The Hidden Geometry of Particle Collisions

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Massachusetts Institute of Technology
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Based on work with Eric Metodiev and Jesse Thaler
[PRL 2019, 2004.04159]

Particle Physics Phenomenology Series
Università di Genova

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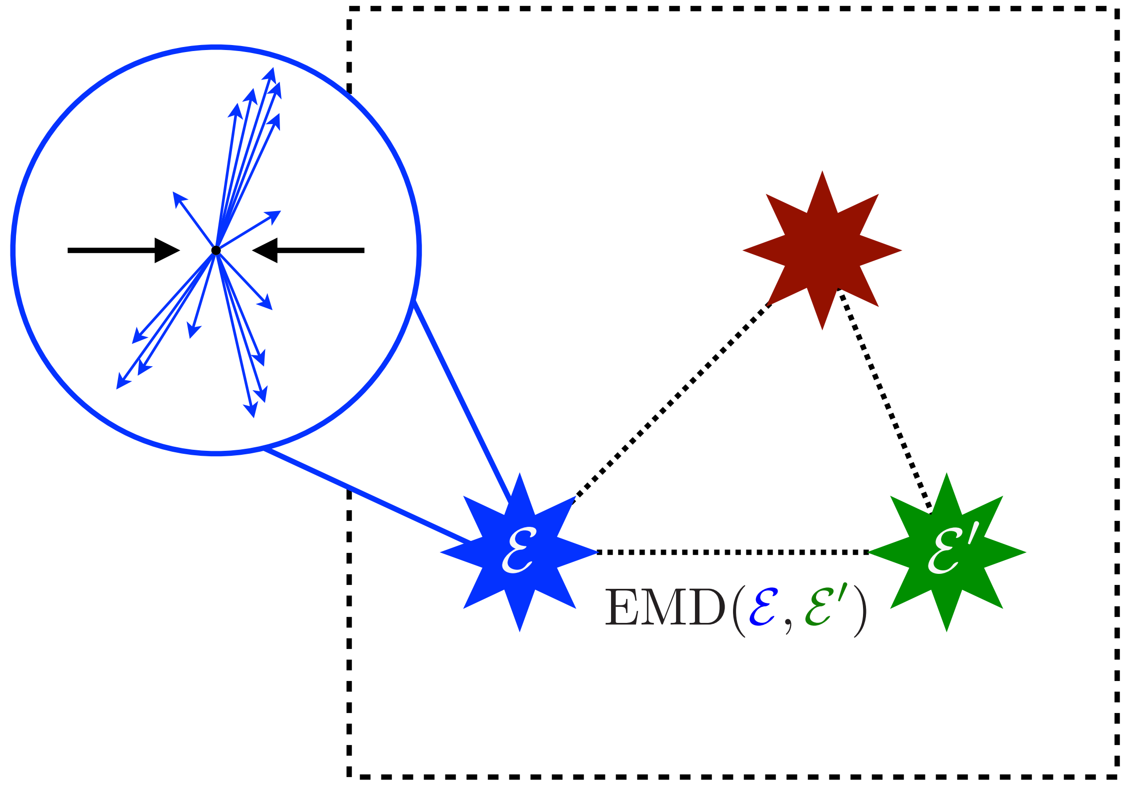
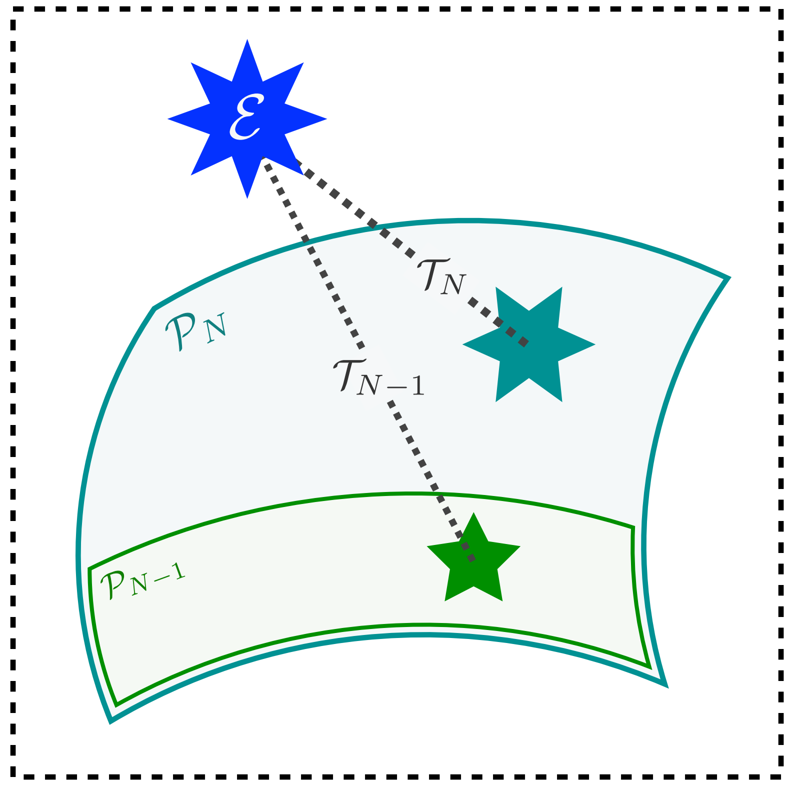
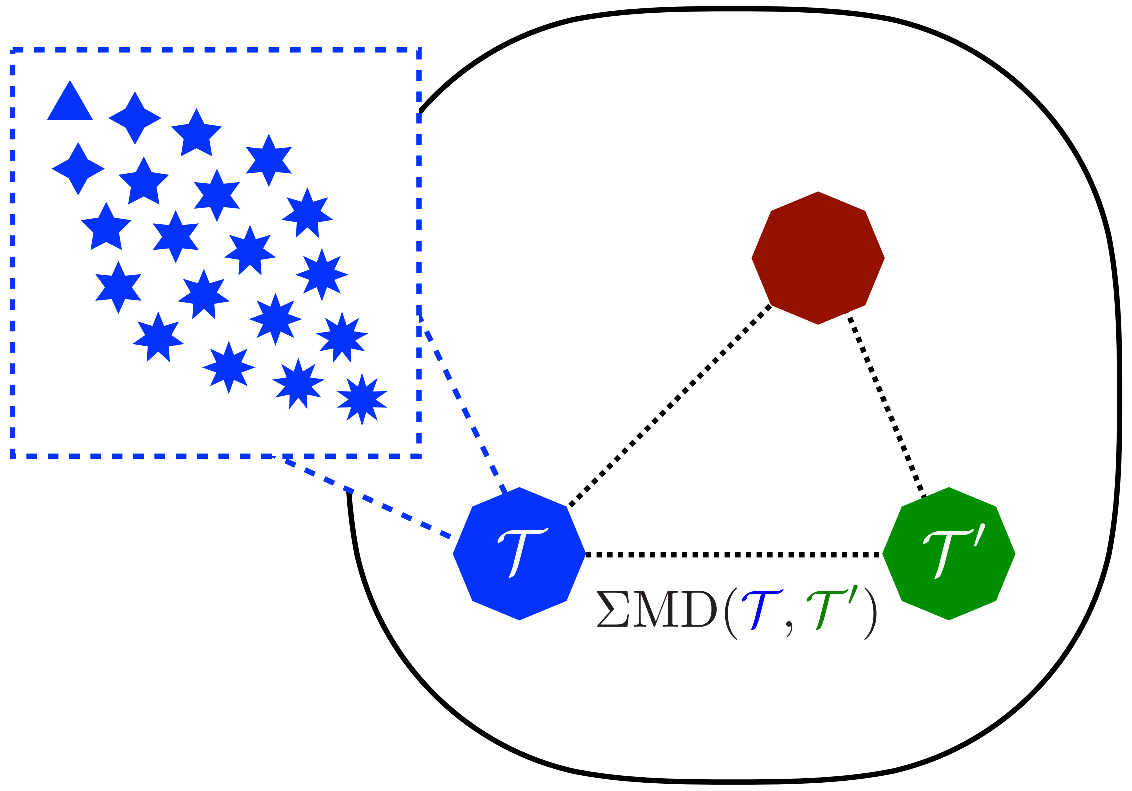


