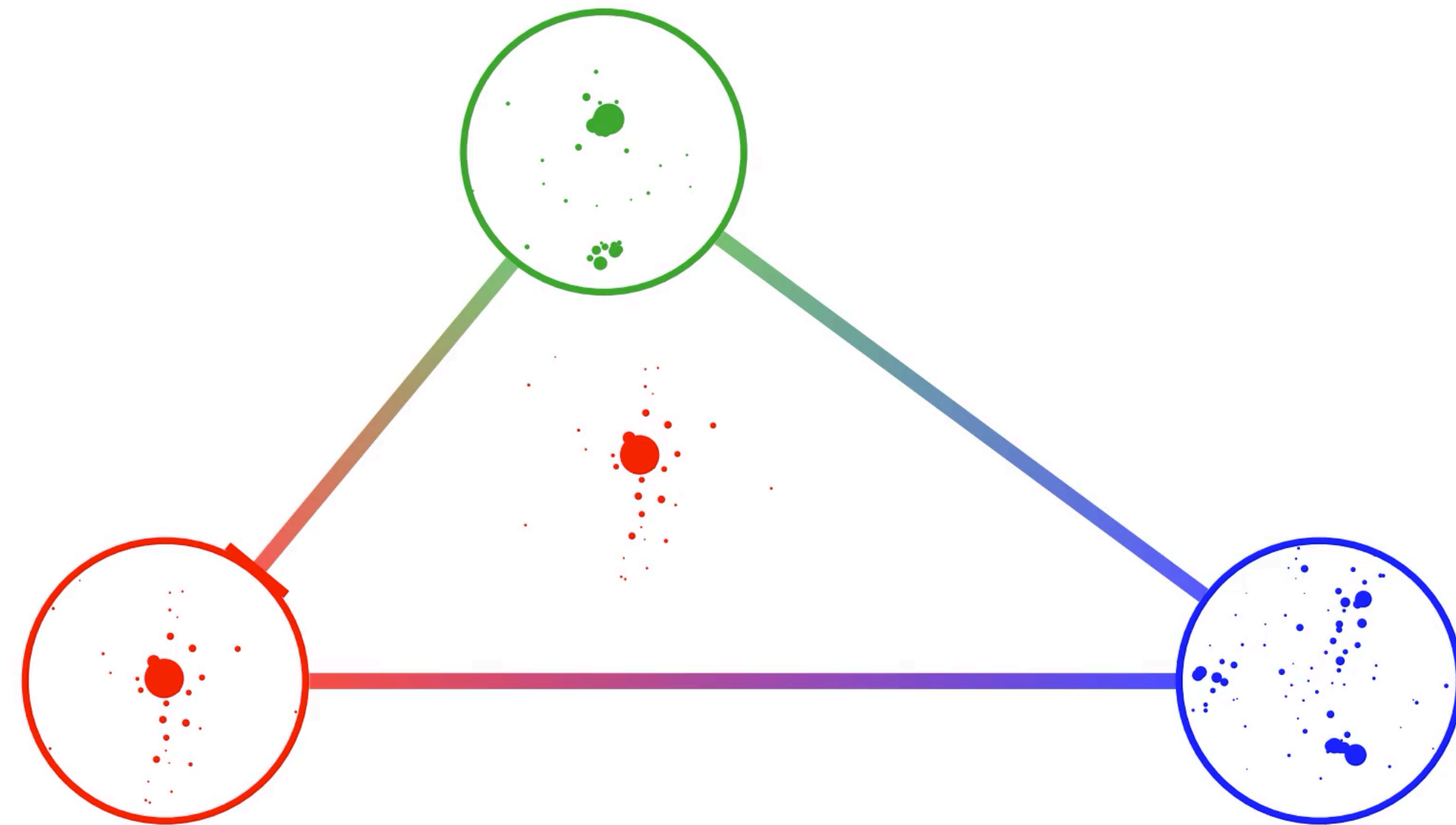
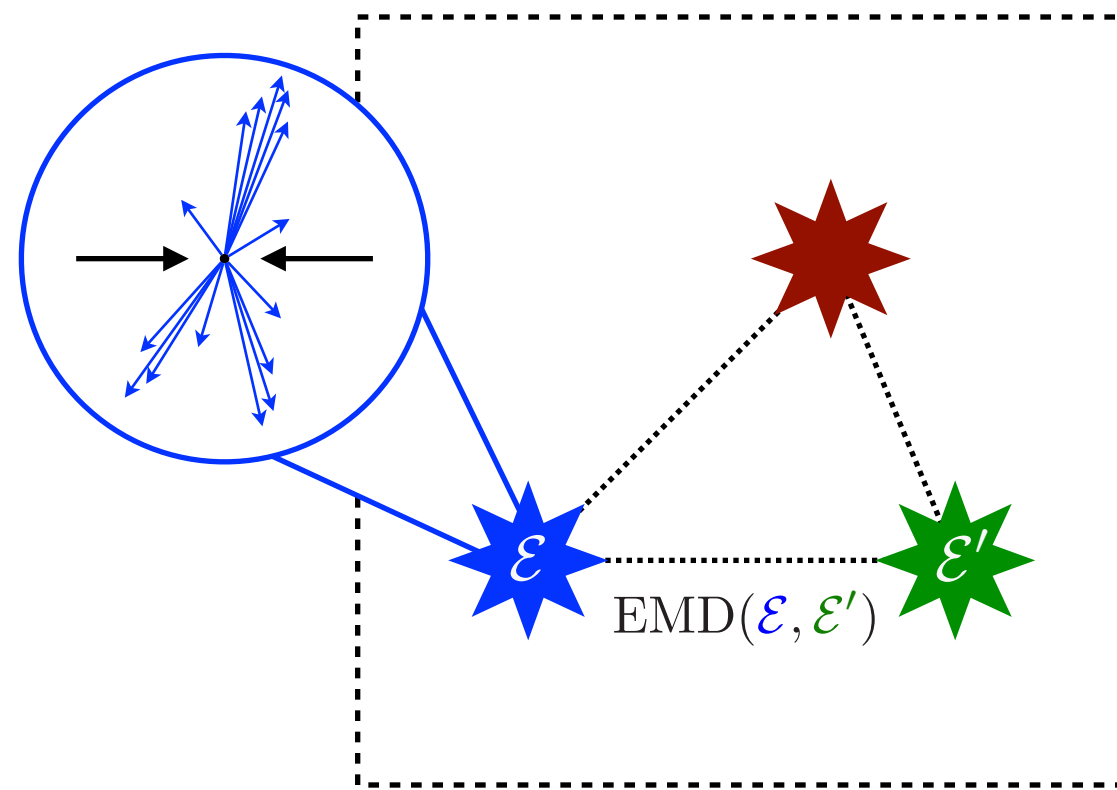
- **Energy** flow is theoretically and experimentally robust
- EMD metrizes the space of **energy** flows (events)
- Manifolds in the space of events can be visualized and quantified



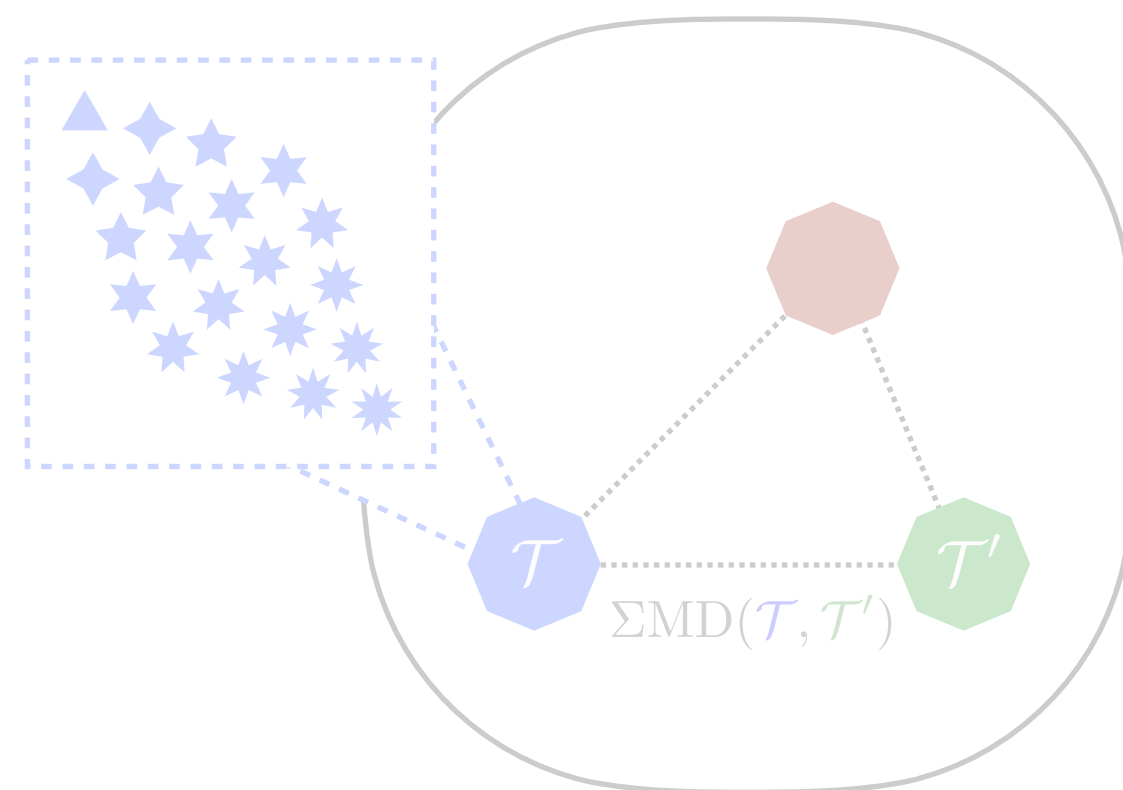
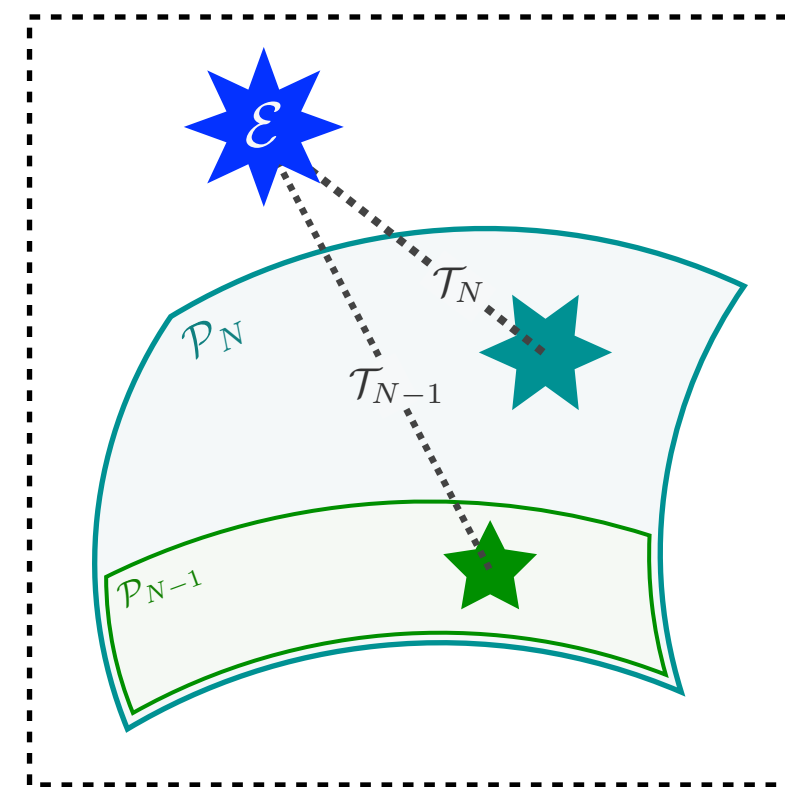
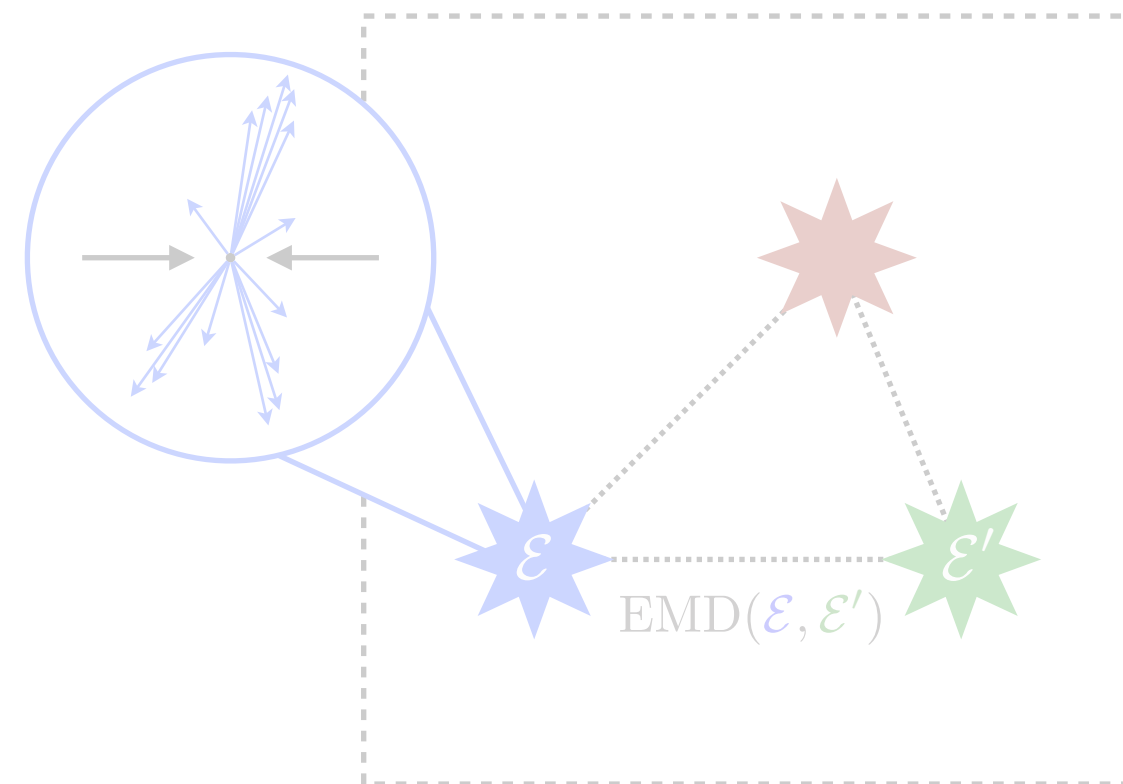
What else can this geometry do for us?

The (Metric) Space of Events

- **Energy** flow is theoretically and experimentally robust
- EMD metrizes the space of **energy** flows (events)
- Manifolds in the space of events can be visualized and quantified



What else can this geometry do for us?



The (Metric) Space of Events

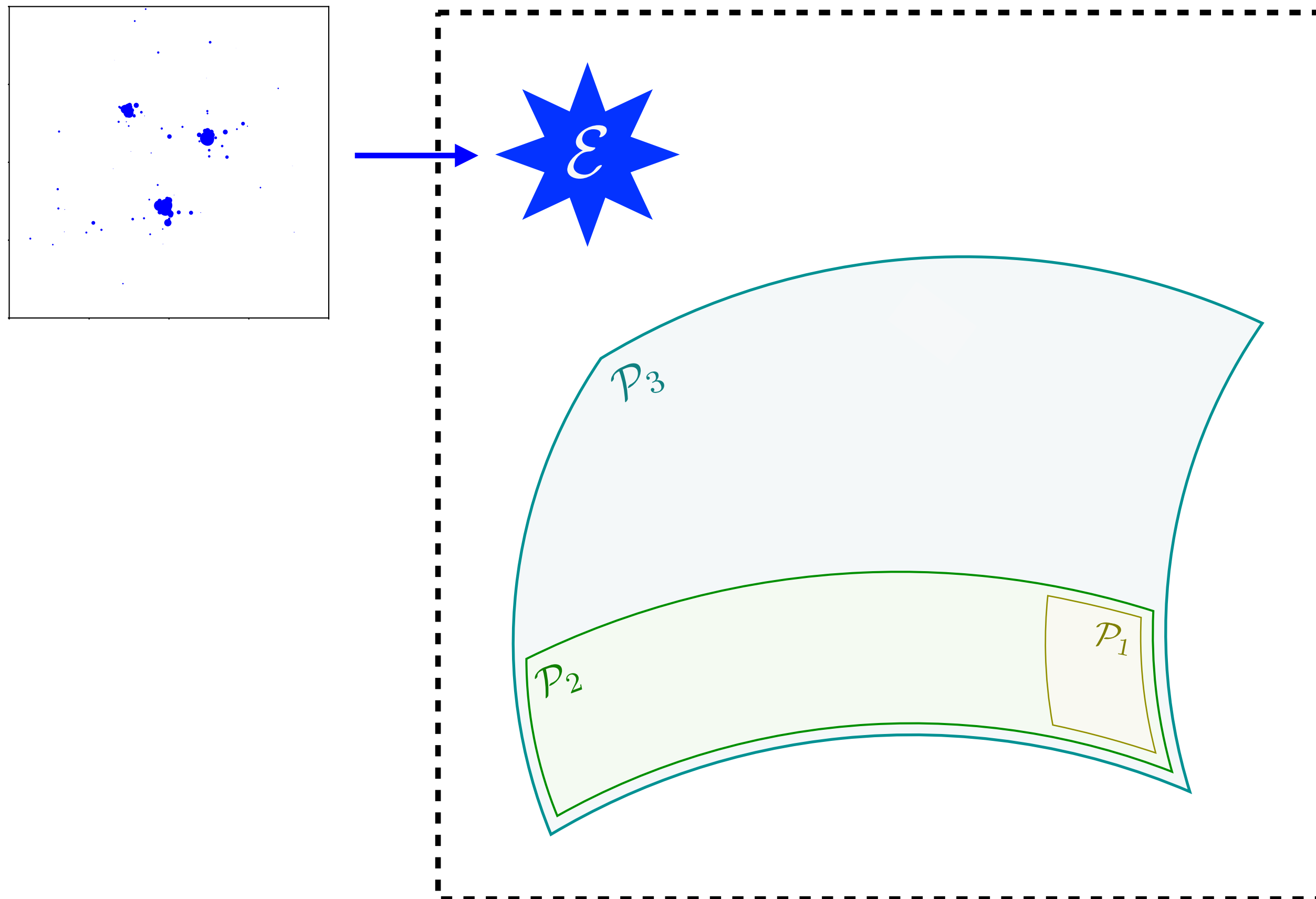
Revealing Hidden Geometry

Theory Space

N-particle Manifolds in the Space of Events

[PTK, Metodiev, Thaler, [2004.04159](#)]

$$\mathcal{P}_N = \text{set of all N-particle configurations} = \left\{ \sum_{i=1}^N \mathbf{E}_i \delta(\hat{n} - \hat{n}_i) \mid \mathbf{E}_i \geq 0 \right\}$$



⋮

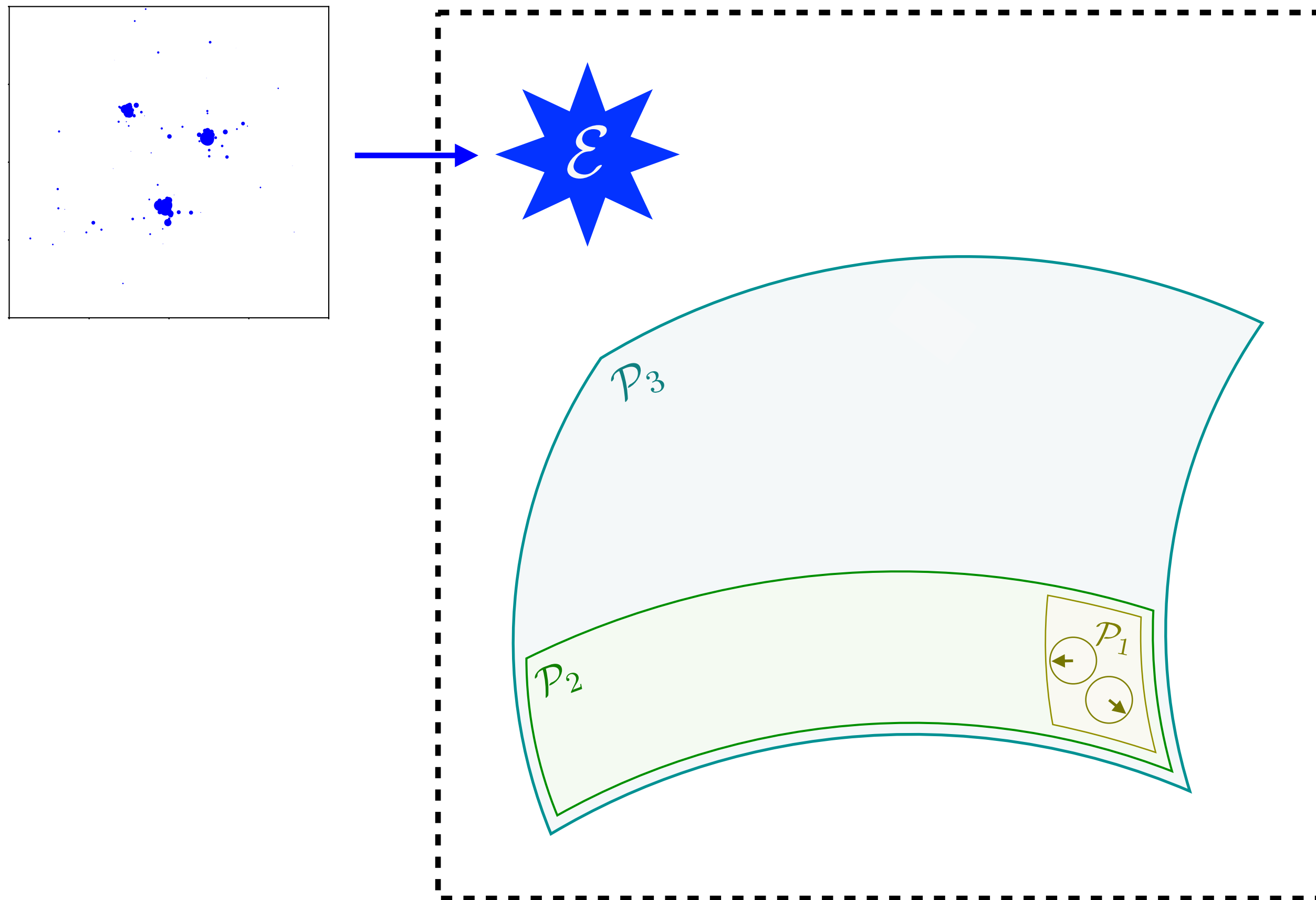
$$\mathcal{P}_N \supset \mathcal{P}_{N-1} \supset \cdots \supset \mathcal{P}_3 \supset \mathcal{P}_2 \supset \mathcal{P}_1$$

by **soft** and **collinear** limits

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\mathcal{P}_1 : manifold of events with one particle

\vdots

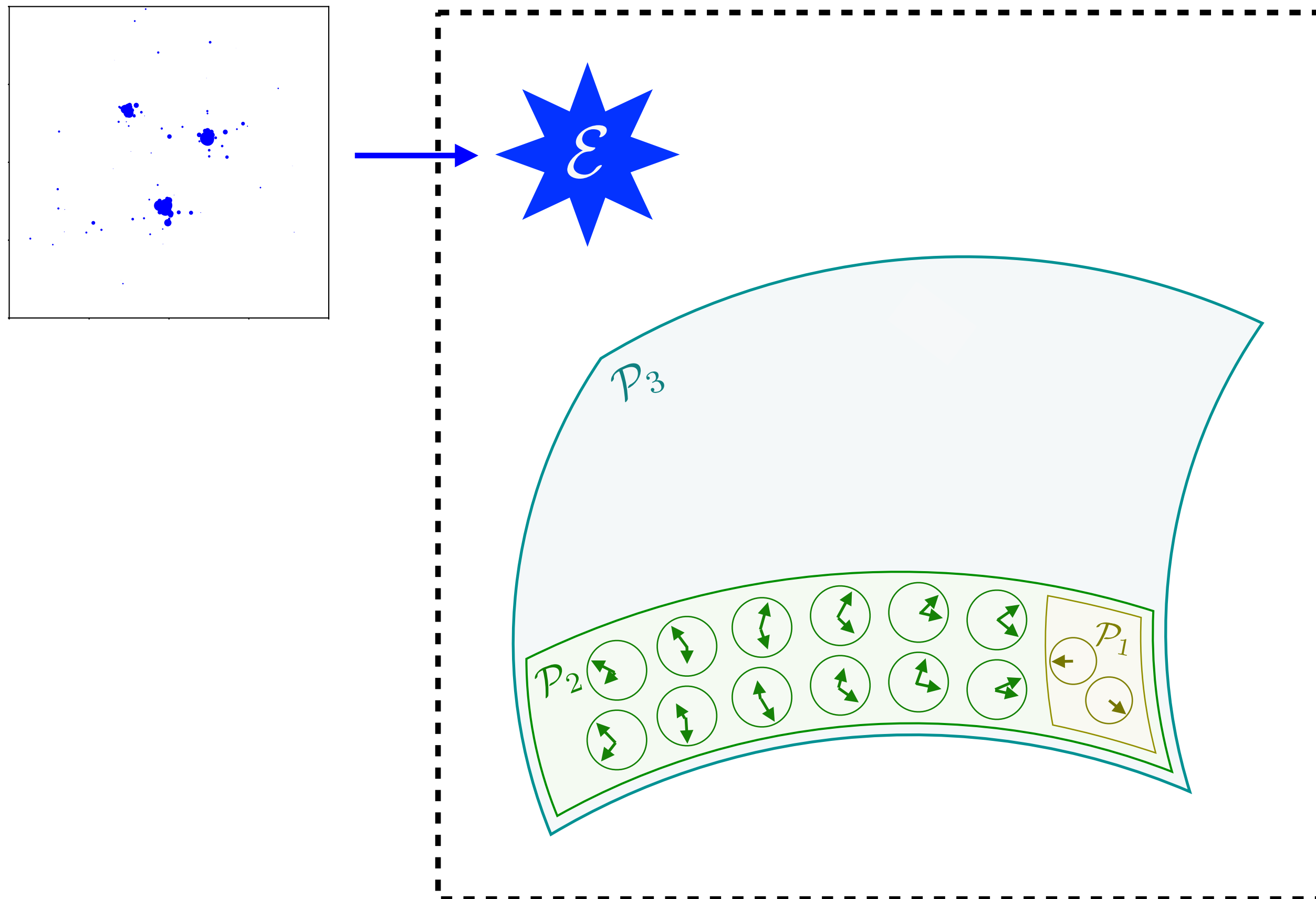
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\mathcal{P}_1 : manifold of events with one particle

\mathcal{P}_2 : manifold of events with two particles

\vdots

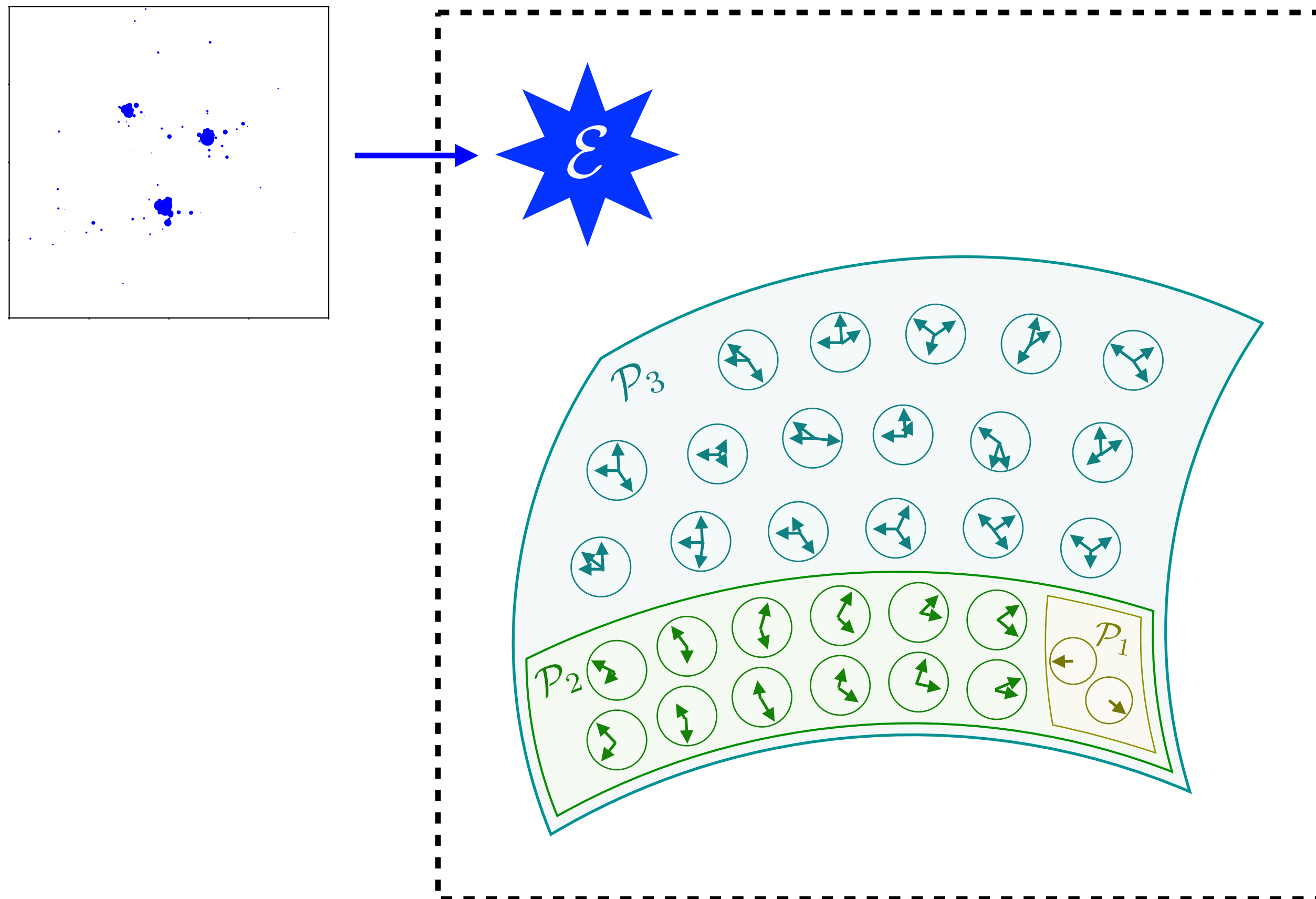
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\mathcal{P}_1 : manifold of events with one particle

\mathcal{P}_2 : manifold of events with two particles

\mathcal{P}_3 : manifold of events with three particles

\vdots

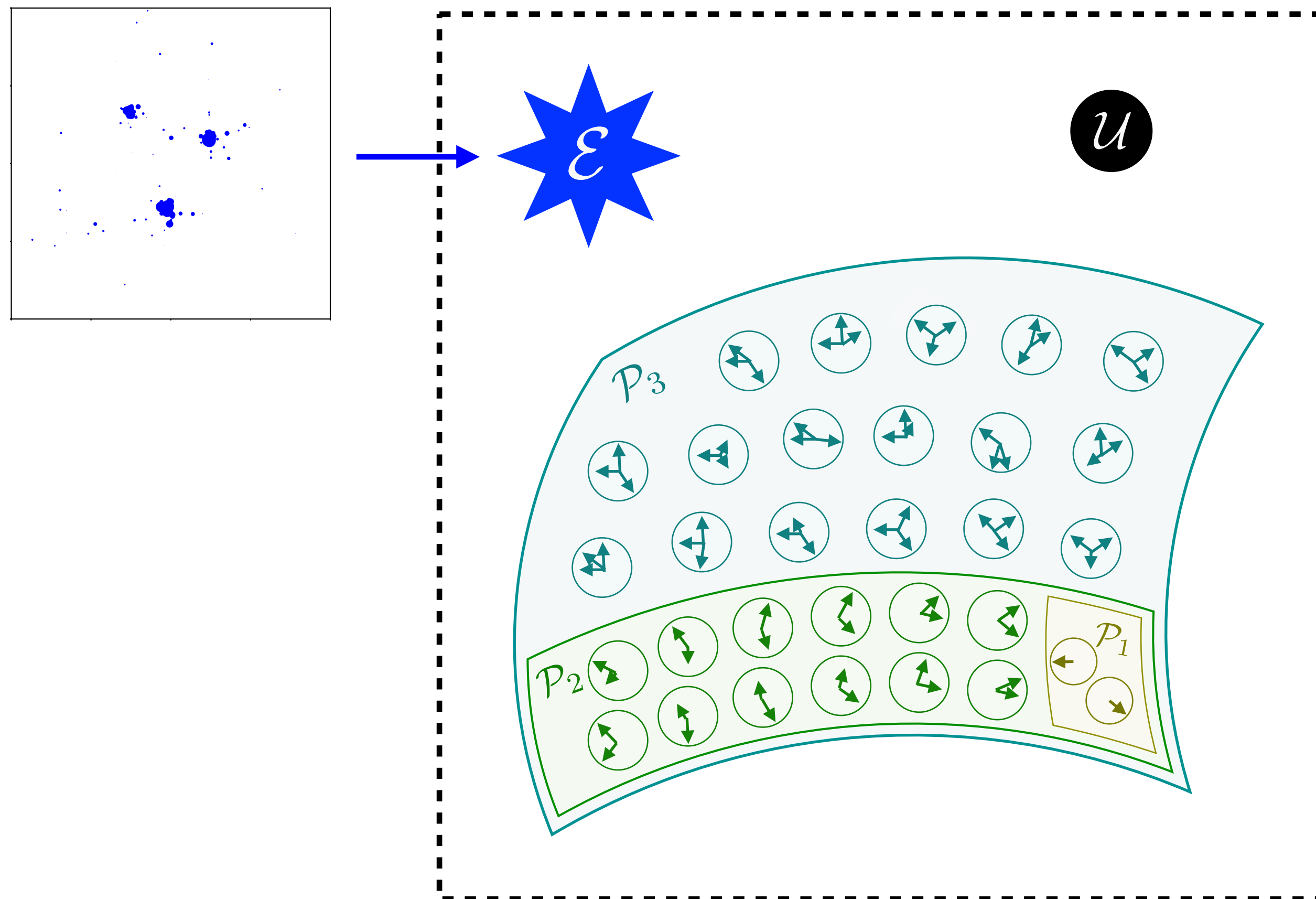
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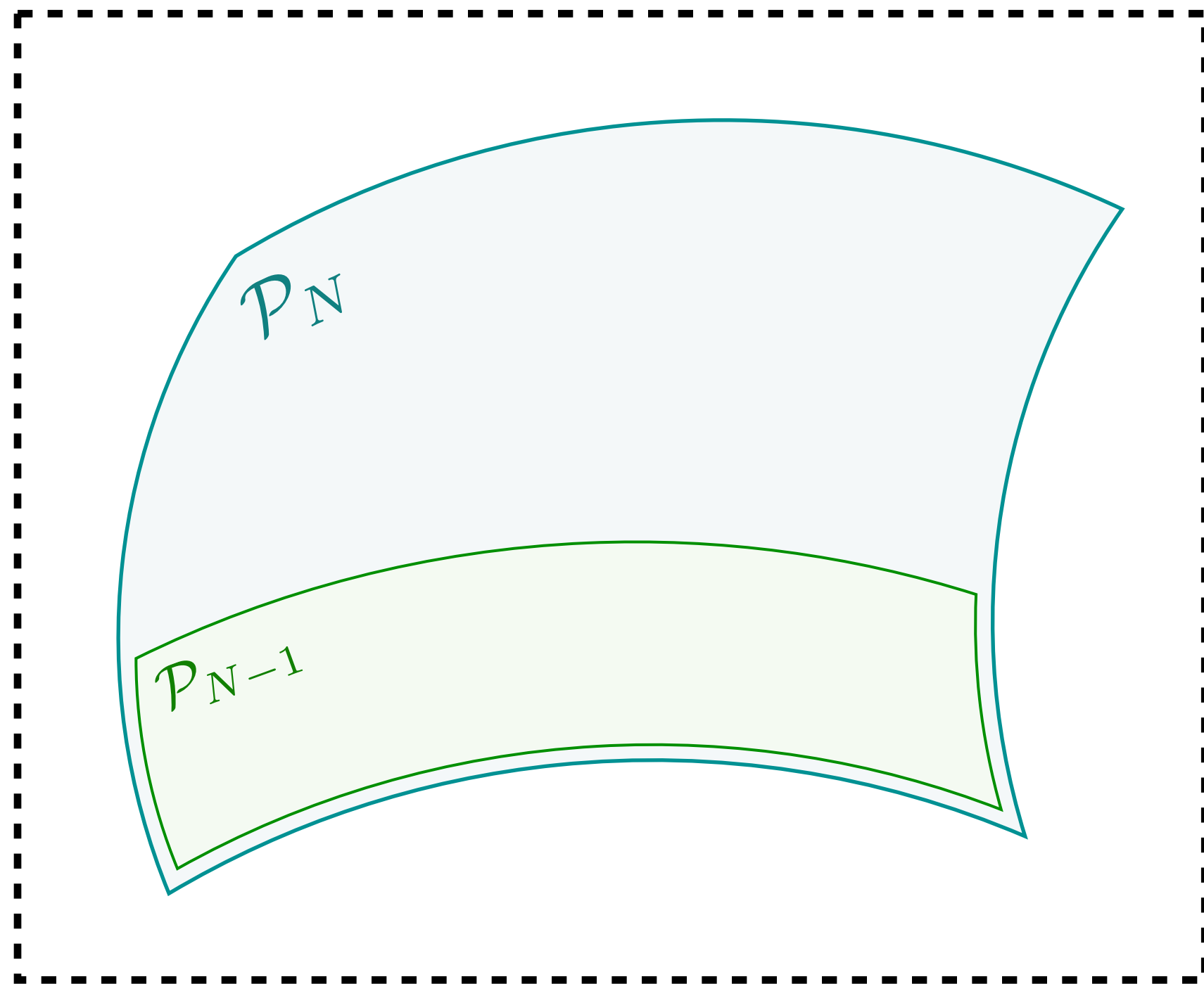
by **soft** and **collinear** limits

\mathcal{U} Uniform event, not contained in any \mathcal{P}_N

N-particle Manifolds in the Space of Events – Infrared Divergences

[PTK, Metodiev, Thaler, [2004.04159](#)]

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$$dP_{i \rightarrow ig} \simeq \frac{2\alpha_s}{\pi} C_a \frac{d\theta}{\theta} \frac{dz}{z}$$

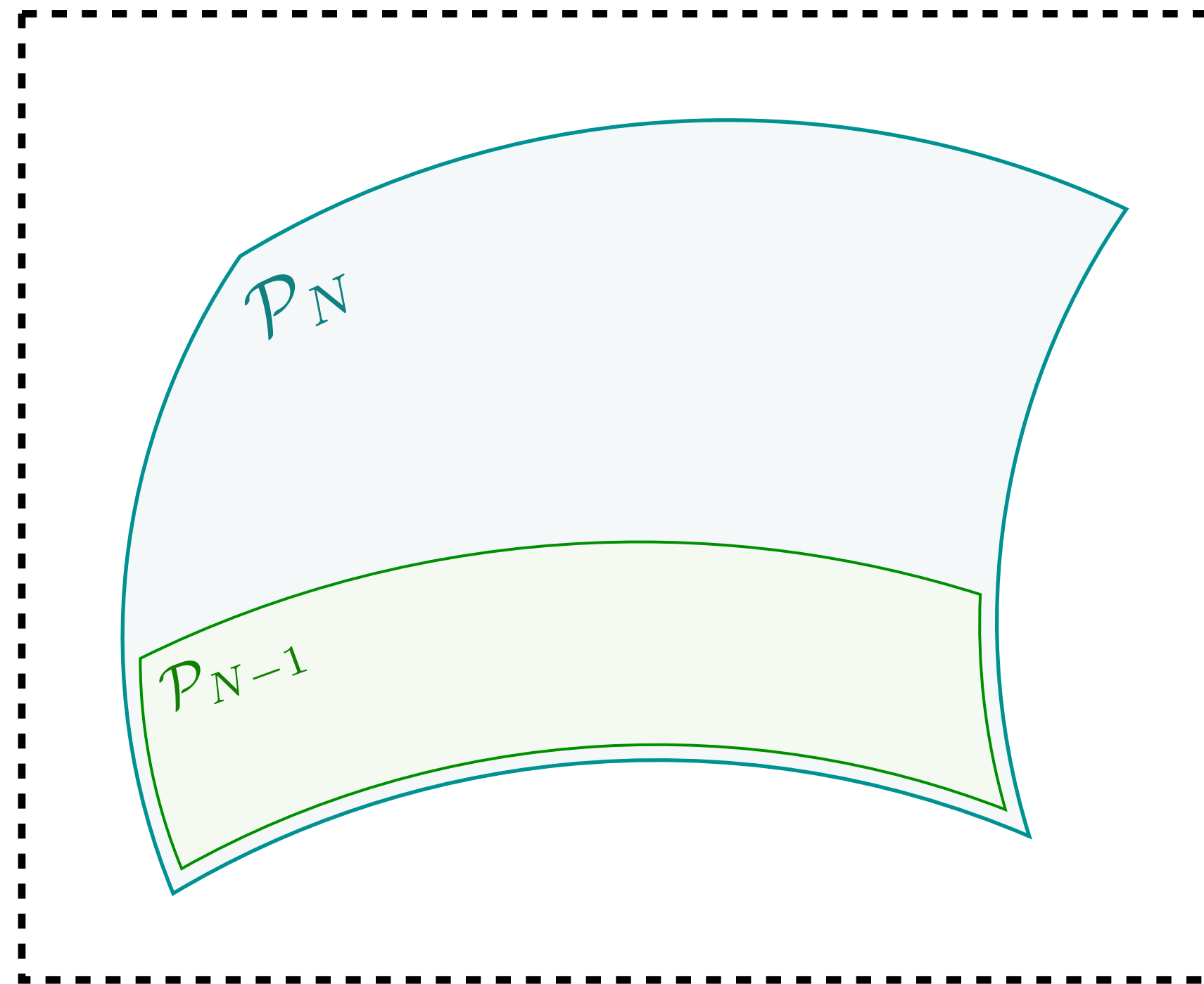
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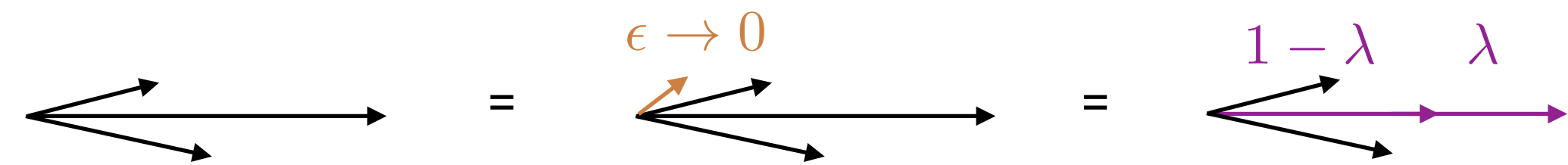
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Energy flow is unchanged by exact soft/collinear emissions



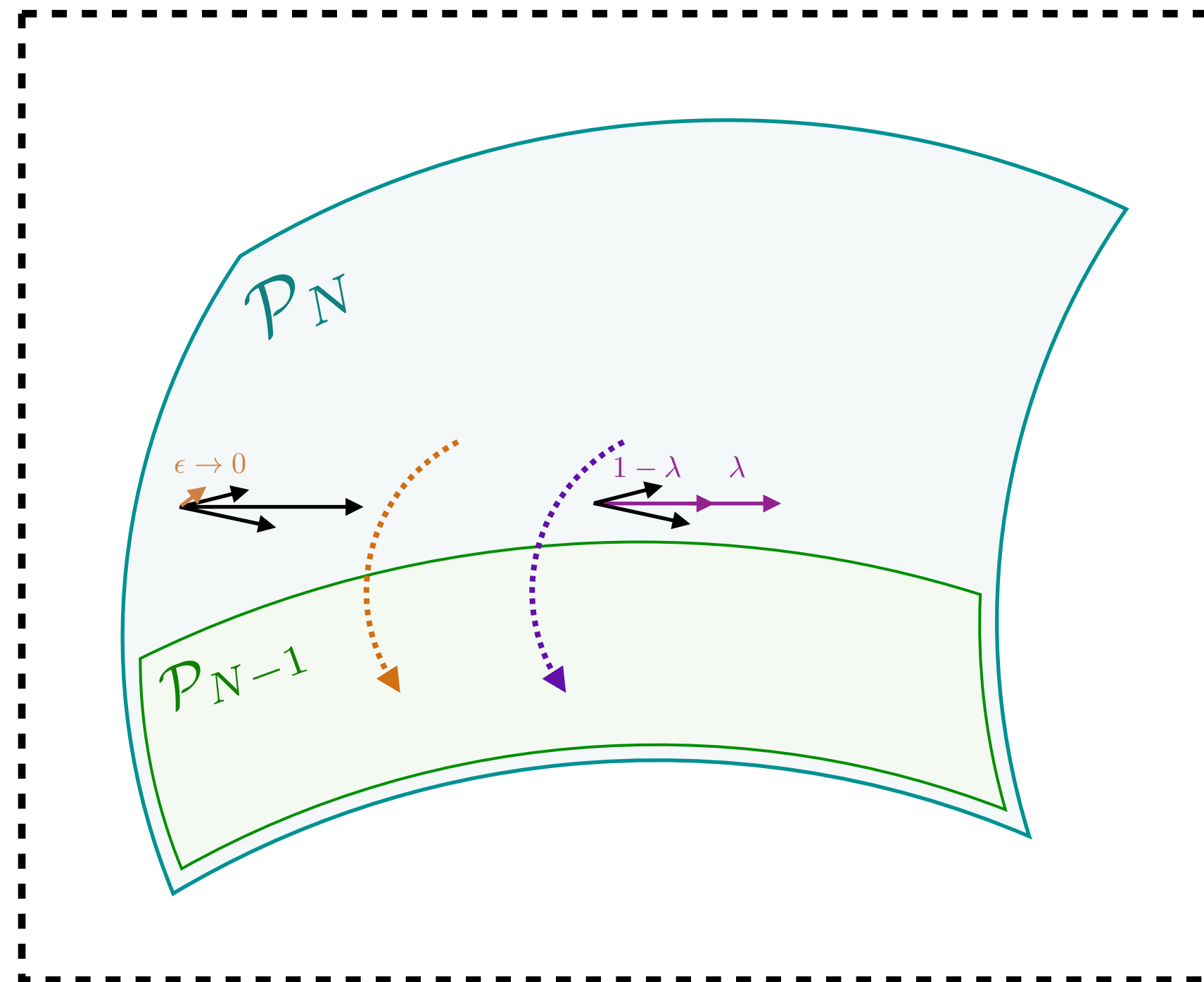
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N-particle Manifolds in the Space of Events – Infrared Divergences

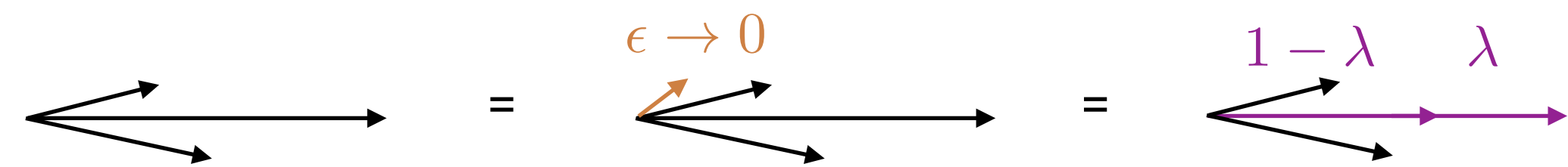
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Energy flow is unchanged by exact soft/collinear emissions



Functions of energy flow automatically satisfy exact IRC invariance!