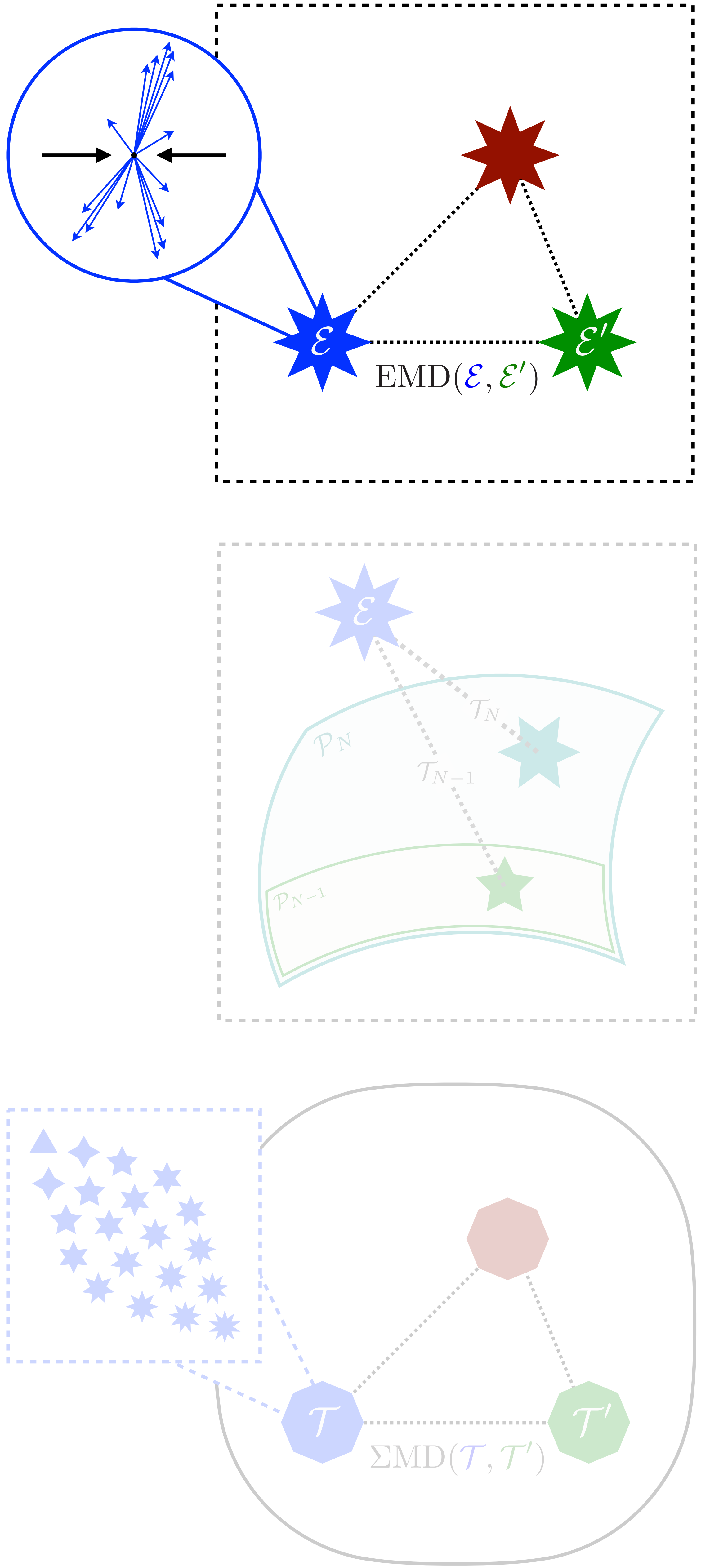




The (Metric) Space of Events

Revealing Hidden Geometry

Theory Space



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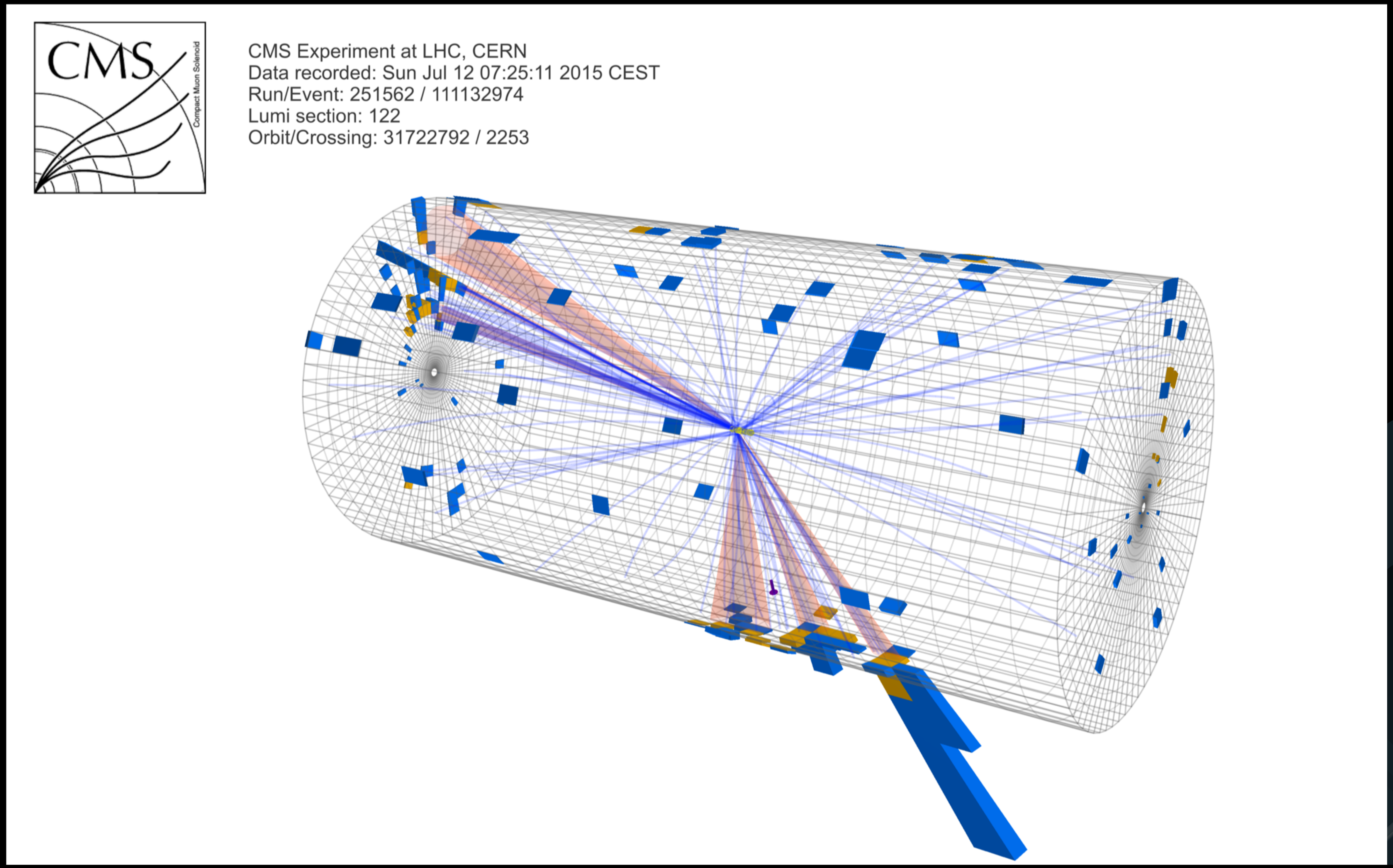
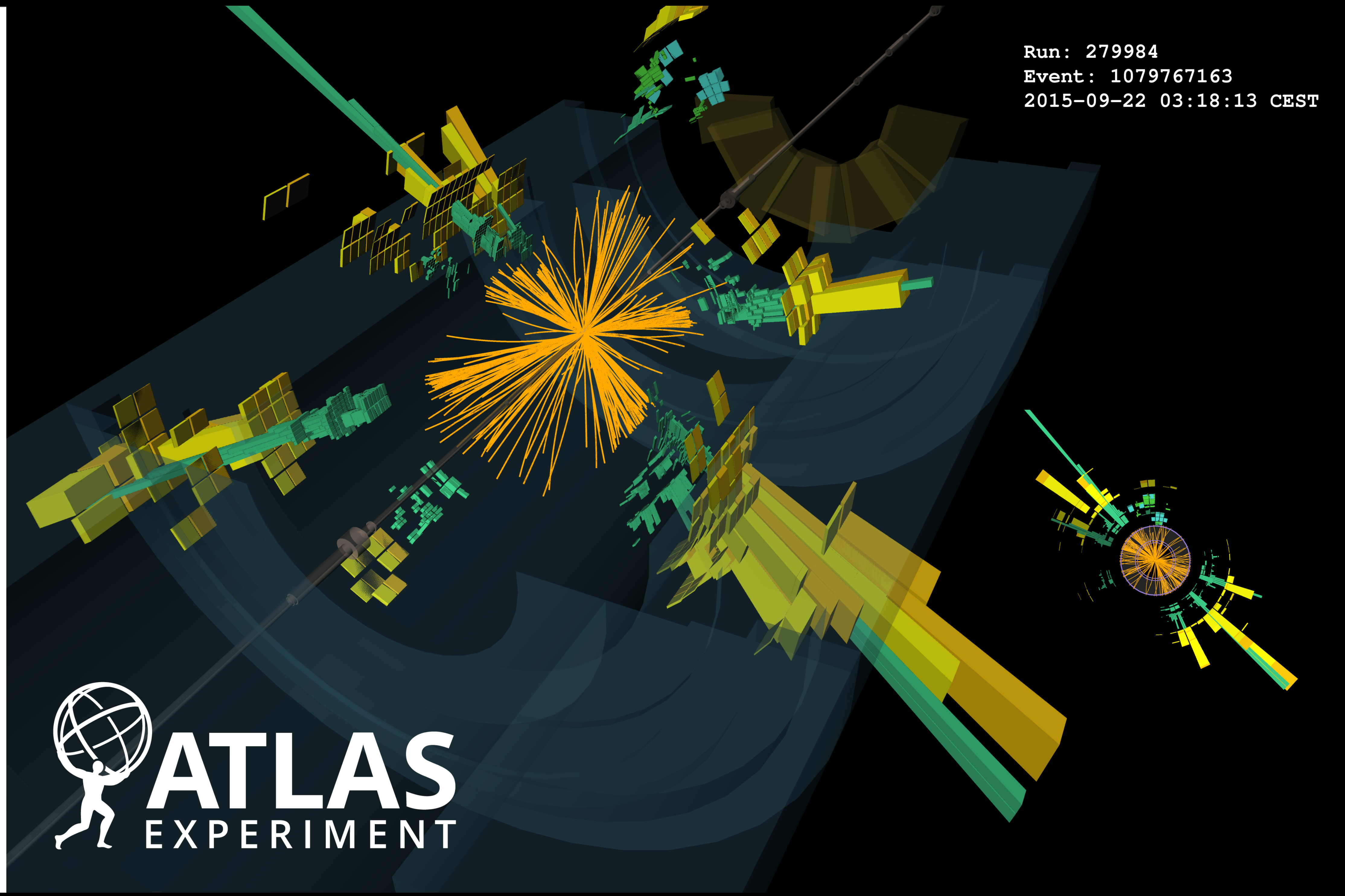
Explicit Geometry – Individual Events at the LHC

High-energy collisions produce final state particles with
energy, *direction*, *charge*, *flavor*, and *other quantum numbers*



CMS hadronic $t\bar{t}$ event



ATLAS high jet-multiplicity event

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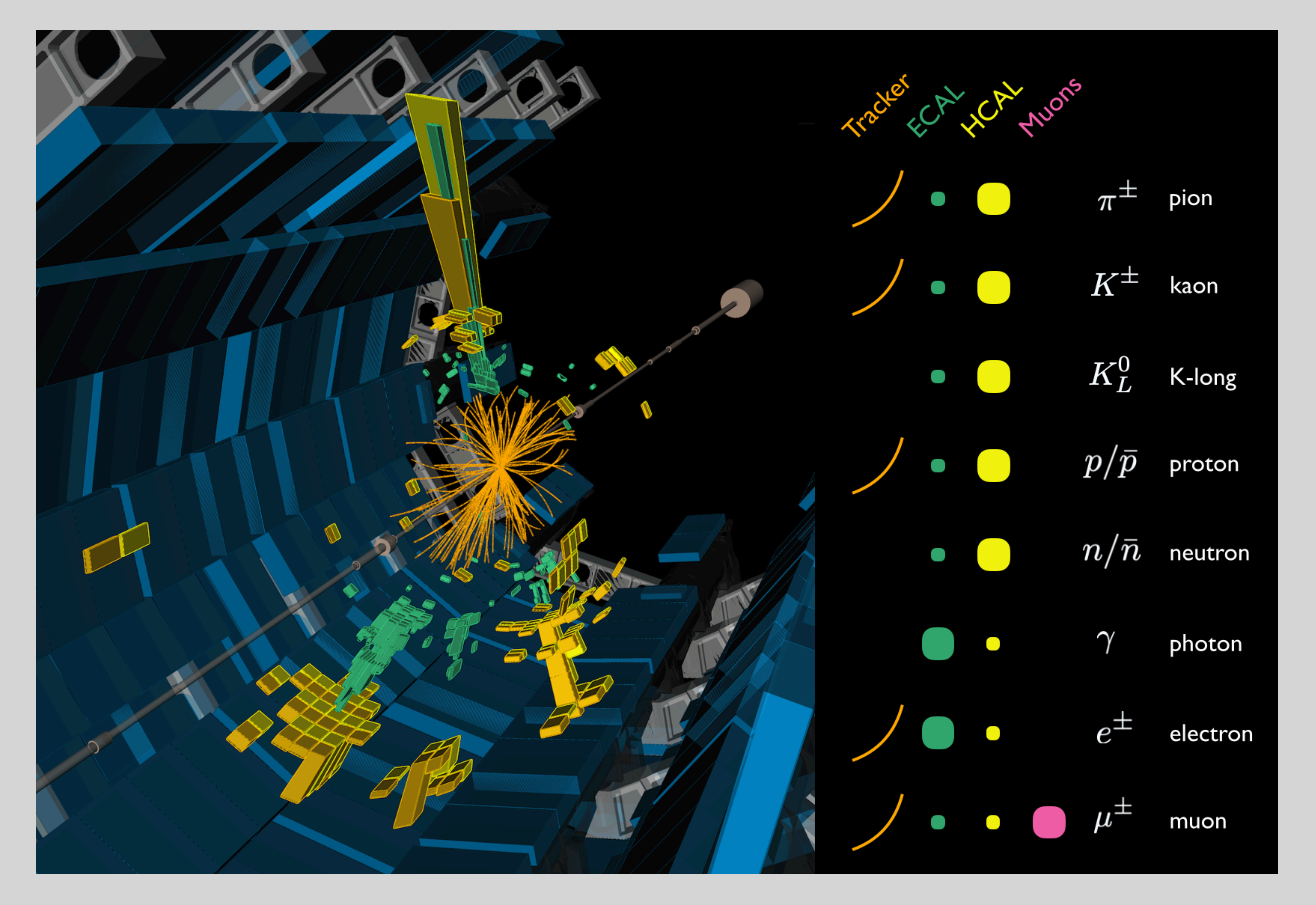

Explicit Geometry – Individual Events in Theory

Hard collision

Good understanding via perturbation theory

Fragmentation

Semi-classical parton shower, effective field theory

Hadronization

Poorly understood (non-perturbative), modeled empirically

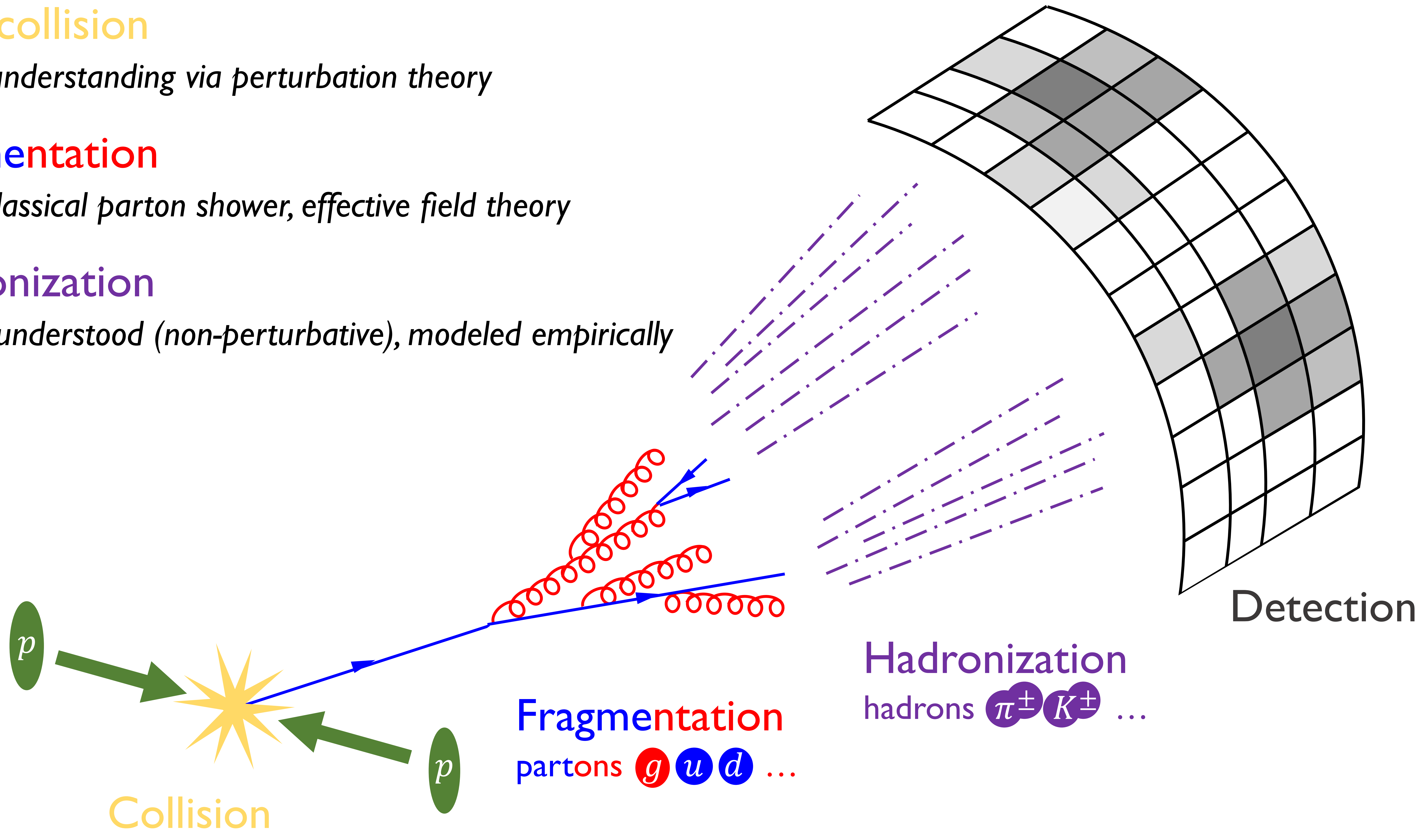

Diagram by Eric Metodiev

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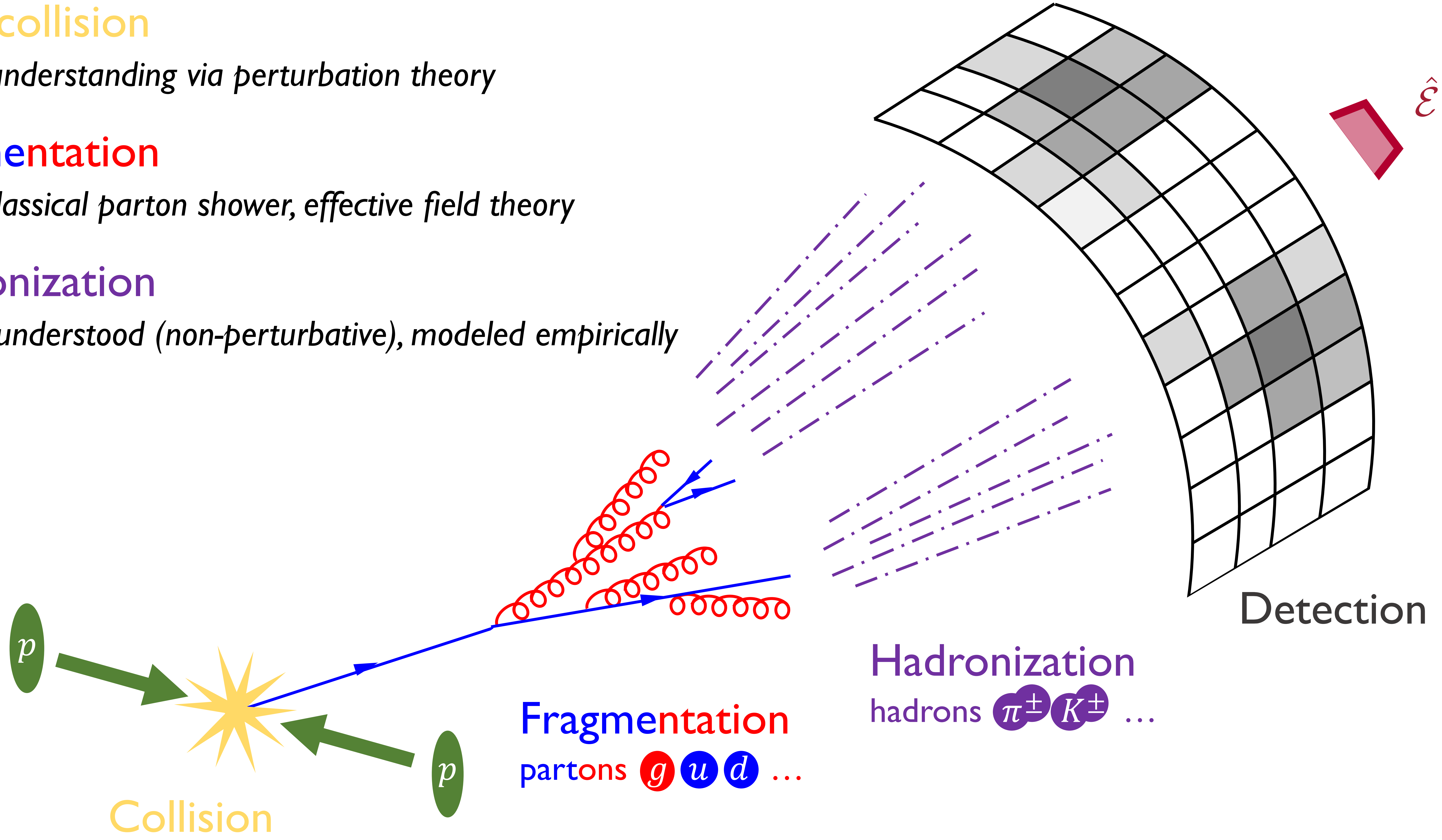


$$\hat{\mathcal{E}} \simeq \lim_{t\to\infty} \hat{n}_i T^{0i}(t, vt\hat{n})$$

Stress-energy flow

Robust to non-perturbative and detector effects

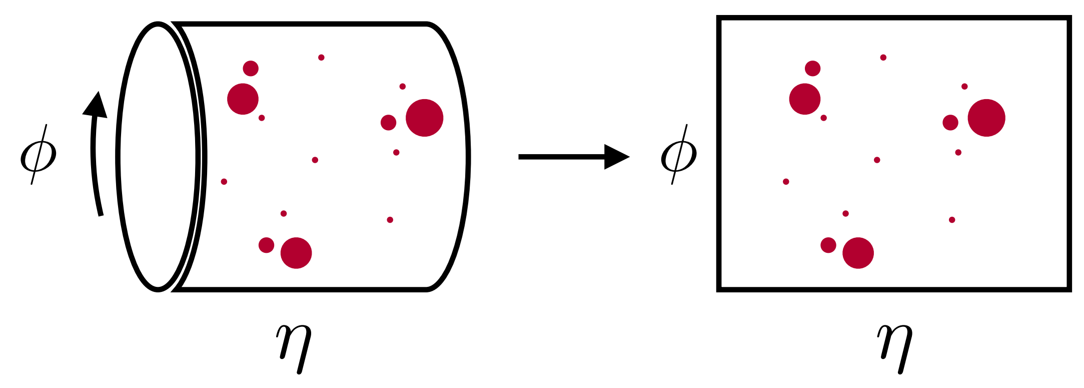
Well-defined for massless gauge theories

Correlation functions calculated in *N*=4 SYM and QCD

Diagram by Eric Metodiev

[Sveshnikov, Tkachov, PLB 1996; Hofman, Maldacena, JHEP 2008; Mateu, Stewart, Thaler, PRD 2013; Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, PRL 2014; Chen, Moult, Zhang, Zhu, 2004.11381; Dixon, PTK, Moult, Thaler, Zhu, *to appear soon*]

Explicit Geometry – Events as Distributions of Energy



[PTK, Metodiev, Thaler, JHEP 2019; PTK, Metodiev, Thaler, 2004.04159]

Energy flow distribution fully captures IRC-safe information

$$\mathcal{E}(\hat{n}) = \sum_{i=1}^{M} E_i\, \delta(\hat{n} - \hat{n}_i)$$

Energy Flow Distribution — Energy (p_T) — Direction (y,φ)

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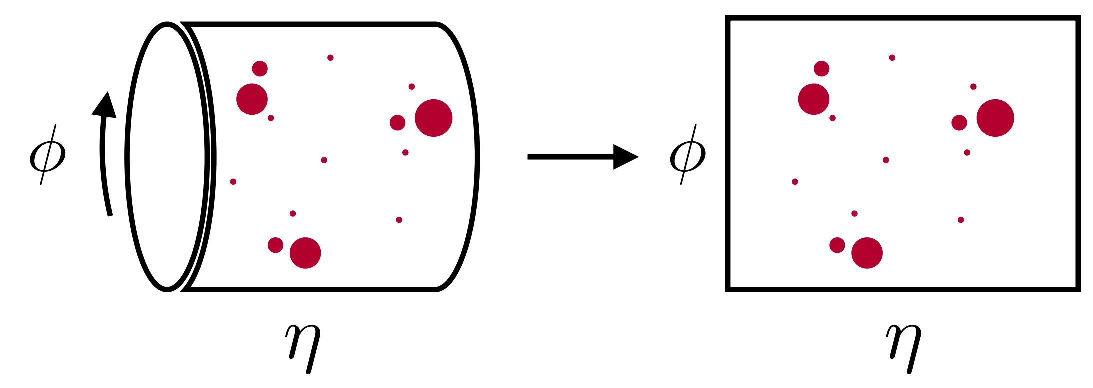