Real and virtual divergences appear naturally together

$$\mathcal{P}_N \supset \mathcal{P}_{N-1} \supset \cdots \supset \mathcal{P}_3 \supset \mathcal{P}_2 \supset \mathcal{P}_1$$

by soft and collinear limits

Defining IRC Safety Precisely

[Sterman, Weinberg, [PRL 1997](#); Sterman, [PRD 1978](#); Banfi, Salam, Zanderighi, [JHEP 2005](#)]

Infrared and collinear safety is a proxy for perturbative calculability of an observable

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Exact IRC invariance

$$\mathcal{O}(p_1^\mu, \dots, p_M^\mu) = \mathcal{O}(0p_0^\mu, p_1^\mu, \dots, p_M^\mu)$$

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Guarantees observable is well-defined on energy flows

Allows for pathological observables, e.g. pseudo-multiplicity

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Eliminates common observables with hard boundaries

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Eliminates common observables with hard boundaries

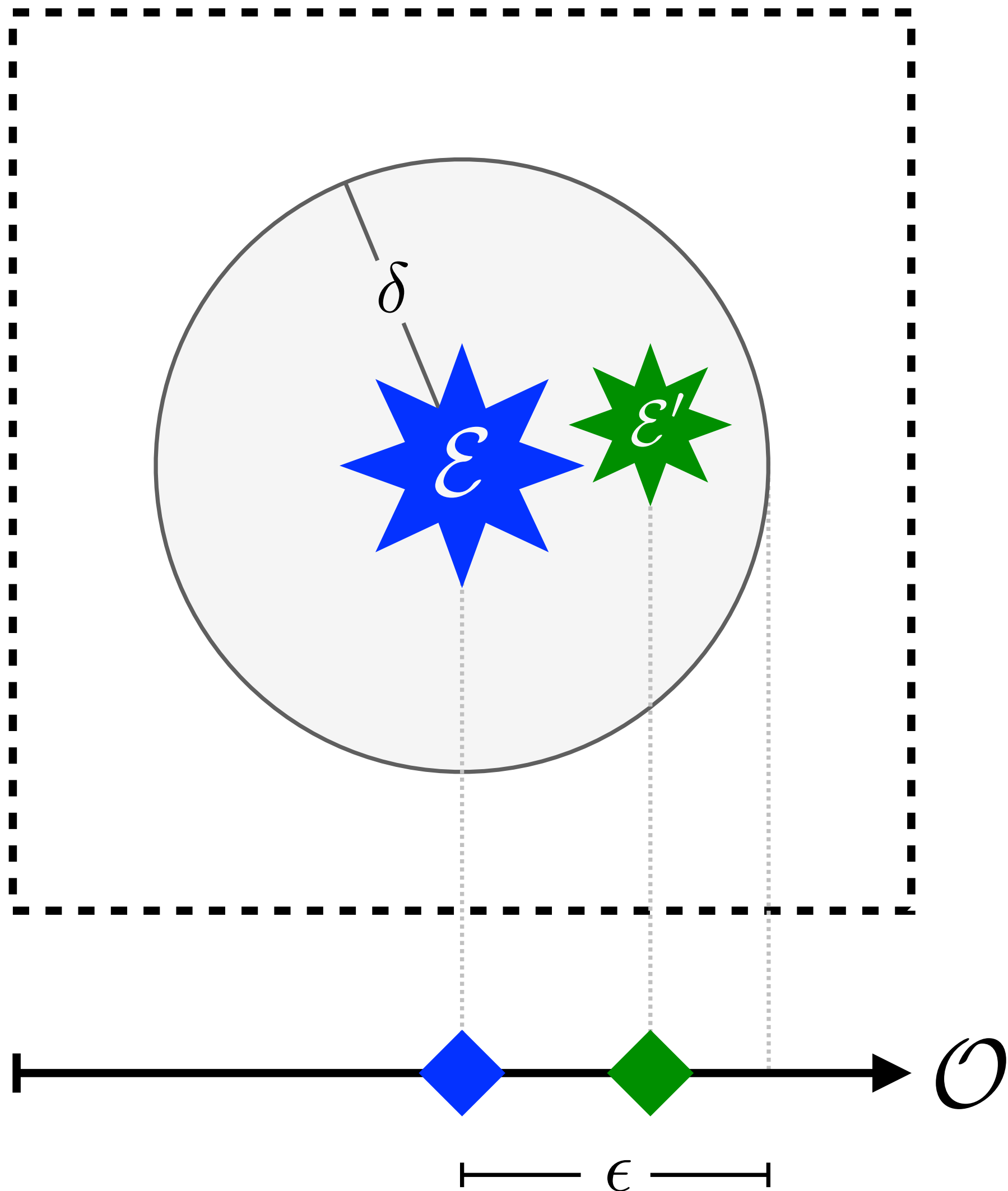
All Observables	Comments
Multiplicity ($\sum_i 1$)	IR unsafe and C unsafe
Momentum Dispersion [65] ($\sum_i E_i^2$)	IR safe but C unsafe
Sphericity Tensor [66] ($\sum_i p_i^\mu p_i^\nu$)	IR safe but C unsafe
Number of Non-Zero Calorimeter Deposits	C safe but IR unsafe

Defined on Energy Flows	
Pseudo-Multiplicity ($\min\{N \mid \mathcal{T}_N = 0\}$)	Robust to exact IR or C emissions

Infrared & Collinear Safe	
Jet Energy ($\sum_i E_i$)	Disc. at jet boundary
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Soft-Dropped Jet Mass [38, 68]	Disc. at grooming threshold
Calorimeter Activity [69] (N_{95})	Disc. at cell boundary

More EMD Geometry – Continuity in the Space of Events

[PTK, Metodiev, Thaler, 2004.04159]



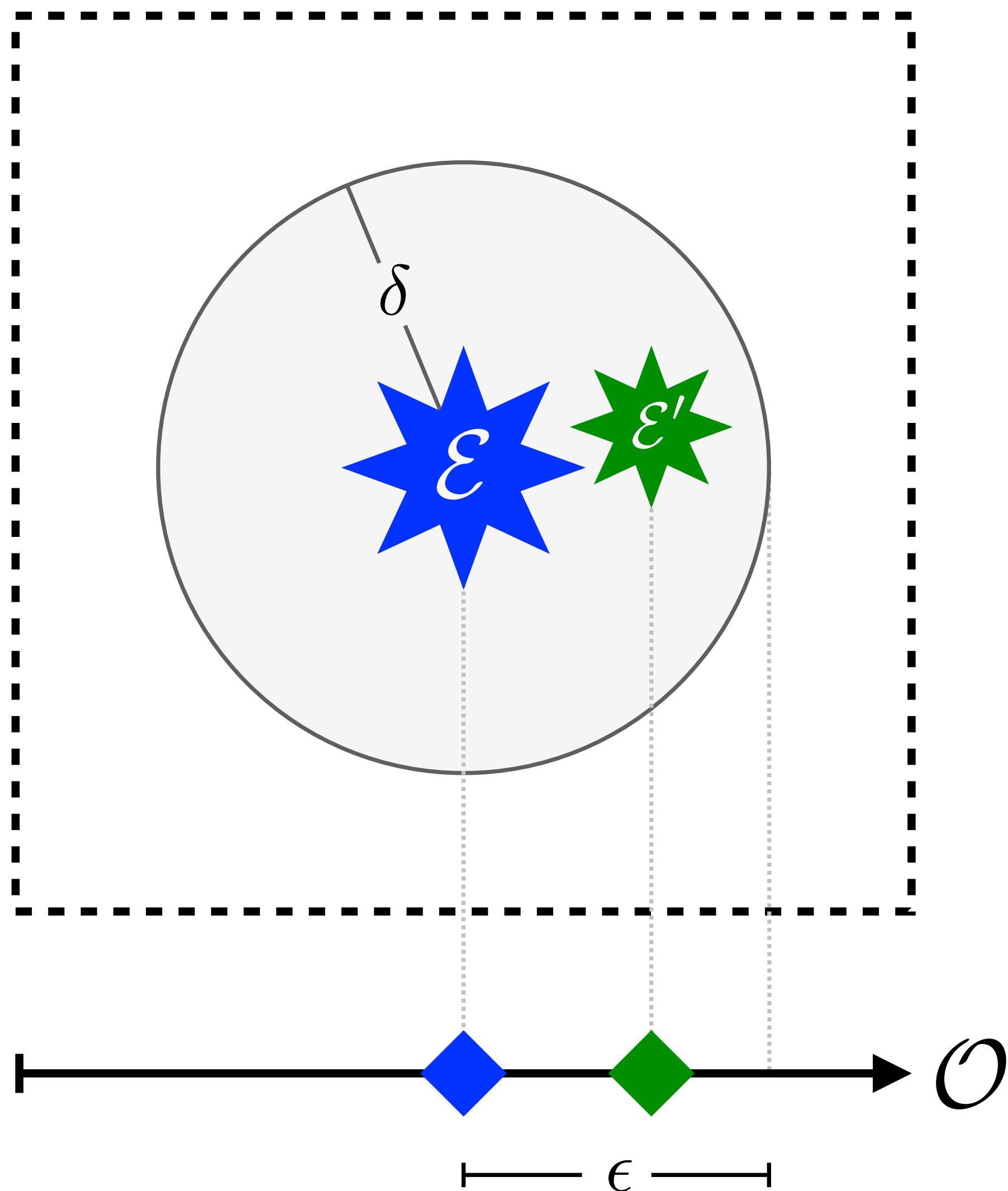
Classic $\epsilon - \delta$ definition of continuity in a metric space

An observable \mathcal{O} is **EMD continuous** at an event \mathcal{E} if, for any $\epsilon > 0$, there exists a $\delta > 0$ such that for all events \mathcal{E}' :

$$\text{EMD}(\mathcal{E}, \mathcal{E}') < \delta \implies |\mathcal{O}(\mathcal{E}) - \mathcal{O}(\mathcal{E}')| < \epsilon.$$

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Towards a geometric definition of **IRC** Safety

$$\text{IRC Safety} = \text{EMD Continuity}^*$$

*on all but a negligible set[‡] of events

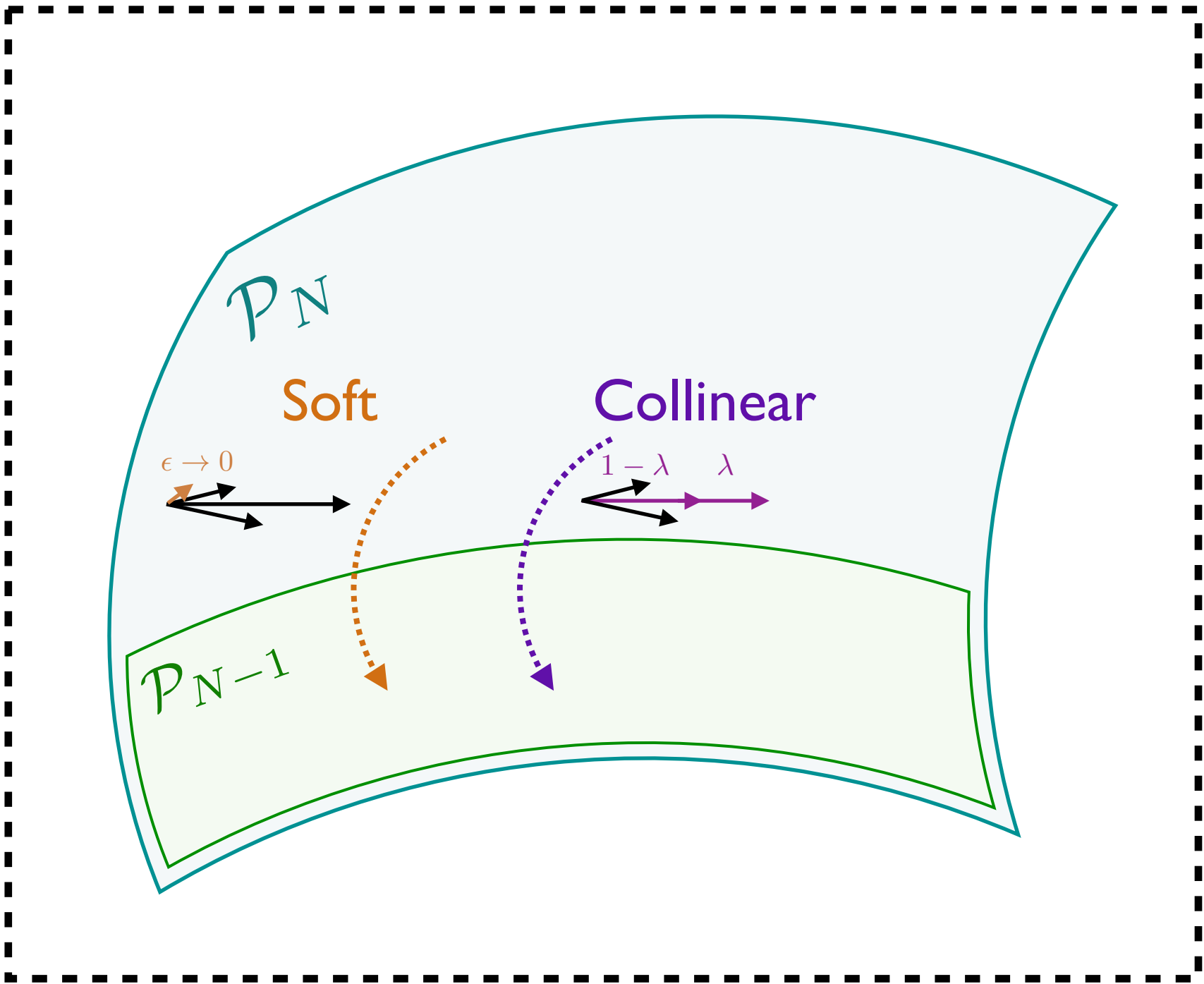
[‡]a negligible set is one that contains no positive-radius EMD-ball

⋮

Perturbation Theory in the Space of Events

[PTK, Metodiev, Thaler, [2004.04159](#)]

Infrared singularities of massless gauge theories appear on each \mathcal{P}_N



Perturbation Theory in the Space of Events

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Sudakov safety

[Larkoski, Thaler, [JHEP 2014](#); Larkoski, Marzani, Thaler, [PRD 2015](#)]

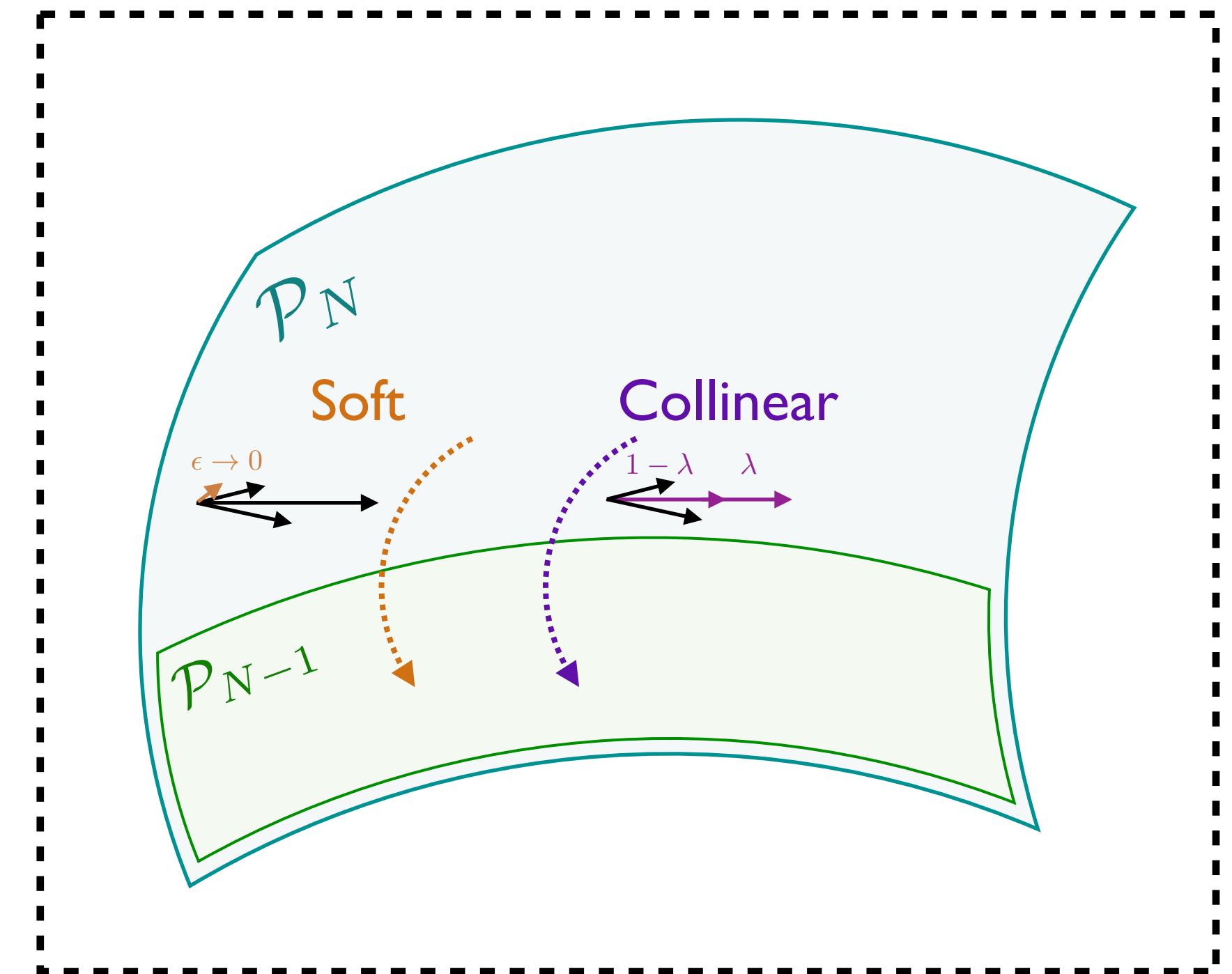
Some observables have discontinuities on \mathcal{P}_N for some N

A resummed IRC-safe companion can mitigate the divergences

$$p(\mathcal{O}_{\text{Sudakov}}) = \int d\mathcal{O}_{\text{Comp.}} p(\mathcal{O}_{\text{Sudakov}} | \mathcal{O}_{\text{Comp.}}) p(\mathcal{O}_{\text{Comp.}})$$

Event geometry suggests N -(sub)jettiness as universal companion

Infrared singularities of massless gauge theories appear on each \mathcal{P}_N



Perturbation Theory in the Space of Events

[PTK, Metodiev, Thaler, [2004.04159](#)]

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Fixed-order calculability

[Sterman, [PRD 1979](#); Banfi, Salam, Zanderighi, [JHEP 2005](#)]

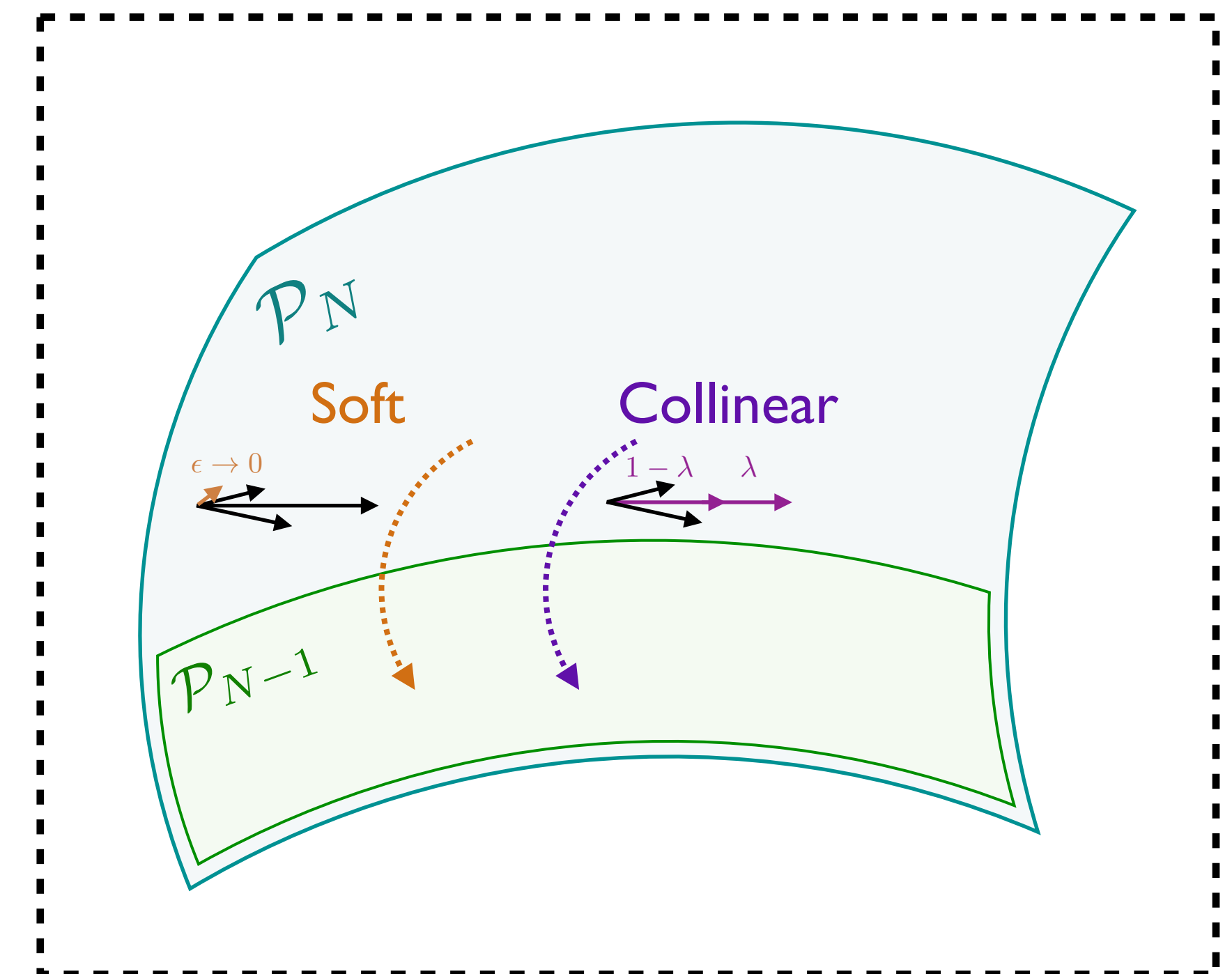
Is a statement of integrability on each \mathcal{P}_N

EMD continuity must be upgraded to EMD-Hölder continuity on each \mathcal{P}_N

$$\lim_{\mathcal{E} \rightarrow \mathcal{E}'} \frac{\mathcal{O}(\mathcal{E}) - \mathcal{O}(\mathcal{E}')}{\text{EMD}(\mathcal{E}, \mathcal{E}')^c} = 0, \quad c > 0$$

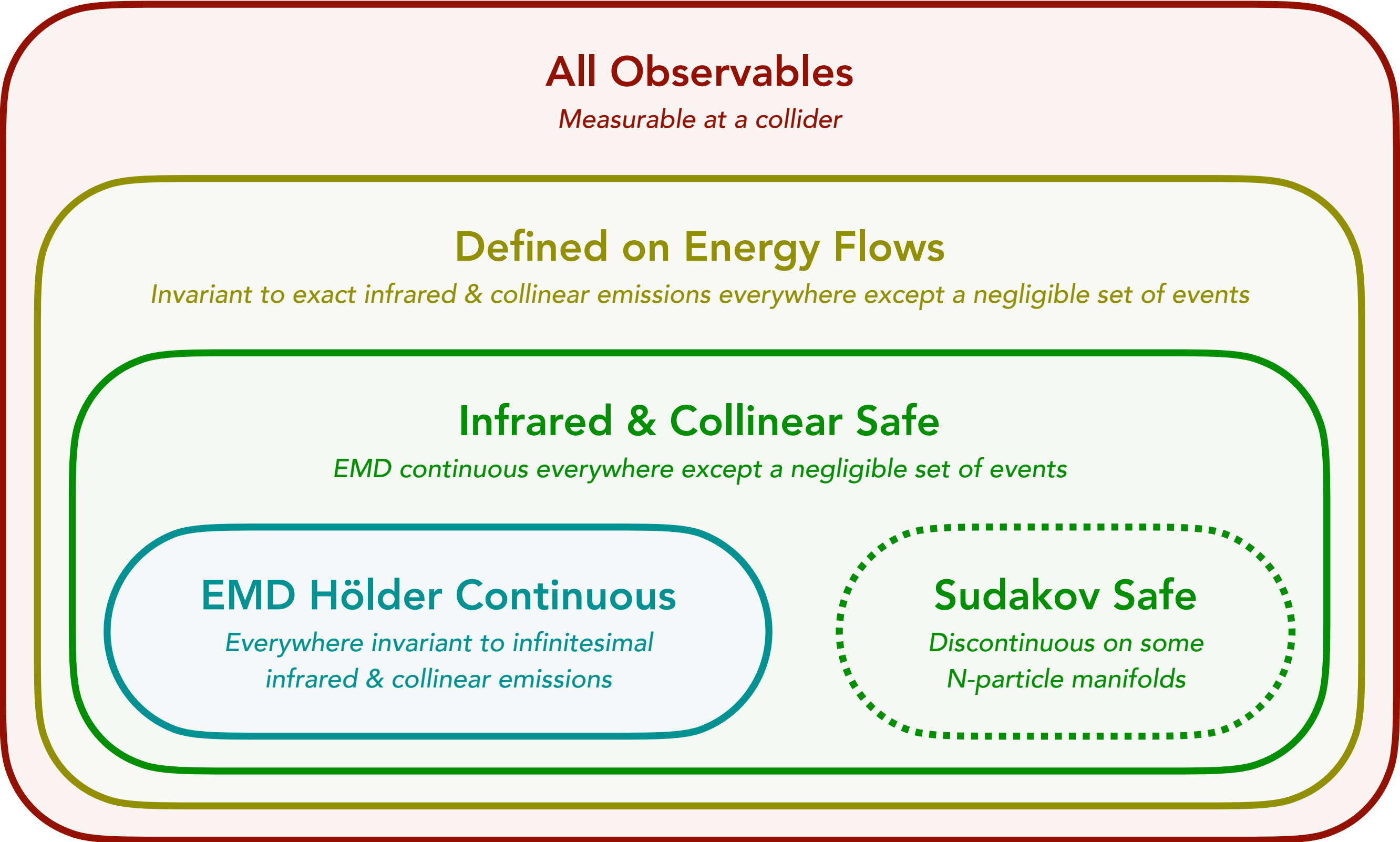
Example: $V(\mathcal{E}) = \mathcal{T}_2(\mathcal{E}) \left(1 + \frac{1}{\ln E(\mathcal{E})/\mathcal{T}_3(\mathcal{E})} \right)$ is EMD continuous but not EMD Hölder continuous (it is Sudakov safe)

Infrared singularities of massless gauge theories appear on each \mathcal{P}_N



Hierarchy of IRC Safety Definitions

[PTK, Metodiev, Thaler, 2004.04159]



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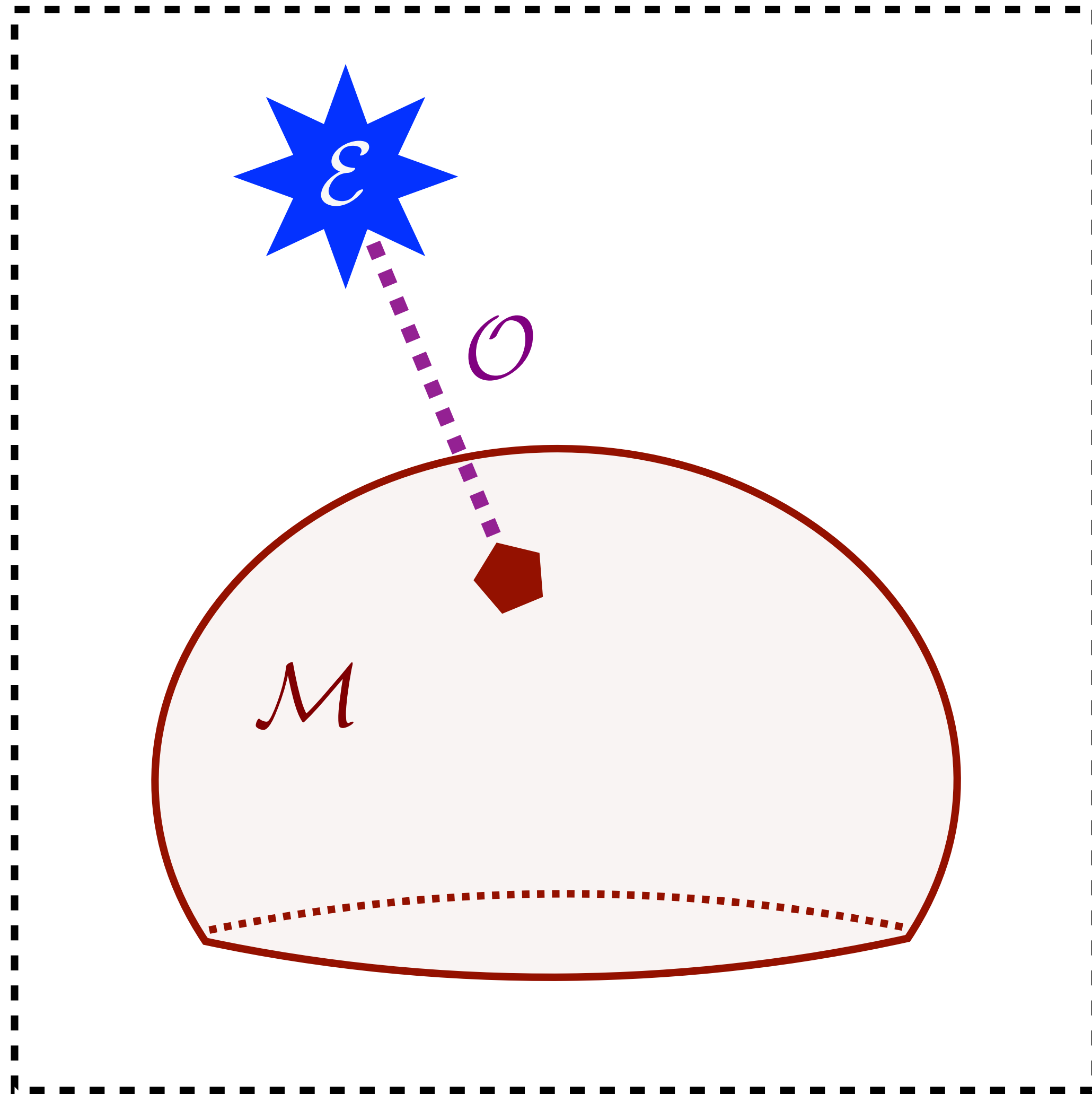
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Sudakov Safe	
Groomed Momentum Fraction [39] (z_g)	Disc. on 1-particle manifold
Jet Angularity Ratios [37]	Disc. on 1-particle manifold
N-subjettiness Ratios [47, 48] (τ_{N+1}/τ_N)	Disc. on N-particle manifold
V parameter [36] (Eq. (2.11))	Hölder disc. on 3-particle manifold

EMD Hölder Continuous Everywhere	
Thrust [40, 41]	
Sphericity [42]	
Angularities [70]	
N-jettiness [44] (\mathcal{T}_N)	
C parameter [71–74]	Resummation beneficial at $C = \frac{3}{4}$
Linear Sphericity [72] ($\sum_i E_i n_i^\mu n_i^\nu$)	
Energy Correlators [36, 75–77]	
Energy Flow Polynomials [15, 17]	

Defining Observables via Event Space Geometry

[PTK, Metodiev, Thaler, 2004.04159]

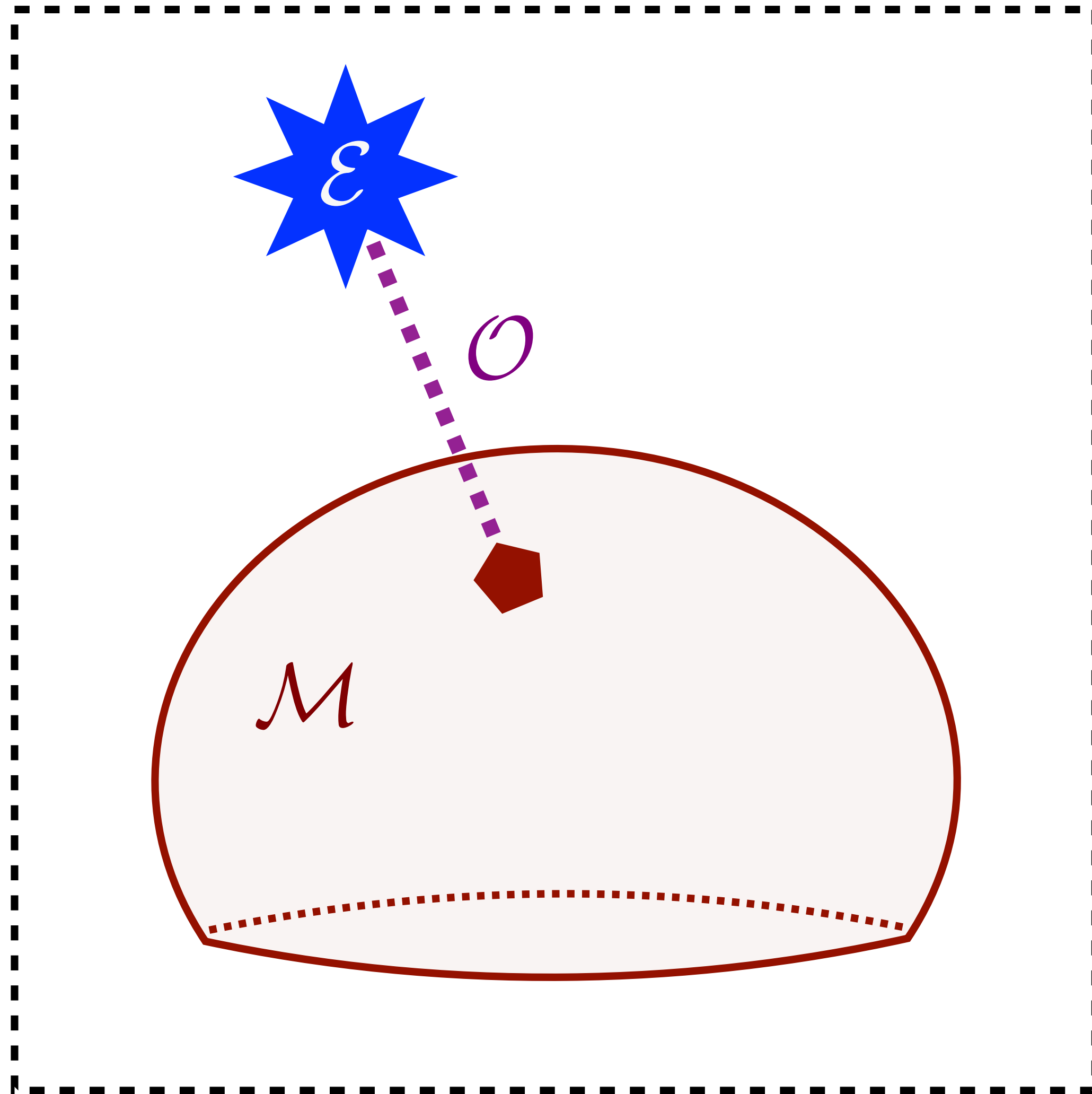


Many common *observables* are distance of closest approach from event to a specific *manifold*

$$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \text{EMD}_{\beta, R}(\mathcal{E}, \mathcal{E}')$$

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[PTK, Metodiev, Thaler, 2004.04159]



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$$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \text{EMD}_{\beta, R}(\mathcal{E}, \mathcal{E}')$$

EMD variant for equal-energy events

$$\text{EMD}_{\beta}(\mathcal{E}, \mathcal{E}') = \lim_{R \rightarrow \infty} R^{\beta} \text{EMD}_{\beta, R}(\mathcal{E}, \mathcal{E}') = \min_{\{f_{ij} \geq 0\}} \sum_{i=1}^M \sum_{j=1}^{M'} f_{ij} \theta_{ij}^{\beta}$$

Enforces equal energy (else infinity)
on equal-energy events

Defining Observables via Event Space Geometry

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[PTK, Metodiev, Thaler, [2004.04159](#)]

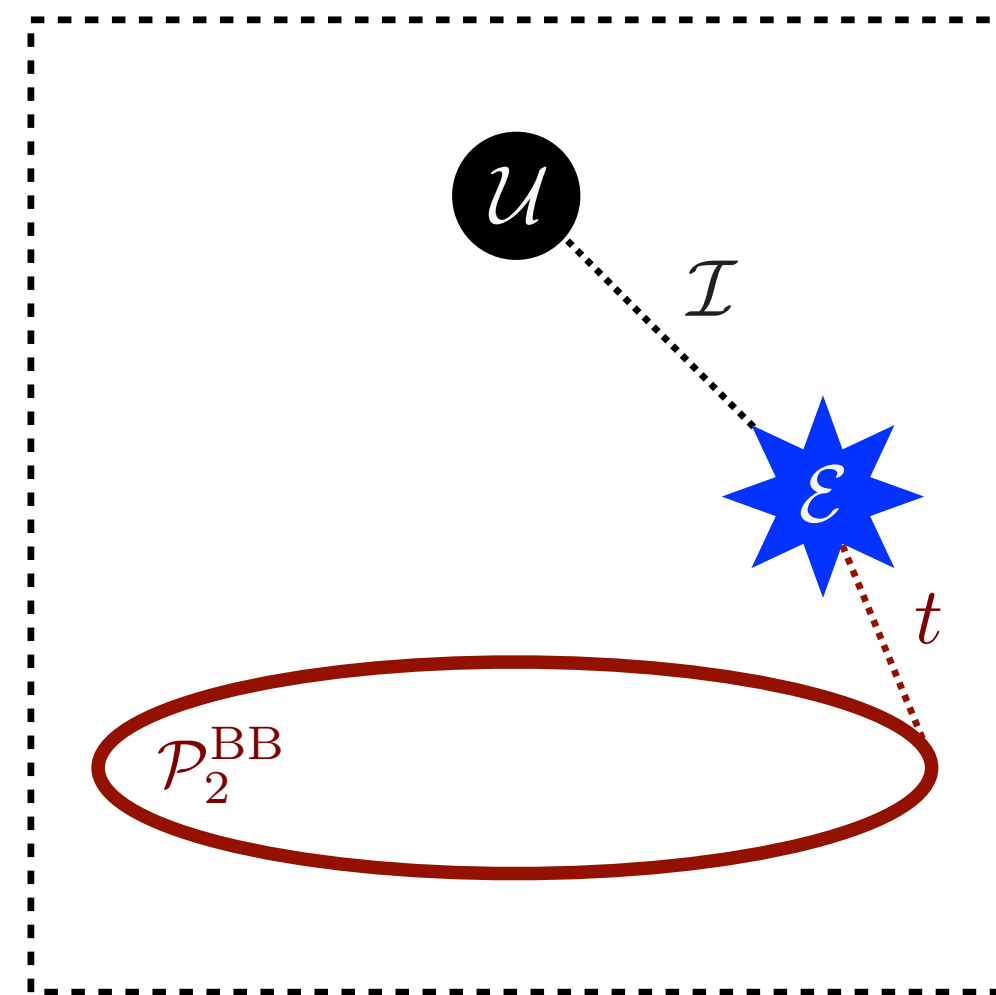
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[PTK, Metodiev, Thaler, 2004.04159]