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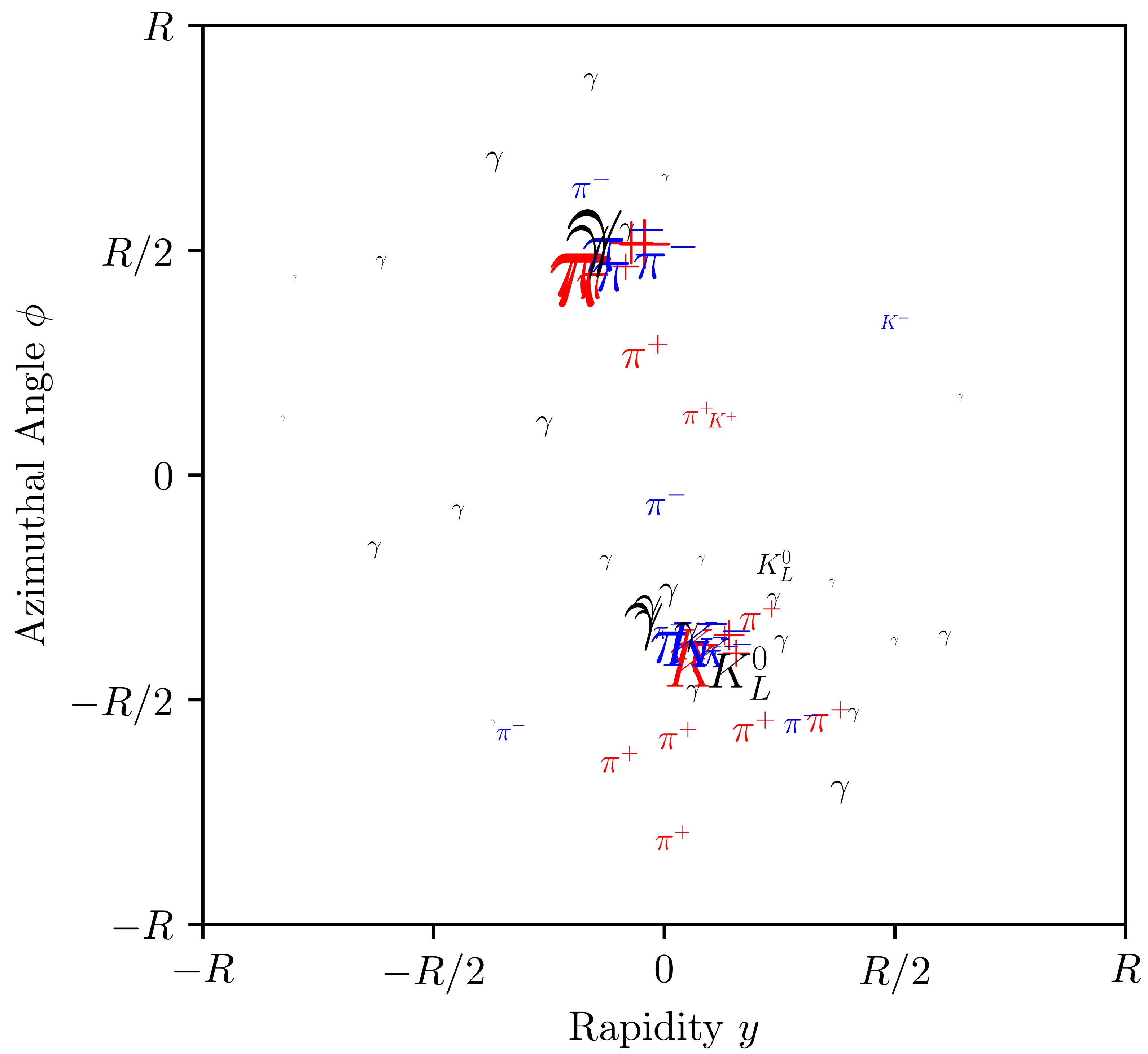
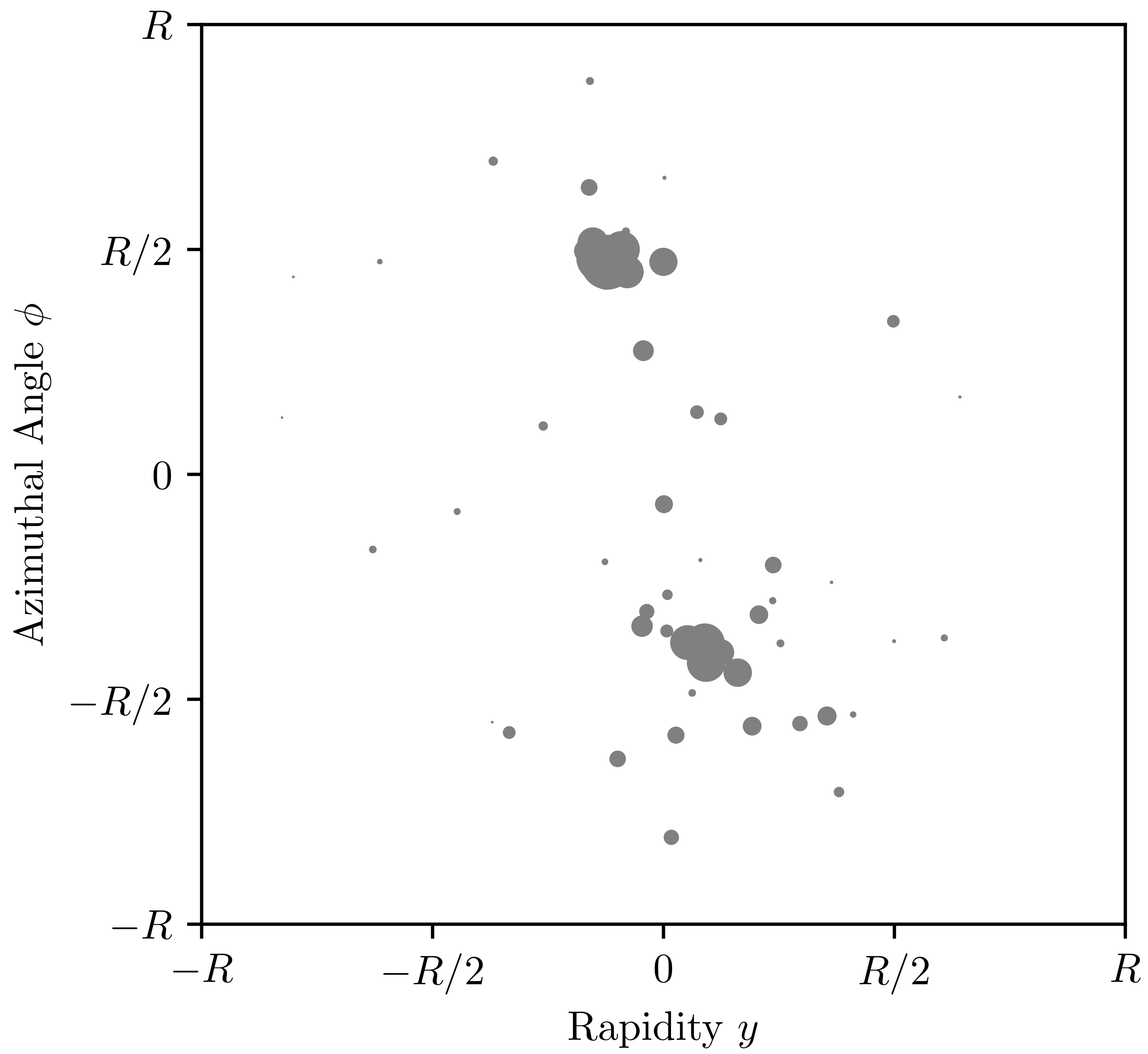
↑ Energy Flow Distribution ↑ Energy (p_T) ↑ Direction (y,φ)

Full event is a set of particles having momentum and charge/flavor



⇒

The energy flow is unpixelized and ignores charge/flavor information



When are Two Distributions Similar?

Optimal transport minimizes the "work" (stuff x distance) required to transport supply to demand

[Monge, 1781; Vaseršteĭn, 1969; Peleg, Werman, Rom, IEEE 1989;
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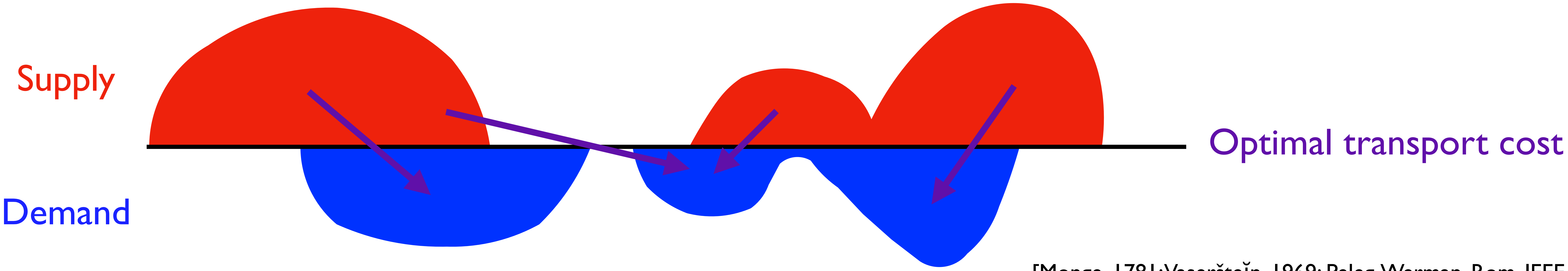

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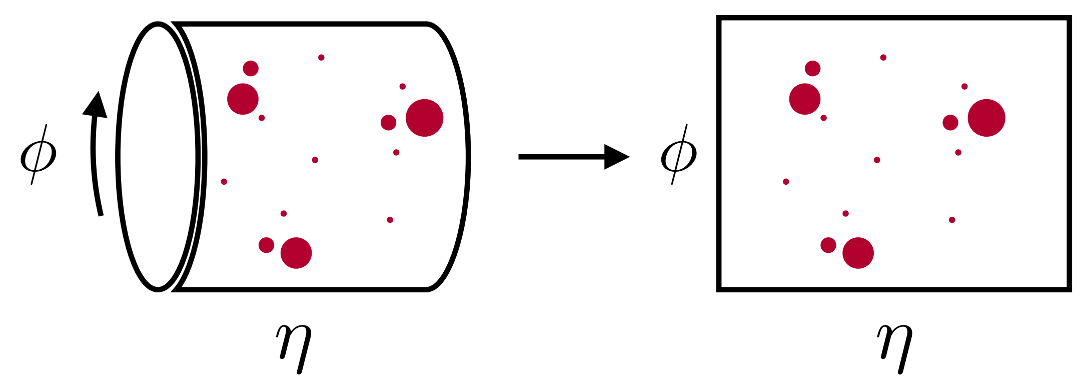
Towards a Hidden Geometry – When are two events similar?



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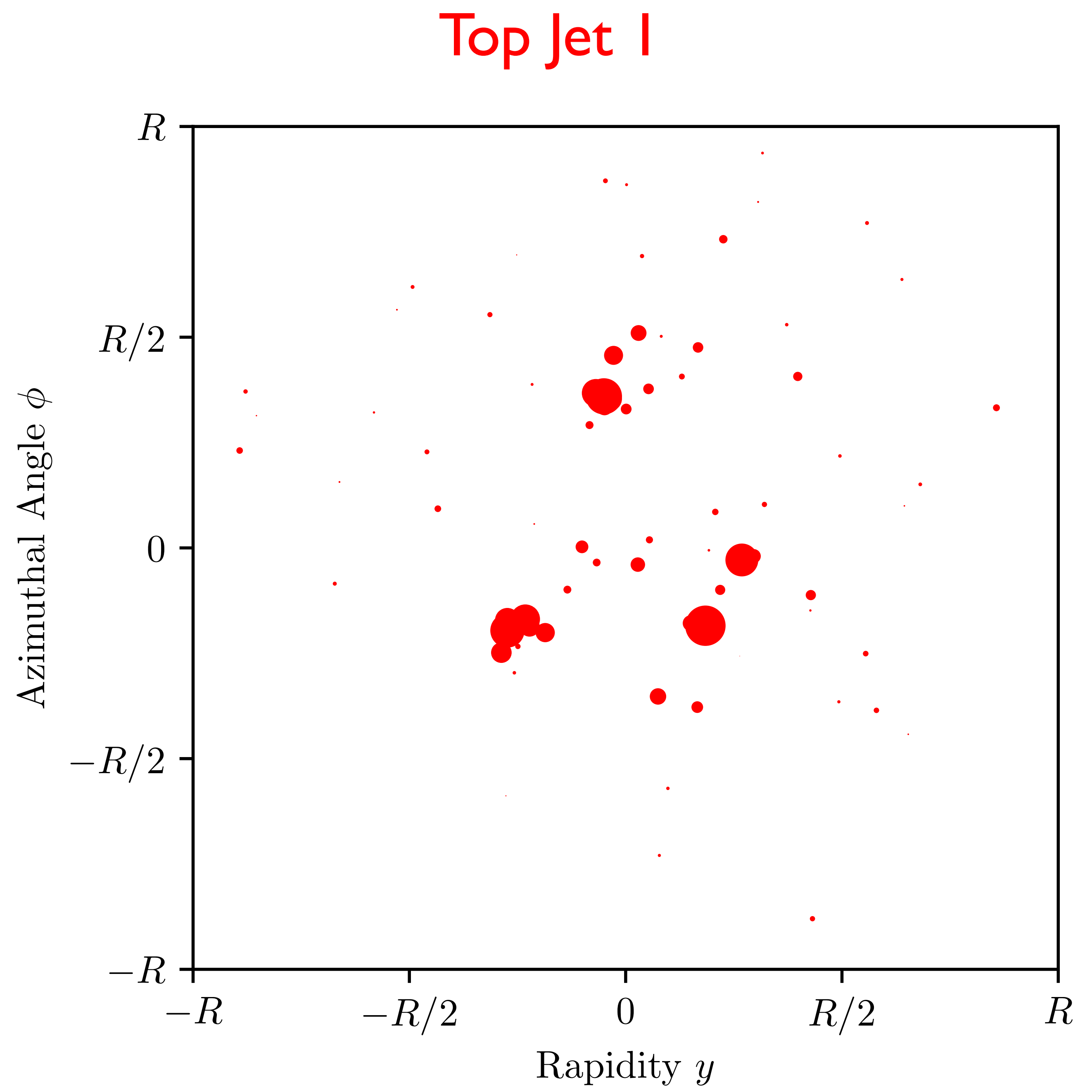