Thrust, sphericity, isotropy*

*Distance of closest approach
to a specific manifold*



$$t(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{P}_2^{\text{BB}}} \text{EMD}_2(\mathcal{E}, \mathcal{E}')$$

$$\sqrt{s(\mathcal{E})} = \min_{\mathcal{E}' \in \mathcal{P}_2^{\text{BB}}} \text{EMD}_1(\mathcal{E}, \mathcal{E}')$$

$$\mathcal{I}^{(\beta)}(\mathcal{E}) = \min_{\mathcal{E}' \in M_{\mathcal{U}}} \text{EMD}_{\beta}(\mathcal{E}, \mathcal{E}')$$

[Farhi, PRL 1977; Georgi, Machacek, PRL 1977]

*New! [Cesarotti, Thaler, 2004.06125]

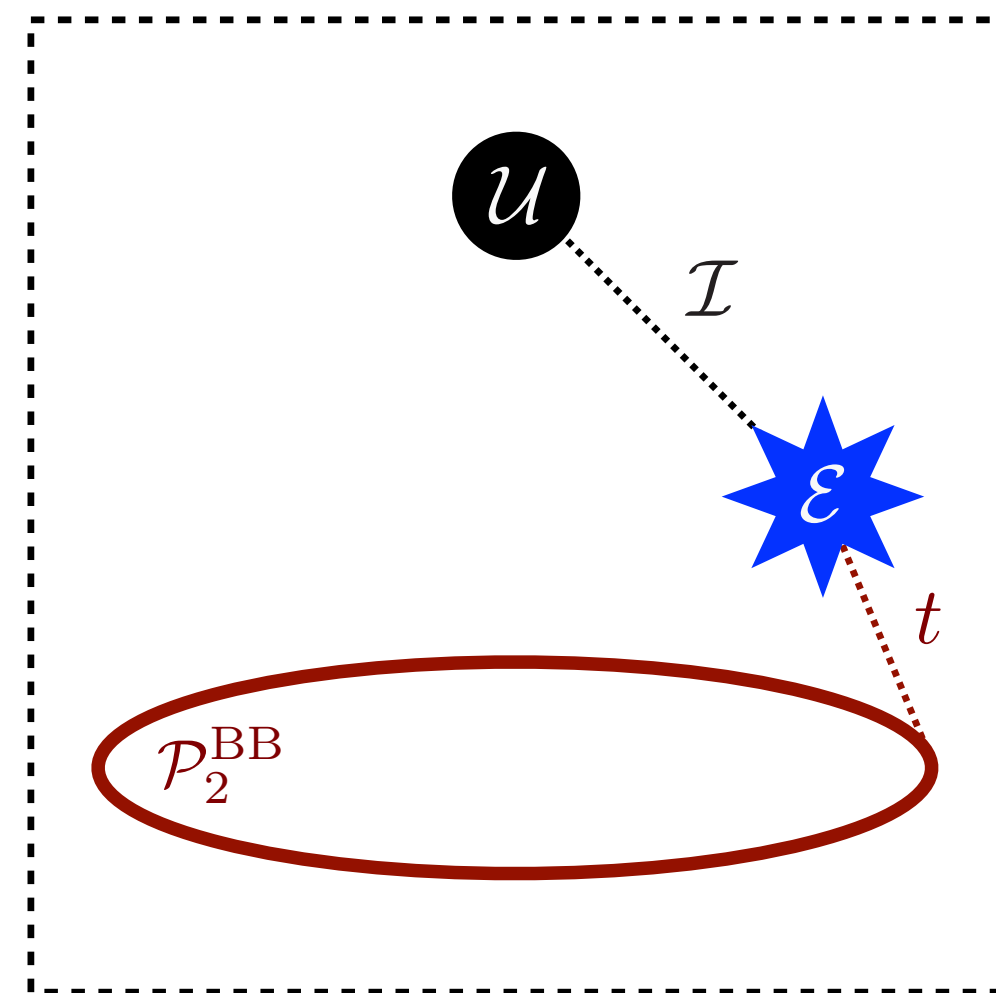
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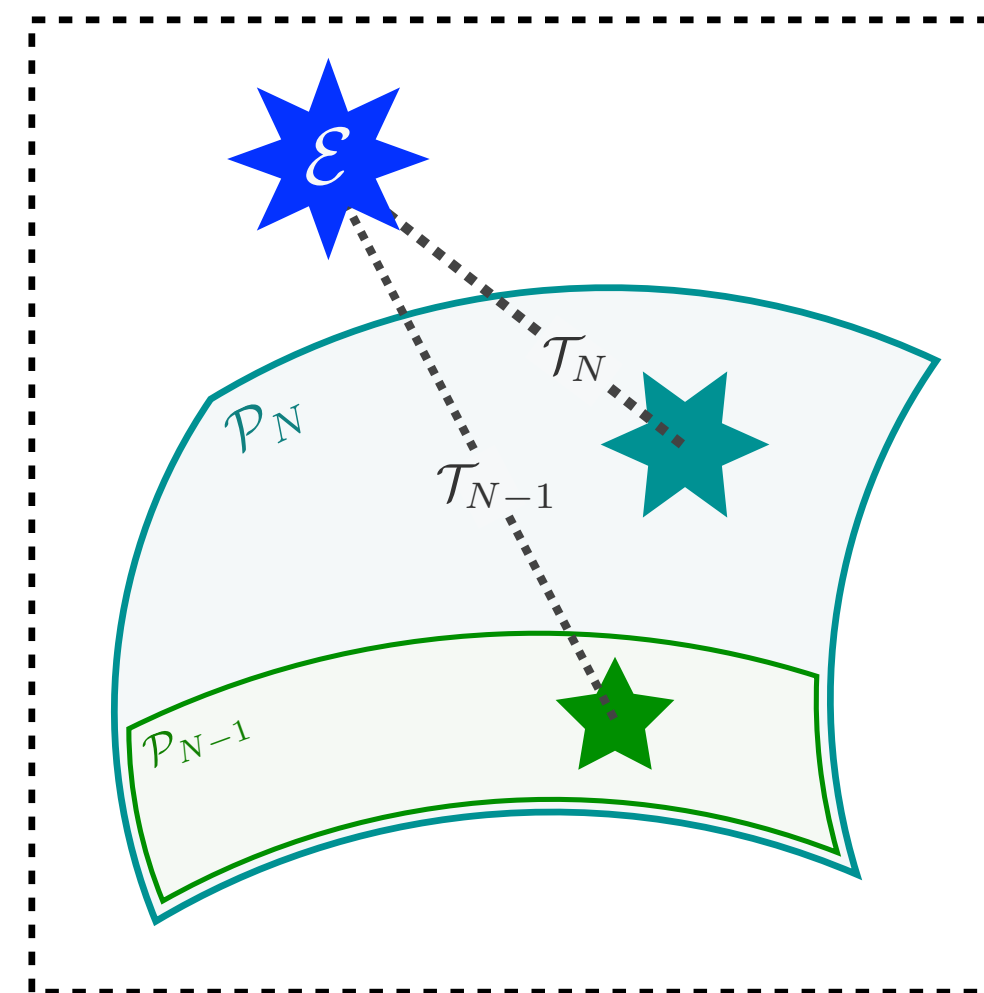
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[Farhi, PRL 1977; Georgi, Machacek, PRL 1977]

*New! [Cesarotti, Thaler, 2004.06125]

N-jettiness

*Minimum distance from event
to N-particle manifold*



without beam region

$$\mathcal{T}_N^{(\beta)}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{P}_N} \text{EMD}_{\beta}(\mathcal{E}, \mathcal{E}')$$

with constant beam distance R^{β}

$$\mathcal{T}_N^{(\beta, R)}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{P}_N} \text{EMD}_{\beta, R}(\mathcal{E}, \mathcal{E}')$$

[Brandt, Dahmen, Z. Phys 1979;

Stewart, Tackmann, Waalewijn, PRL 2010]

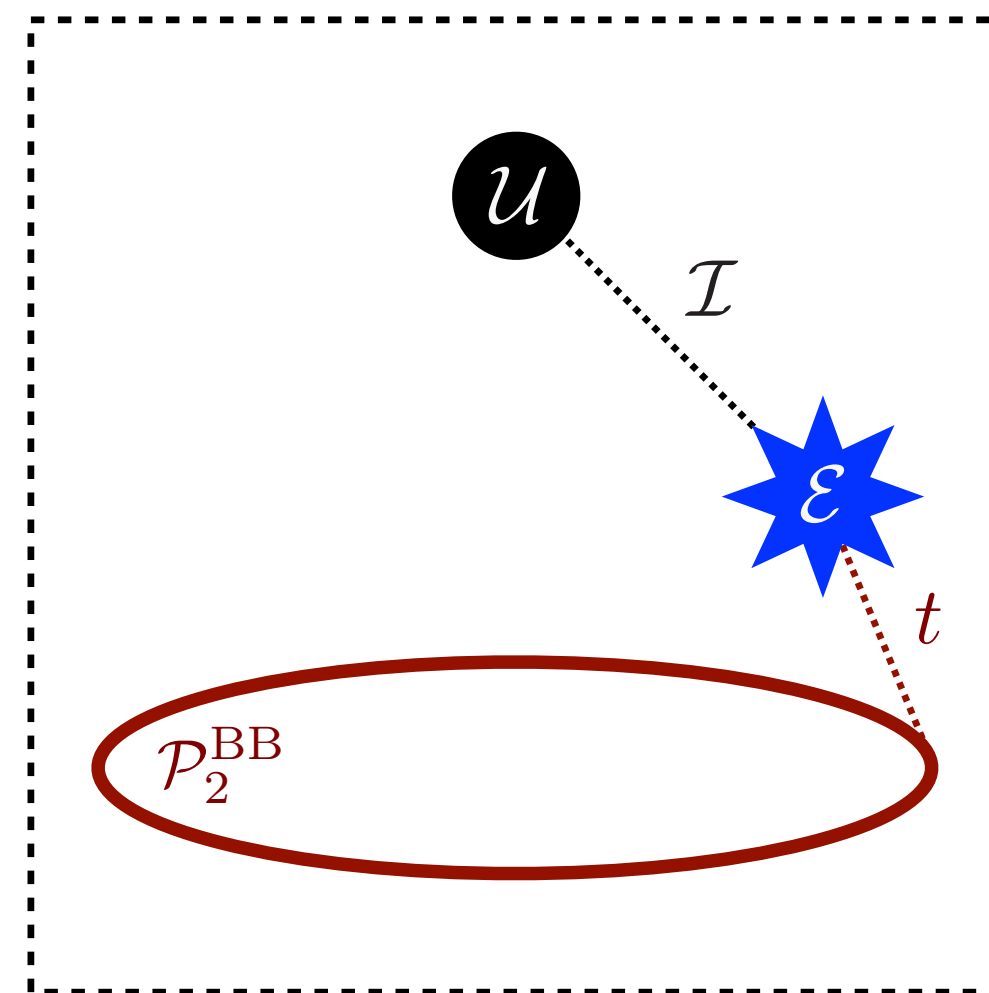
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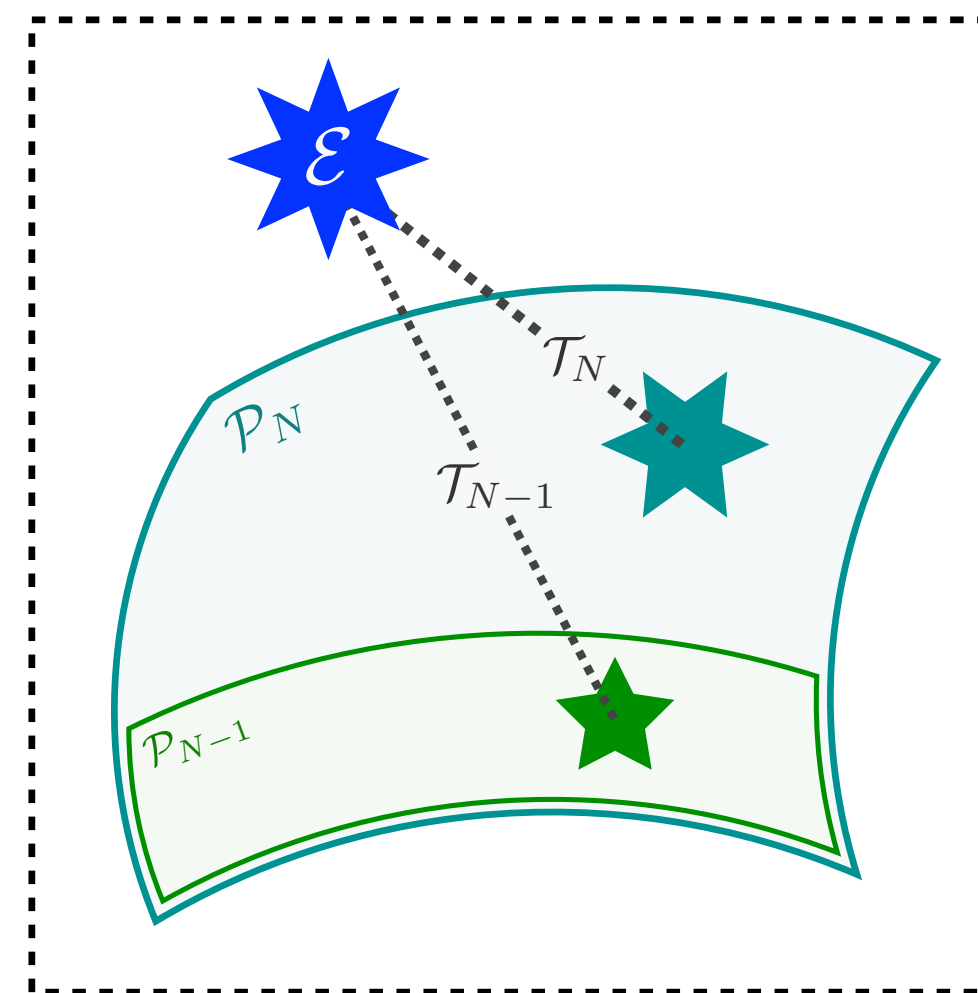
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N-jettiness

*Minimum distance from event
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with constant beam distance R^{β}

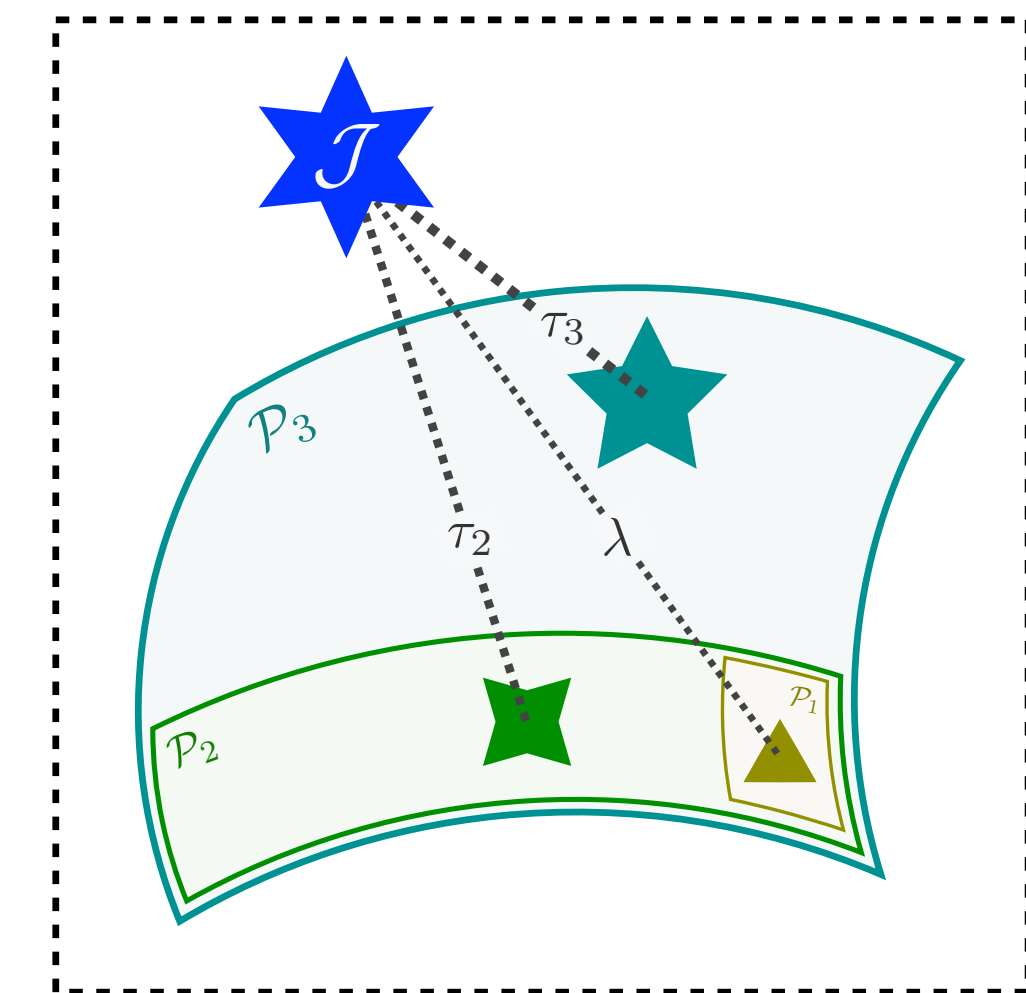
$$\mathcal{T}_N^{(\beta,R)}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{P}_N} \text{EMD}_{\beta,R}(\mathcal{E}, \mathcal{E}')$$

[Brandt, Dahmen, Z. Phys 1979;

Stewart, Tackmann, Waalewijn, PRL 2010]

N-subjettiness, angularities

*Smallest distance from jet to
N-particle manifold*



for recoil-free angularity

$$\lambda_{\beta}(\mathcal{J}) = \min_{\mathcal{J}' \in \mathcal{P}_1} \text{EMD}_{\beta}(\mathcal{J}, \mathcal{J}')$$

$$\tau_N^{(\beta)}(\mathcal{J}) = \min_{\mathcal{J}' \in \mathcal{P}_N} \text{EMD}_{\beta}(\mathcal{J}, \mathcal{J}')$$

[Ellis, Vermilion, Walsh, Hornig, Lee, JHEP 2010;

Thaler, Van Tilburg, JHEP 2011, JHEP 2012]

Jets in the Space of Events – The Closest N -particle Description of an M -particle Event

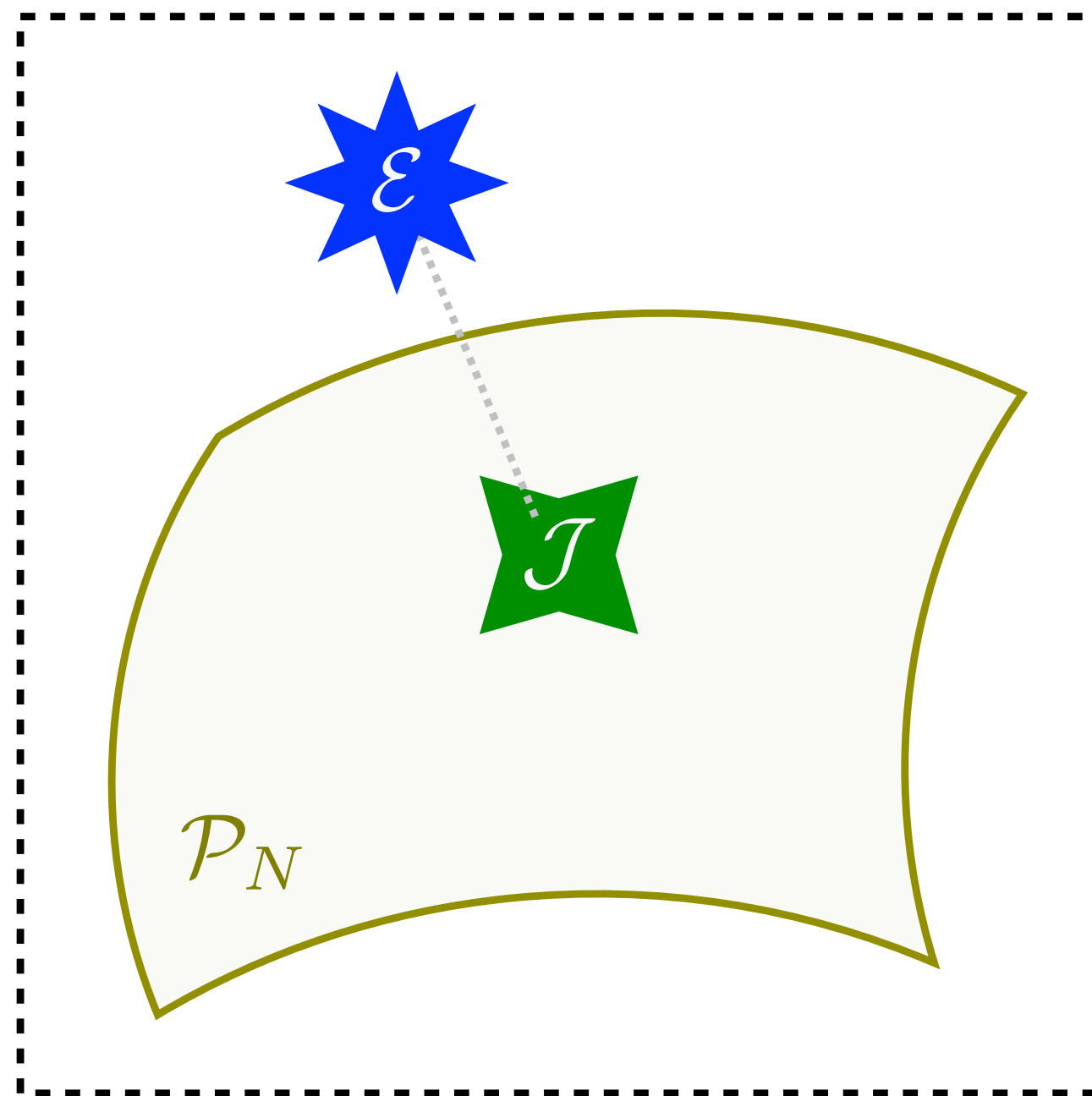
[PTK, Metodiev, Thaler, [2004.04159](#)]

Jets in the Space of Events – The Closest N -particle Description of an M -particle Event

[PTK, Metodiev, Thaler, [2004.04159](#)]

Exclusive cone finding

*XCone finds N jets by
minimizing N -jettiness*



$$\mathcal{J}_{N,\beta,R}^{\text{XCone}}(\mathcal{E}) = \arg \min_{\mathcal{J} \in \mathcal{P}_N} \text{EMD}_{\beta,R}(\mathcal{E}, \mathcal{J})$$

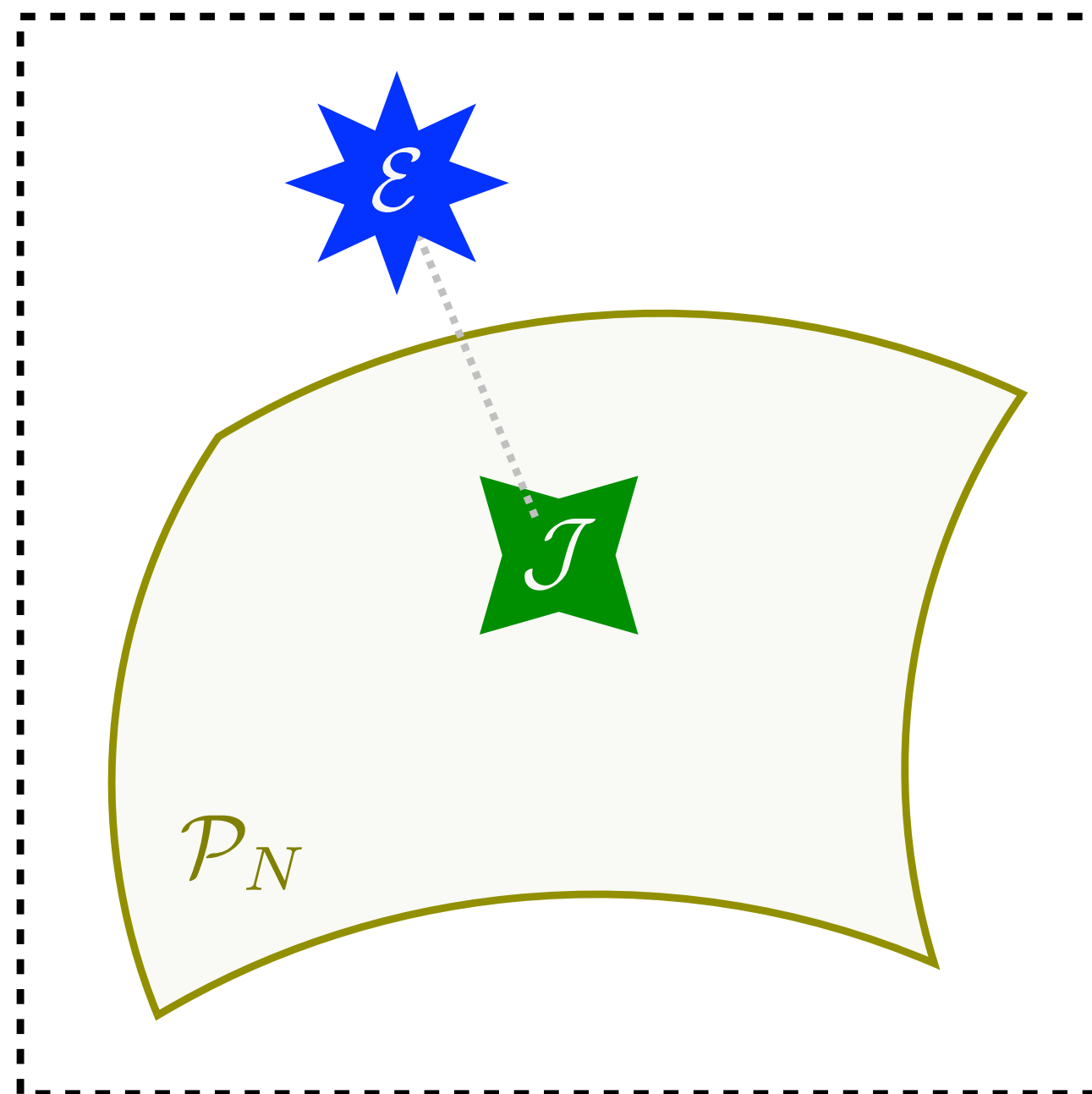
[Stewart, Tackmann, Thaler, Vermilion, Wilkason, [JHEP 2015](#);
Thaler, Wilkason, [JHEP 2015](#)]

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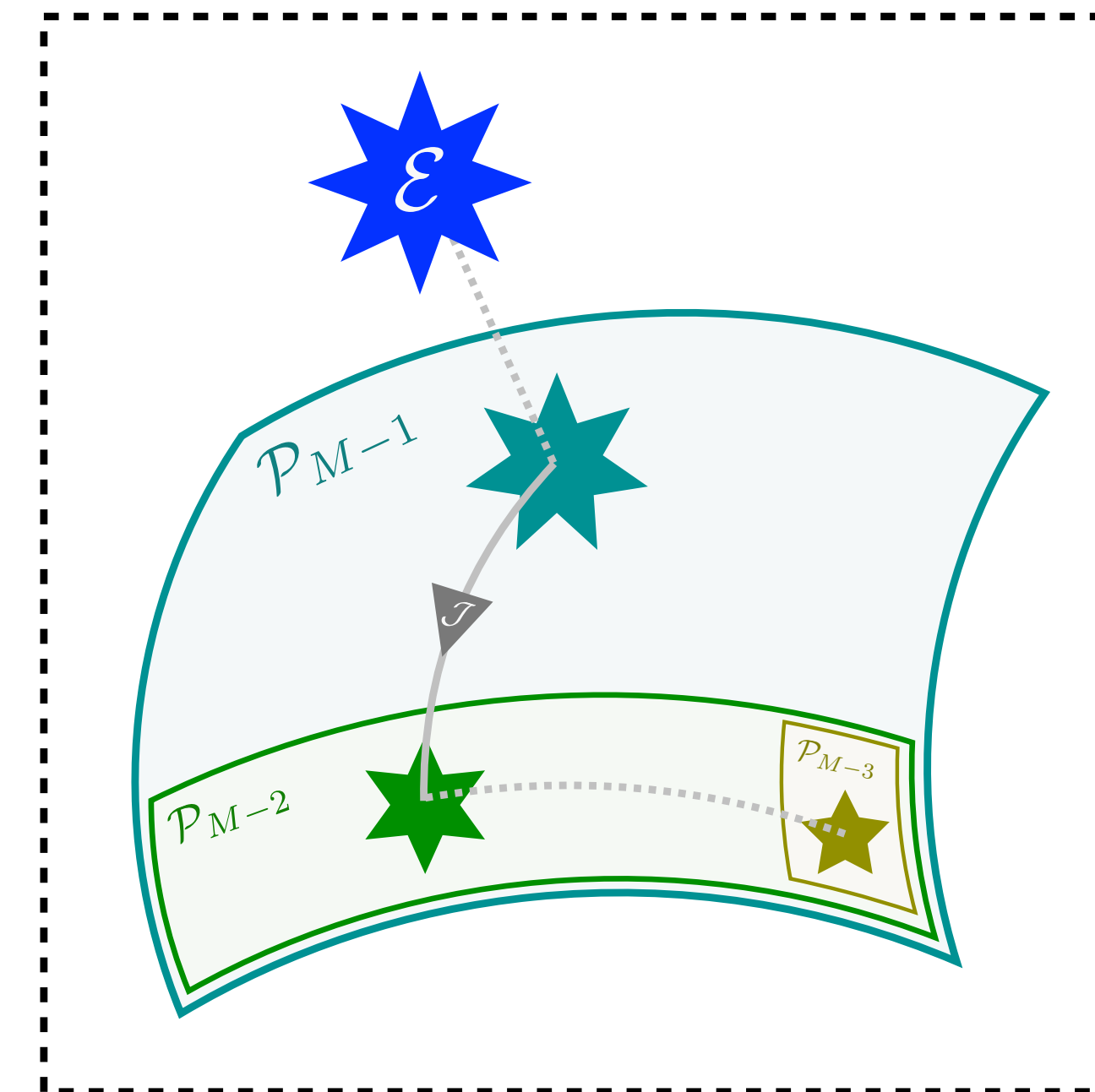


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[Stewart, Tackmann, Thaler, Vermilion, Wilkason, [JHEP 2015](#);
Thaler, Wilkason, [JHEP 2015](#)]

Sequential recombination

Iteratively merges particles or identifies a jet



“destroying” energy
corresponds to identifying a jet

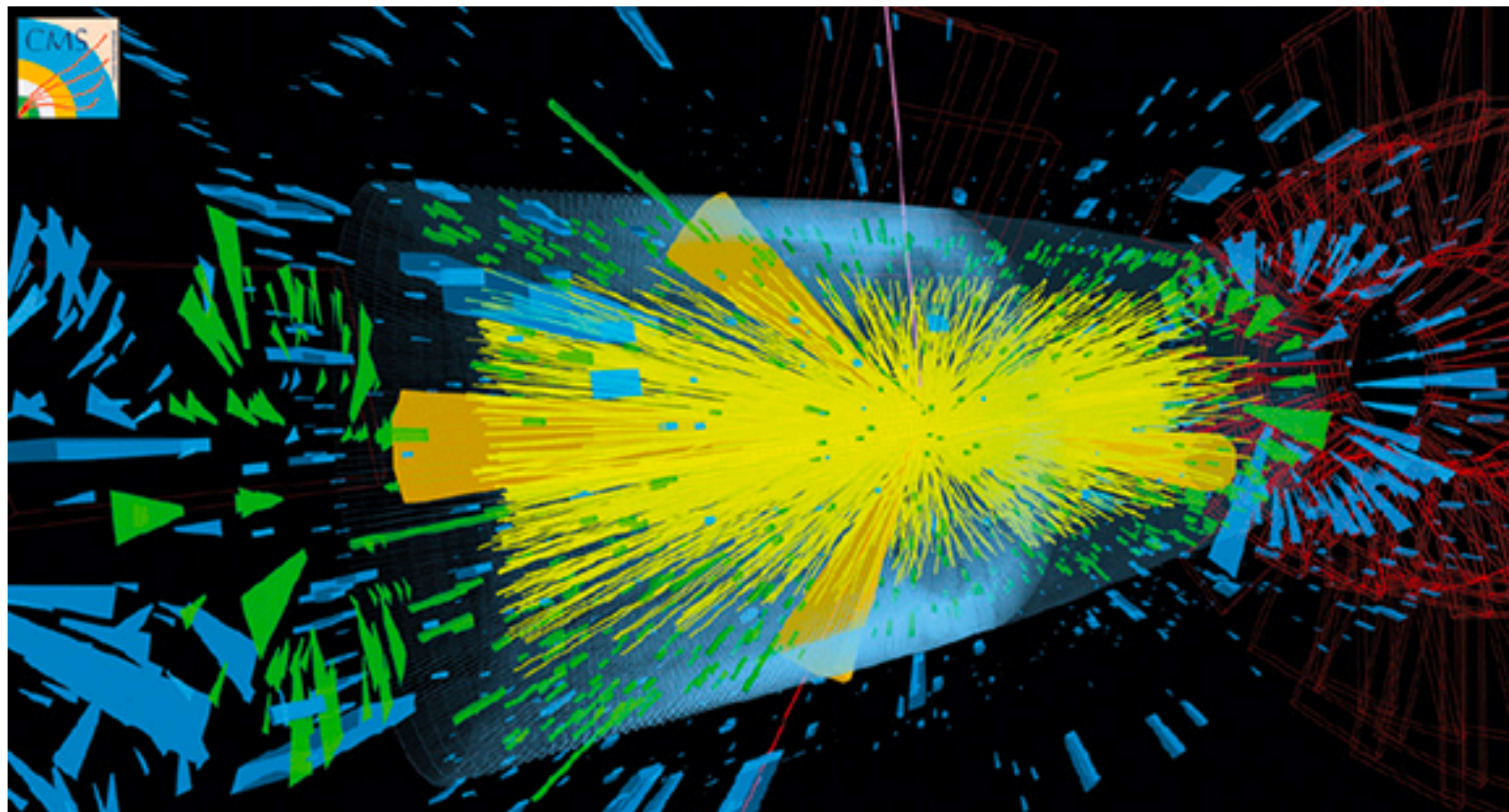
event with one fewer particle after one step

$$\mathcal{E}_{M-1}^{(\beta,R)}(\mathcal{E}_M) = \arg \min_{\mathcal{E}'_{M-1} \in \mathcal{P}_{M-1}} \text{EMD}_{\beta,R}(\mathcal{E}_M, \mathcal{E}'_{M-1})$$

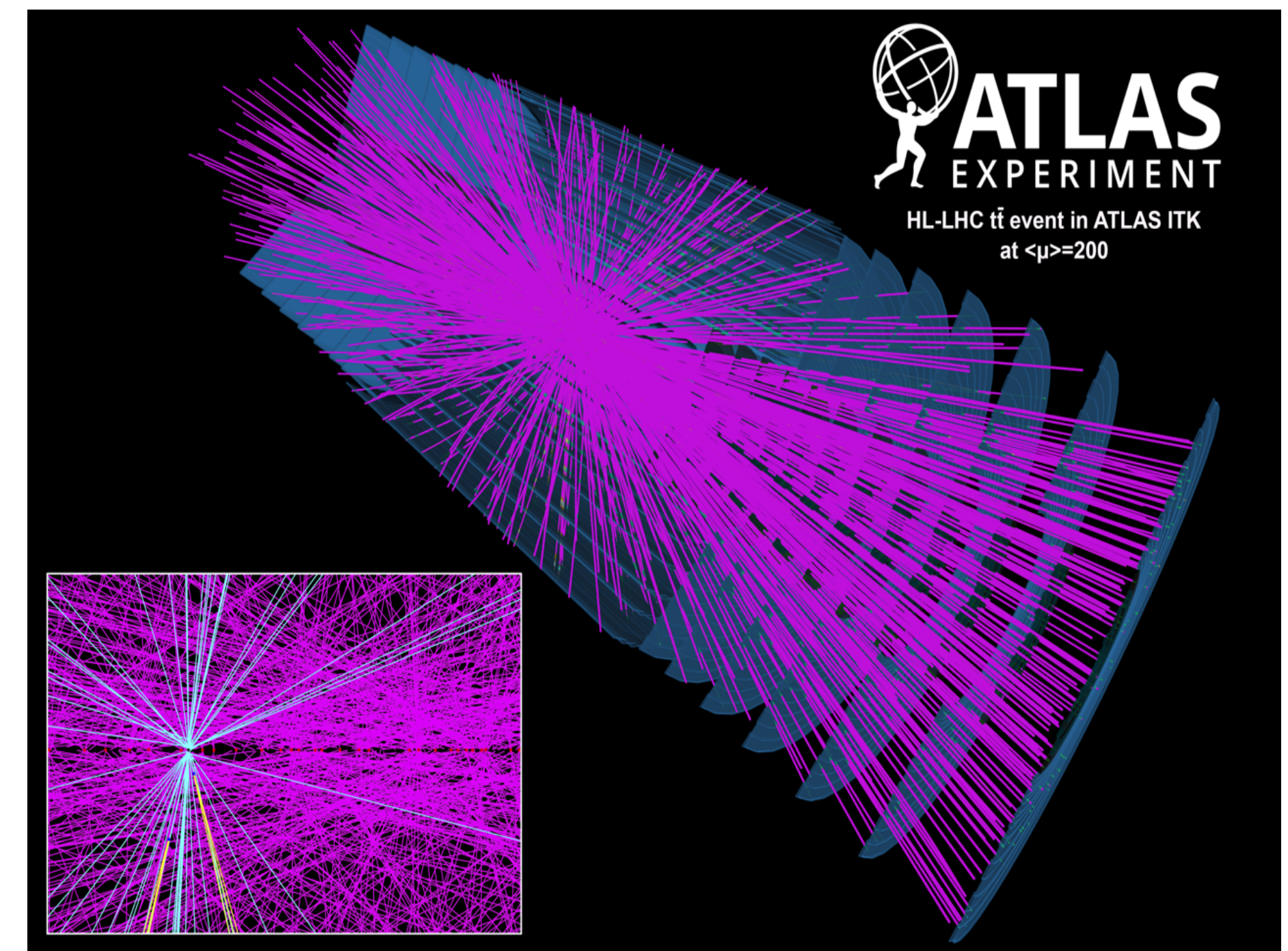
[Catani, Dokshitzer, Seymour, Webber, [Nucl. Phys. B 1993](#);
Ellis, Soper, [PRD 1993](#);
Dokshitzer, Leder, Moretti, Webber, [JHEP 1997](#);
Cacciari, Salam, Soyez, [JHEP 2008](#)]

Pileup at the (HL-)LHC

Pileup is uniform (on average) radiation from additional proton-proton collisions



VBF Higgs + 200 pileup vertices

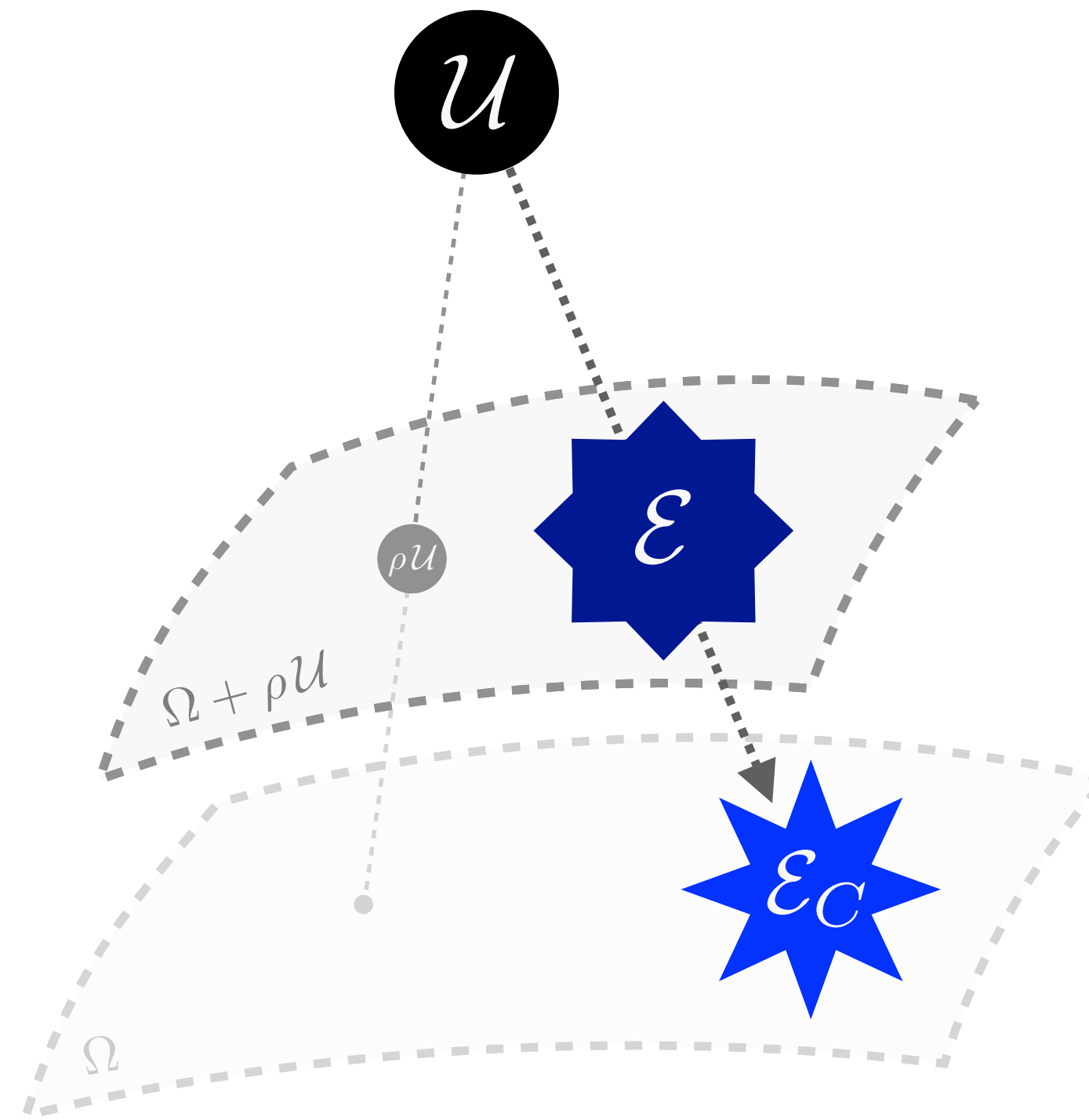


$t\bar{t}$ + 200 pileup vertices

Pileup Mitigation in Event Space

Pileup: uniform (on average) radiation from additional proton-proton collisions

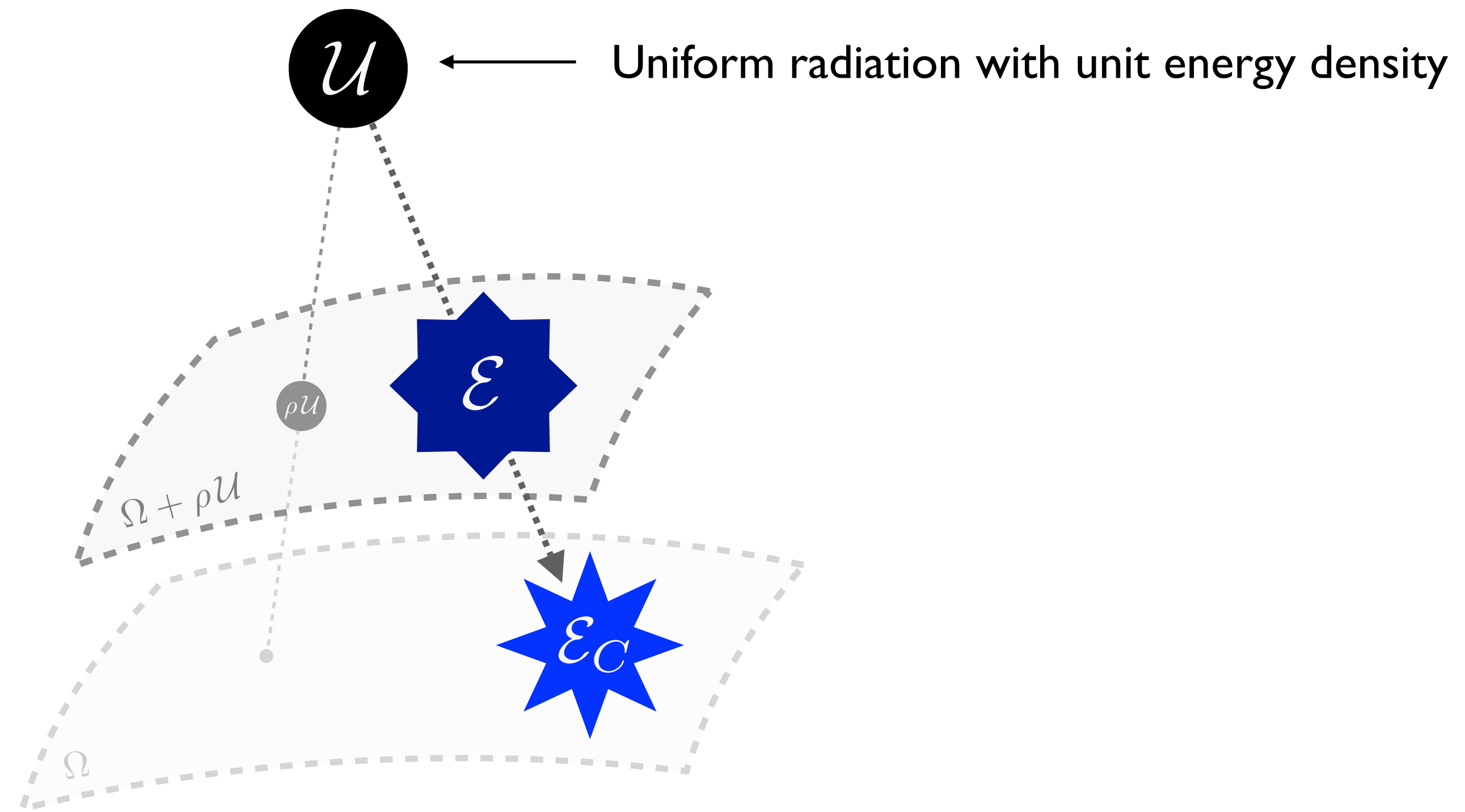
Pileup mitigation: “moving away” from the uniform event