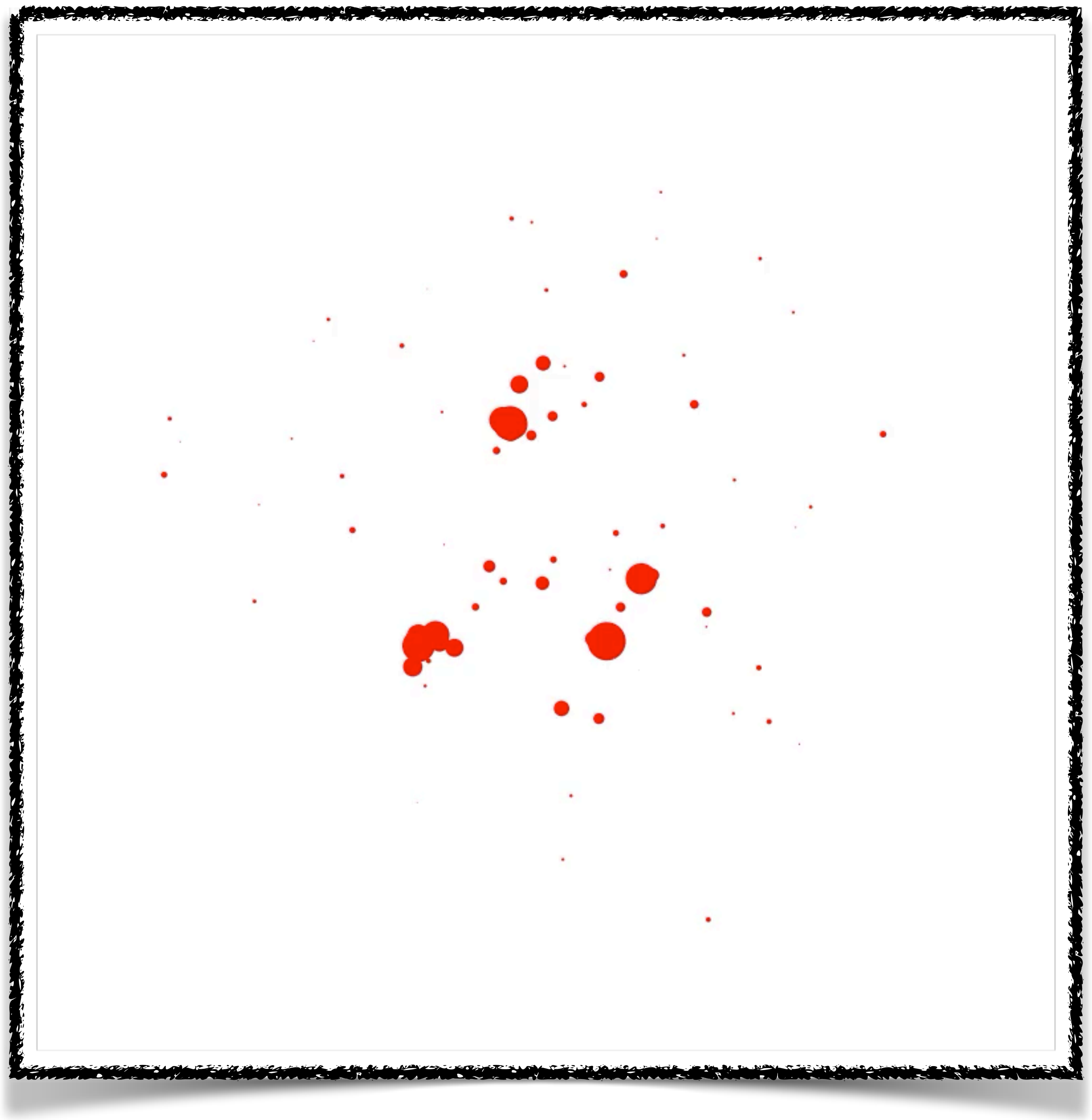

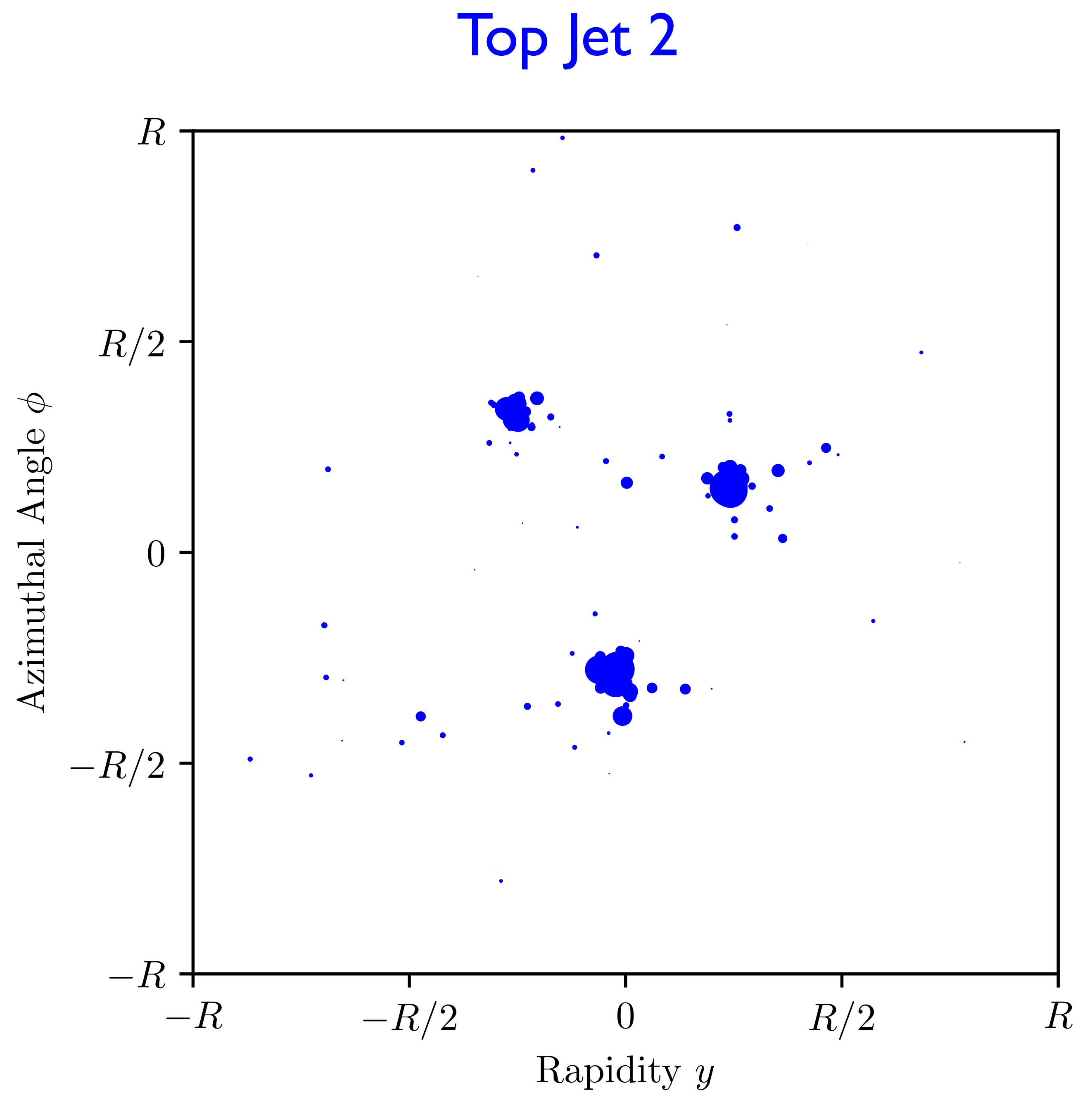

$$\mathcal{E}(\hat{n}) = \sum_{i=1}^{M} E_i \, \delta(\hat{n} - \hat{n}_i)$$

Provides a metric on normalized distributions in a space with a ground distance measure

↳ symmetric, non-negative, triangle inequality, zero iff identical

$$\theta_{ij} = \sqrt{\Delta y_{ij}^2 + \Delta \phi_{ij}^2}$$

[Peleg, Werman, Rom, IEEE 1989; Rubner, Tomasi, Guibas, ICCV 1998, ICJV 2000; Pele, Werman, ECCV 2008; Pele, Taskar, GSI 2013]

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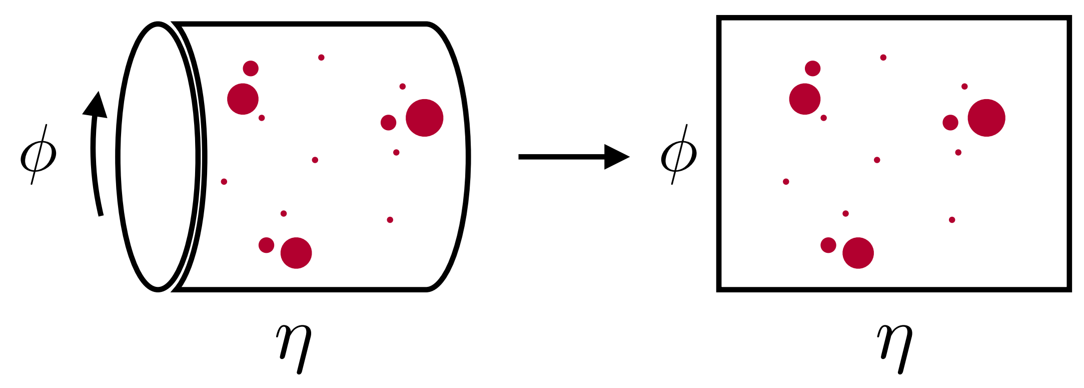

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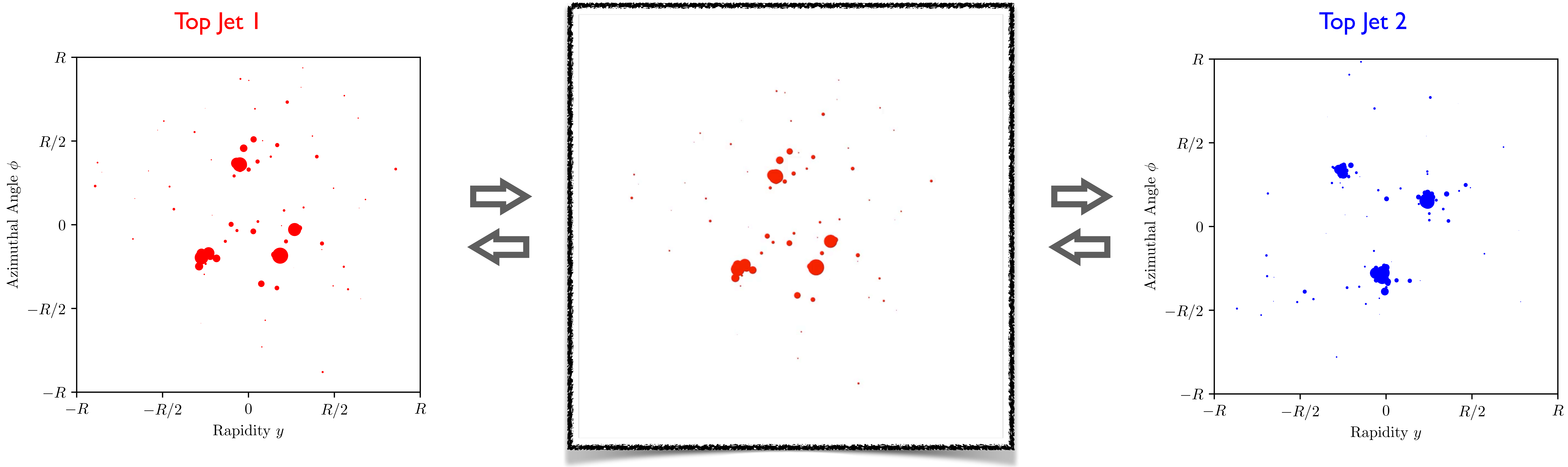