Pileup Mitigation in Event Space

Pileup: uniform (on average) radiation from additional proton-proton collisions

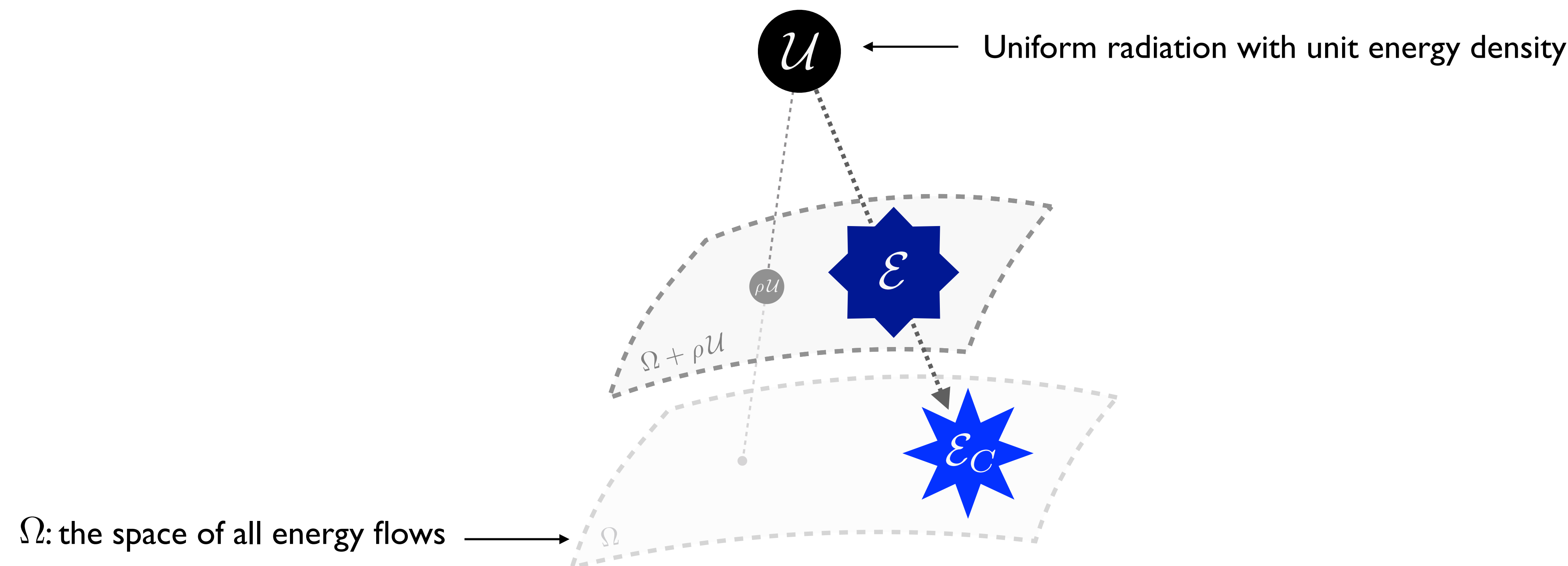
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Pileup Mitigation in Event Space

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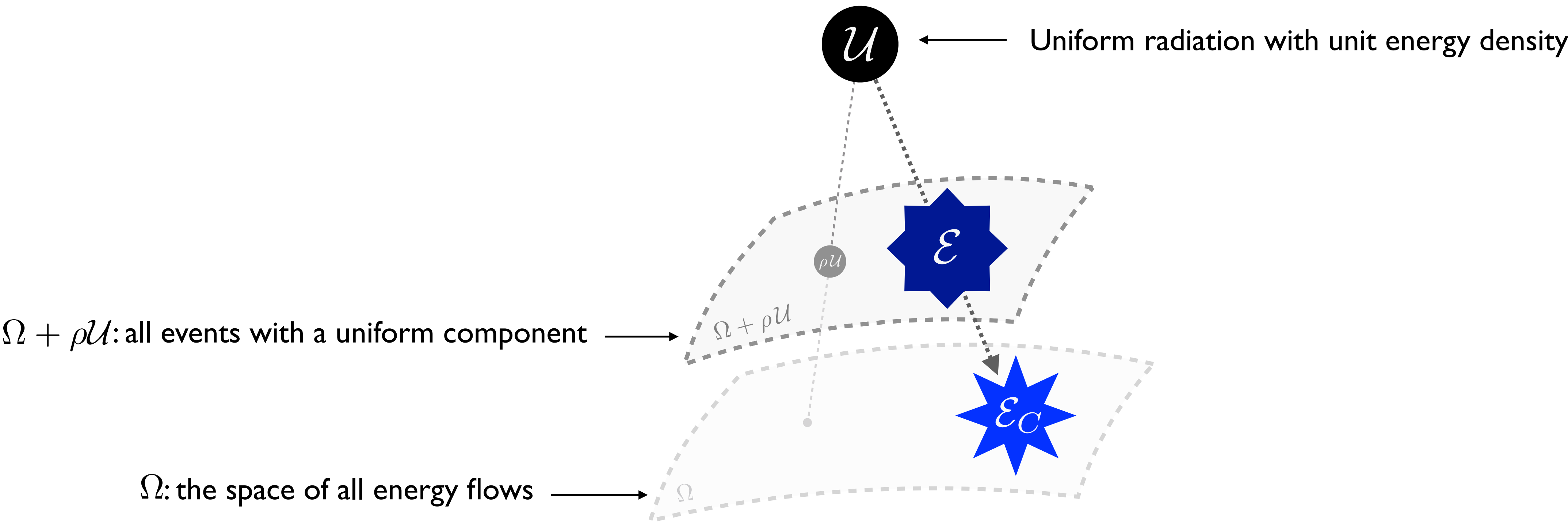
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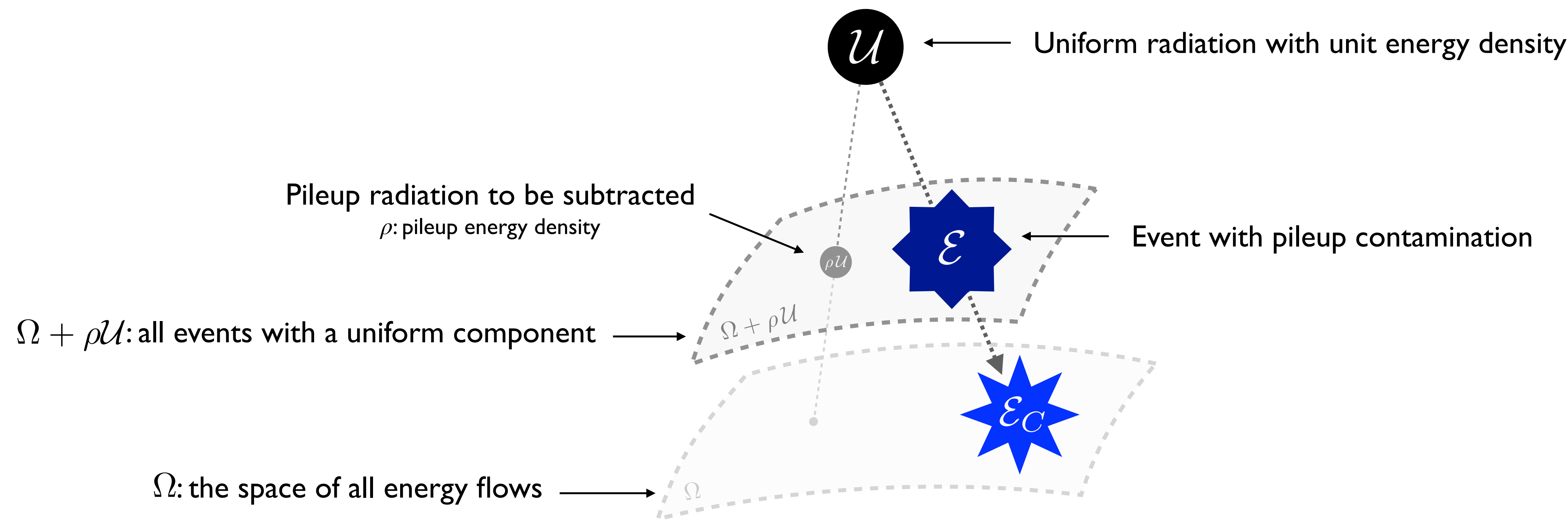
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Pileup Mitigation in Event Space

Pileup: uniform (on average) radiation from additional proton-proton collisions

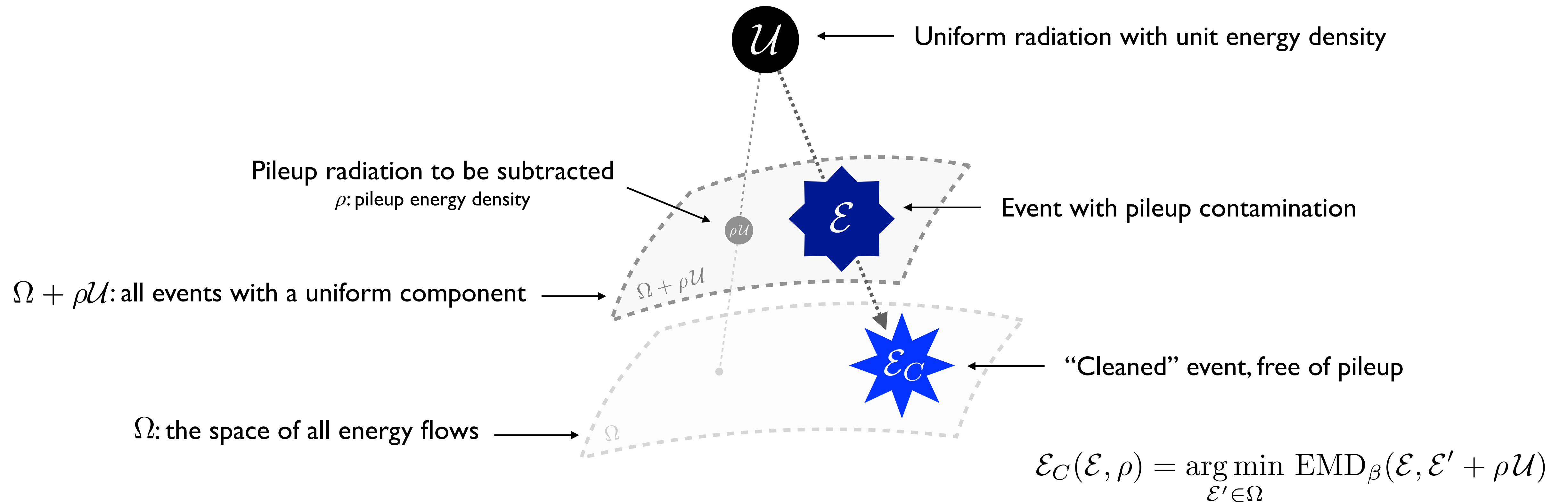
Pileup mitigation: “moving away” from the uniform event



Pileup Mitigation in Event Space

Pileup: uniform (on average) radiation from additional proton-proton collisions

Pileup mitigation: “moving away” from the uniform event

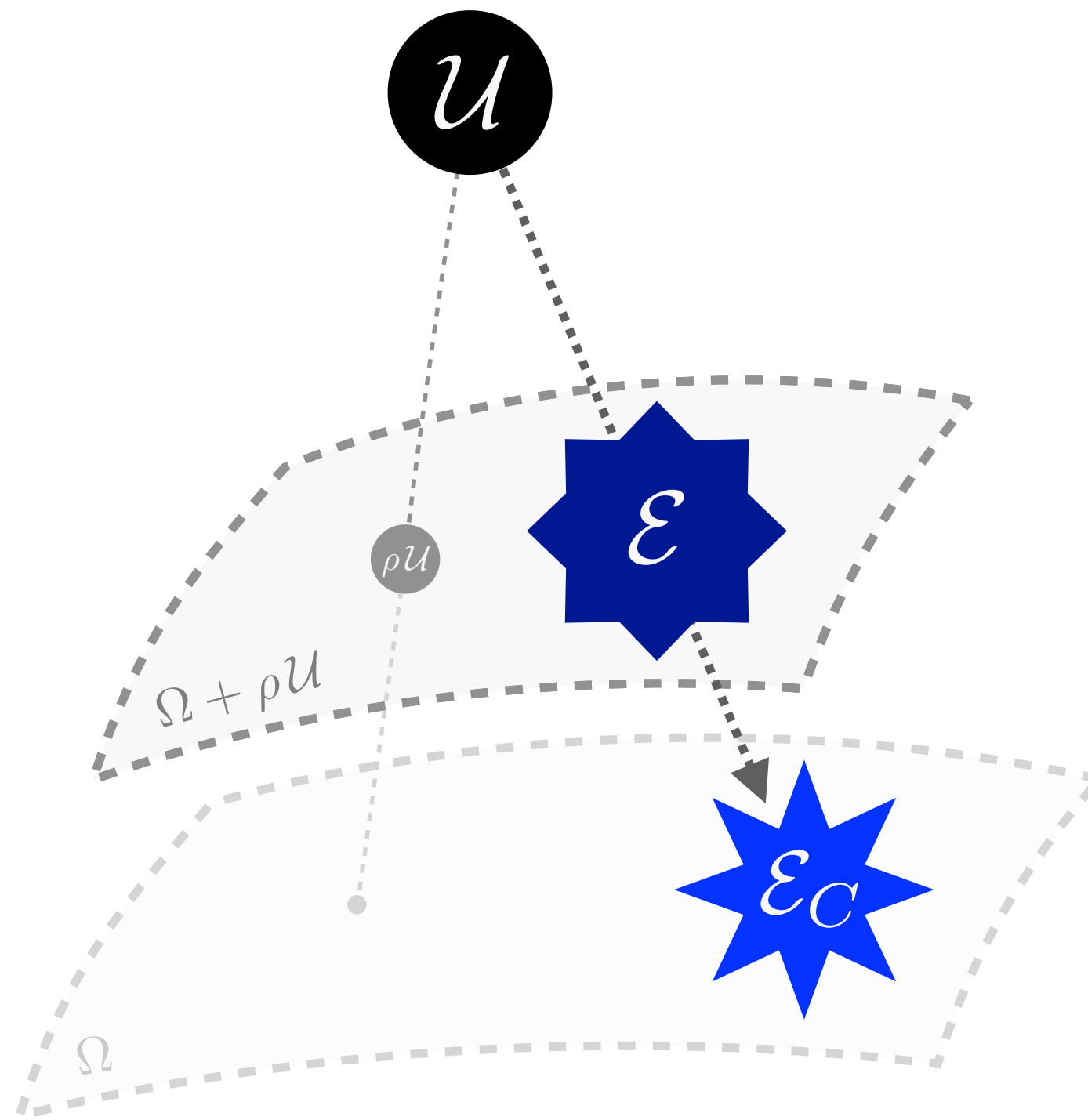


Pileup Mitigation in Event Space

Area subtraction

Particle energies corrected proportional to area of associated region

$$\mathcal{E}_C(\mathcal{E}, \rho) = \arg \min_{\mathcal{E}' \in \Omega} \text{EMD}_\beta(\mathcal{E}, \mathcal{E}' + \rho \mathcal{U})$$

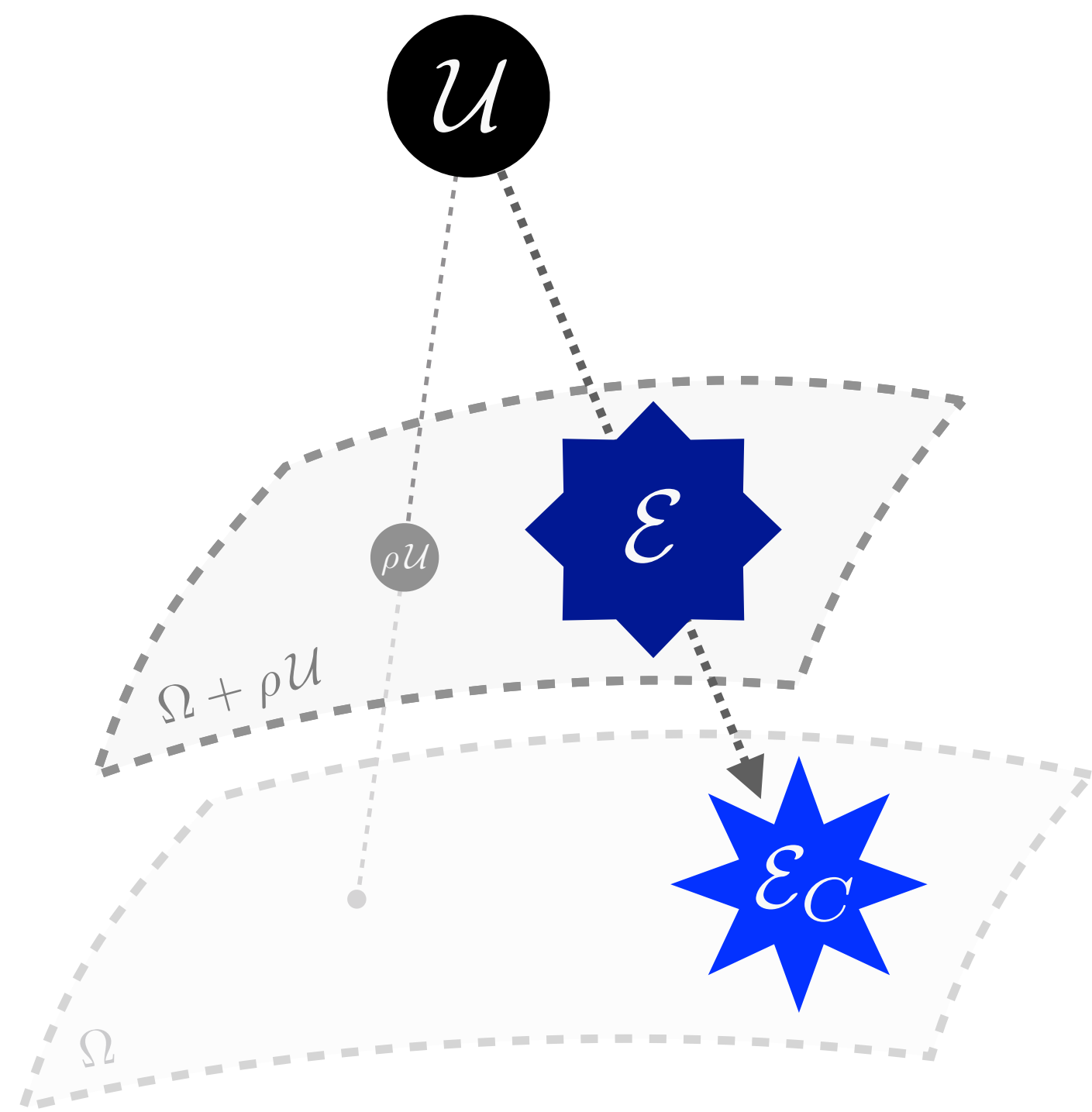


Pileup Mitigation in Event Space

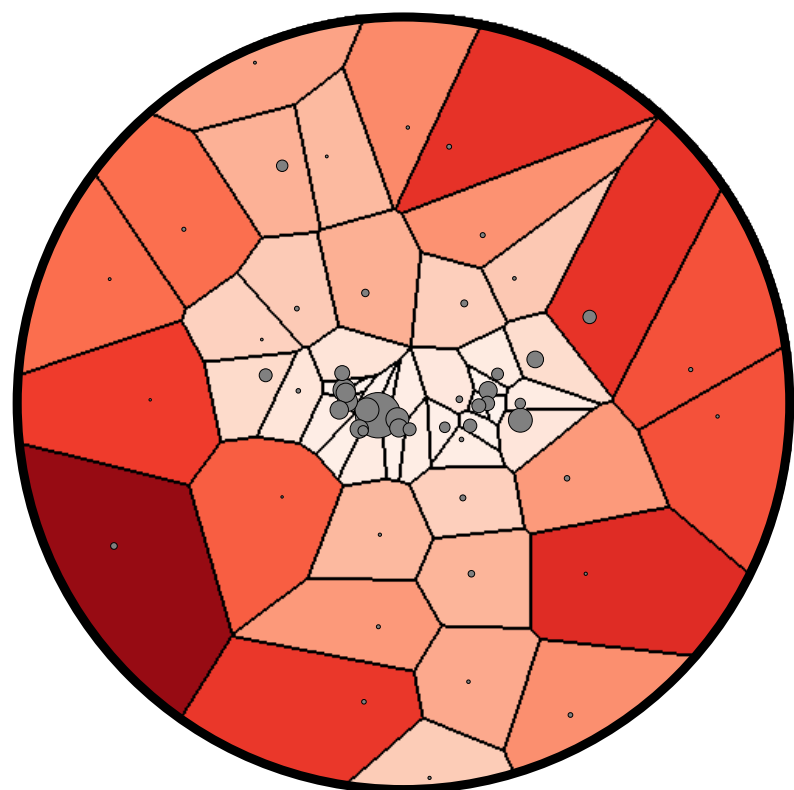
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Voronoi



[Cacciari, Salam, Soyez, JHEP 2008]

Voronoi regions IRC unsafe

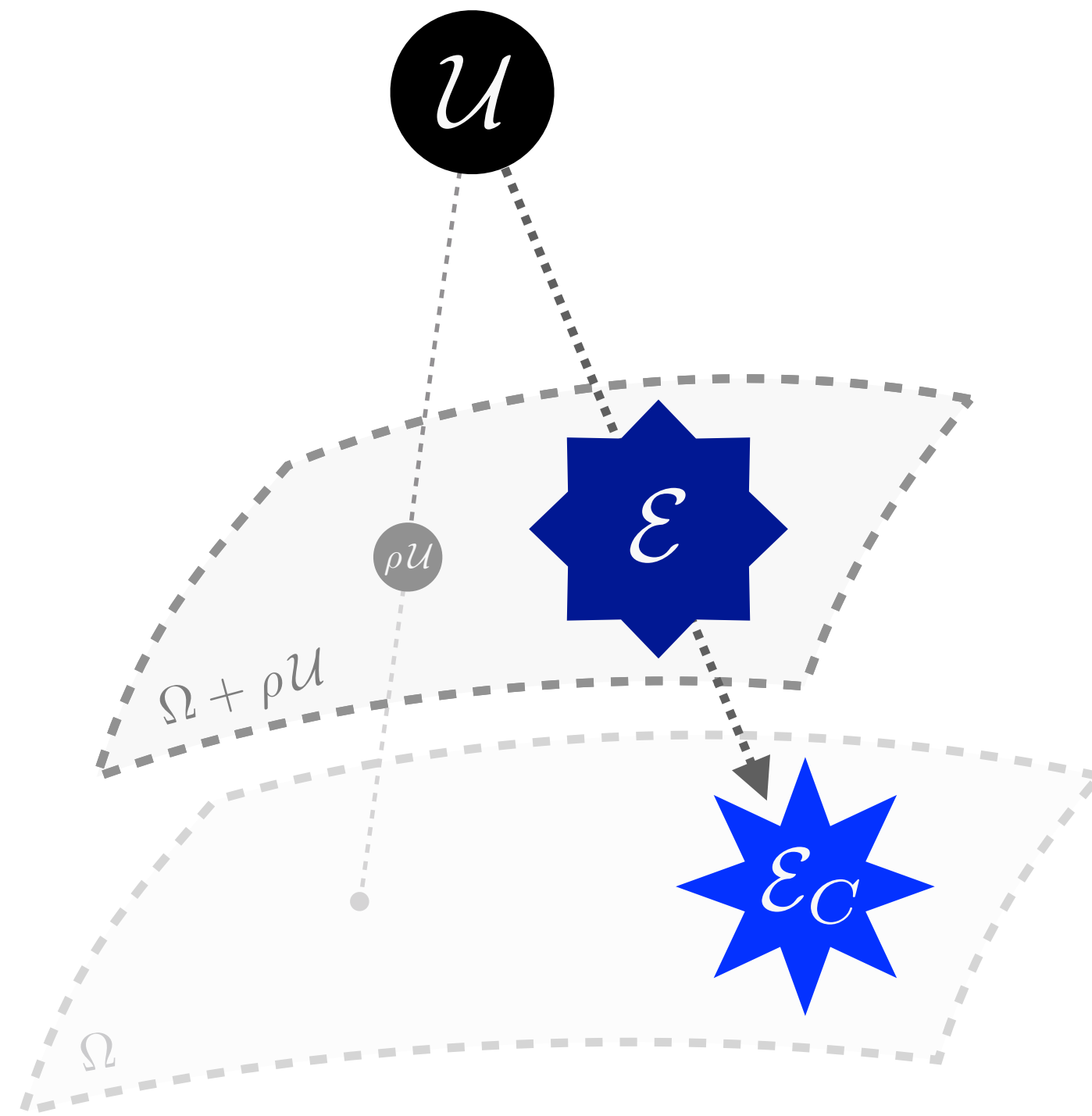
Sensitive to small modifications

Pileup Mitigation in Event Space

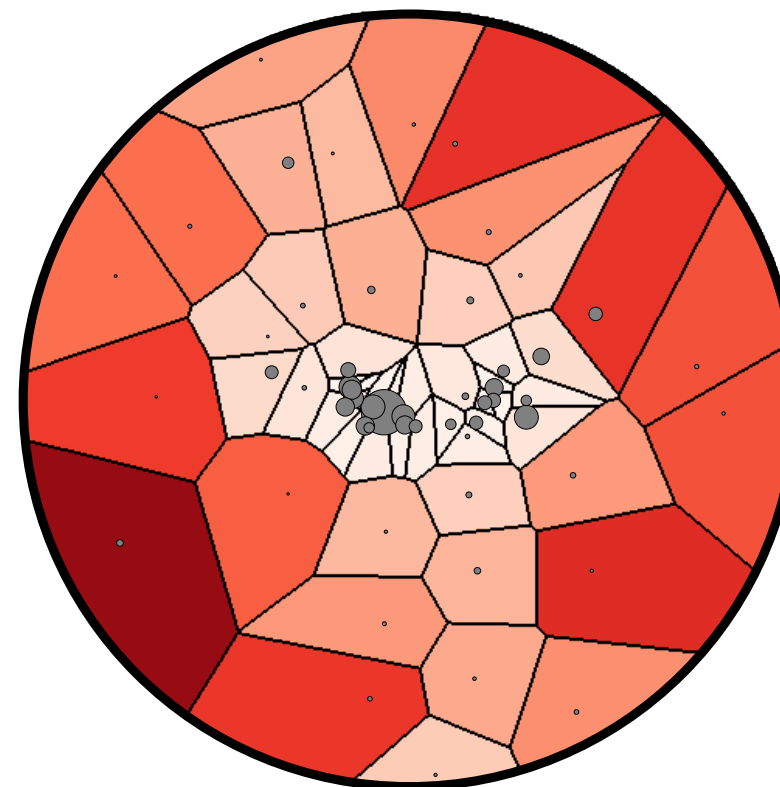
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Voronoi

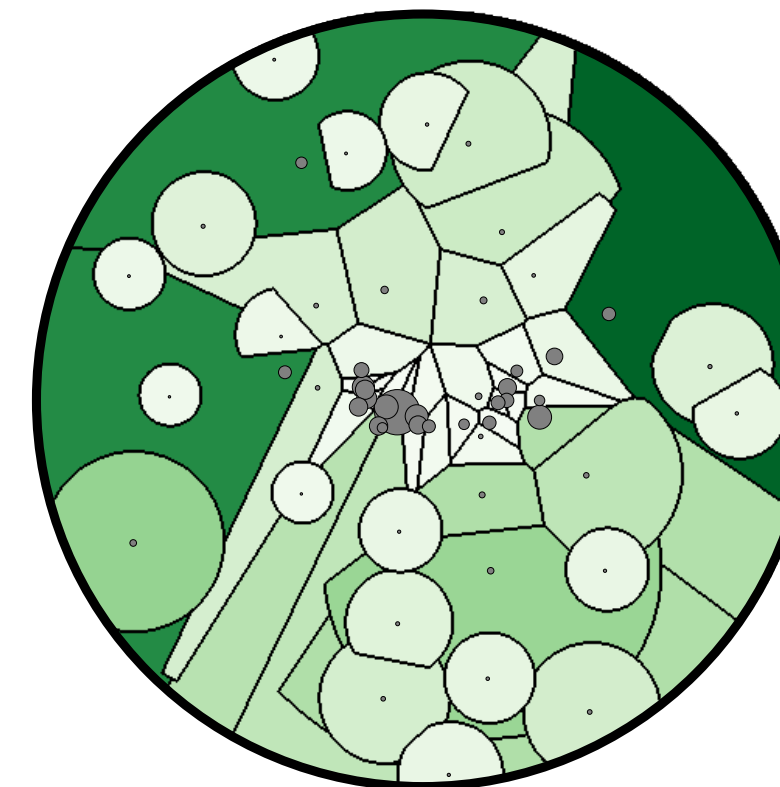


[Cacciari, Salam, Soyez, JHEP 2008]

Voronoi regions IRC unsafe

Sensitive to small modifications

Constituent subtraction



[Berta, Spouta, Miller, Leitner, JHEP 2008]

Lays down grid of “ghost” particles

Ghosts associate to nearest particle

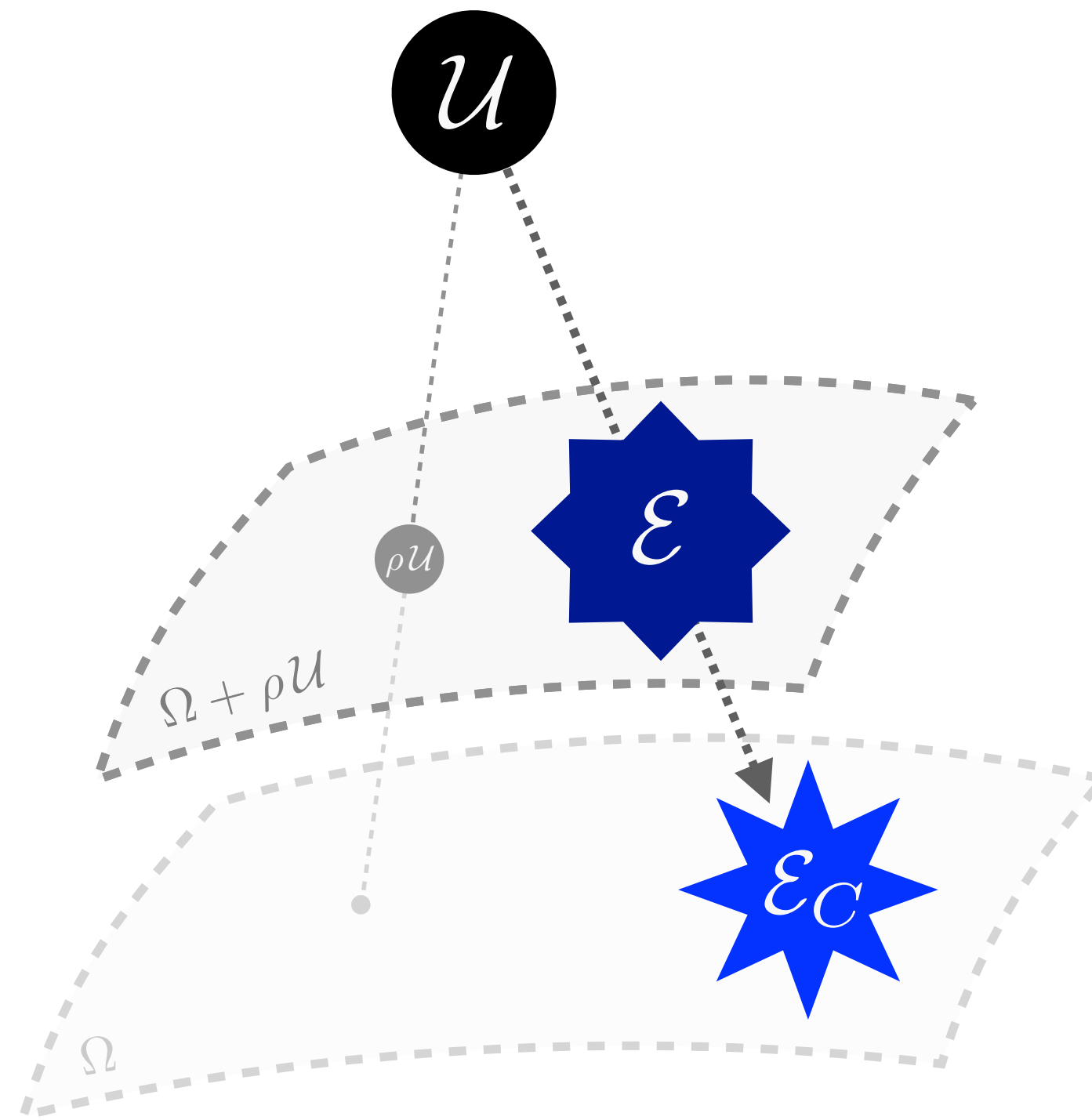
Vanished particles don’t attract ghosts

Pileup Mitigation in Event Space

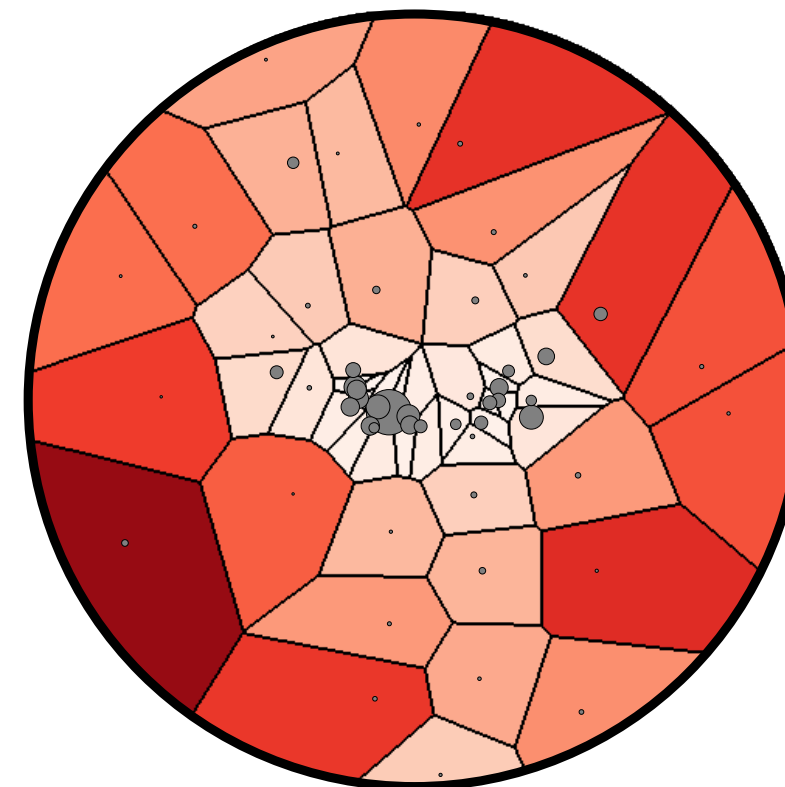
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Voronoi

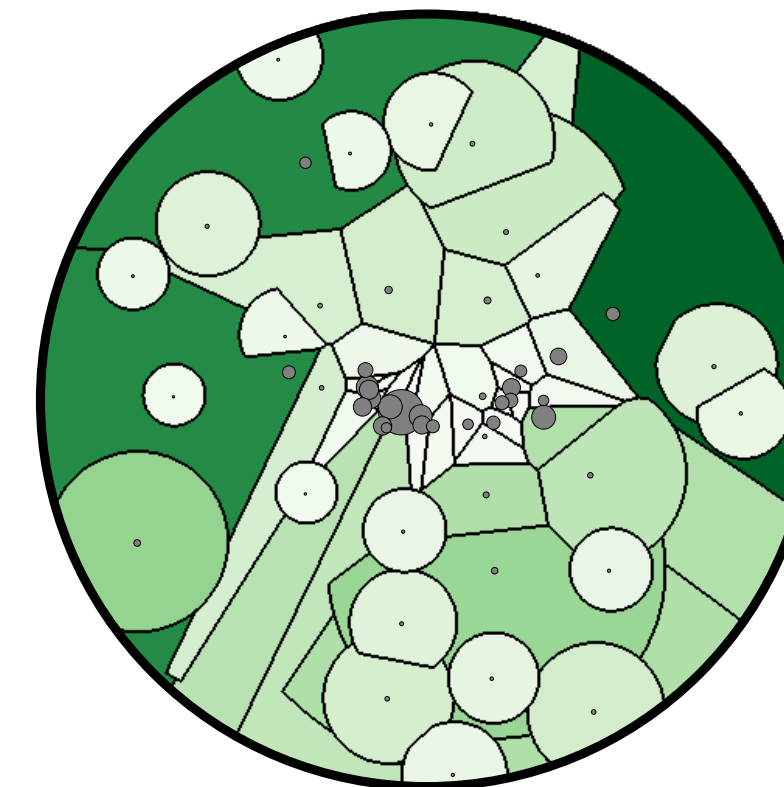


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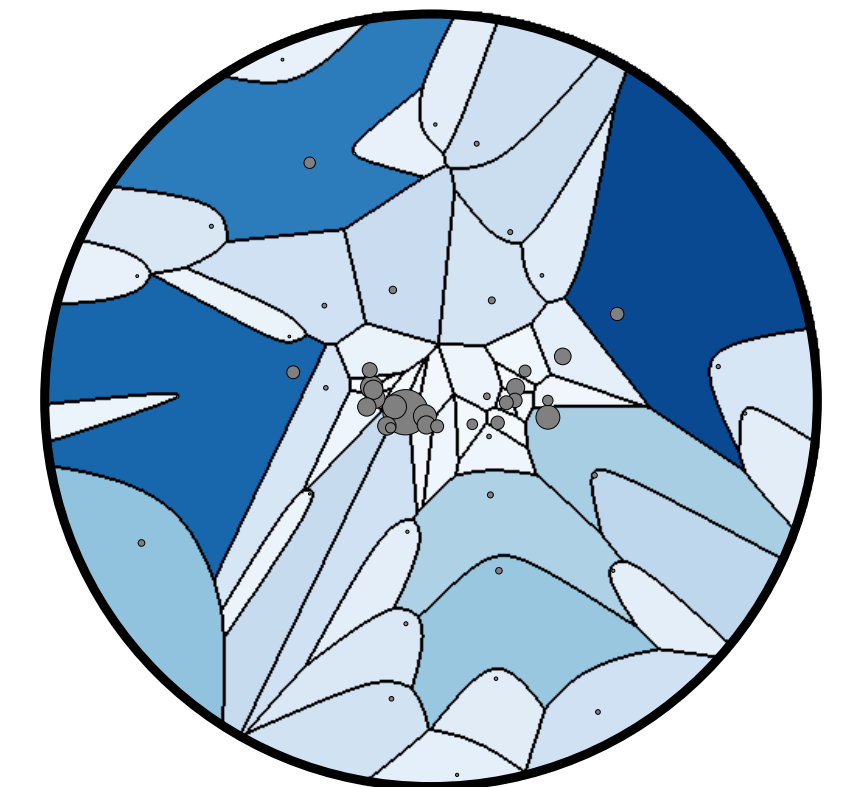
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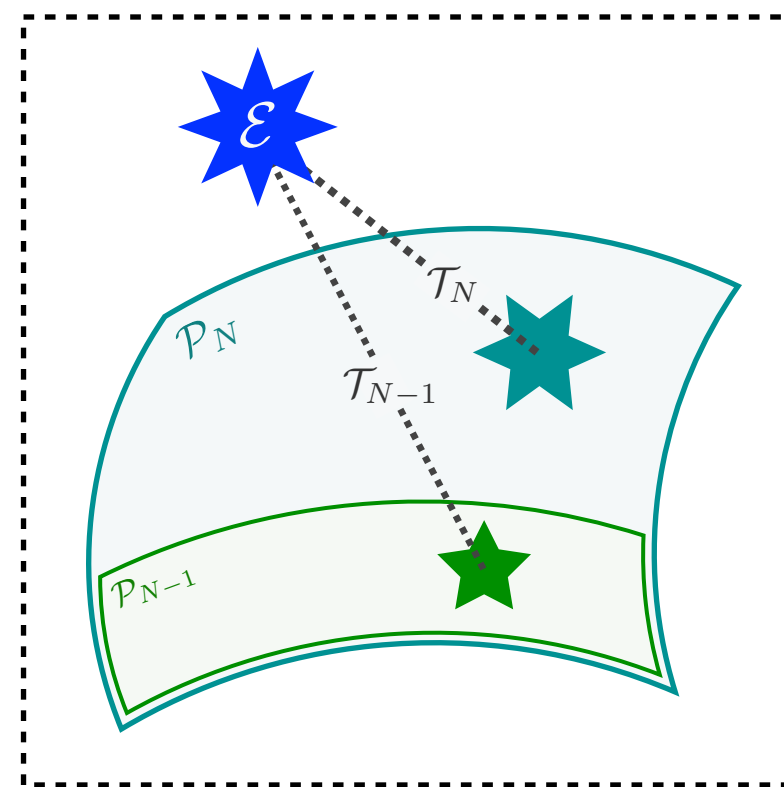
Apollonius



[PTK, Metodiev, Thaler, 2004.04159]

Ghosts are optimally assigned to particles by minimizing EMD

Apollonius regions have an understood continuum limit

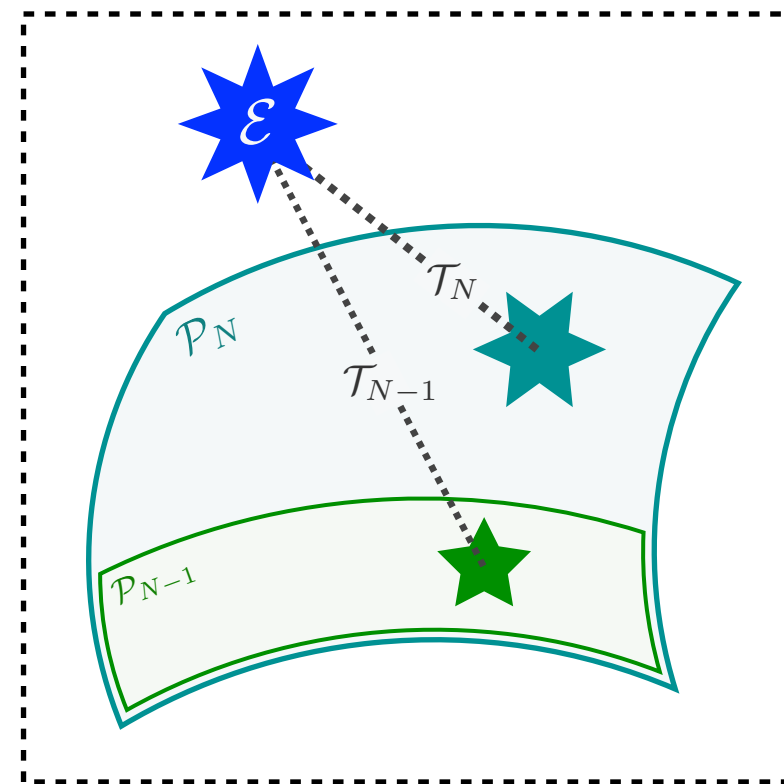


Revealing Hidden Geometry

- Event space exhibits a rich geometry that can be probed using the **EMD**
- Decades worth of collider techniques are naturally described in this geometry
- Many new techniques are suggested, and new light is shed on old ones

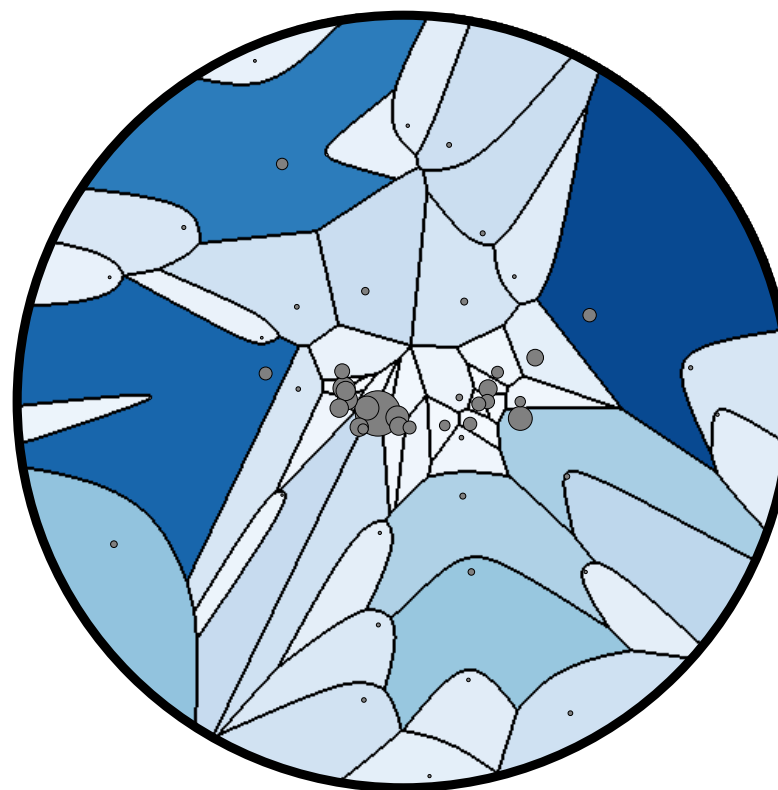
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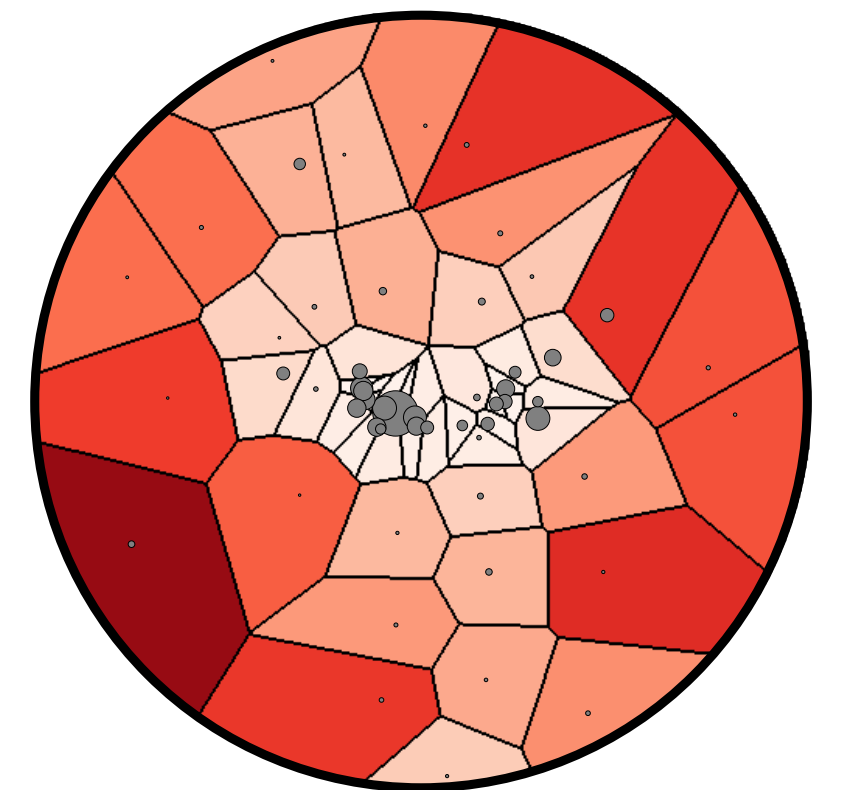


Can pileup mitigators be effective jet groomers?