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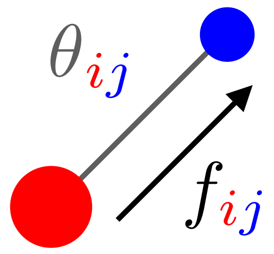
The Energy Mover's Distance (EMD)

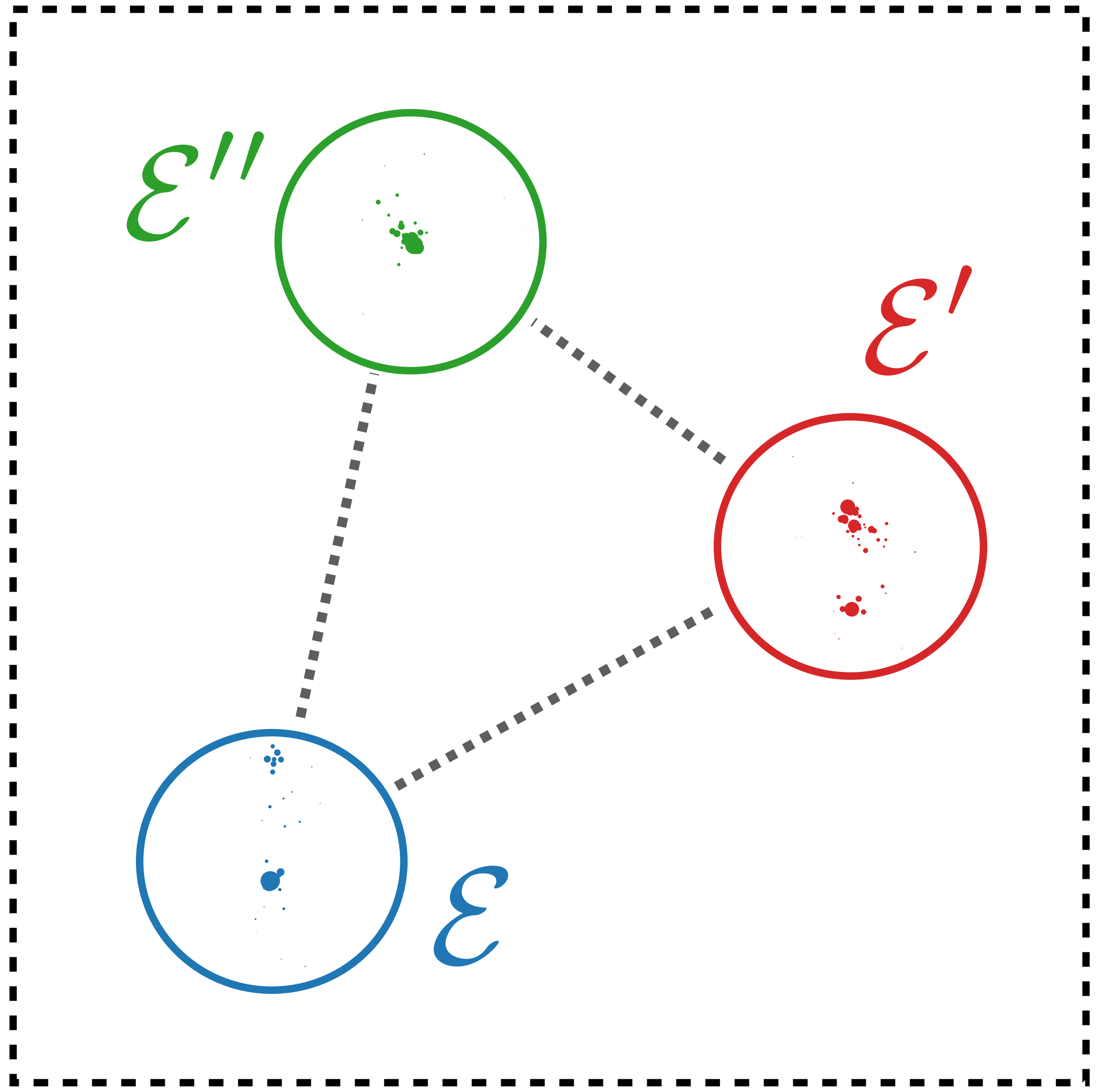
[PTK, Metodiev, Thaler, PRL 2019]

EMD between energy flows defines a metric on the space of events

$$\mathrm{EMD}_{\beta,R}(\mathcal{E},\mathcal{E}') = \underbrace{\min_{\{f_{ij}\geq 0\}} \sum_i \sum_j f_{ij} \left(\frac{\theta_{ij}}{R}\right)^\beta}_{\text{Cost of optimal transport}} + \underbrace{\left|\sum_i E_i - \sum_j E'_j\right|}_{\text{Cost of energy creation}}$$

$$\underbrace{\sum_j f_{ij} \leq E_i, \quad \sum_i f_{ij} \leq E'_j, \quad \sum_{ij} f_{ij} = \min\left(\sum_i E_i, \sum_j E'_j\right)}_{\text{Capacity constraints to ensure proper transport}}$$





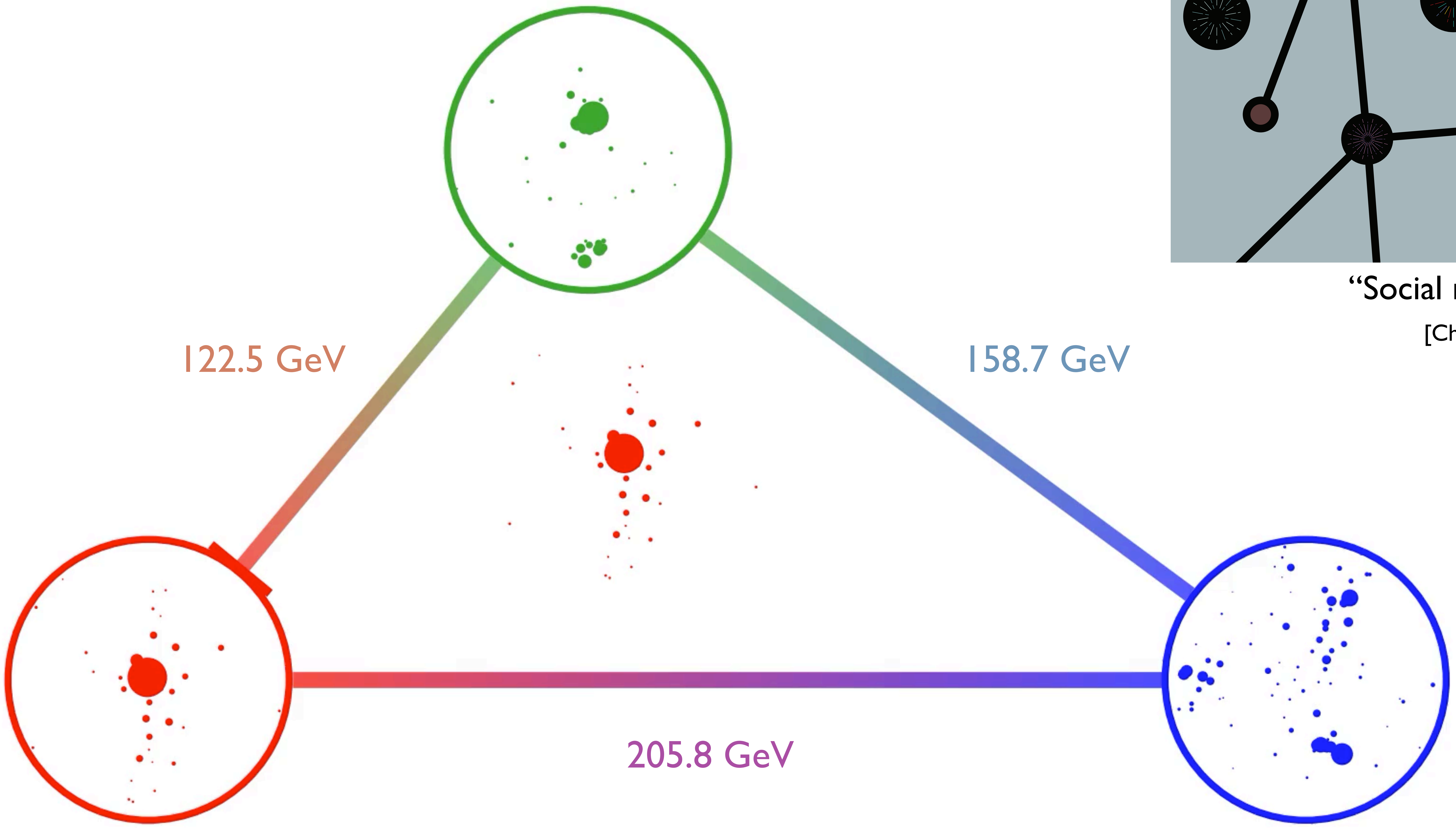
R: controls cost of transporting energy vs. destroying/creating it
β: angular weighting exponent

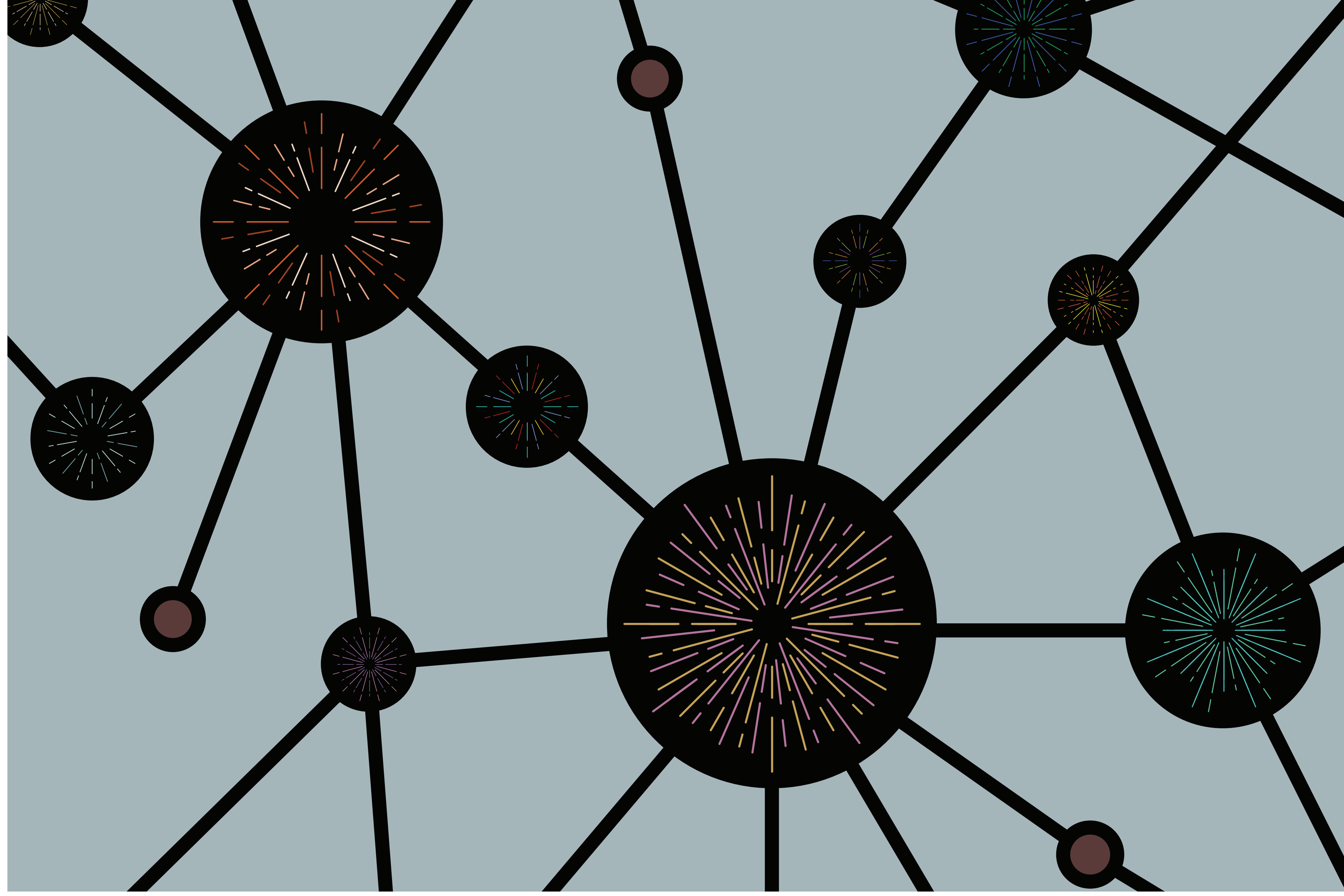
Triangle inequality satisfied for $R \geq d_{\max}/2$

$$0 \leq \mathrm{EMD}(\mathcal{E},\mathcal{E}') \leq \mathrm{EMD}(\mathcal{E},\mathcal{E}'') + \mathrm{EMD}(\mathcal{E}'',\mathcal{E}')$$

i.e. $R \geq$ jet radius for conical jets

Geodesics in the Space of Events





"Social networking of jets"