Apollonius

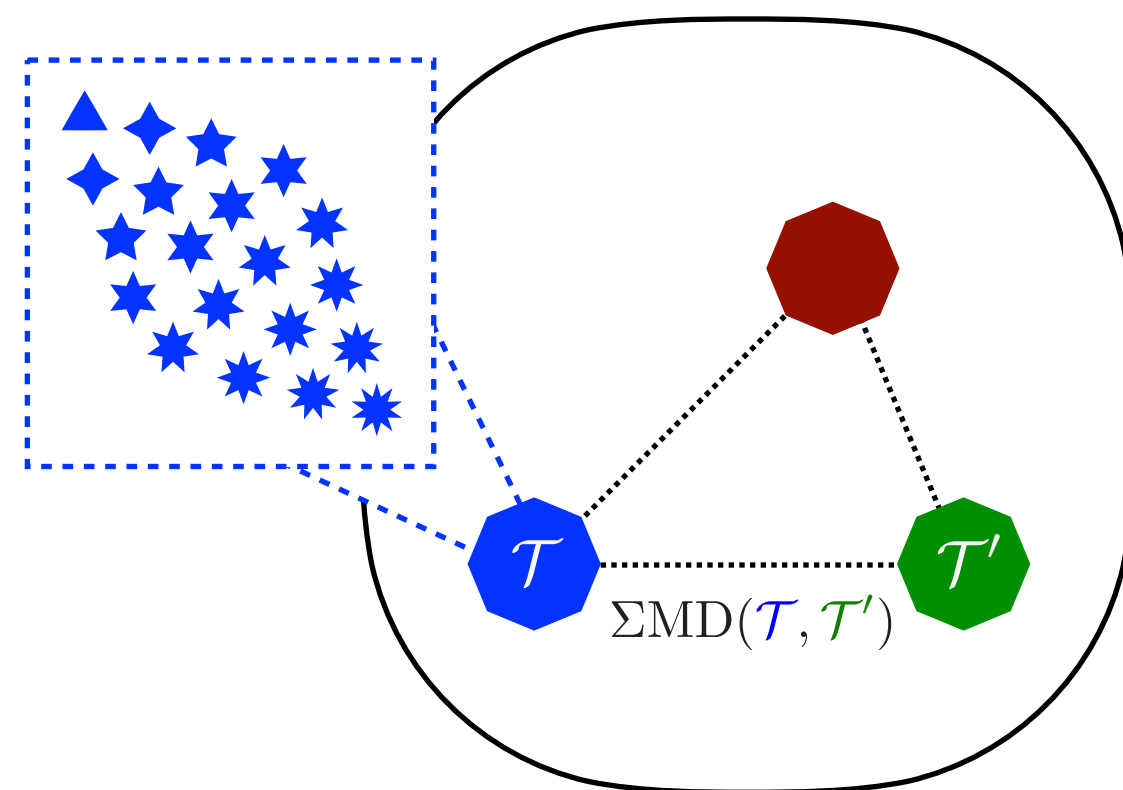
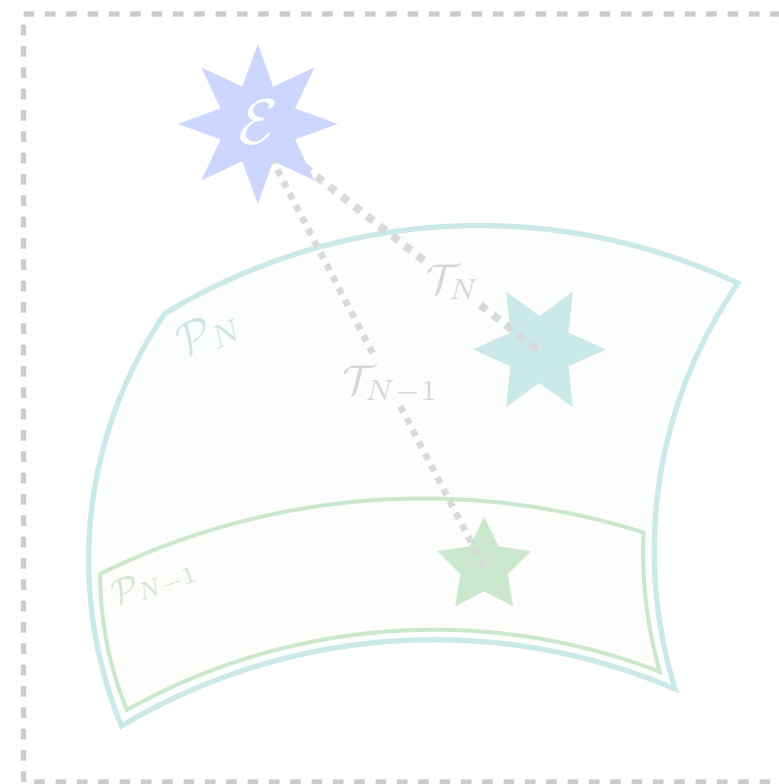
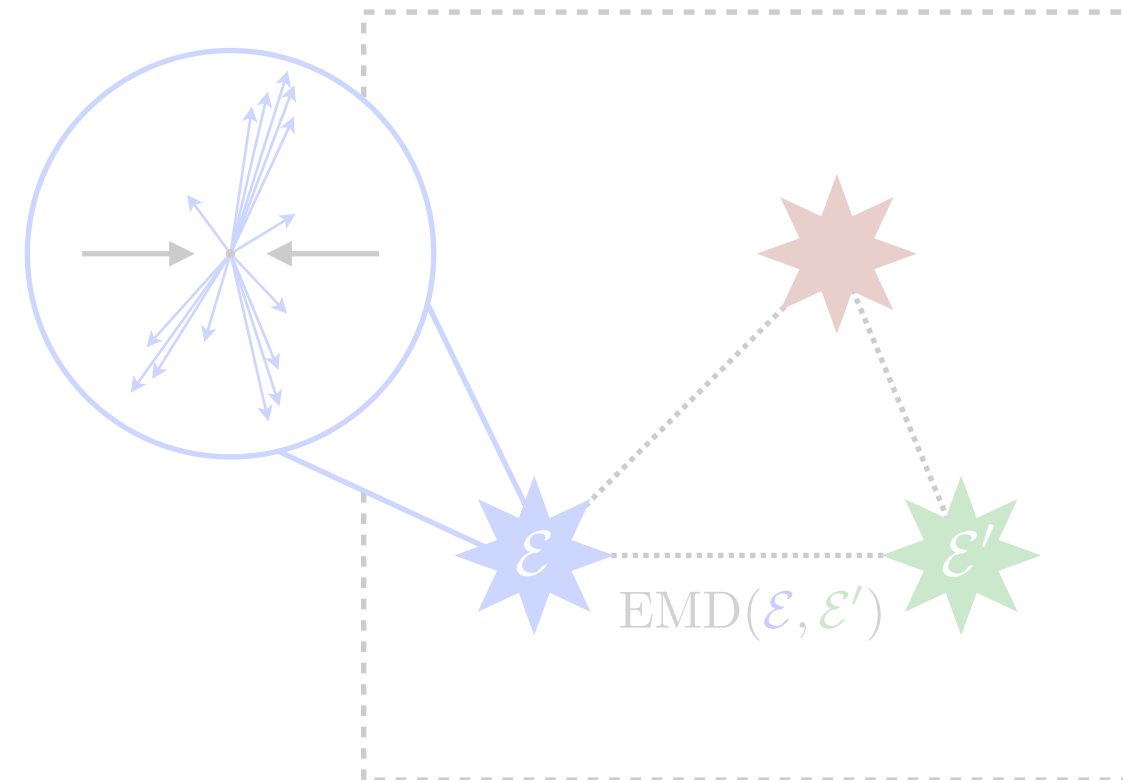


Voronoi



can be approximated by iterating

[Alipour-fard, PTK, Metodiev, Thaler, to appear soon]



The (Metric) Space of Events

Revealing Hidden Geometry

Theory Space

Templated Metric Construction

Inputs

- “Points” that live in the ground metric space
- “Ground metric” that measures point distances
- “Weights” associated to each point

Output

- A new metric for collections of weighted points
- A metric space where these distributions live

p-Wasserstein metric from optimal transport theory

$$W_p(\mu, \nu) = \left(\inf_{J \in \mathcal{J}(\mu, \nu)} \int_{M \times M} d(x, y)^p dJ(x, y) \right)^{1/p}$$

(M, d) , metric space
 $\mathcal{J}(\mu, \nu)$, space of joint distributions
with marginals μ, ν

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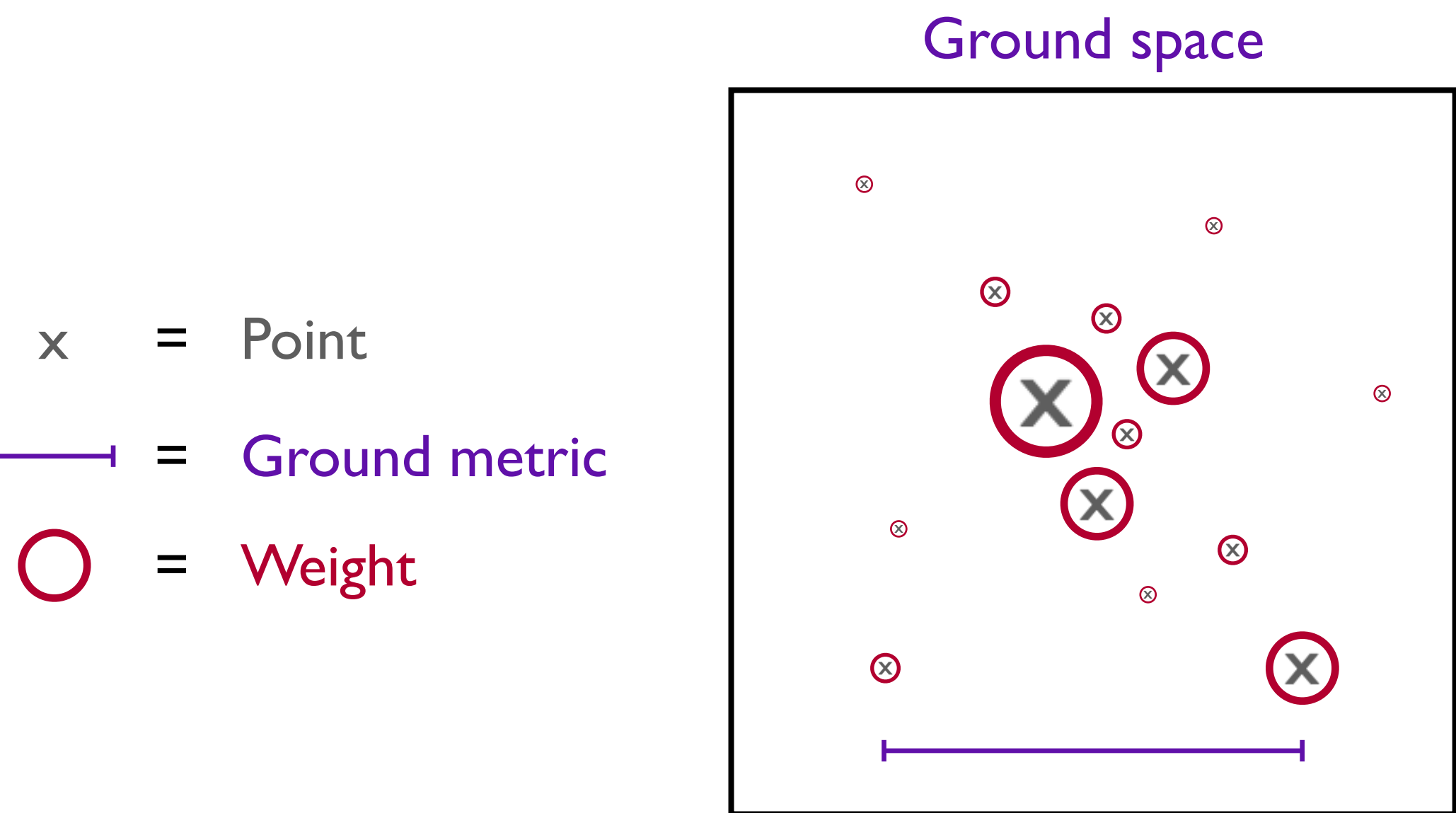
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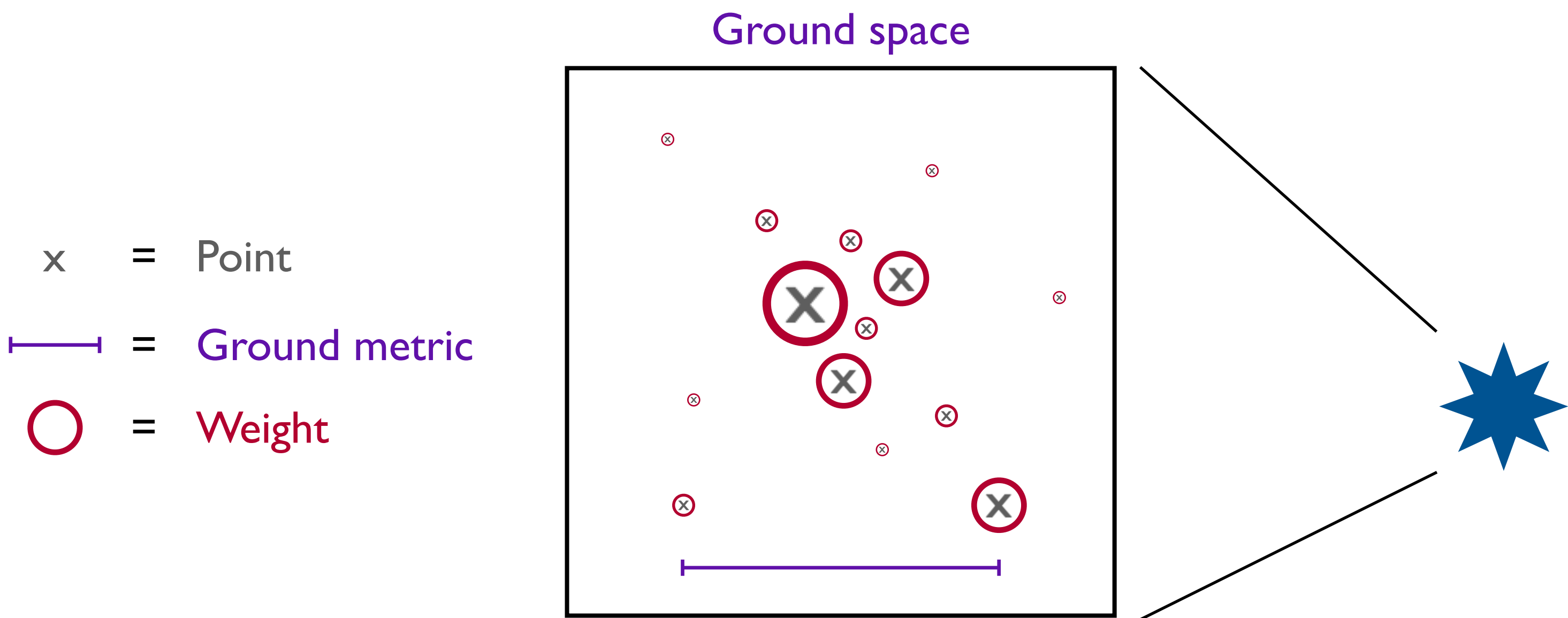
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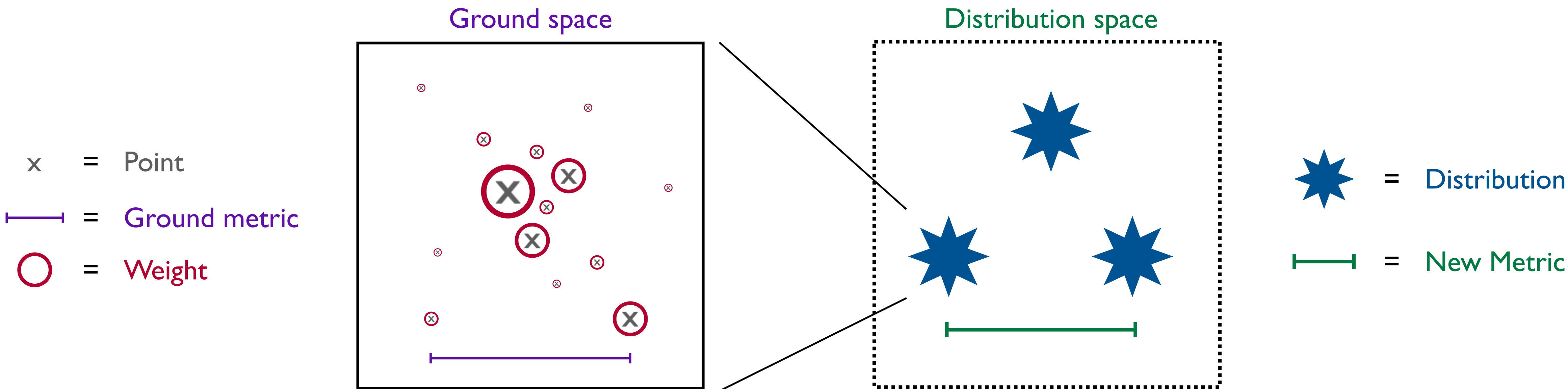
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Templated Metric Construction – Energy Mover’s Distance

[PTK, Metodiev, Thaler, [PRL 2019](#)]

Inputs

- “Points” that live in the ground metric space
- “Ground metric” that measures point distances
- “Weights” associated to each point

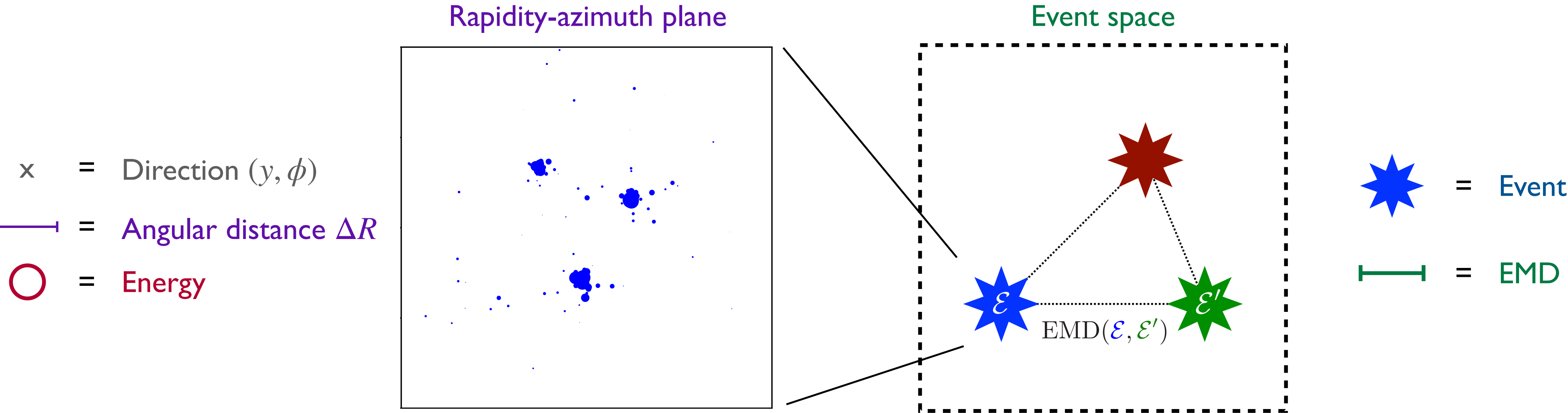
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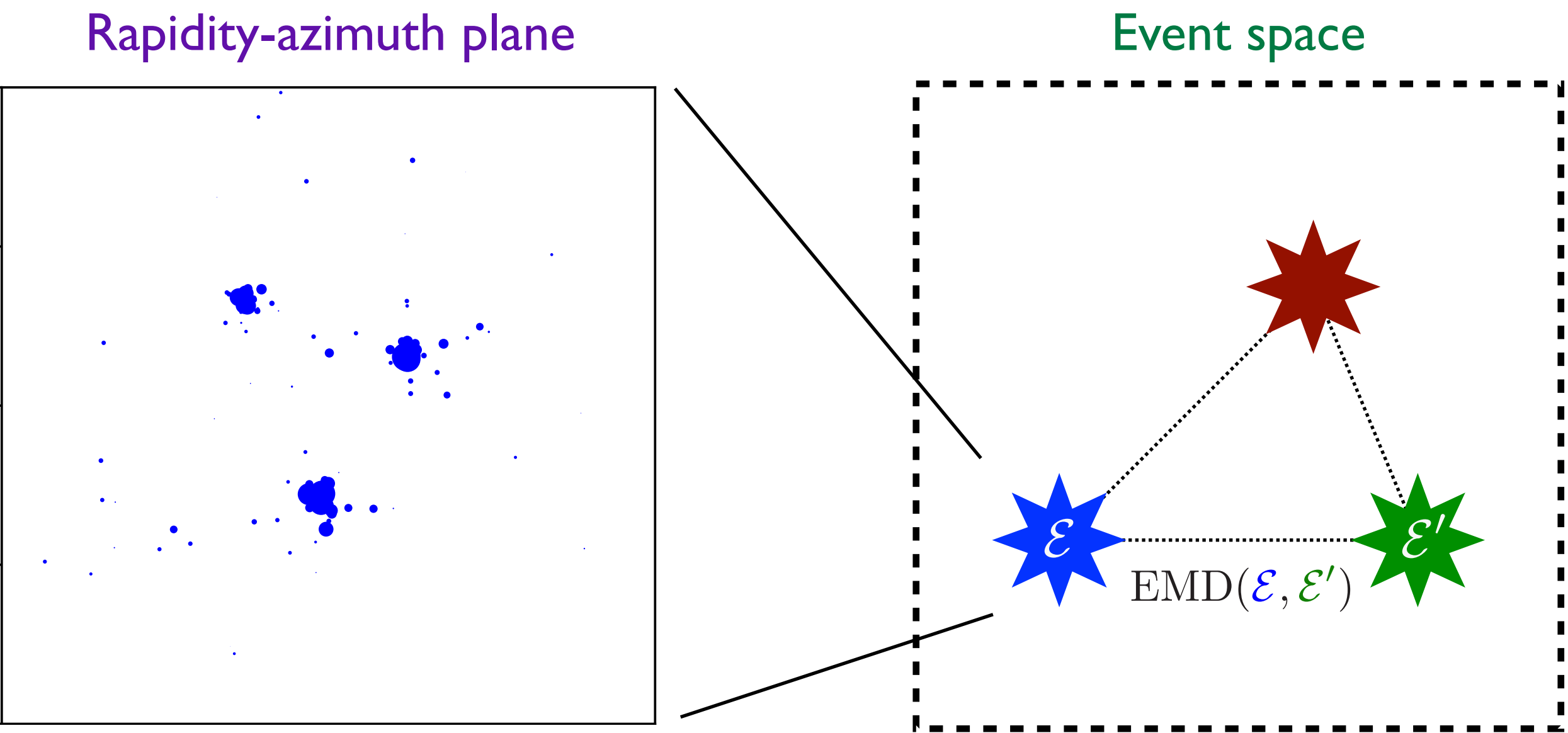
(M, d) , metric space
 $\mathcal{J}(\mu, \nu)$, space of joint distributions
with marginals μ, ν



Bootstrapping to the Cross-Section Mover's Distance (Σ MD)

[PTK, Metodiev, Thaler, 2004.04159]

- x = Direction (y, ϕ)
- —|— = Angular distance ΔR
- \bigcirc = Energy

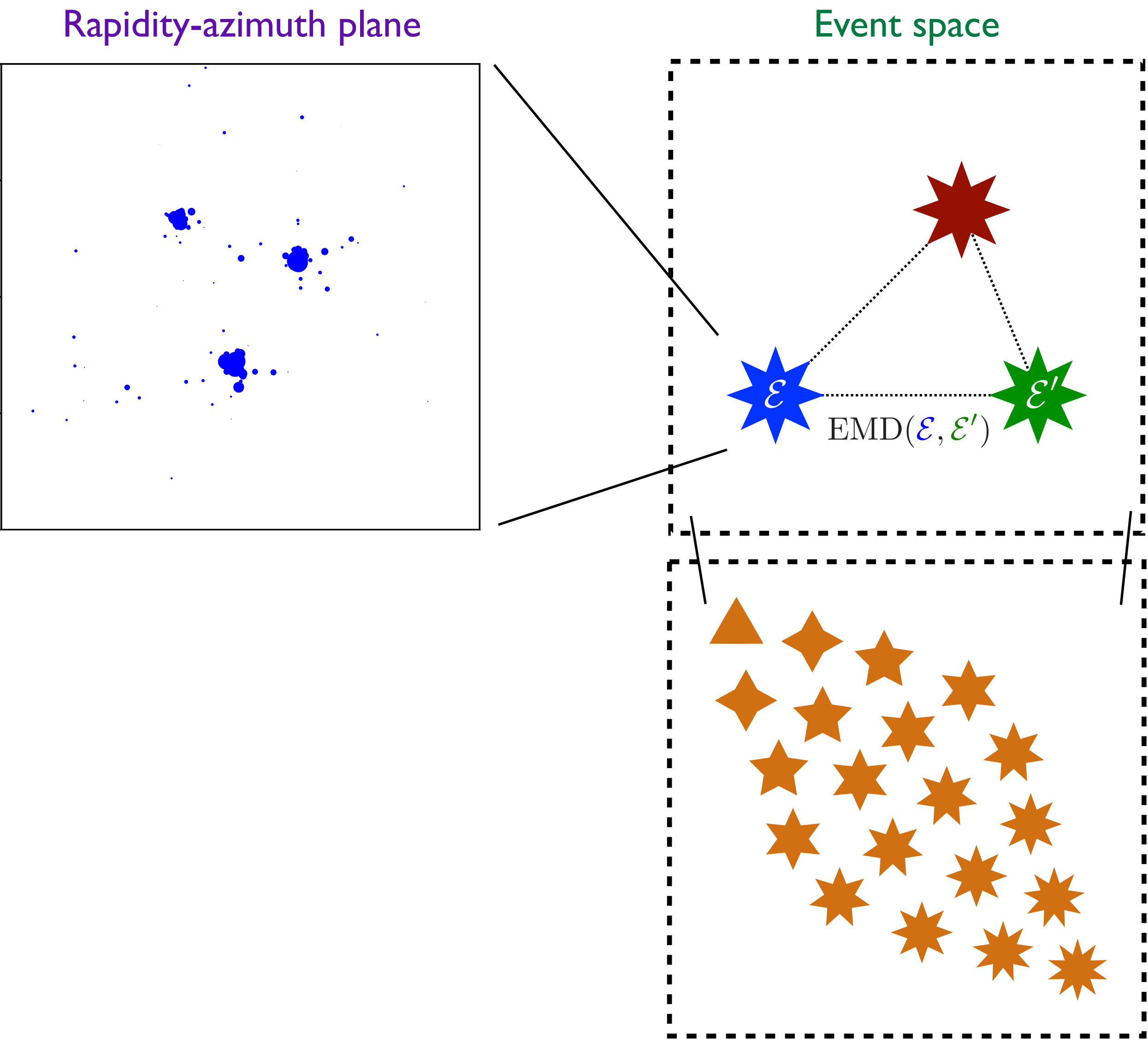


- \star = Event
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Bootstrapping to the Cross-Section Mover's Distance (Σ MD)

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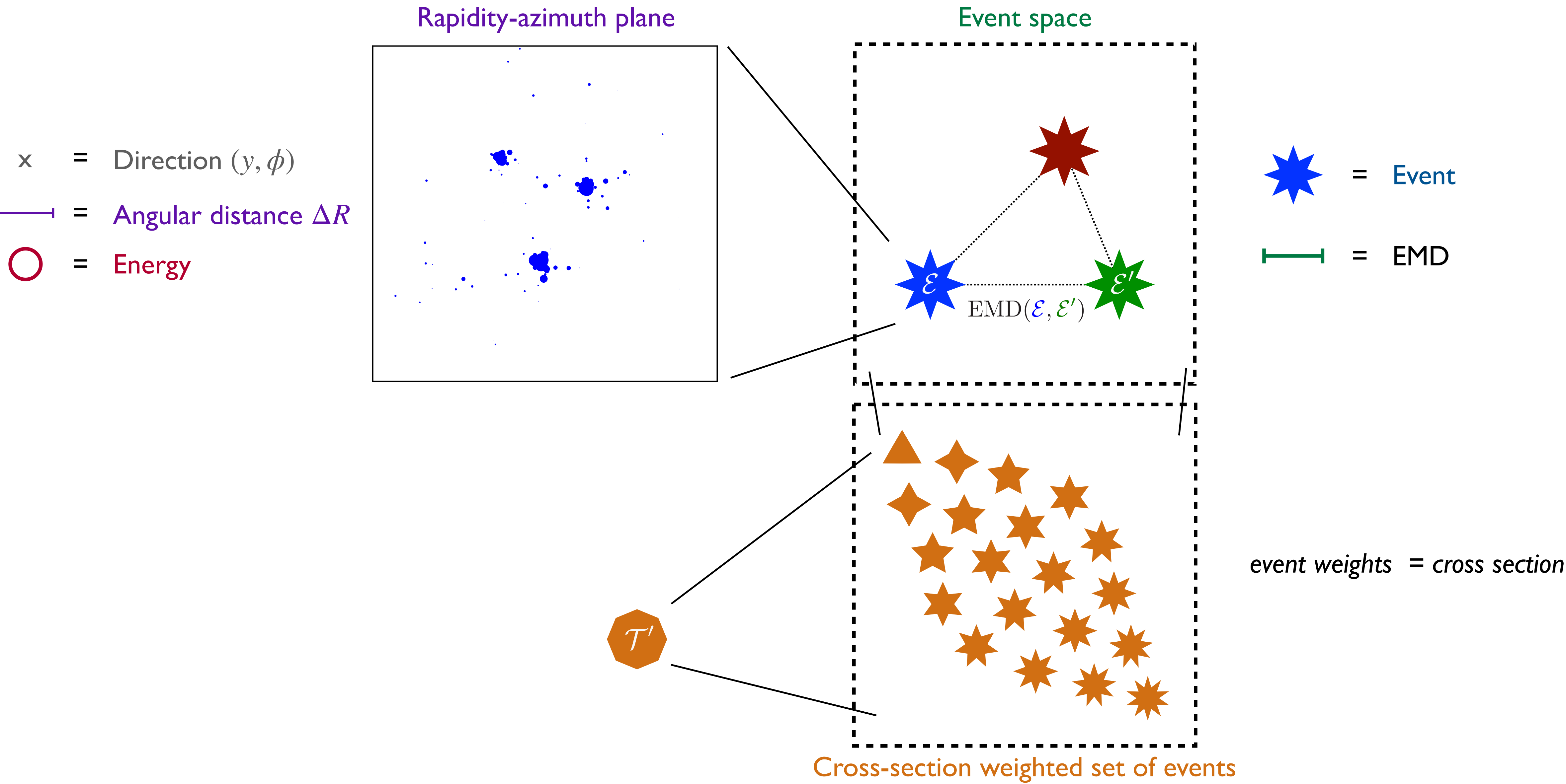
- \star = Event
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event weights = cross section

Cross-section weighted set of events

Bootstrapping to the Cross-Section Mover's Distance (Σ MD)

[PTK, Metodiev, Thaler, 2004.04159]

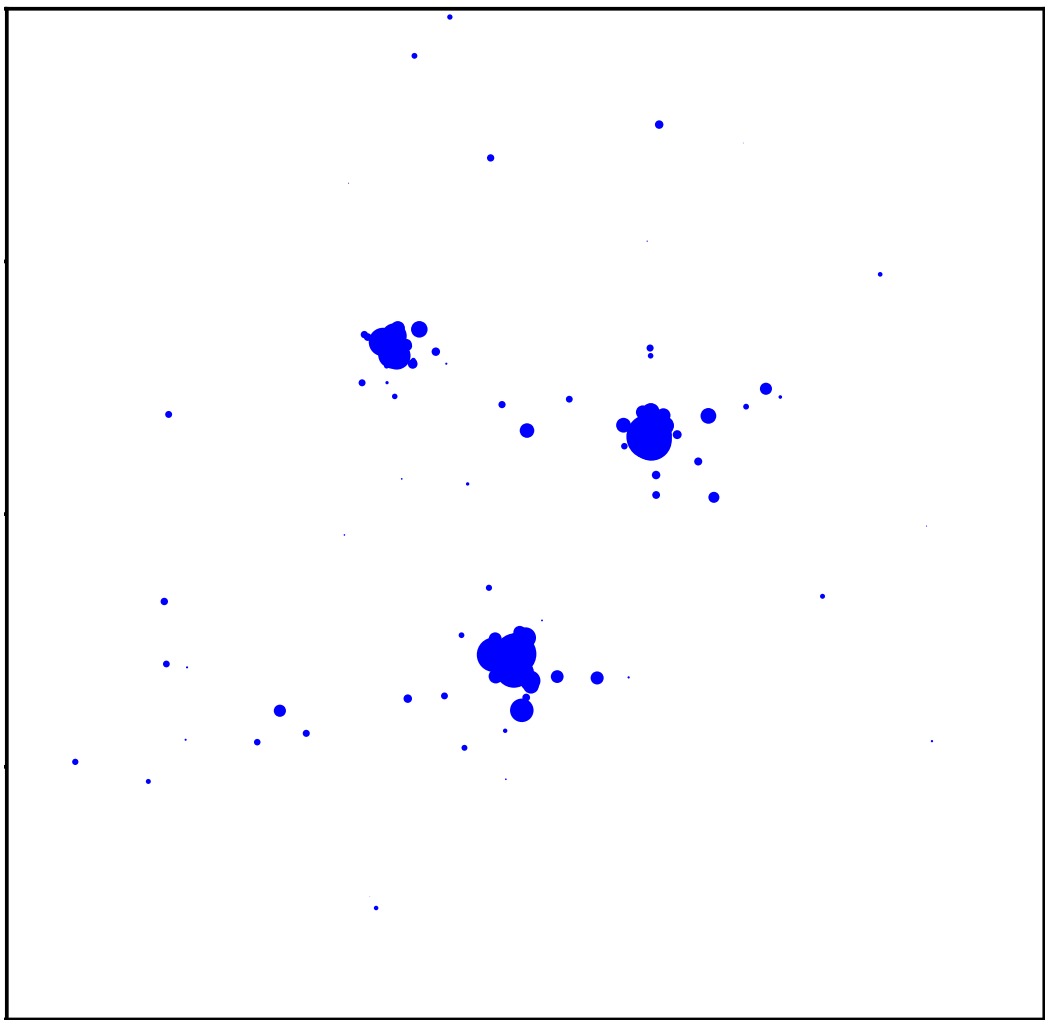


Bootstrapping to the Cross-Section Mover's Distance (ΣMD)

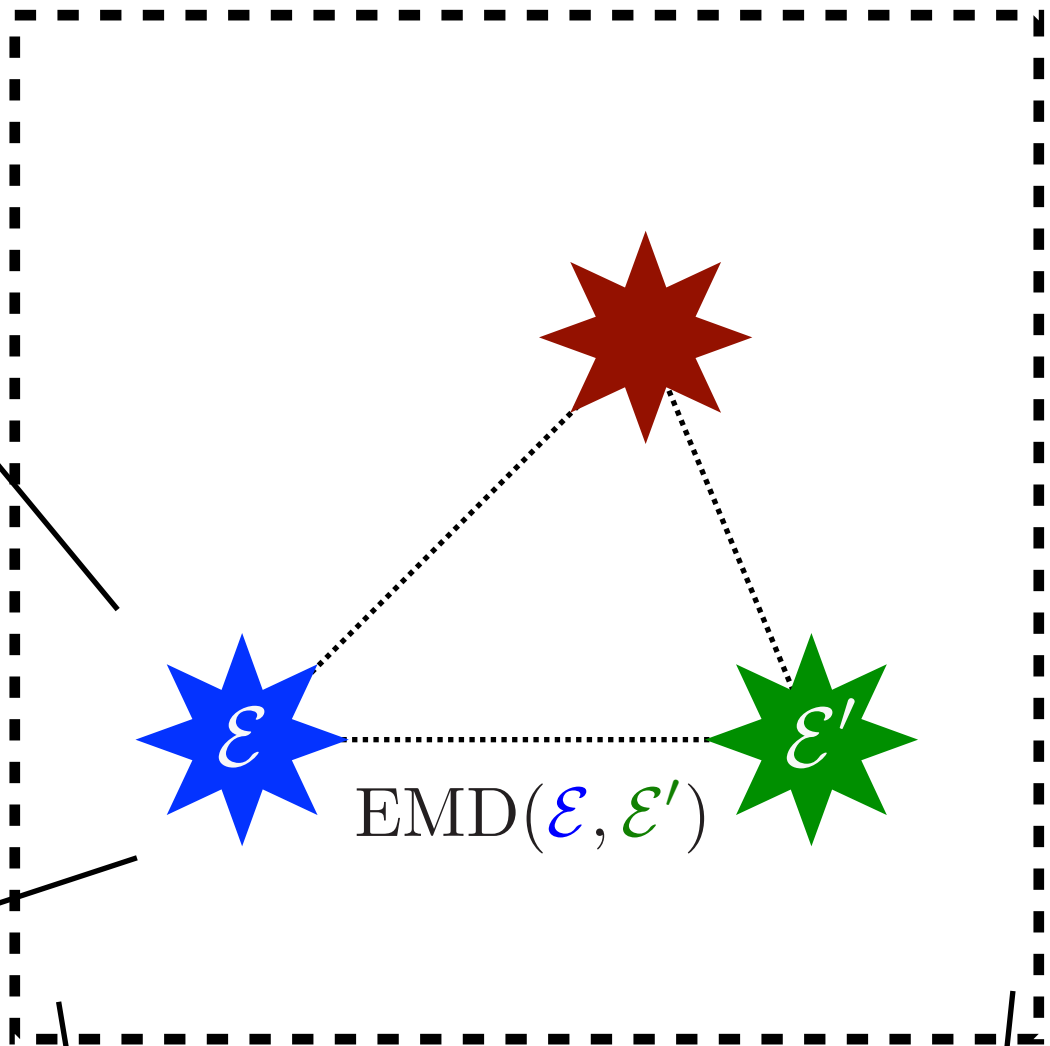
[PTK, Metodiev, Thaler, 2004.04159]

- x = Direction (y, ϕ)
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Rapidity-azimuth plane

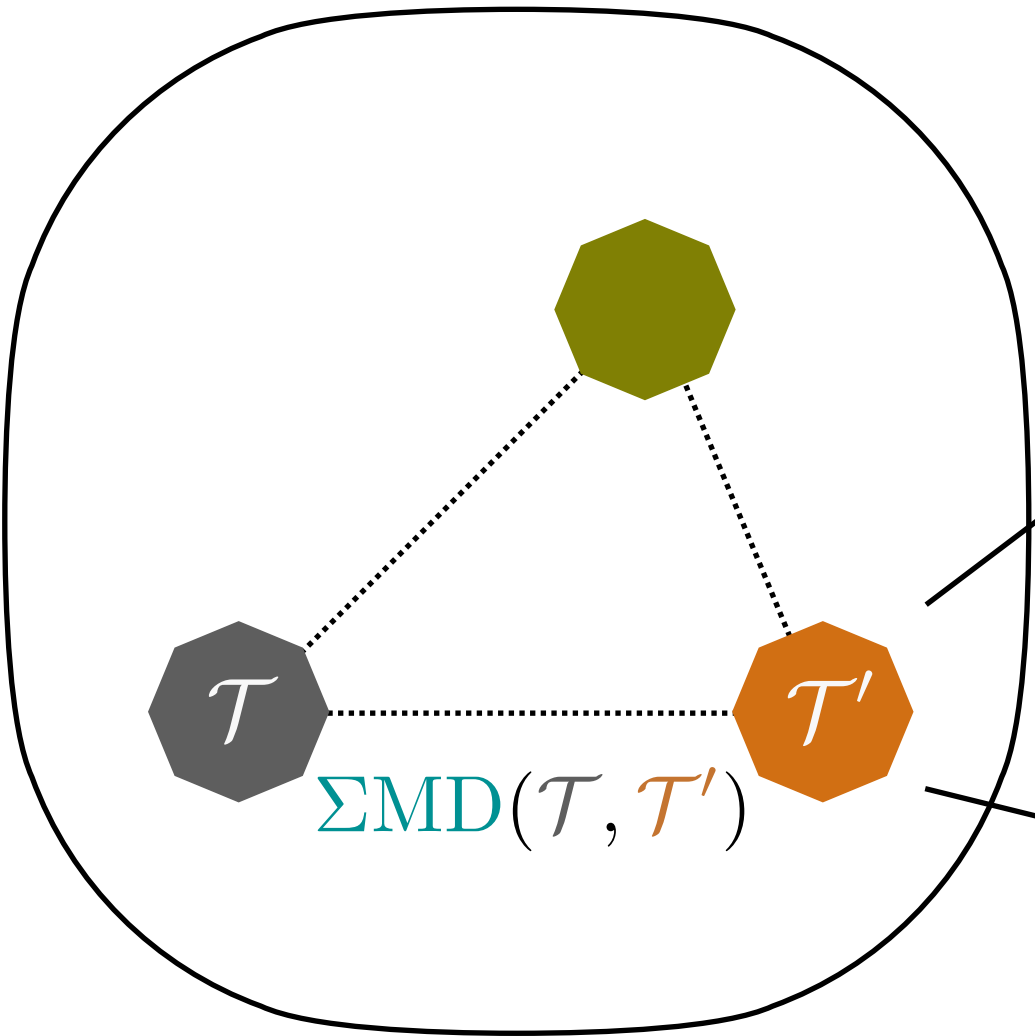


Event space

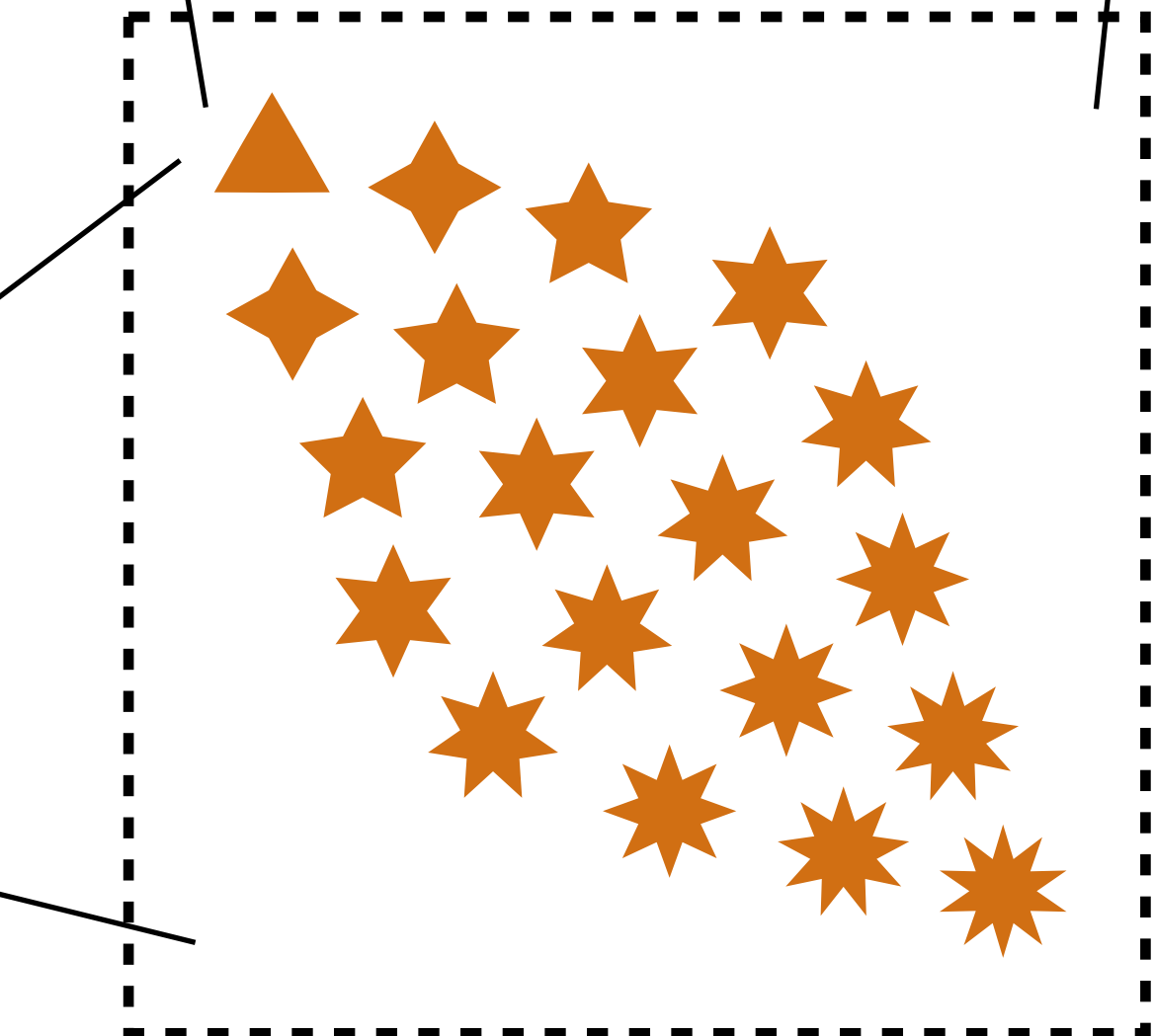


- = Event
- = EMD

ΣMD is a metric between theories!



Theory space



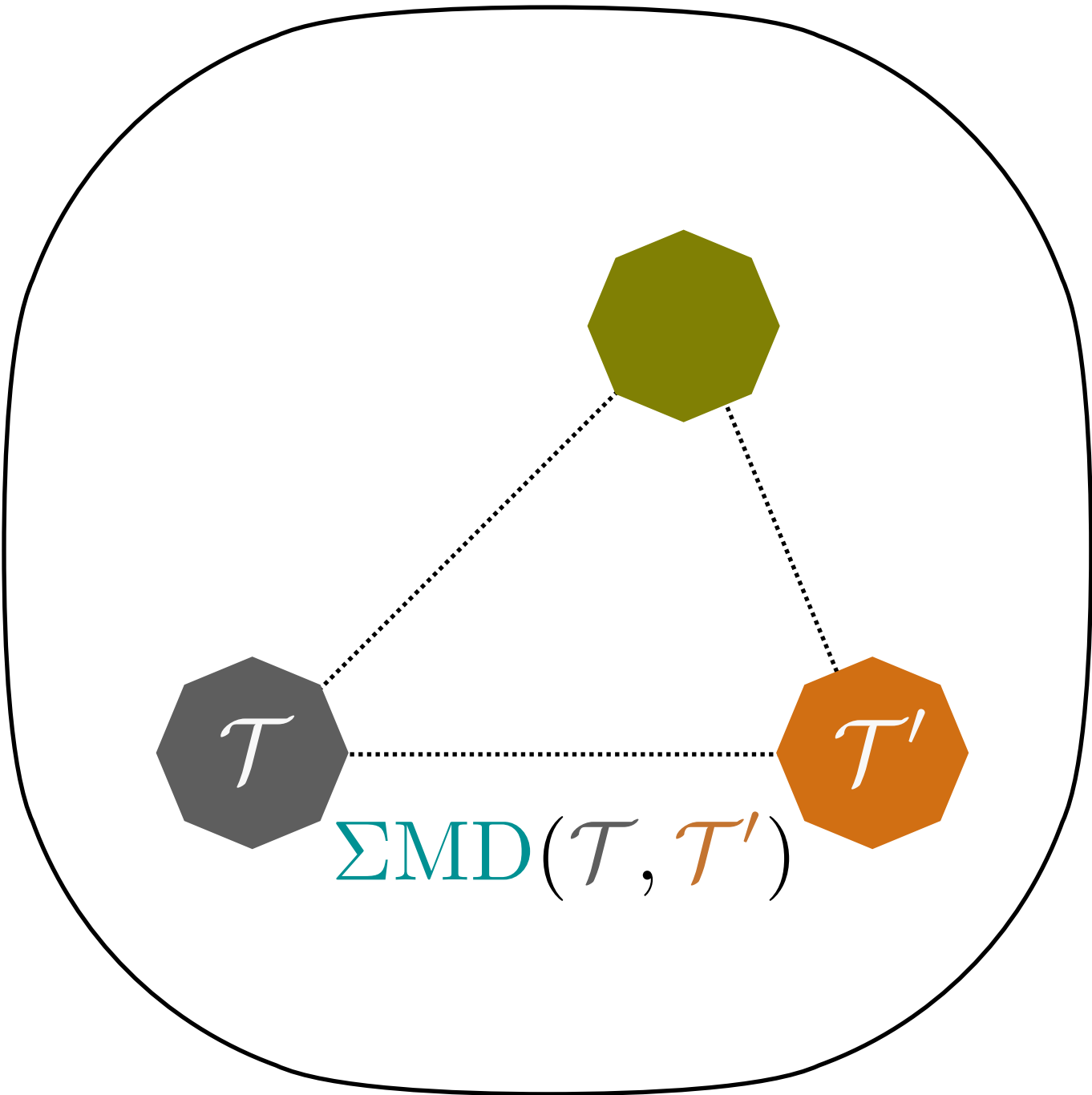
Cross-section weighted set of events

event weights = cross section

The Cross-Section Mover's Distance (Σ MD)

[PTK, Metodiev, Thaler, 2004.04159]

Σ MD uses EMD as the ground metric and event cross sections as weights



$$\Sigma\text{MD}_{\gamma,S;\beta,R}(\mathcal{T}, \mathcal{T}') = \min_{\mathcal{F}_{ij} \geq 0} \sum_{i=1}^N \sum_{j=1}^{N'} \mathcal{F}_{ij} \left(\frac{\text{EMD}_{\beta,R}(\mathcal{E}_i, \mathcal{E}'_j)}{S} \right)^\gamma + \left| \sum_{i=1}^N \sigma_i - \sum_{j=1}^{N'} \sigma'_j \right|$$

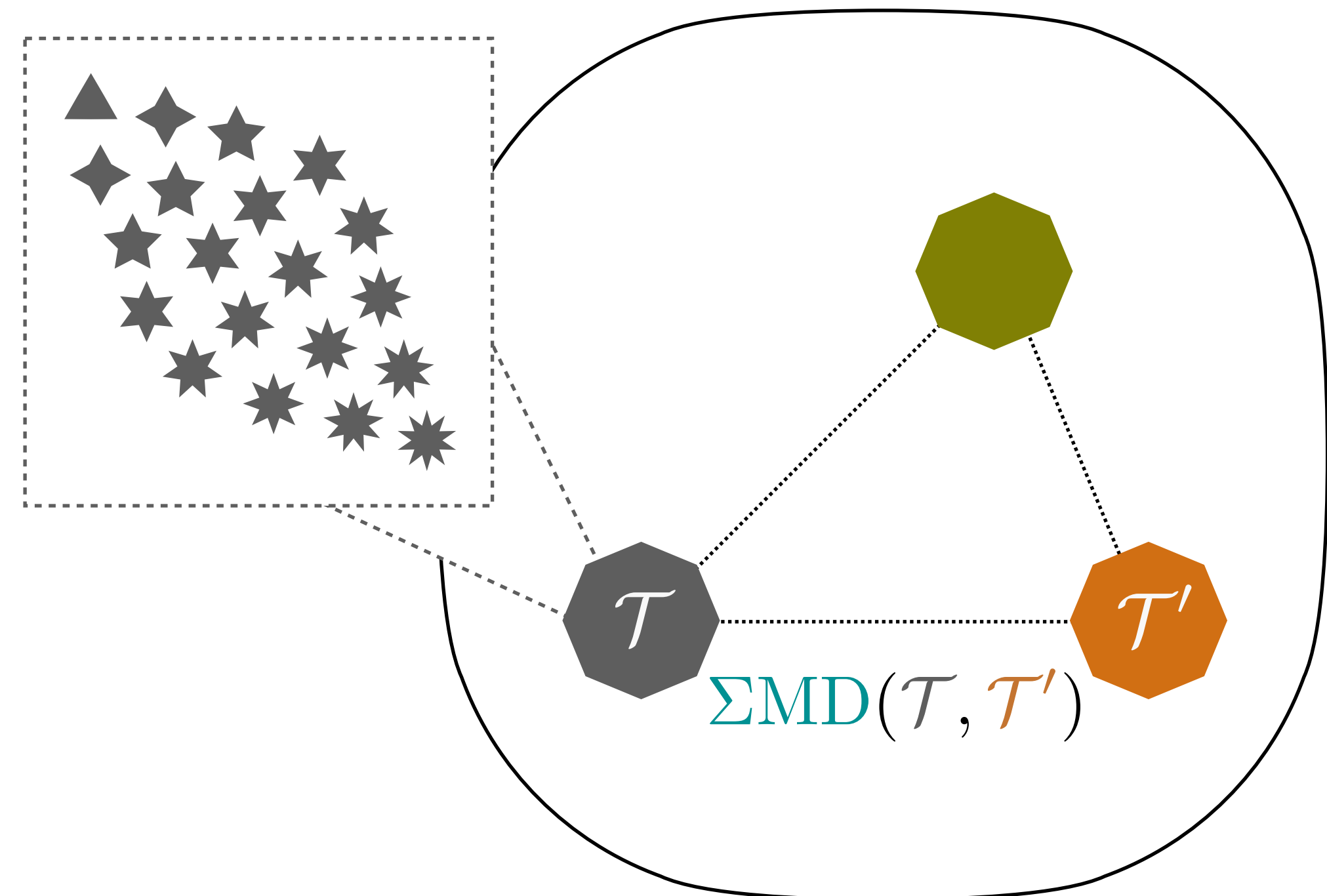
$$\underbrace{\sum_{i=1}^N \mathcal{F}_{ij} \leq \sigma'_j, \quad \sum_{j=1}^{N'} \mathcal{F}_{ij} \leq \sigma_i, \quad \sum_{i=1}^N \sum_{j=1}^{N'} \mathcal{F}_{ij} = \min \left(\sum_{i=1}^N \sigma_i, \sum_{j=1}^{N'} \sigma'_j \right)}_{\text{Usual constraints to ensure proper transport}}$$

	Energy Mover's Distance	Cross Section Mover's Distance
Symbol	EMD	Σ MD
Description	Distance between events	Distance between theories
Weight	Particle energies E_i	Event cross sections σ_i
Ground Metric	Particle distances θ_{ij}	Event distances $\text{EMD}(\mathcal{E}_i, \mathcal{E}_j)$

The Space of Theories

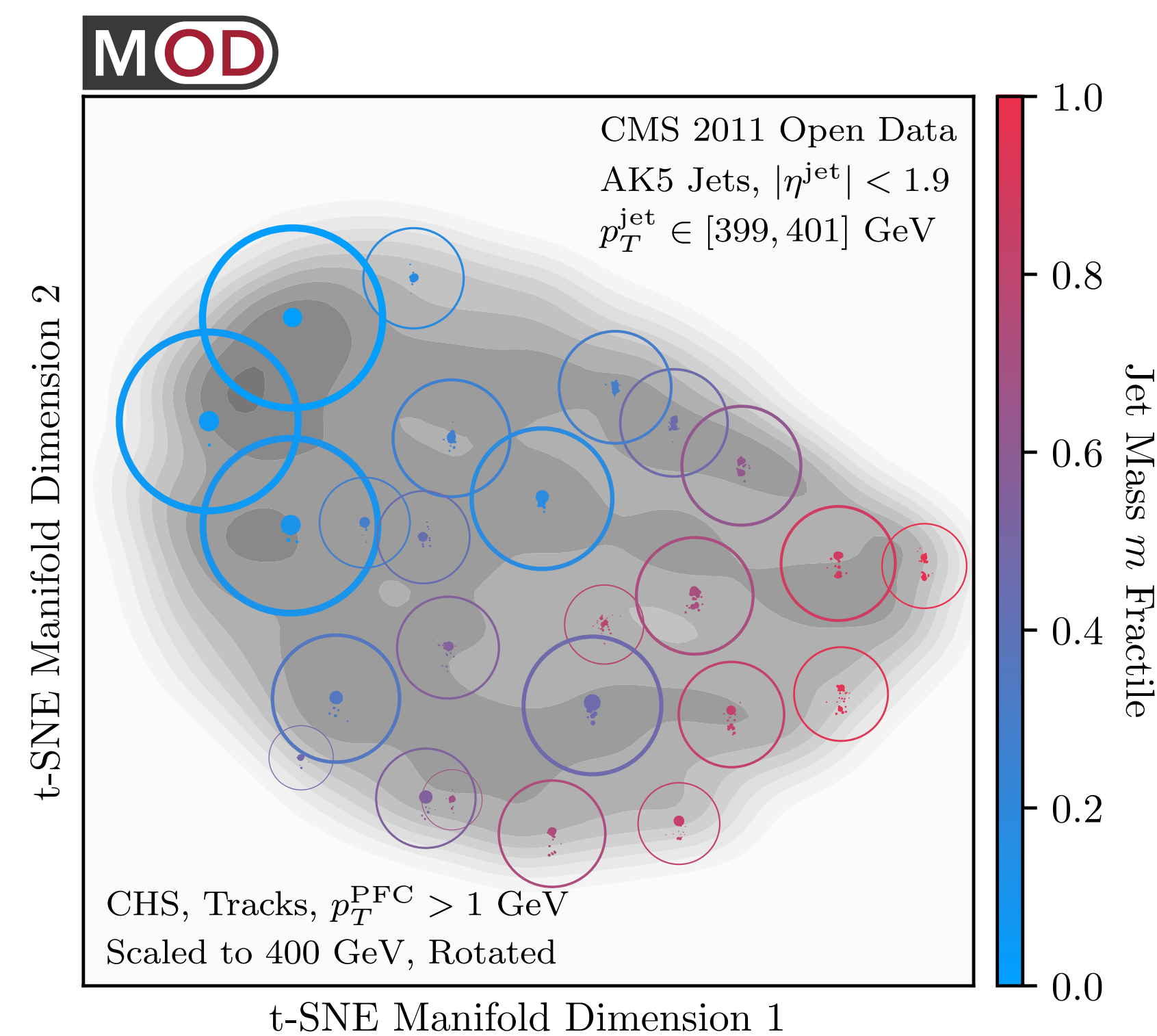
[PTK, Metodiev, Thaler, [2004.04159](#)]

ΣMD provides a rigorous construction of theory space

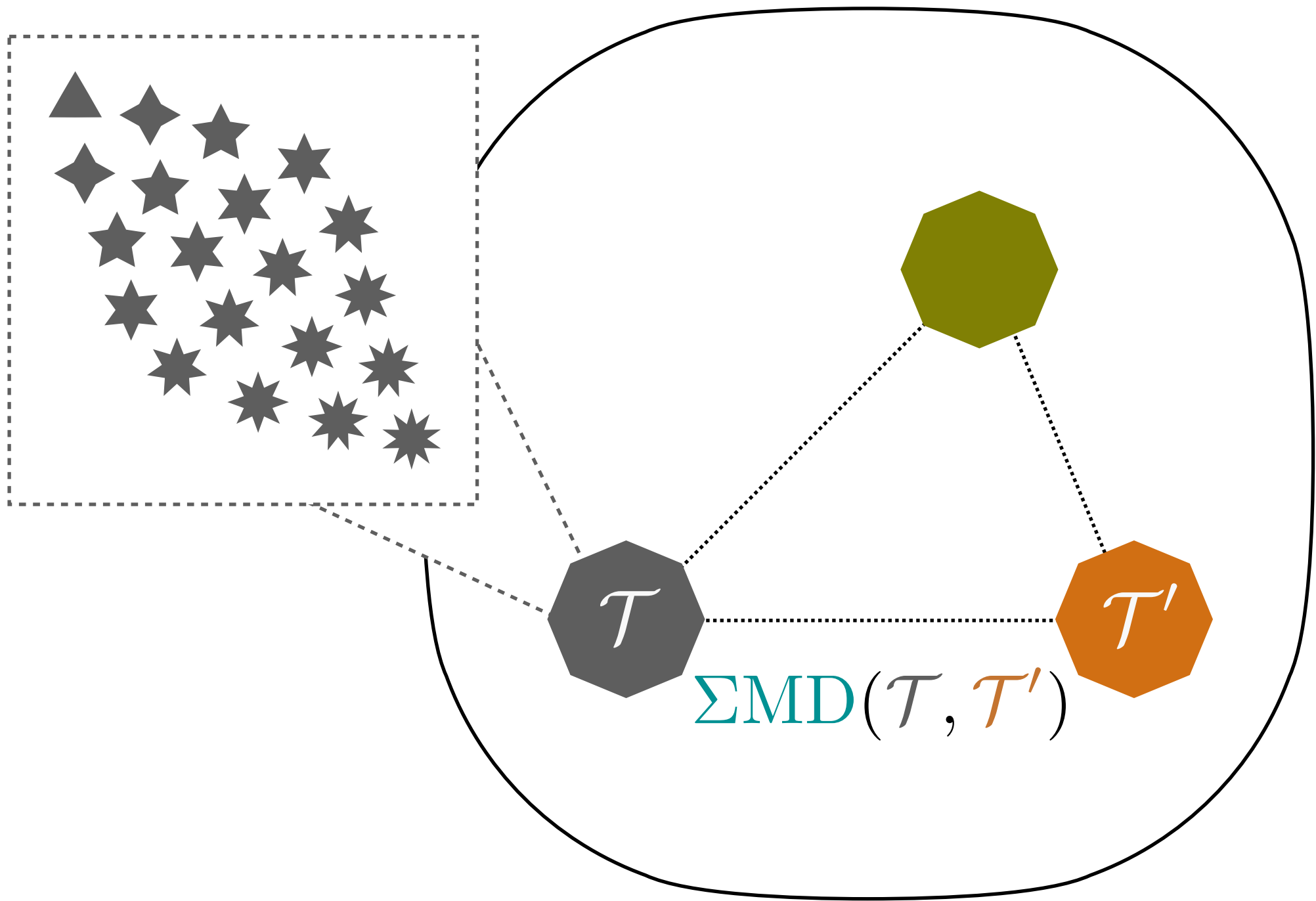


*Theories are distinguished by their **energy** flows only

ΣMD provides a rigorous construction of theory space



=

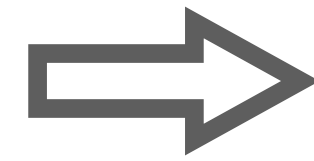


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Applications of Σ MD and the Space of Theories

Applications of Σ MD and the Space of Theories

N -(sub)jettiness



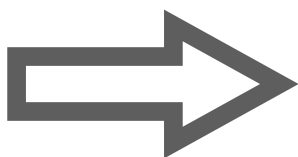
k -eventiness defined

$$\mathcal{V}_k^{(\gamma)}(\{\sigma_i, \mathcal{E}_i\}) = \min_{\mathcal{K}_1, \dots, \mathcal{K}_k} \sum_{i=1}^N \sigma_i \min \{\text{EMD}(\mathcal{E}_i, \mathcal{K}_1), \dots, \text{EMD}(\mathcal{E}_i, \mathcal{K}_k)\}^\gamma$$

$$\mathcal{V}_k^{(\gamma)}(\mathcal{T}) = \min_{|\mathcal{T}'|=k} \Sigma\text{MD}_\gamma(\mathcal{T}, \mathcal{T}')$$

Applications of Σ MMD and the Space of Theories

N -(sub)jettiness

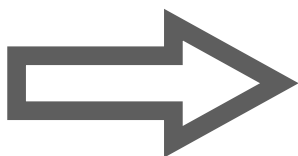


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$$\mathcal{V}_k^{(\gamma)}(\mathcal{T}) = \min_{|\mathcal{T}'|=k} \Sigma\text{MMD}_\gamma(\mathcal{T}, \mathcal{T}')$$

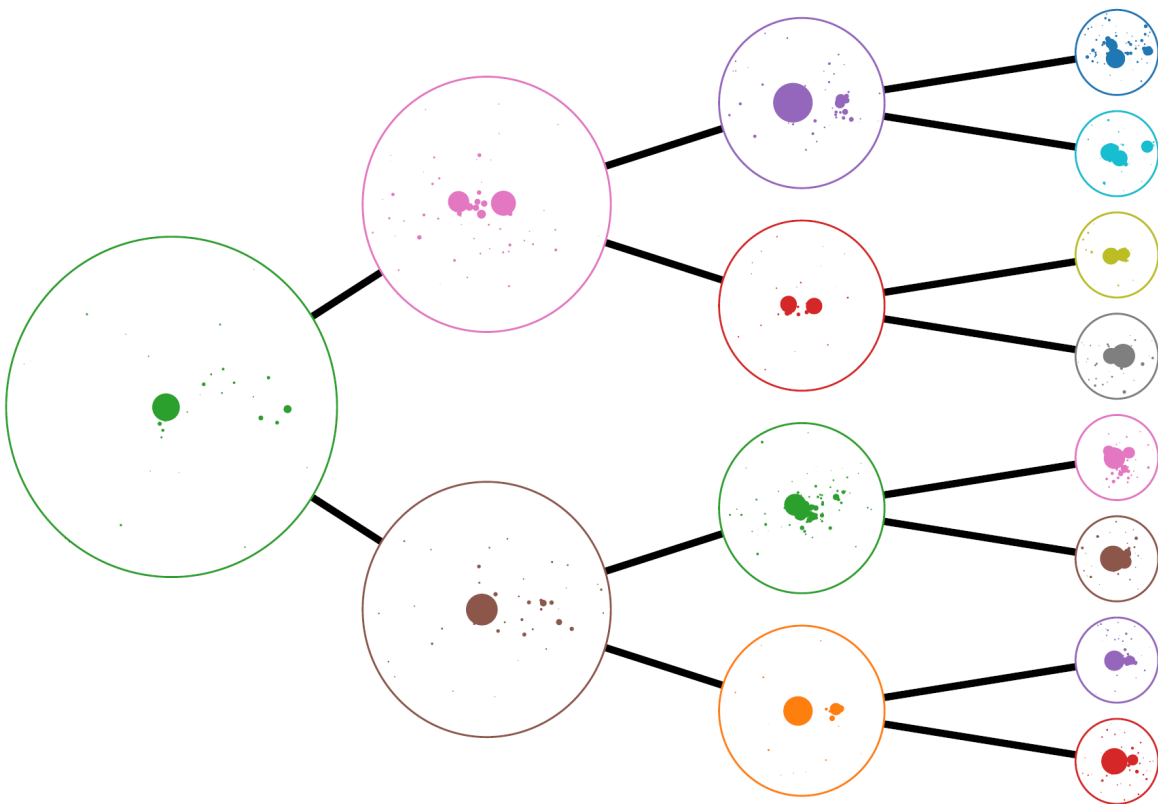
k -eventiness defined

Jet clustering



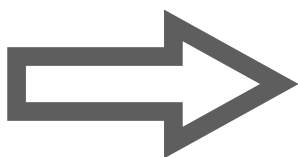
Event clustering enabled

- Exclusive cone finding
- Sequential recombination



Applications of Σ MD and the Space of Theories

N -(sub)jettiness

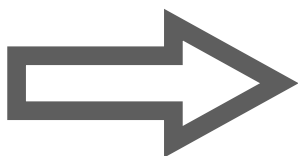


k -eventiness defined

$$\mathcal{V}_k^{(\gamma)}(\{\sigma_i, \mathcal{E}_i\}) = \min_{\mathcal{K}_1, \dots, \mathcal{K}_k} \sum_{i=1}^N \sigma_i \min \{ \text{EMD}(\mathcal{E}_i, \mathcal{K}_1), \dots, \text{EMD}(\mathcal{E}_i, \mathcal{K}_k) \}^\gamma$$

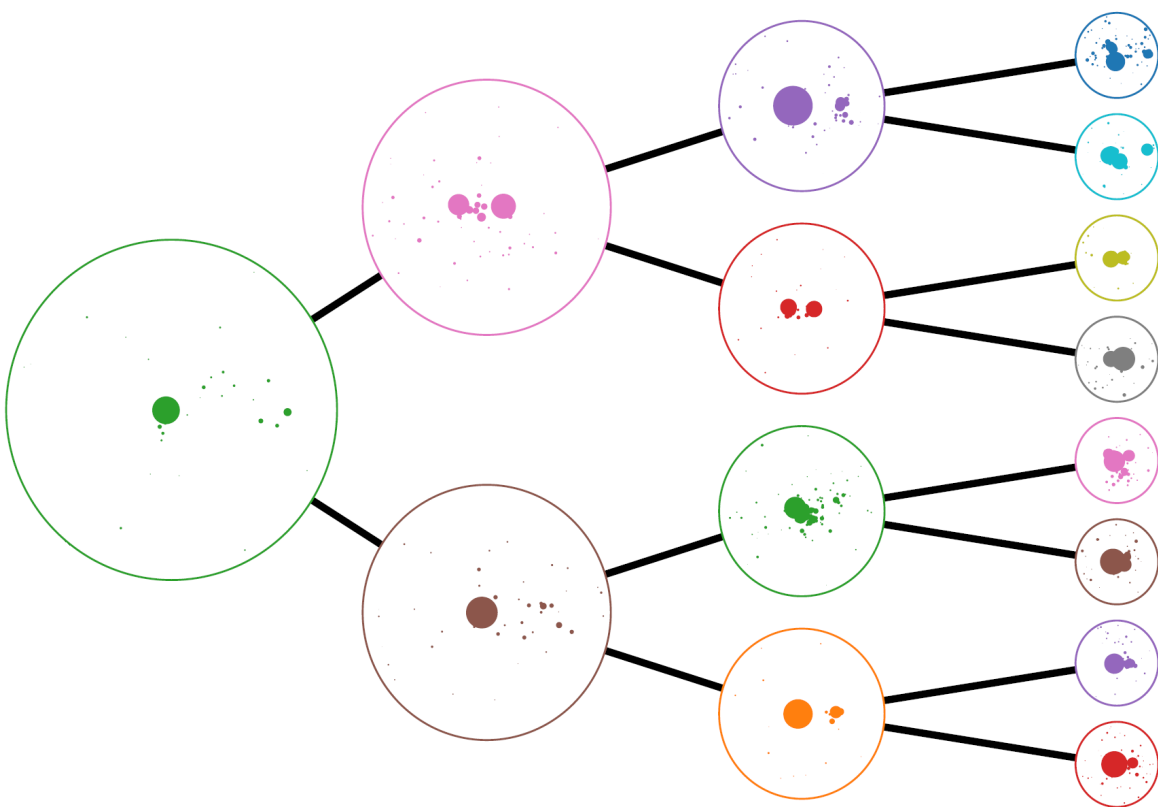
$$\mathcal{V}_k^{(\gamma)}(\mathcal{T}) = \min_{|\mathcal{T}'|=k} \Sigma\text{MD}_\gamma(\mathcal{T}, \mathcal{T}')$$

Jet clustering



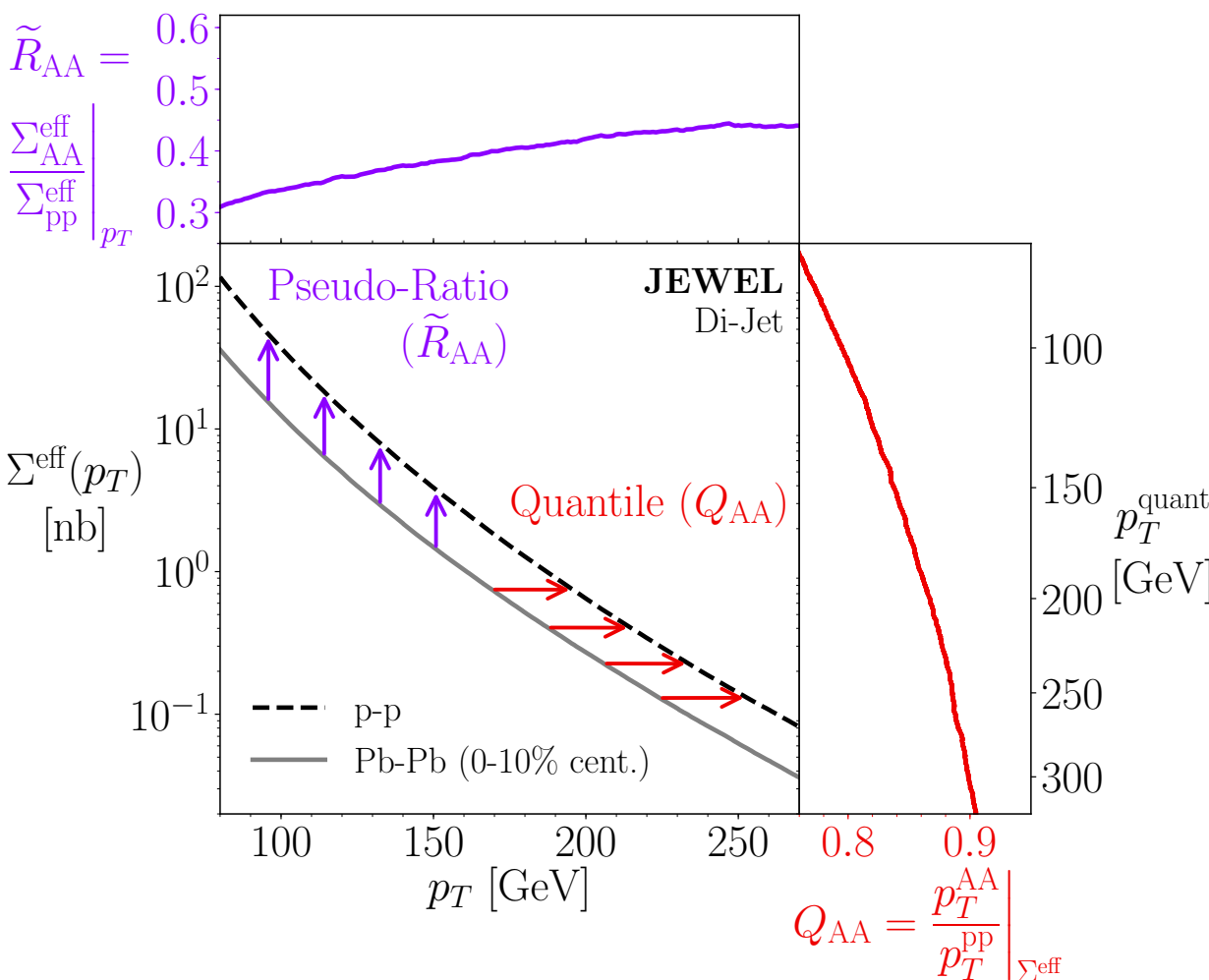
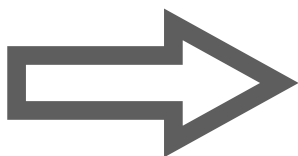
Event clustering enabled

- Exclusive cone finding
- Sequential recombination



Jet quenching in HI collisions

[Brewer, Milhano, Thaler, [PRL 2019](#)]

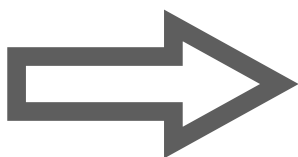


Quantile matching:

$$\Sigma_{pp}^{eff}(p_T^{\text{quant}}) \equiv \Sigma_{AA}^{eff}(p_T^{AA})$$

Applications of Σ MD and the Space of Theories

N -(sub)jettiness

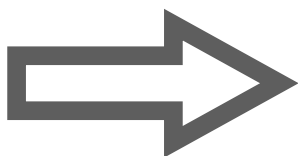


k -eventiness defined

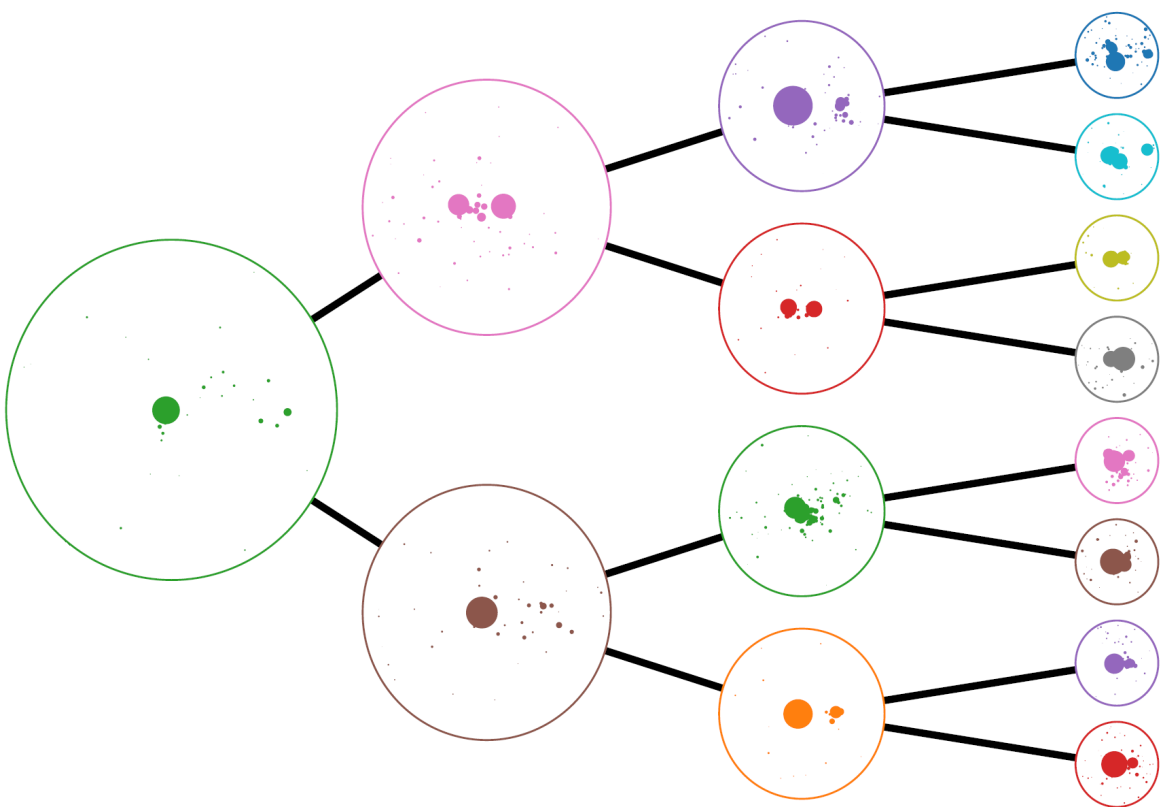
$$\mathcal{V}_k^{(\gamma)}(\{\sigma_i, \mathcal{E}_i\}) = \min_{\mathcal{K}_1, \dots, \mathcal{K}_k} \sum_{i=1}^N \sigma_i \min \{ \text{EMD}(\mathcal{E}_i, \mathcal{K}_1), \dots, \text{EMD}(\mathcal{E}_i, \mathcal{K}_k) \}^\gamma$$

$$\mathcal{V}_k^{(\gamma)}(\mathcal{T}) = \min_{|\mathcal{T}'|=k} \Sigma\text{MD}_\gamma(\mathcal{T}, \mathcal{T}')$$

Jet clustering

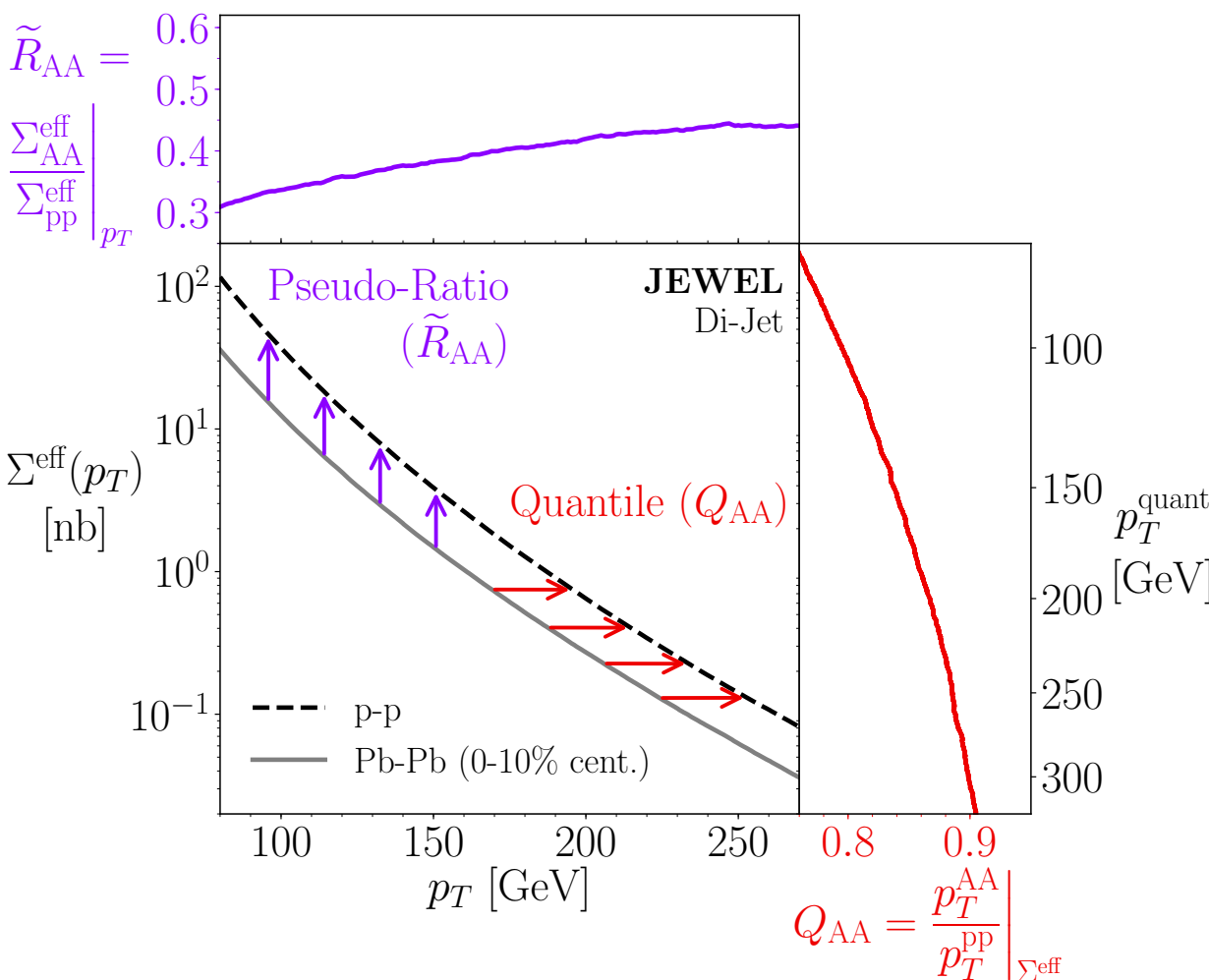
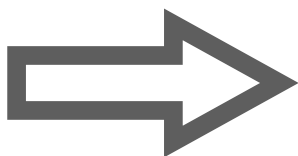