[Chu, MIT News 2019]

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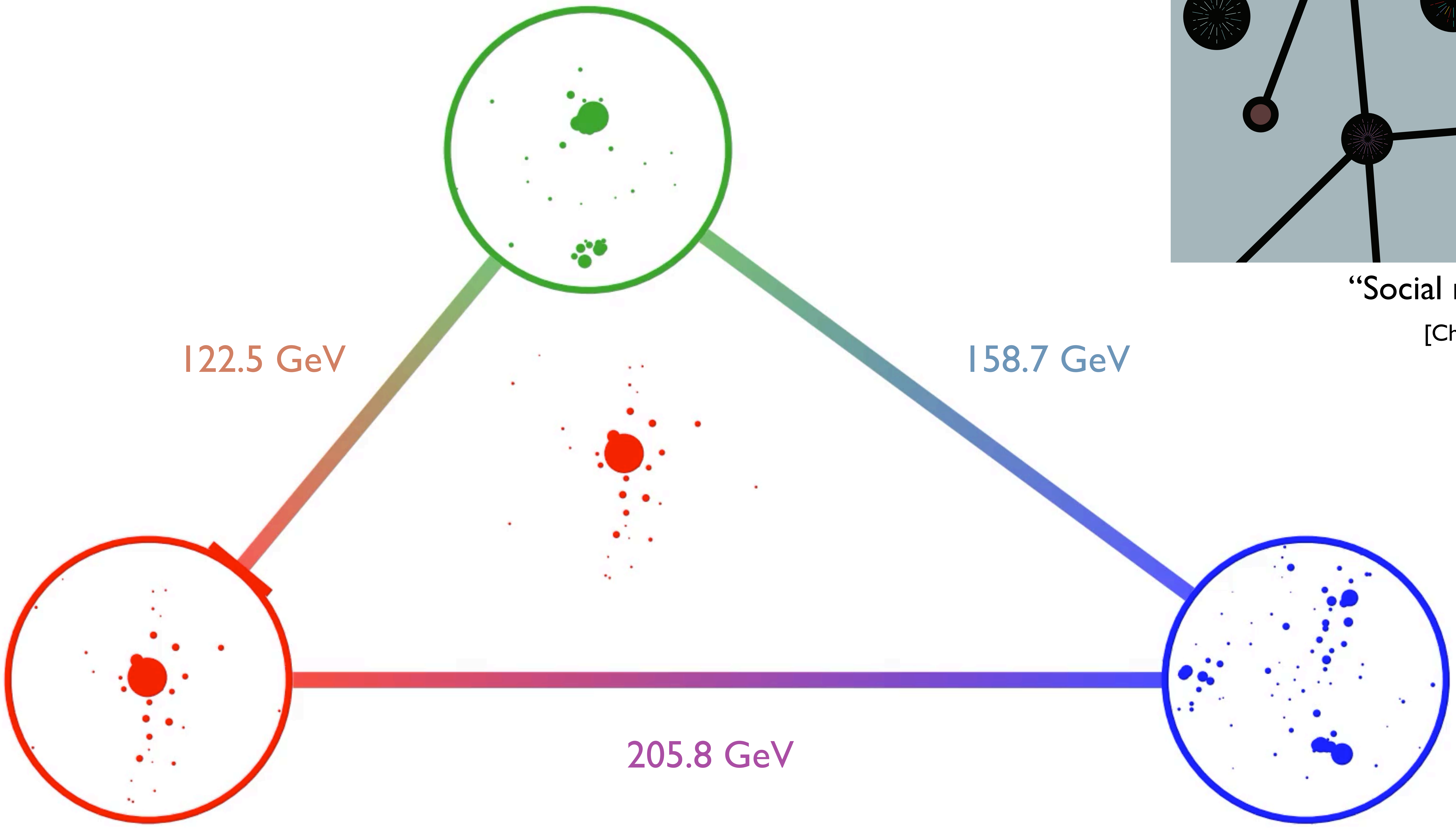

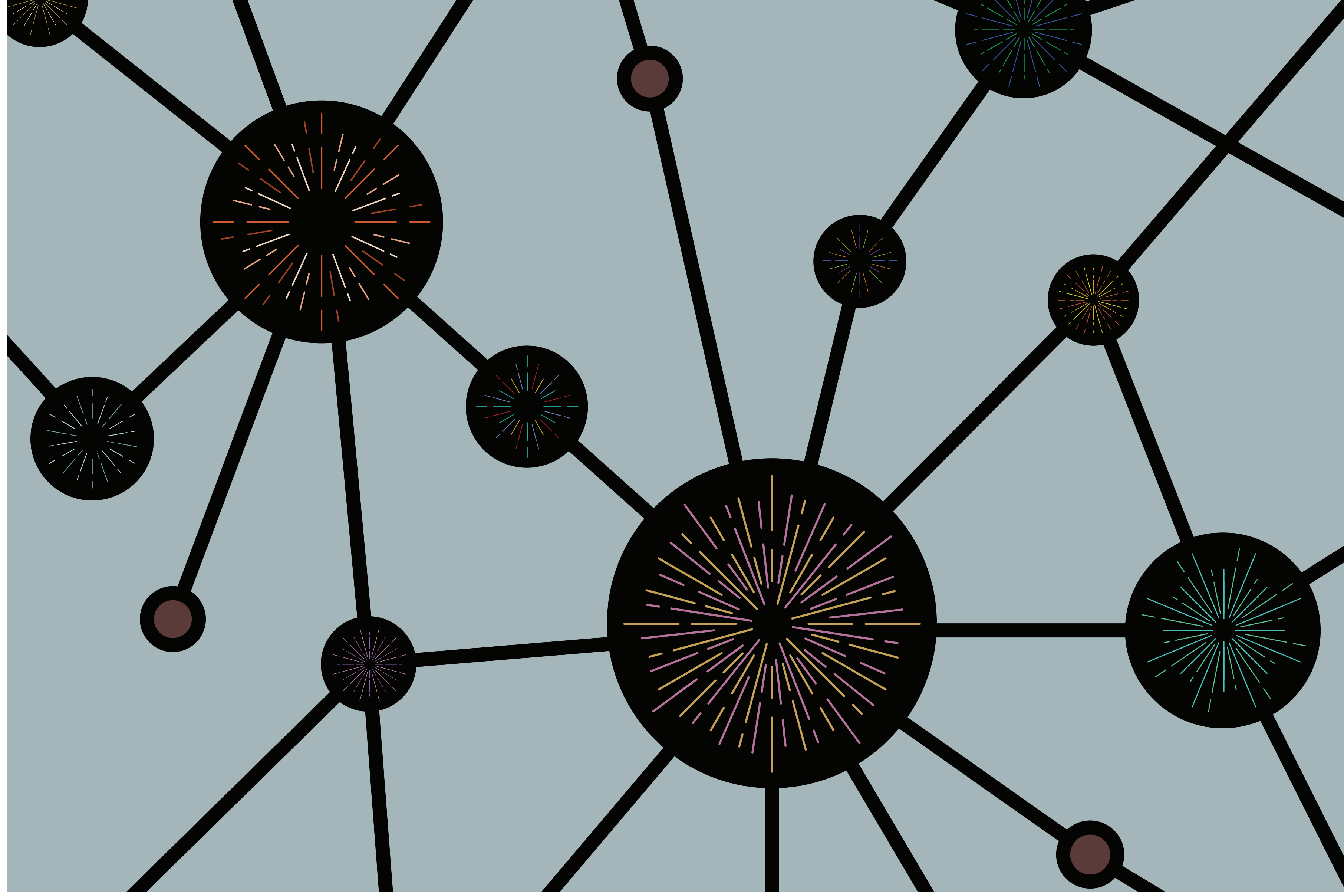

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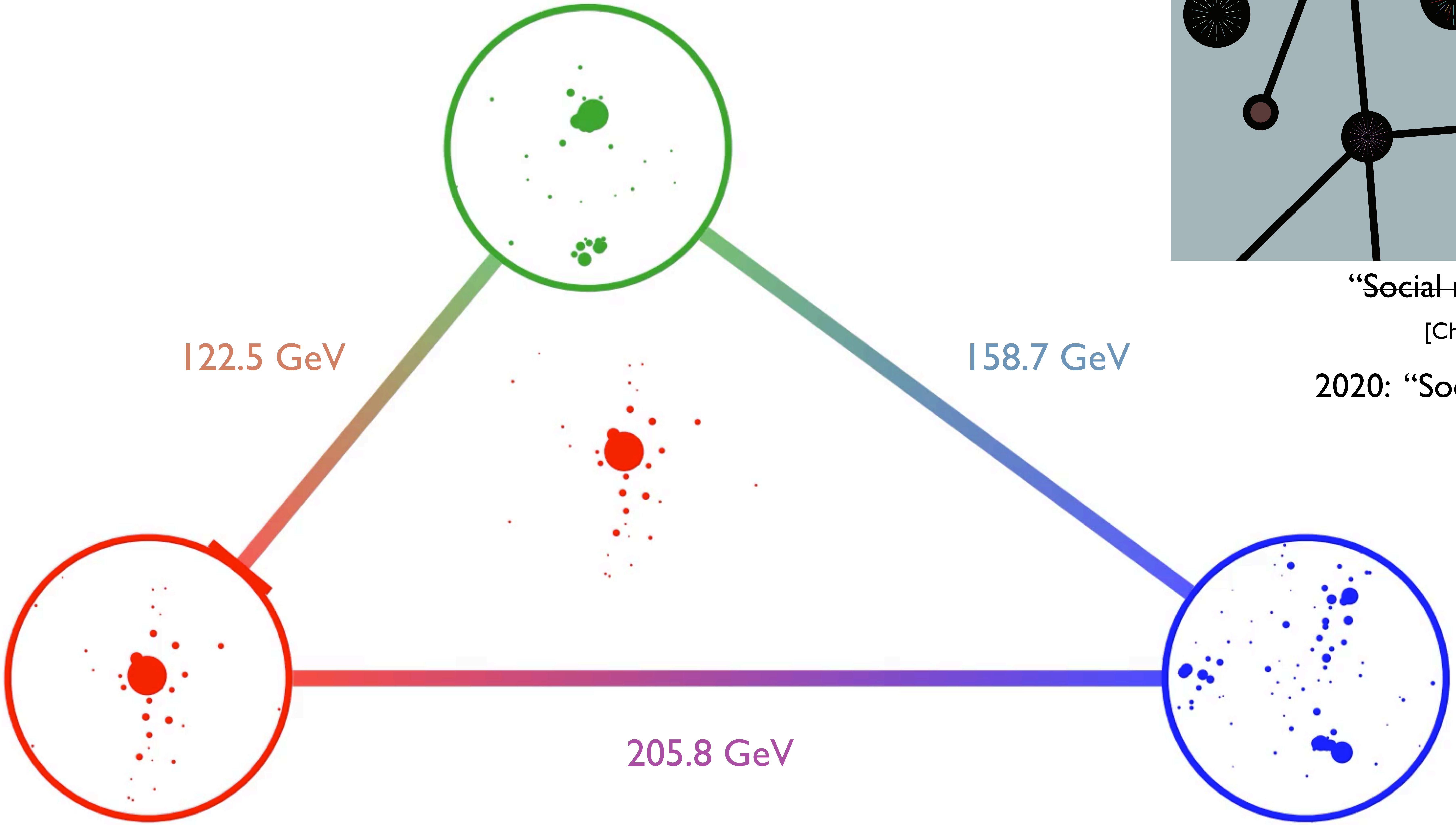