Event clustering enabled
 – Exclusive cone finding
 – Sequential recombination



Jet quenching in HI collisions

[Brewer, Milhano, Thaler, PRL 2019]



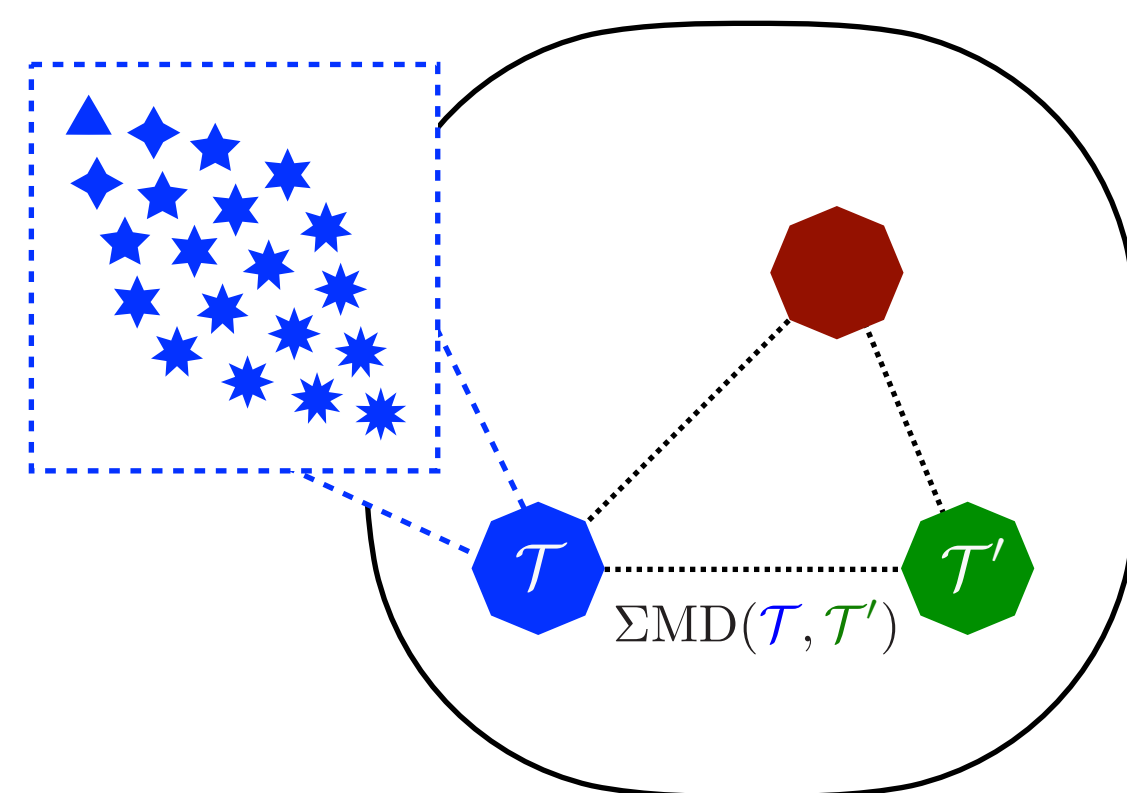
Quantile matching:

$$\Sigma_{pp}^{eff}(p_T^{\text{quant}}) \equiv \Sigma_{AA}^{eff}(p_T^{AA})$$

...is exactly a theory moving problem!

$$p_T^{\text{quant}} = \text{TM}(\mathcal{T}_{AA}, \mathcal{T}_{pp})[p_T^{AA}]$$

↑
optimal p_T -only theory movement



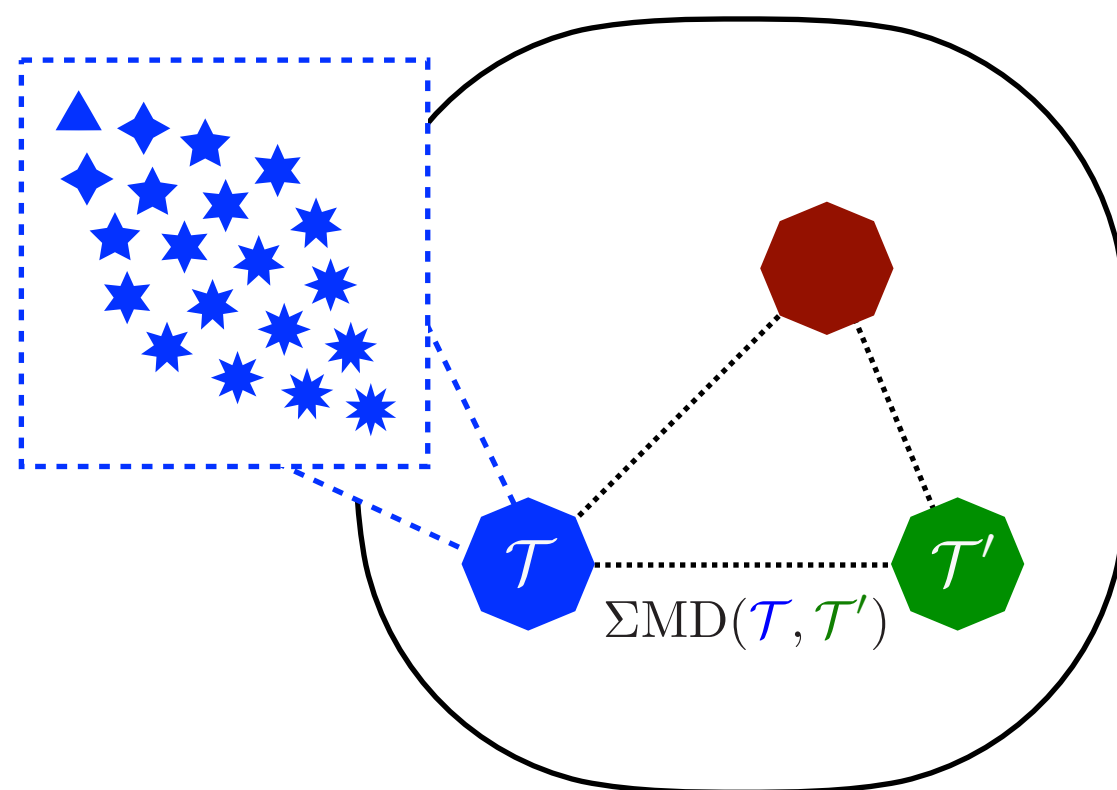
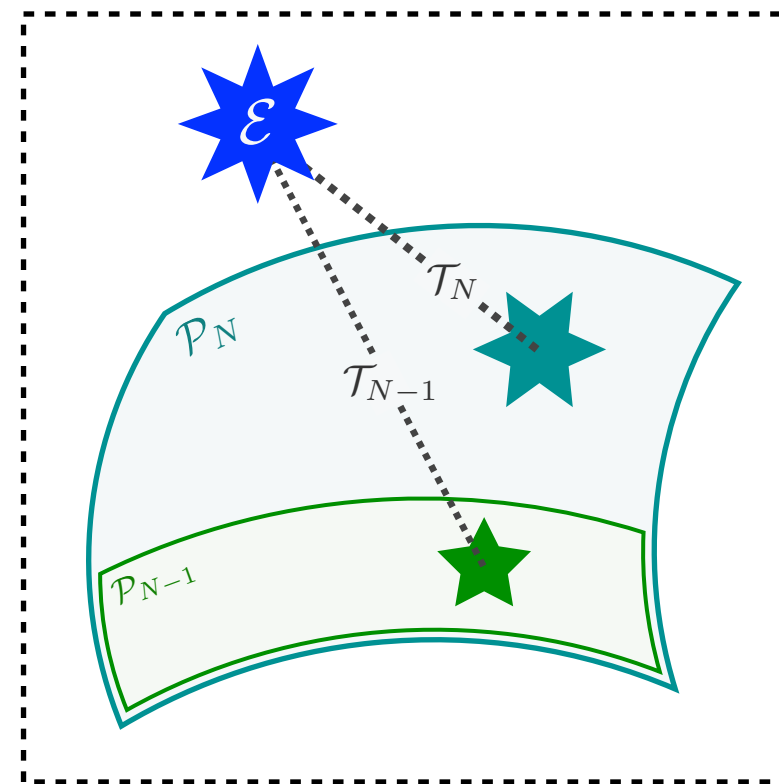
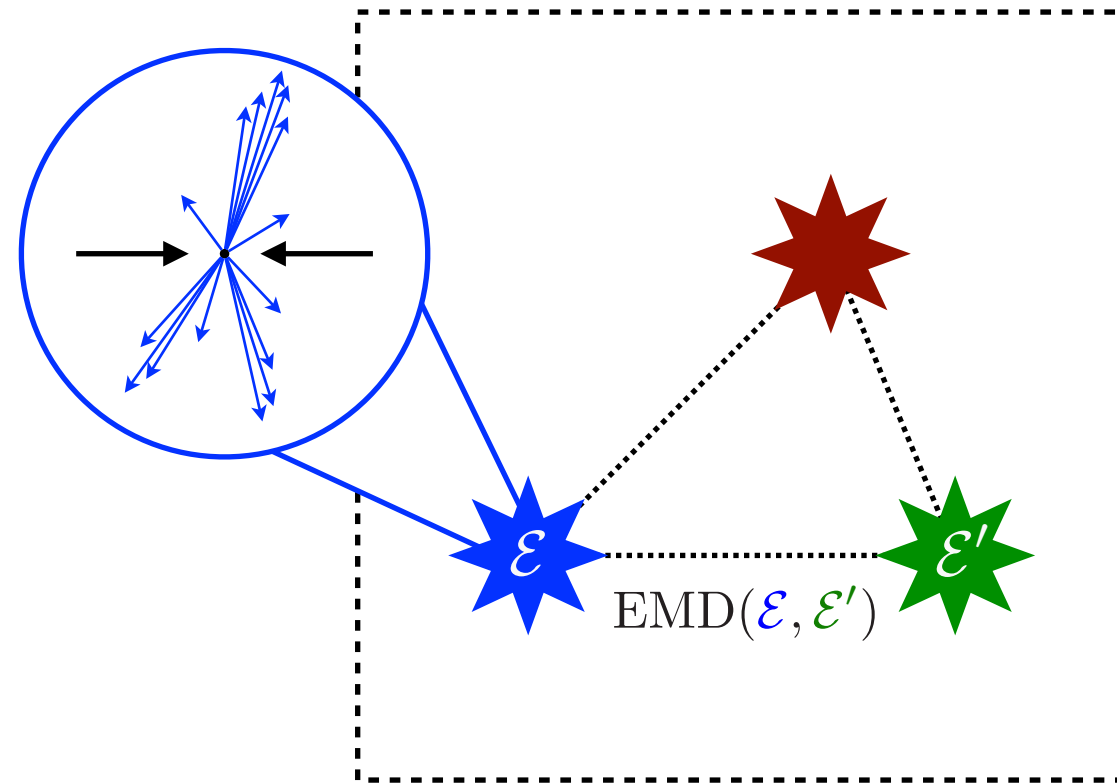
Theory Space

- Is rigorously constructed using the cross-section mover's distance ΣMD
- ΣMD uses the EMD as ground metric and cross sections as weights
- Allows for theories to be explored with tools developed for events

	Energy Mover's Distance	Cross Section Mover's Distance
Symbol	EMD	ΣMD
Description	Distance between events	Distance between theories
Weight	Particle energies E_i	Event cross sections σ_i
Ground Metric	Particle distances θ_{ij}	Event distances $EMD(\mathcal{E}_i, \mathcal{E}_j)$

How else can ΣMD and theory space be utilized?

Perhaps Monte Carlo tuning/benchmarking...



The (Metric) Space of Events

- **Energy** flow is theoretically and experimentally robust
- EMD metrizes the space of energy flows (events)
- Manifolds in the space of events can be visualized and quantified

Revealing Hidden Geometry


- Event space exhibits a rich geometry that can be probed using the **EMD**
- Decades worth of collider techniques are naturally described in this geometry
- Many new techniques are suggested, and new light is shed on old ones

Theory Space

- Is rigorously constructed using the cross-section mover's distance **ΣMD**
- **ΣMD** uses the EMD as ground metric and cross sections as weights
- Allows for theories to be explored with tools developed for events


```
pip3 install energyflow
```

Detailed [examples](#), [demos](#), and [documentation](#)



EnergyFlow

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Energy Flow Moments

Energy Flow Polynomials

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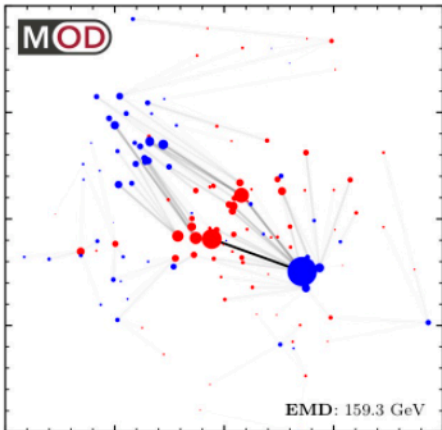
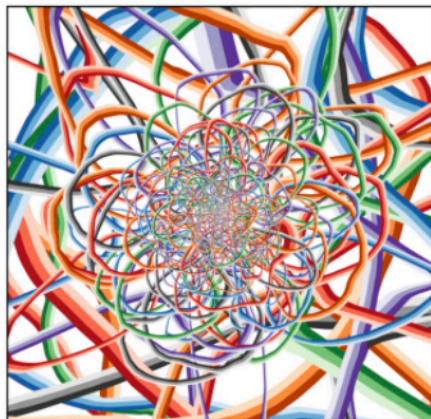
Multigraph Generation

Observables

Utils

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Welcome to EnergyFlow




EnergyFlow is a Python package containing a suite of particle physics tools:

- Energy Flow Polynomials:** EFPs are a collection of jet substructure observables which form a complete linear basis of IRC-safe observables. EnergyFlow provides tools to compute EFPs on events for several energy and angular measures as well as custom measures.
- Energy Flow Networks:** EFNs are infrared- and collinear-safe models designed for learning from collider events as unordered, variable-length sets of particles. EnergyFlow contains customizable Keras implementations of EFNs. Available from version `0.10.0` onward.
- Particle Flow Networks:** PFNs are general models designed for learning from collider events as unordered, variable-length sets of particles, based on the `Deep Sets` framework. EnergyFlow contains customizable Keras implementations of PFNs. Available from version `0.10.0` onward.
- Energy Mover's Distance:** The EMD is a common metric between probability distributions that has been adapted for use as a metric between collider events. EnergyFlow contains code to facilitate the computation of the EMD between events based on an underlying implementation provided by the `Python Optimal Transport (POT)` library. Available from version `0.11.0` onward.
- Energy Flow Moments:** EFM are moments built out of particle energies and momenta that can be evaluated in linear time in the number of particles. They provide a highly efficient means of implementing $\beta = 2$ EFPs and are also very useful for reasoning about linear redundancies that appear between EFPs. Available from version `1.0.0` onward.

The EnergyFlow package also provides easy access to particle physics datasets and useful supplementary features:

- CMS Open Data in MOD HDF5 Format:** Reprocessed datasets from the `CMS Open Data`,

 GitHub

Next »

[hub.gke.mybinder.org/user/pkomiske-energyflow-Ols423ee/notebooks/demos/EMD%20Demo.ipynb](#)

Jupyter EMD Demo (autosaved)

File Edit View Insert Cell Kernel Widgets Help Not Trusted Python 3 O

EnergyFlow website

In this tutorial, we demonstrate how to compute EMD values for particle physics events. The core of the computation is done using the [Python Optimal Transport](#) library with EnergyFlow providing a convenient interface to particle physics events. Batching functionality is also provided using the builtin multiprocessing library to distribute computations to worker processes.

Energy Mover's Distance

The Energy Mover's Distance was introduced in [1902.02346](#) as a metric between particle physics events. Closely related to the Earth Mover's Distance, the EMD solves an optimal transport problem between two distributions of energy (or transverse momentum), and the associated distance is the "work" required to transport supply to demand according to the resulting flow. Mathematically, we have

$$\text{EMD}(\mathcal{E}, \mathcal{E}') = \min_{(f_{ij})} \sum_{ij} f_{ij} \frac{\theta_{ij}}{R} + \left| \sum_i E_i - \sum_j E'_j \right|,$$

$$\sum_j f_{ij} \leq E_i, \quad \sum_i f_{ij} \leq E'_j, \quad \sum_{ij} f_{ij} = \min \left(\sum_i E_i, \sum_j E'_j \right).$$

Imports

```
In [1]: import numpy as np
import matplotlib inline
import matplotlib.pyplot as plt
import energyflow as ef
```

Plot Style

```
In [2]: plt.rcParams['figure.figsize'] = (4,4)
plt.rcParams['figure.dpi'] = 120
plt.rcParams['font.family'] = 'serif'
```

Load EnergyFlow Quark/Gluon Jet Samples

```
In [3]: # load quark and gluon jets
X, y = ef.qg_jets.load(2000, pad=False)
num = 750

# the jet radius for these jets
R = 0.4


# process jets
Gs, Qs = [], []
for arr, events in [(Gs, X[y==0]), (Qs, X[y==1])]:
    for i,x in enumerate(events):
        if i >= num:
            break


# ignore padded particles and removed particle id information
x = x[x[:,0] > 0,:3]

# center jet according to pt-centroid
yphi_avg = np.average(x[:,1:3], weights=x[:,0], axis=0)
x[:,1:3] -= yphi_avg

# mask out any particles farther than R=0.4 away from center (rare)
x = x[np.linalg.norm(x[:,1:3], axis=1) <= R]

# add to list
```










[Upload](#)
[Communities](#)

August 8, 2019

[Dataset](#)
[Open Access](#)

CMS 2011A Open Data | Jet Primary Dataset | $p_T > 375$ GeV | MOD HDF5 Format

 Korniske, Patrick;
  Mastandrea, Radha;
  Metodiev, Eric;
  Naik, Preksha;
  Thaler, Jesse








A dataset of 1,785,625 jets from the [Jet Primary Dataset of the CMS 2011A Open Data](#) reprocessed into the MOD HDF5 format. Jets are selected from the hardest two anti- k_T $R=0.5$ jets in events passing the Jet300 High Level Trigger and are required to have $p_T^{jet} > 375$ GeV, where p_T^{jet} includes a jet energy correction factor. Particle Flow Candidates (PFCs) for each jet are provided and include information about the PFC kinematics, PDG ID, and vertex. Additionally, jets have metadata describing their kinematics and provenance in the original CMS AOD files.

For additional details about the dataset, please see the accompanying paper, Exploring the Space of Jets with CMS Open Data. There, jets were further restricted to have $|\eta^{jet}| < 1.9$ to ensure tracking coverage and have "medium" quality to reject fake jets.

The supported method for downloading, reading, and using this dataset is through the [EnergyFlow Python package](#), which has additional documentation about how to read and use this and related datasets. Should any problems be encountered, please [submit an issue on GitHub](#).

There are corresponding datasets of simulated jets organized by hard parton \hat{p}_T also available on Zenodo:

- [SIM/GEN QCD Jets 170-300 GeV](#)
- [SIM/GEN QCD Jets 300-470 GeV](#)
- [SIM/GEN QCD Jets 470-600 GeV](#)
- [SIM/GEN QCD Jets 600-800 GeV](#)
- [SIM/GEN QCD Jets 800-1000 GeV](#)
- [SIM/GEN QCD Jets 1000-1400 GeV](#)
- [SIM/GEN QCD Jets 1400-1800 GeV](#)
- [SIM/GEN QCD Jets 1800- \$\infty\$ GeV](#)

Files (2.0 GB)	
Name	Size
CMS_Jet300_pT375-infGeV_0_compressed.h5	111.2 MB Download
md5:f1d2d4013e1e0026b4f8cc84b9d5f944 	
CMS_Jet300_pT375-infGeV_10_compressed.h5	110.8 MB Download
md5:7f6e5ab36cb7082ab10efff911509e46 	
CMS_Jet300_pT375-infGeV_11_compressed.h5	111.3 MB Download
md5:a3b2c2e2e1855c6c8106e6c6c0a045ce53 	
CMS_Jet300_pT375-infGeV_12_compressed.h5	111.7 MB Download
md5:a37a4e4e9b52cf59ca2e5aad289e563 	
CMS_Jet300_pT375-infGeV_13_compressed.h5	111.3 MB Download
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[open data](#)
[lhc](#)
[jet](#)
[substructure](#)
[hlep](#)
[physics](#)

Related identifiers:
 Supplement to
[arXiv:1908.08542](#)







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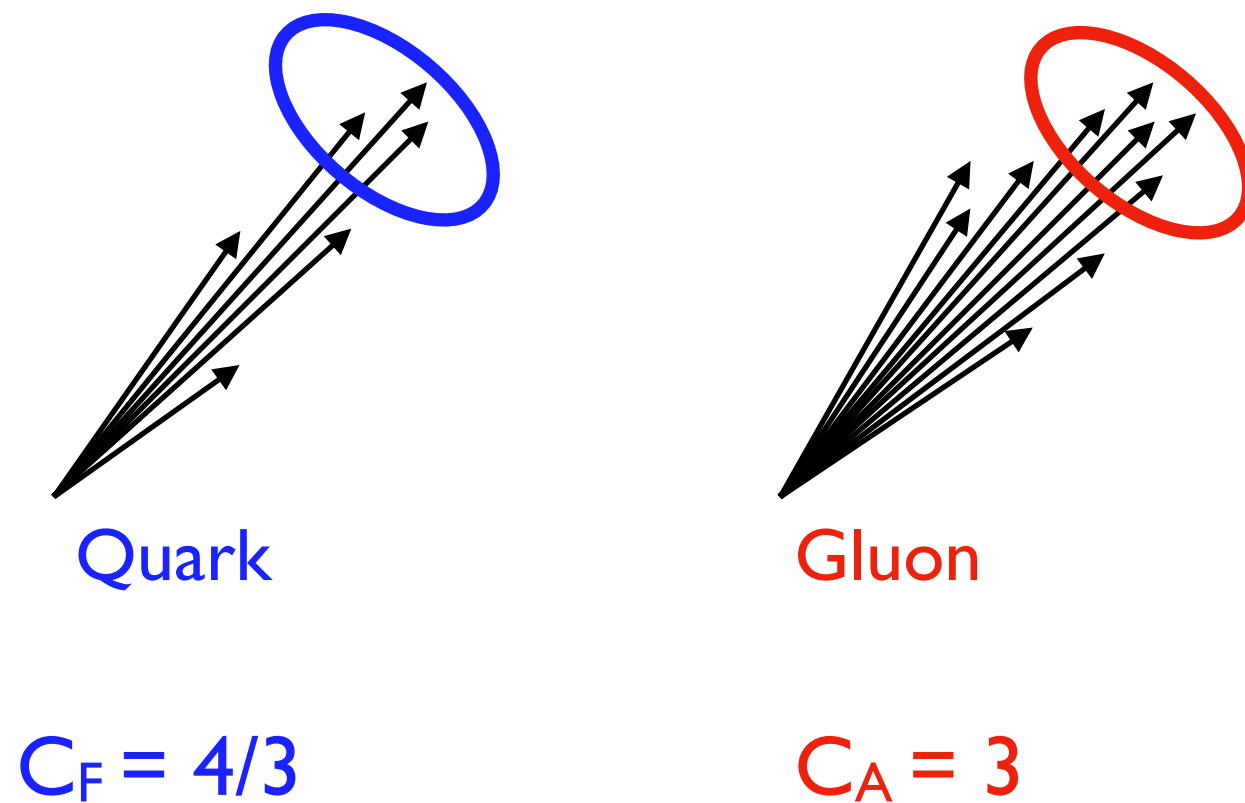
Additional Slides

Quark and Gluon Correlation Dimensions

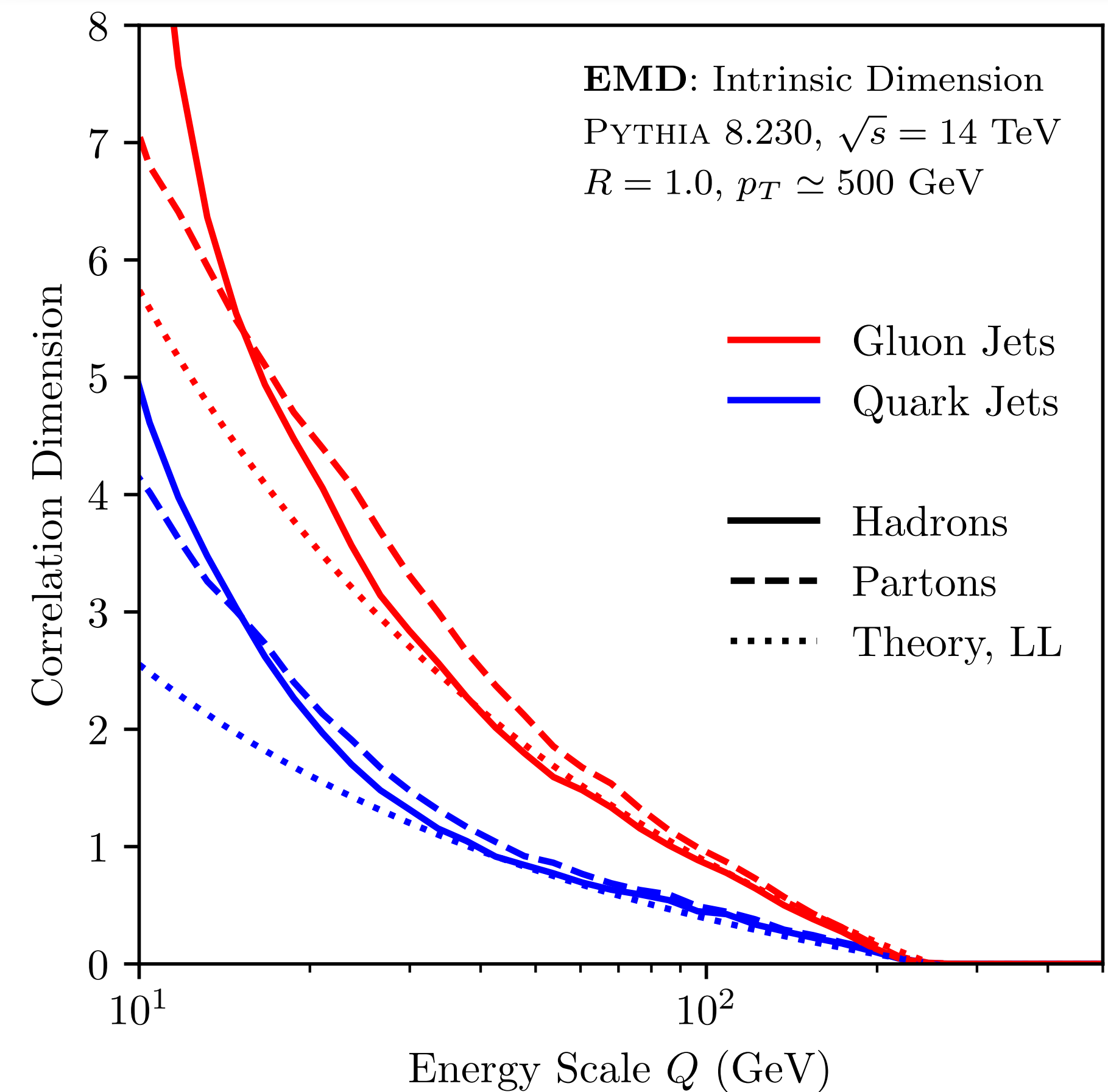
Leading log (single emission) calculation:

$$\dim_i(Q) \simeq -\frac{8\alpha_s}{\pi} C_i \ln \frac{Q}{p_T/2}$$

↑
color factor



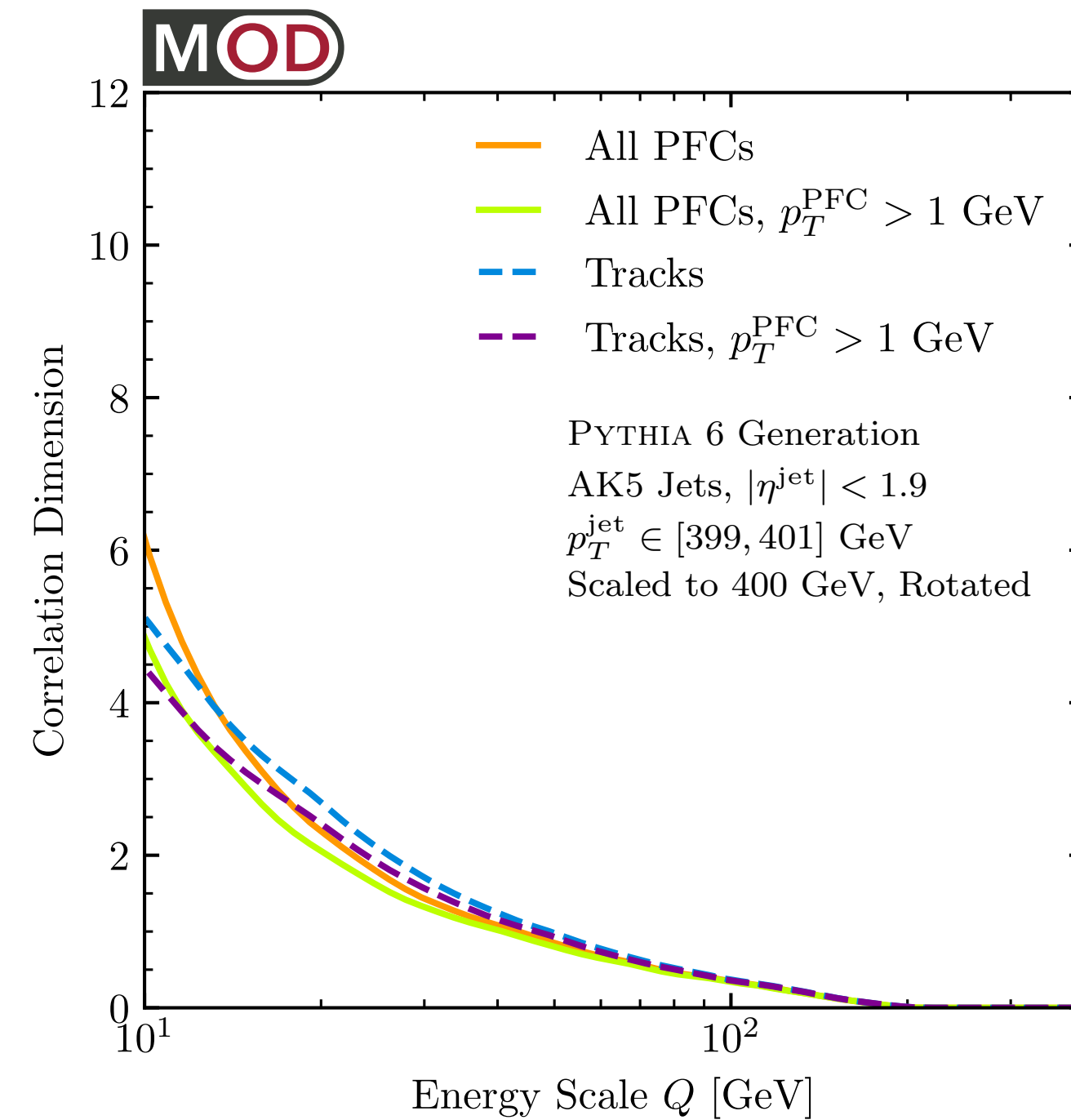
$$\dim(Q) = Q \frac{\partial}{\partial Q} \ln \sum_i \sum_j \Theta(\text{EMD}(\mathcal{E}_i, \mathcal{E}'_j) < Q)$$



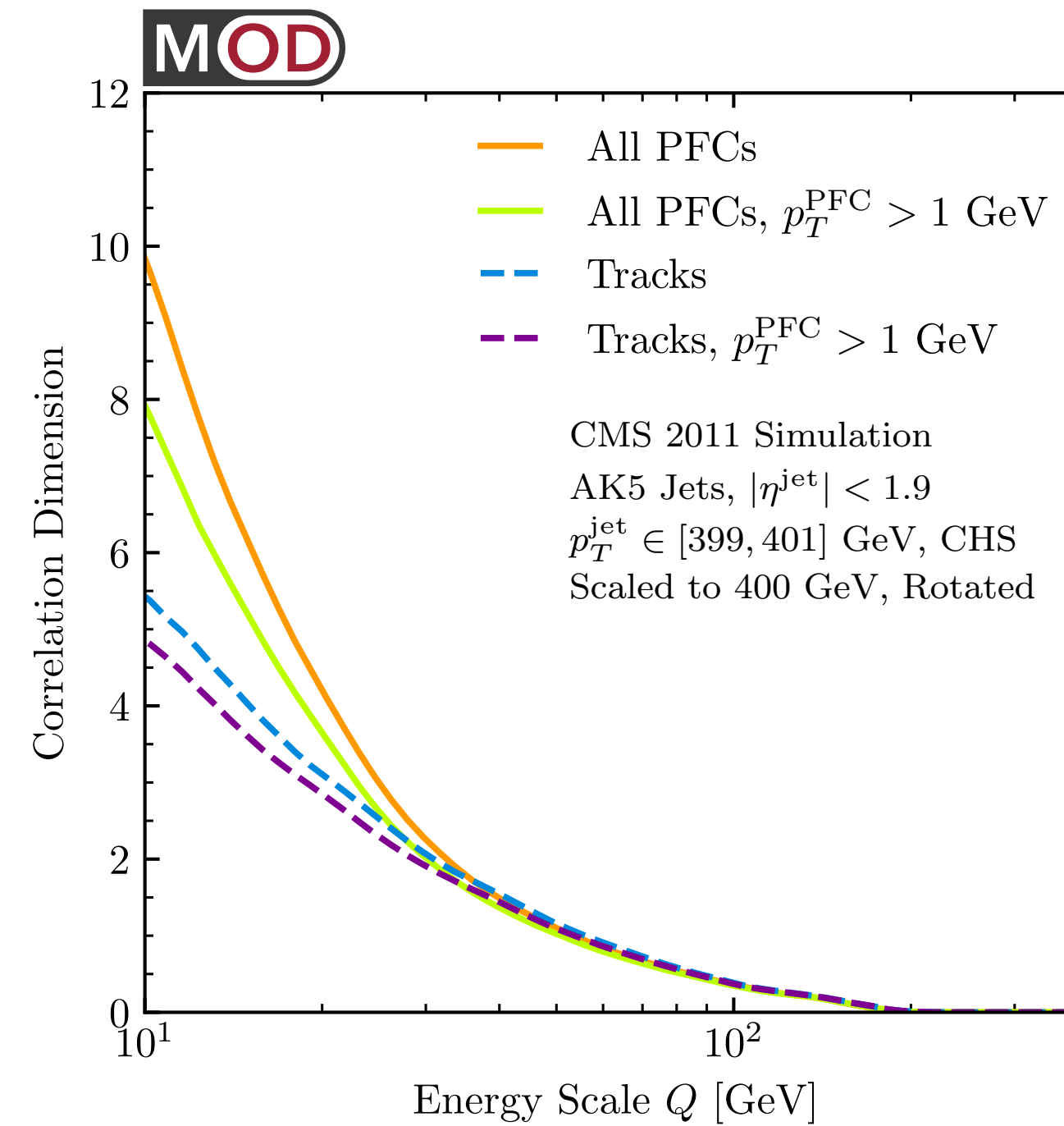
[PTK, Metodiev, Thaler, *to appear soon*]

Correlation Dimension at Particle and Detector Levels

Particle-level (PYTHIA)



Detector-level (PYTHIA + GEANT 4)



CMS Open Data

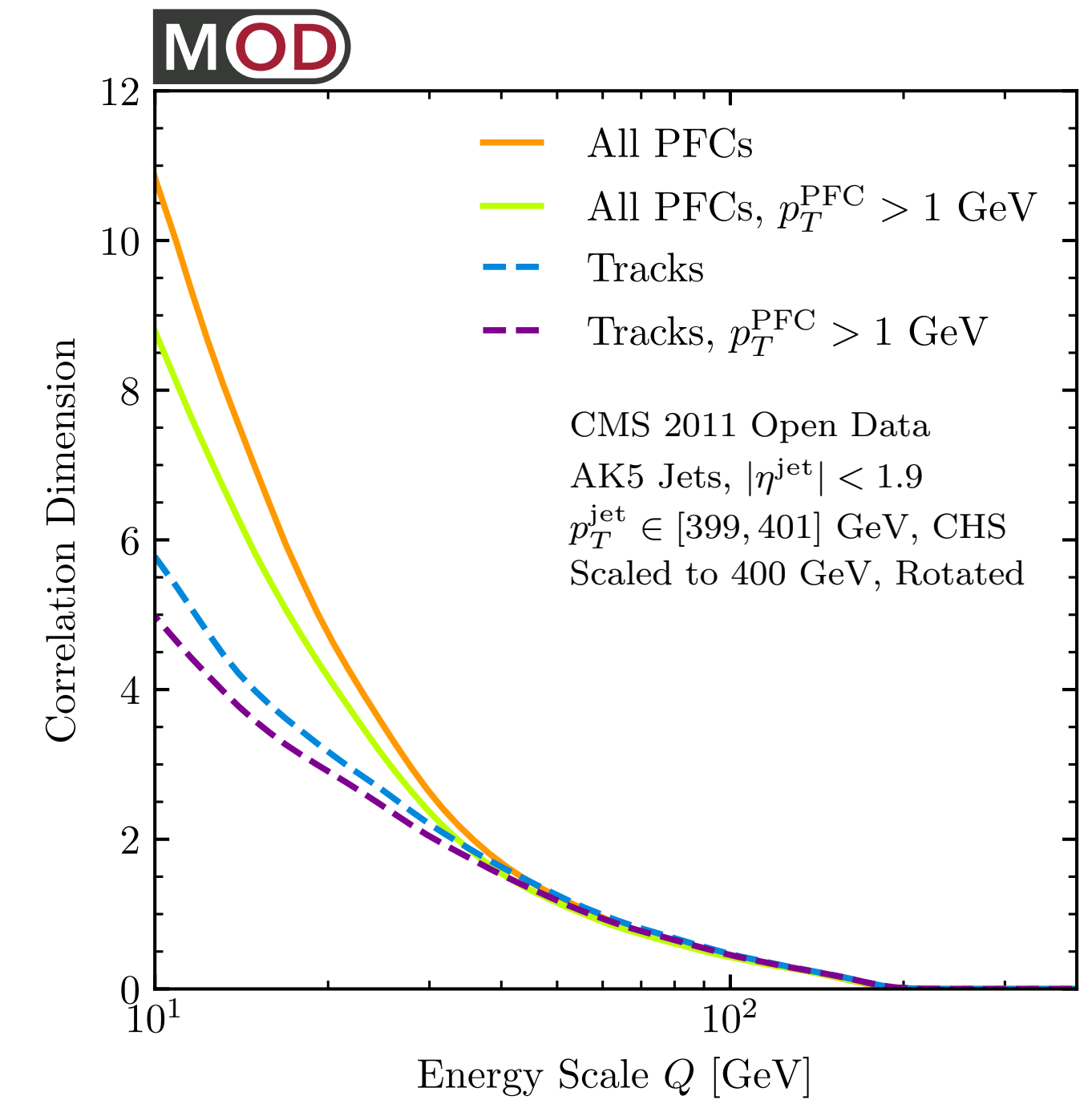
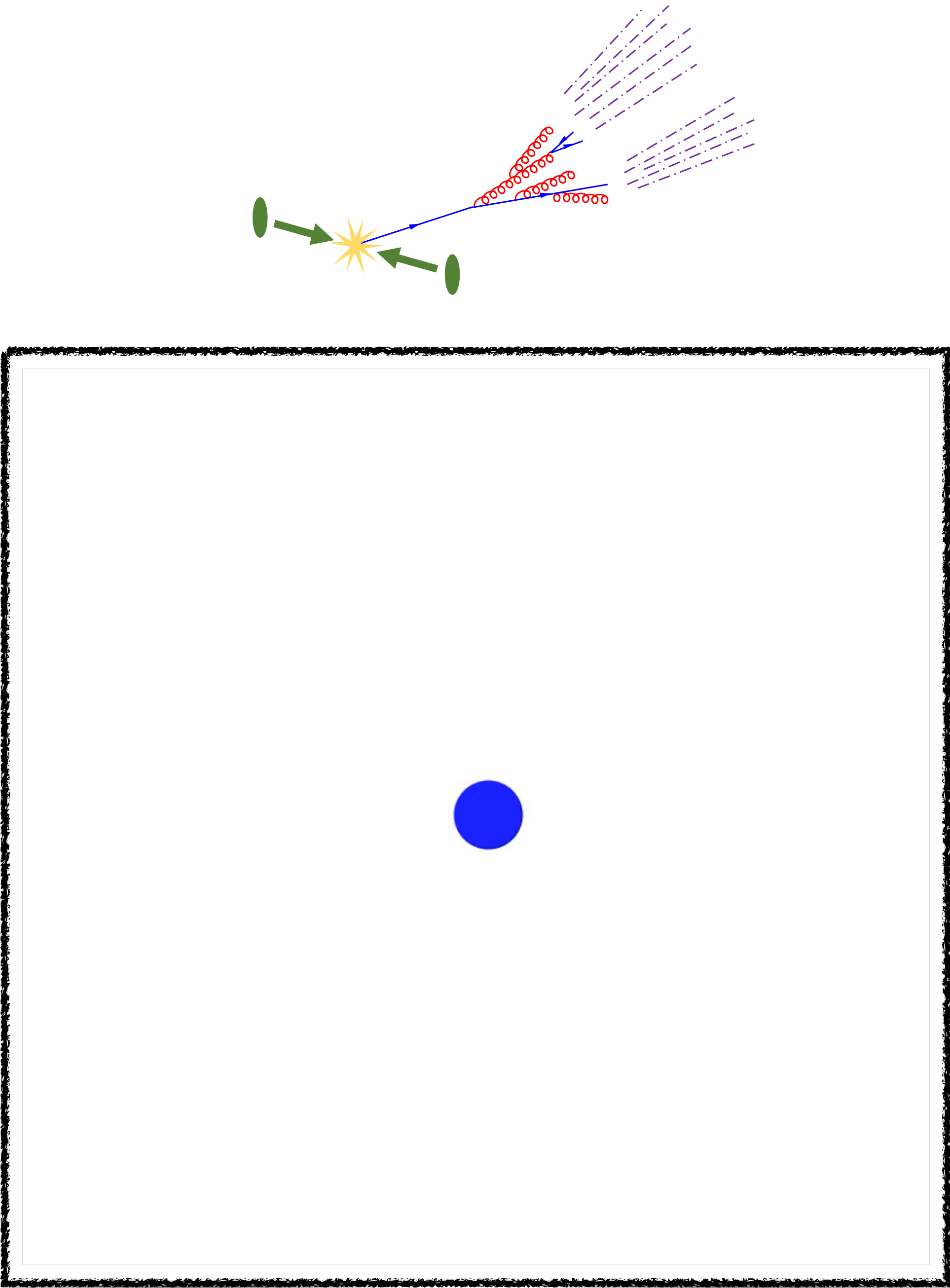
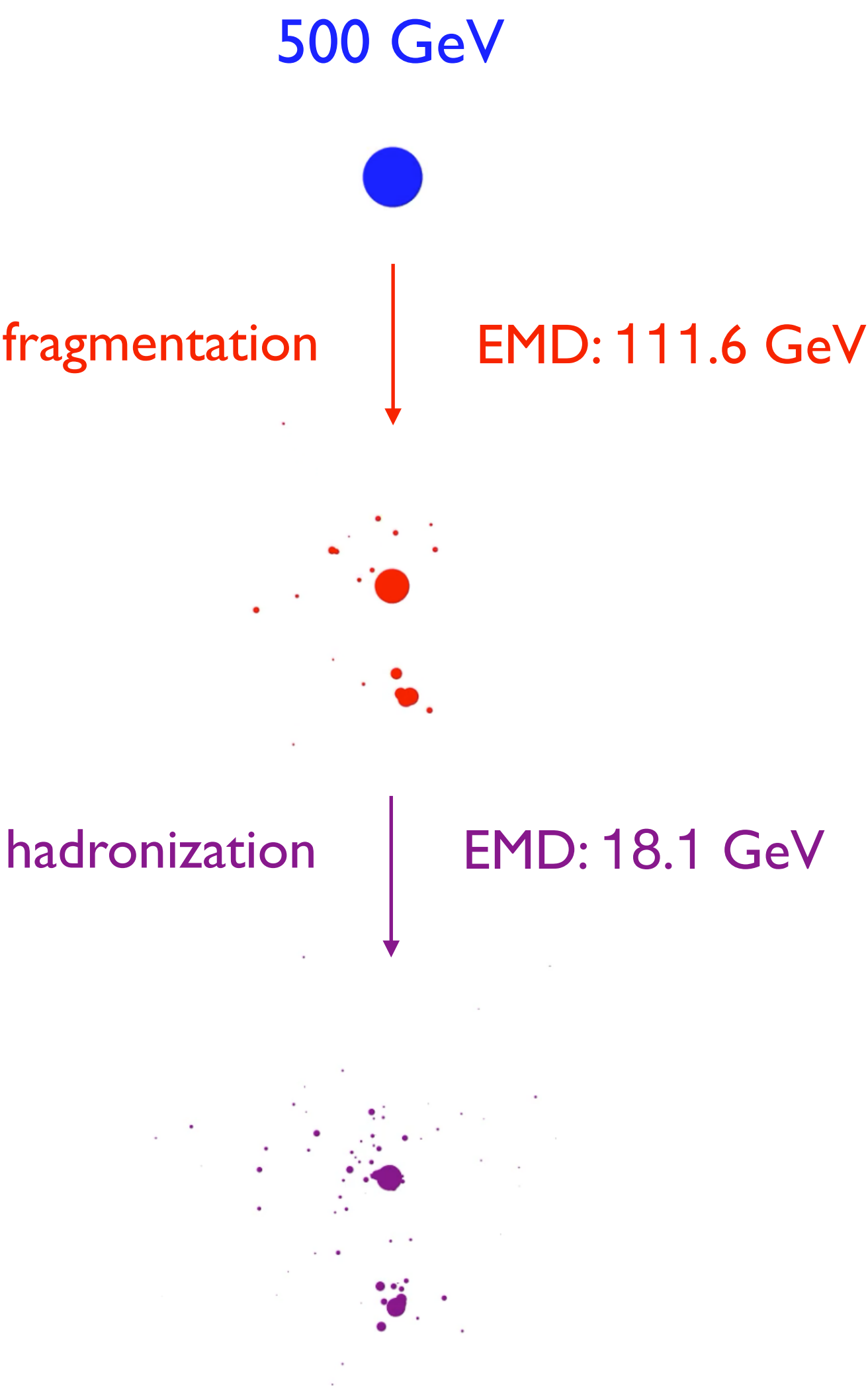


Table of Observables Defined via Event Space Geometry

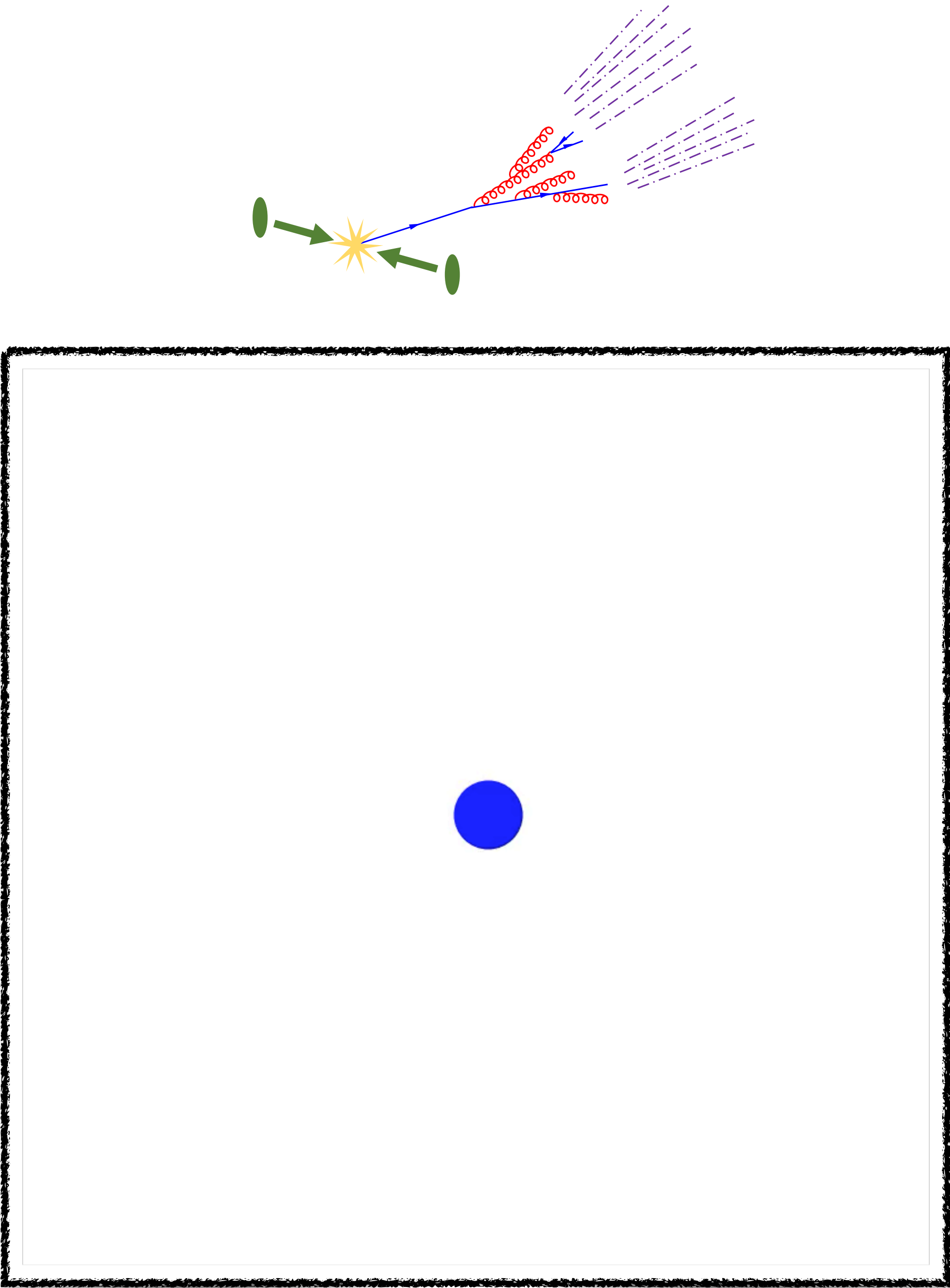
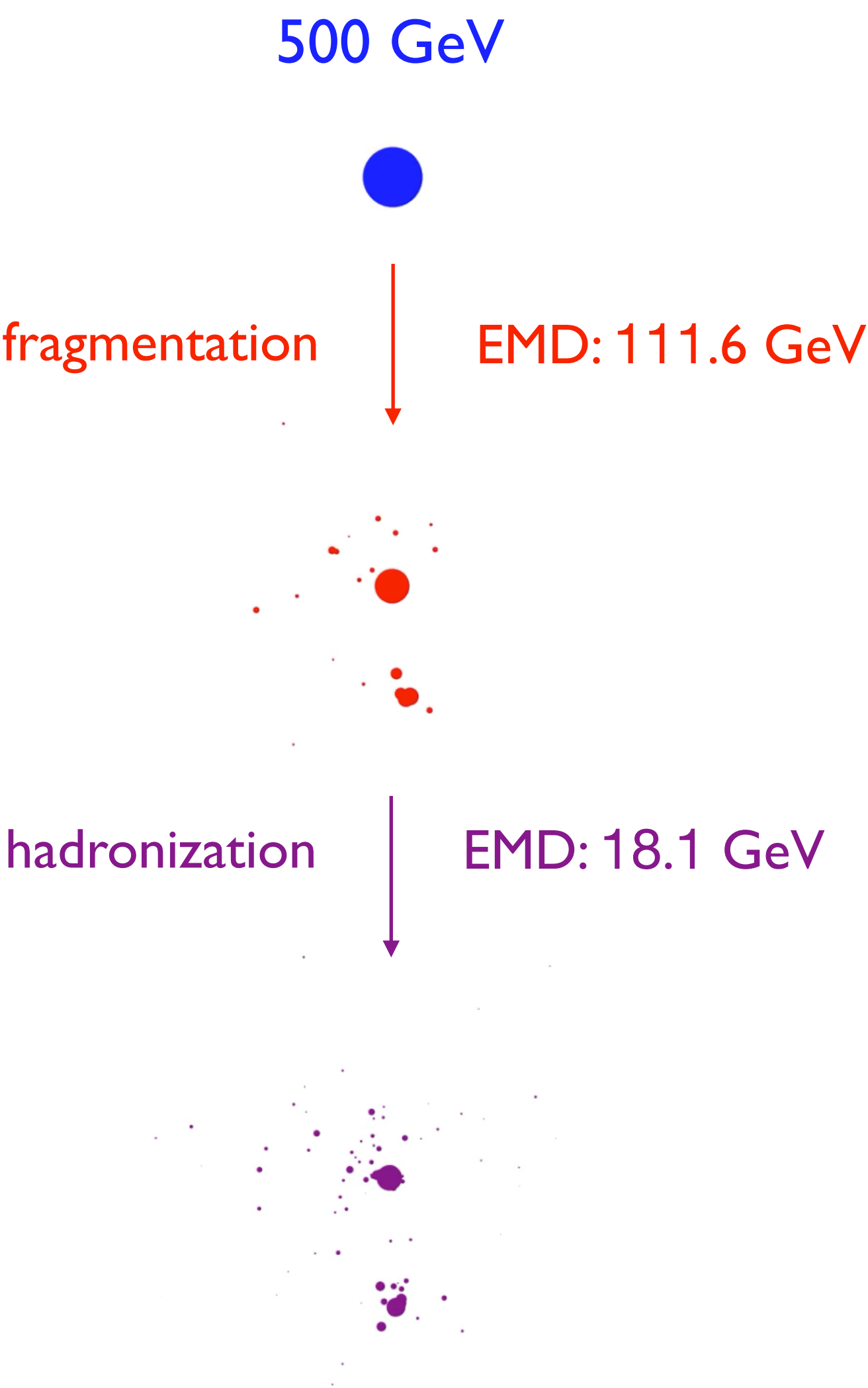
$$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \text{EMD}_{\beta,R}(\mathcal{E}, \mathcal{E}')$$

$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \text{EMD}_{\beta}(\mathcal{E}, \mathcal{E}')$			
Name		β	Manifold \mathcal{M}
Thrust	$t(\mathcal{E})$	2	$\mathcal{P}_2^{\text{BB}}$: 2-particle events, back to back
Spherocity	$\sqrt{s(\mathcal{E})}$	1	$\mathcal{P}_2^{\text{BB}}$: 2-particle events, back to back
Broadening	$b(\mathcal{E})$	1	\mathcal{P}_2 : 2-particle events
N -jettiness	$\mathcal{T}_N^{(\beta)}(\mathcal{E})$	β	\mathcal{P}_N : N -particle events
Isotropy	$\mathcal{I}^{(\beta)}(\mathcal{E})$	β	$\mathcal{M}_{\mathcal{U}}$: Uniform events
Jet Angularities	$\lambda_{\beta}(\mathcal{J})$	β	\mathcal{P}_1 : 1-particle jets
N -subjettiness	$\tau_N^{(\beta)}(\mathcal{J})$	β	\mathcal{P}_N : N -particle jets

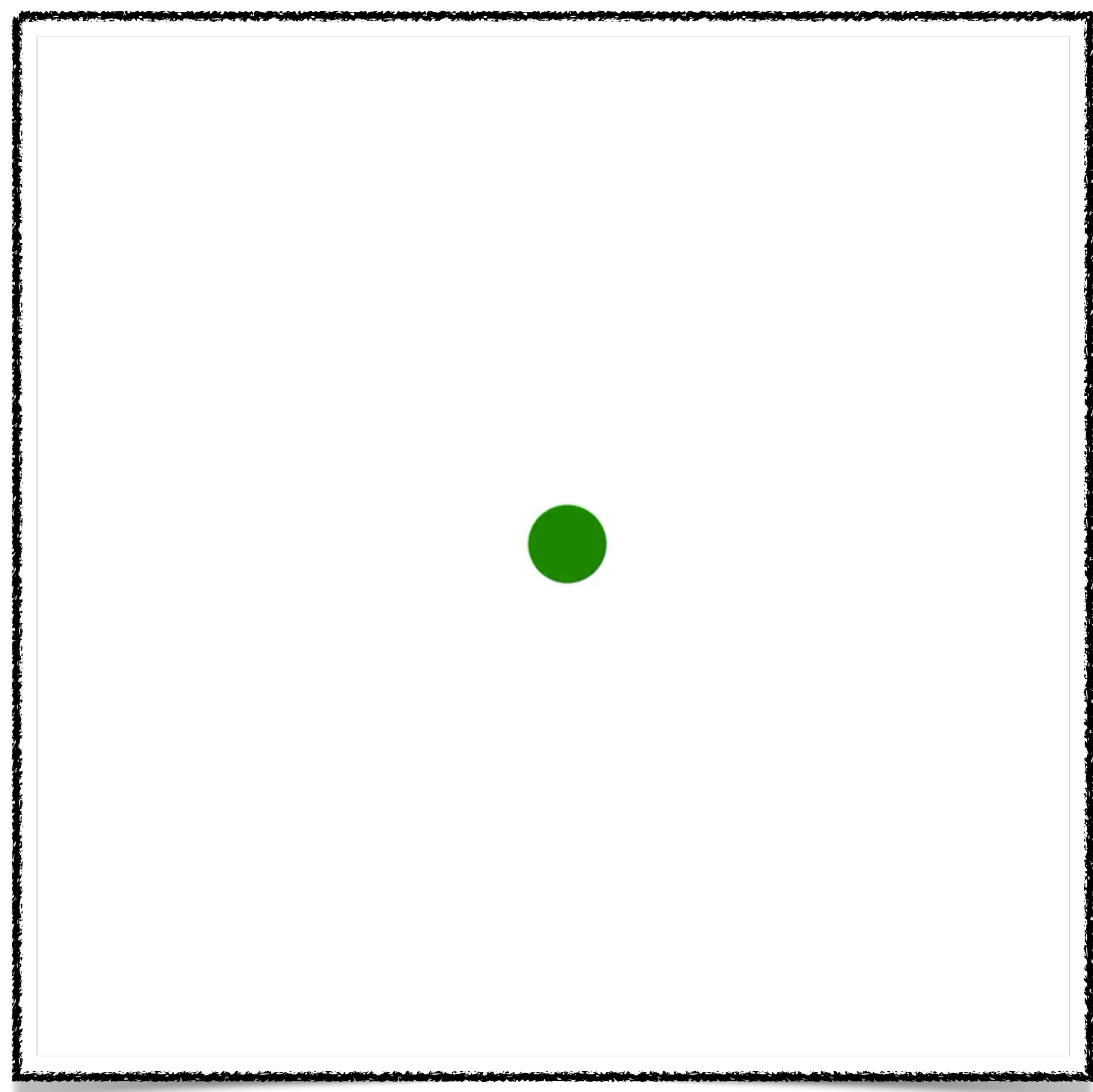
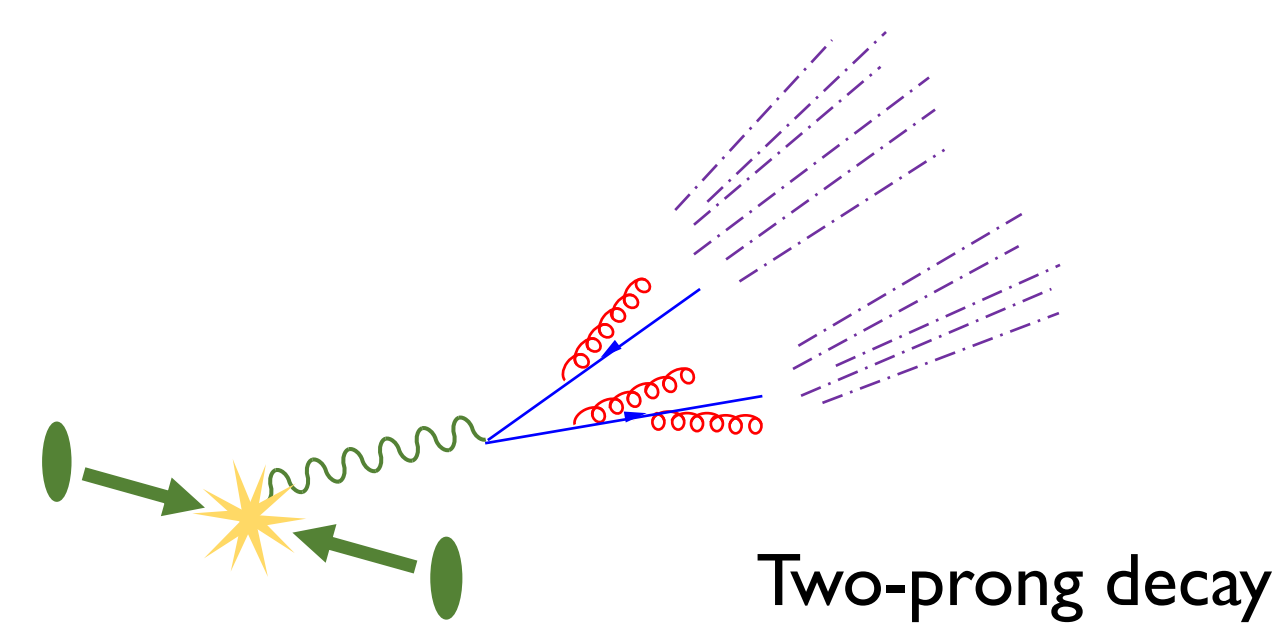
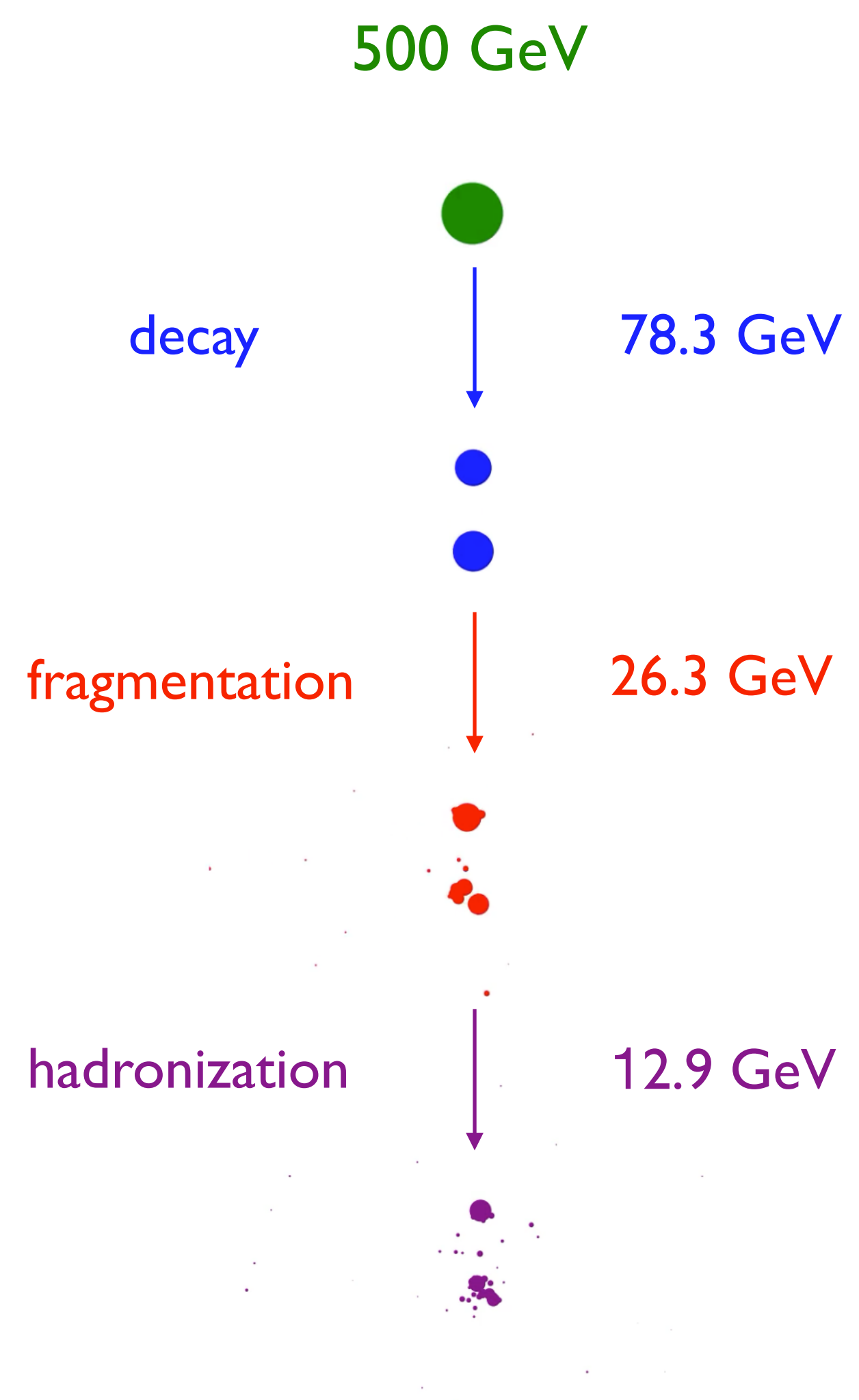
Visualizing Jet Formation – QCD Jets



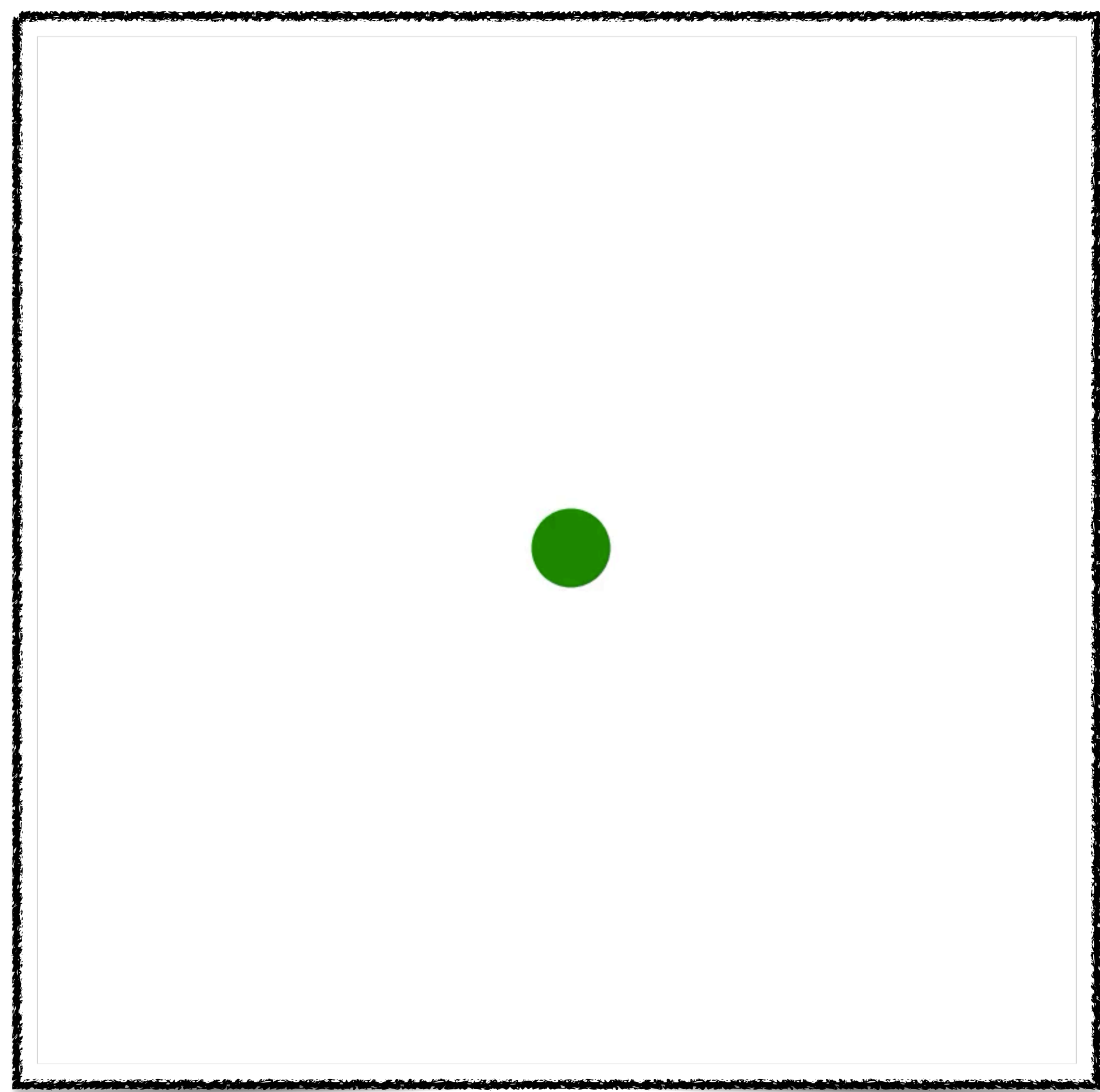
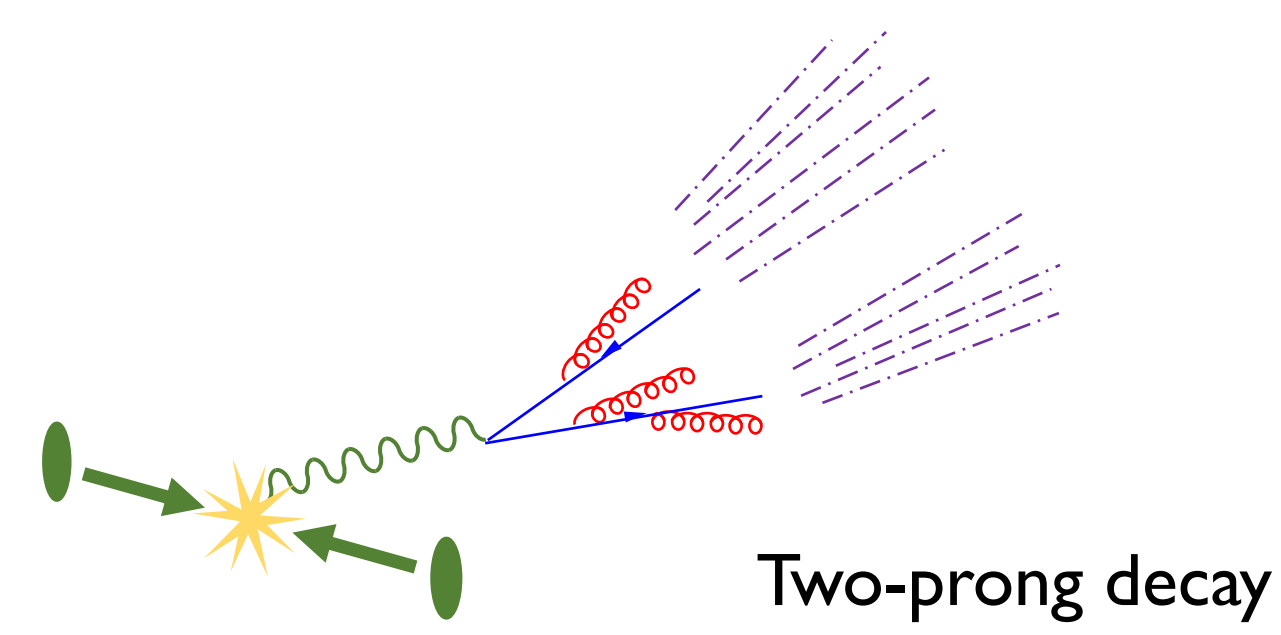
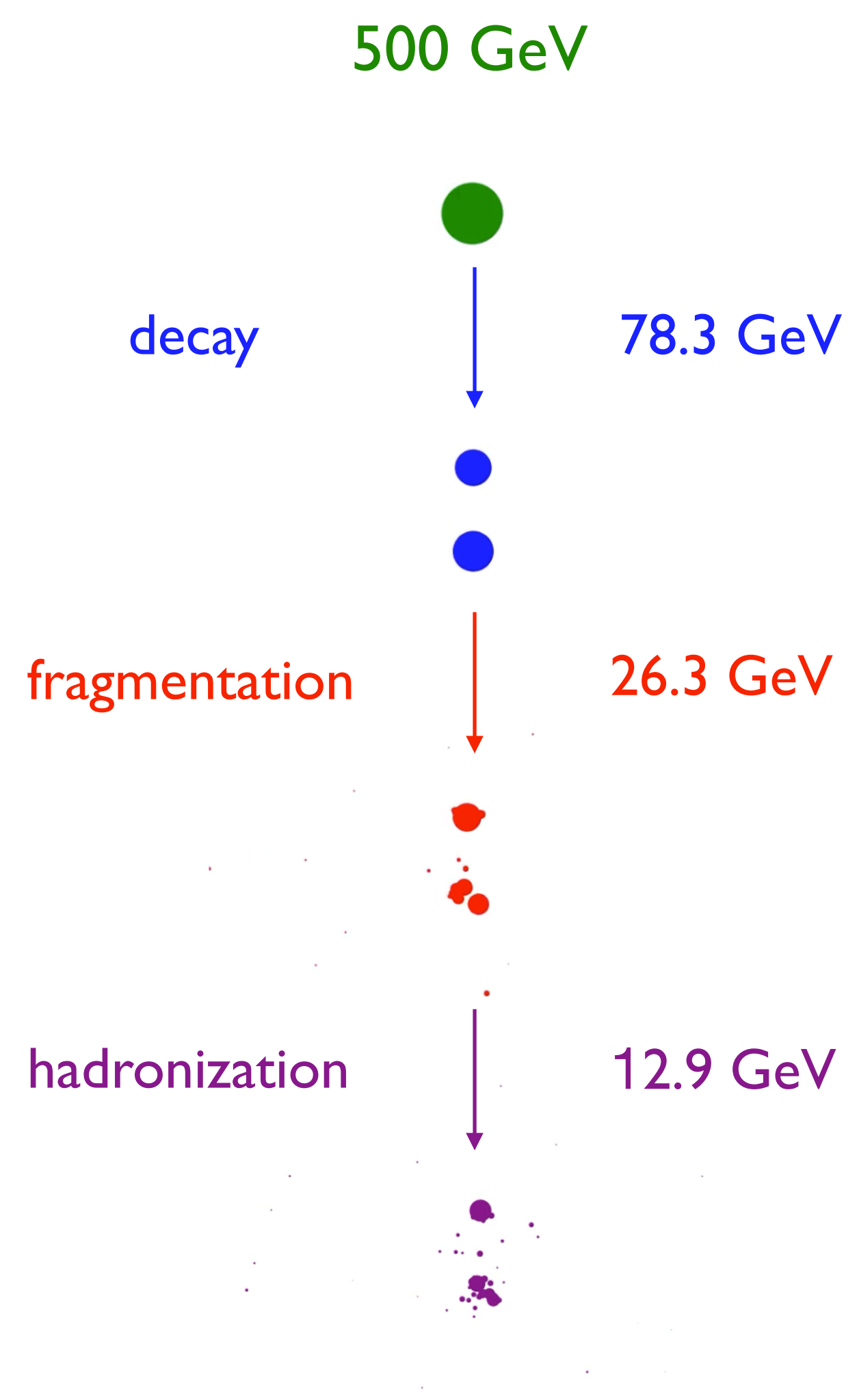
Visualizing Jet Formation – QCD Jets



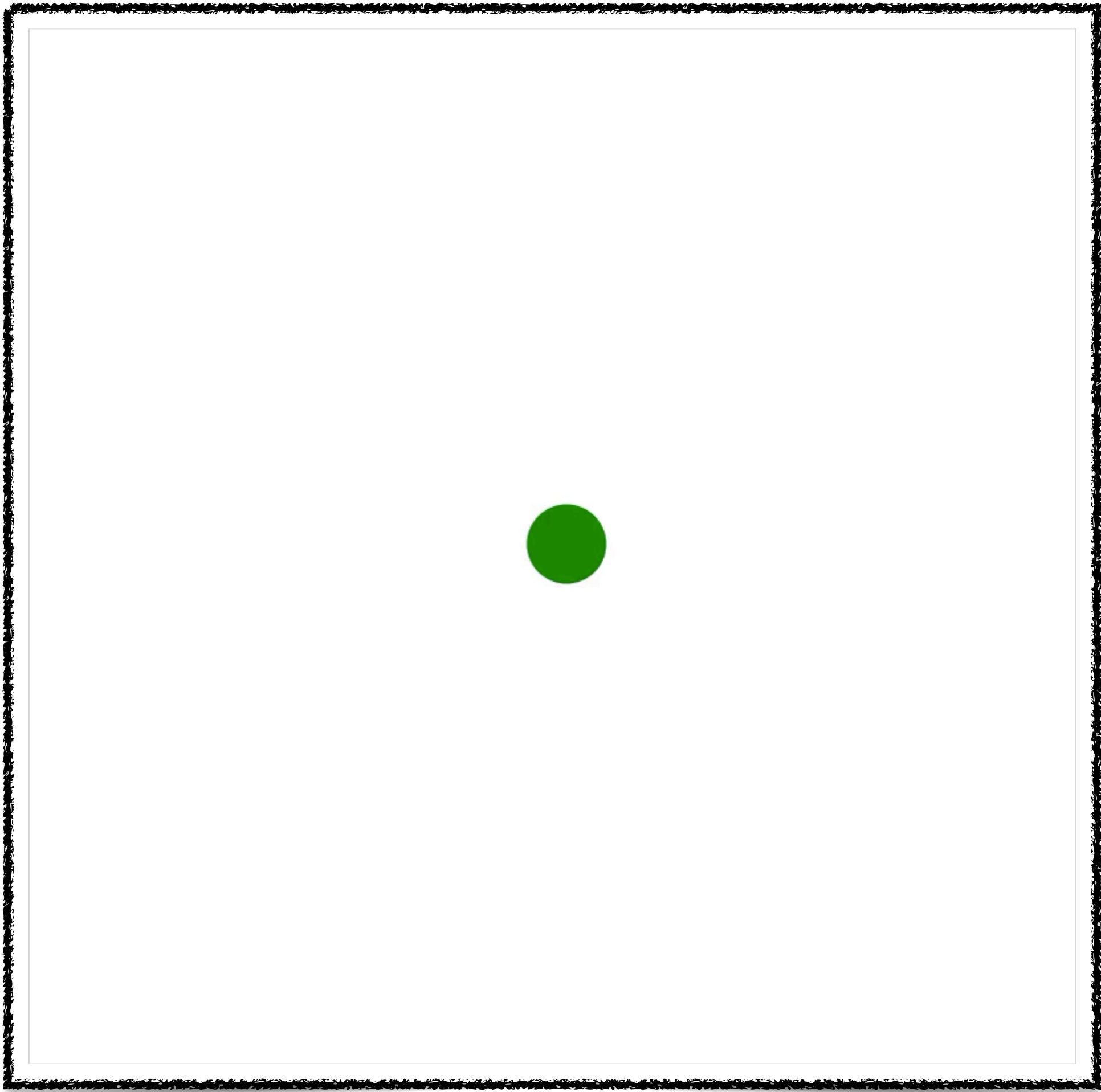
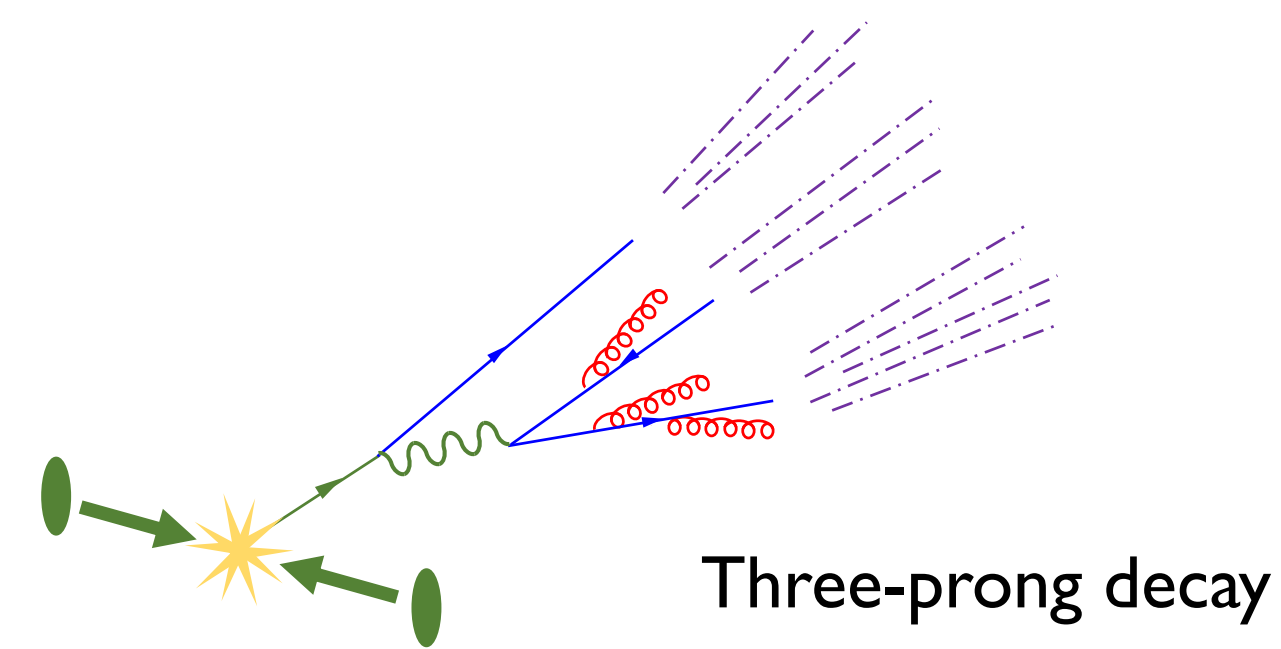
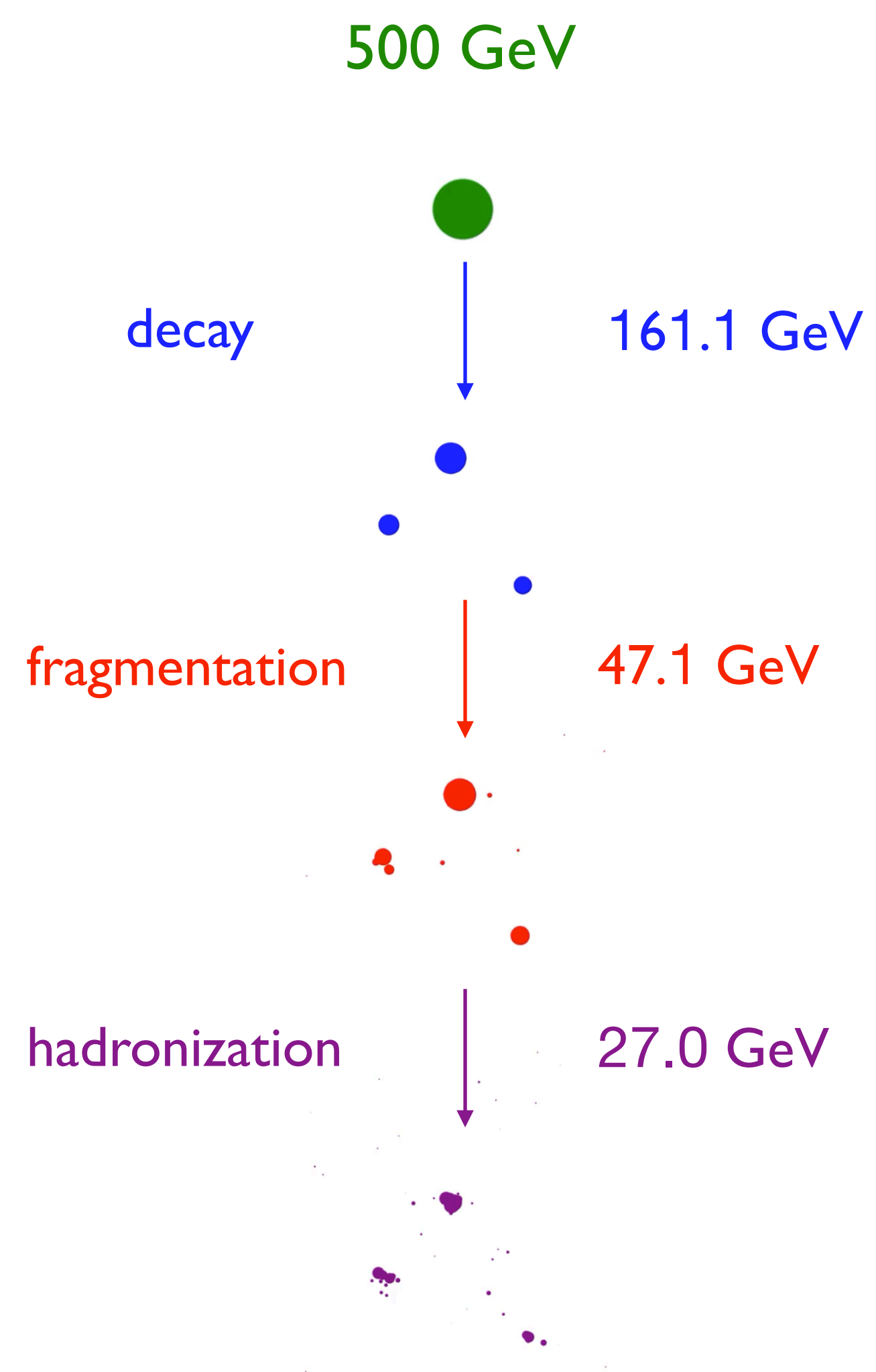
Visualizing Jet Formation – W Jets



Visualizing Jet Formation – W Jets



Visualizing Jet Formation – Top Jets



Visualizing Jet Formation – Top Jets