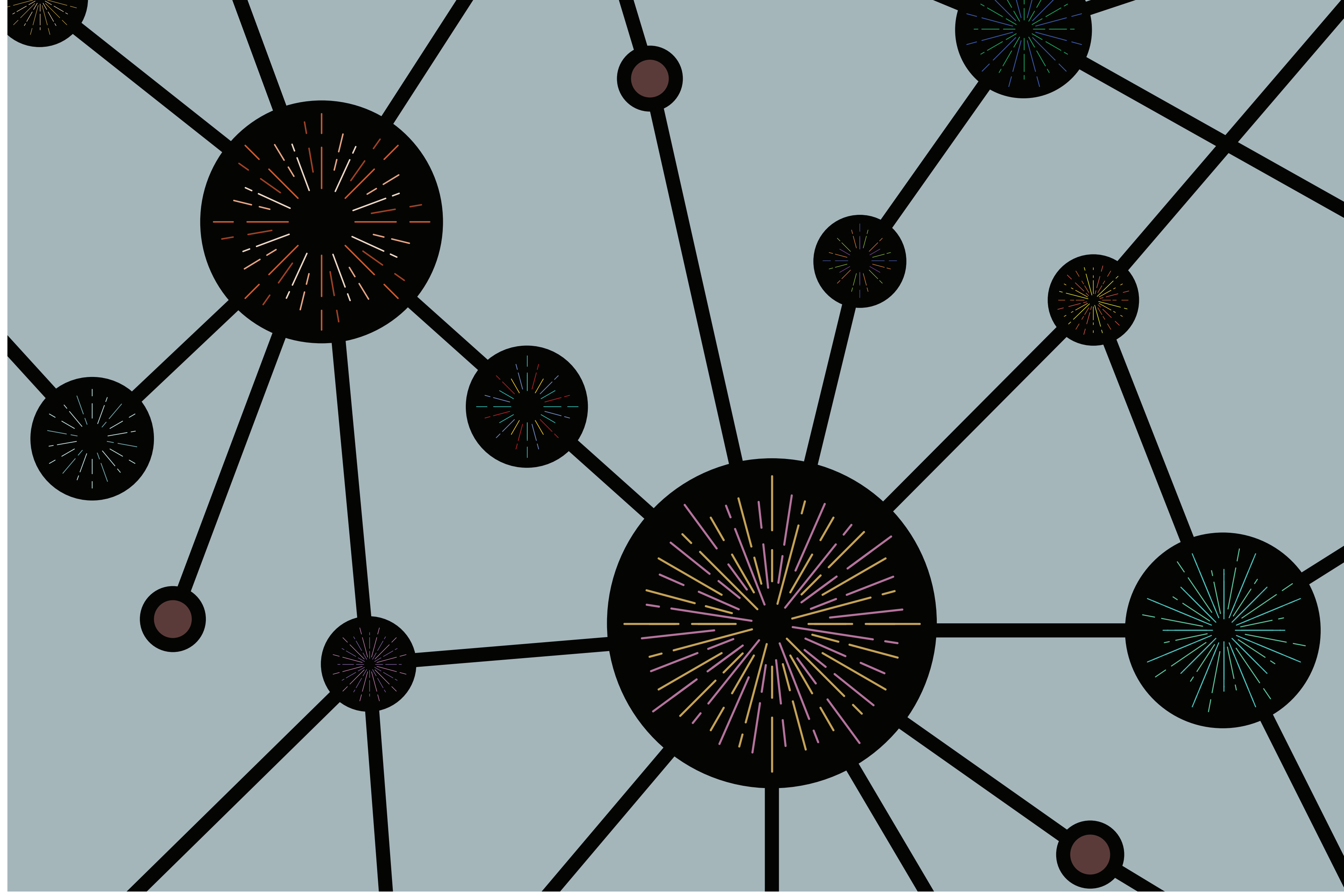

"~~Social networking of jets~~"

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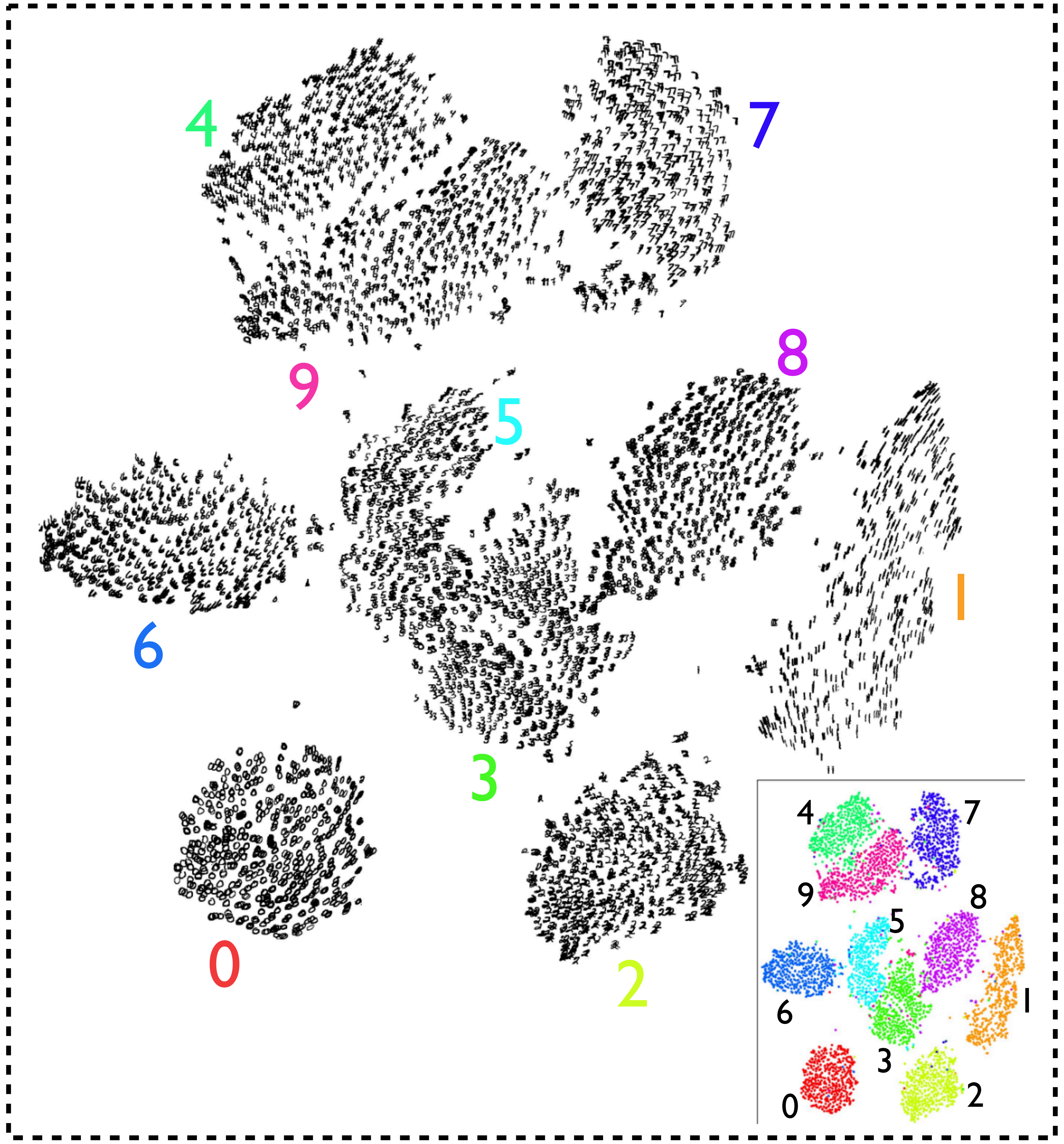
2020: "Social distancing of jets"

Visualizing Geometry in the Space of Events

t-Distributed Stochastic Neighbor Embedding (t-SNE)

MNIST handwritten digits



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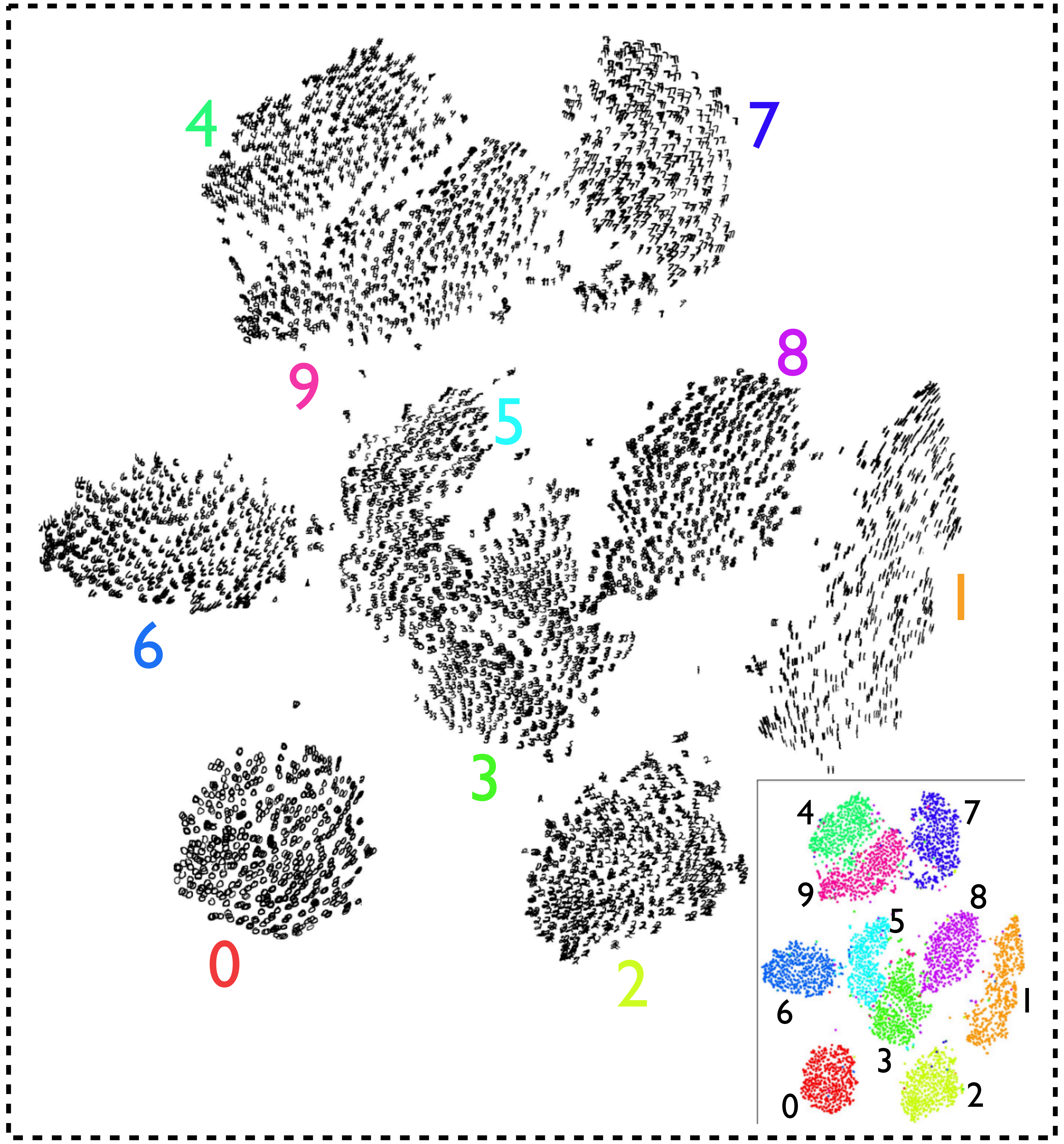

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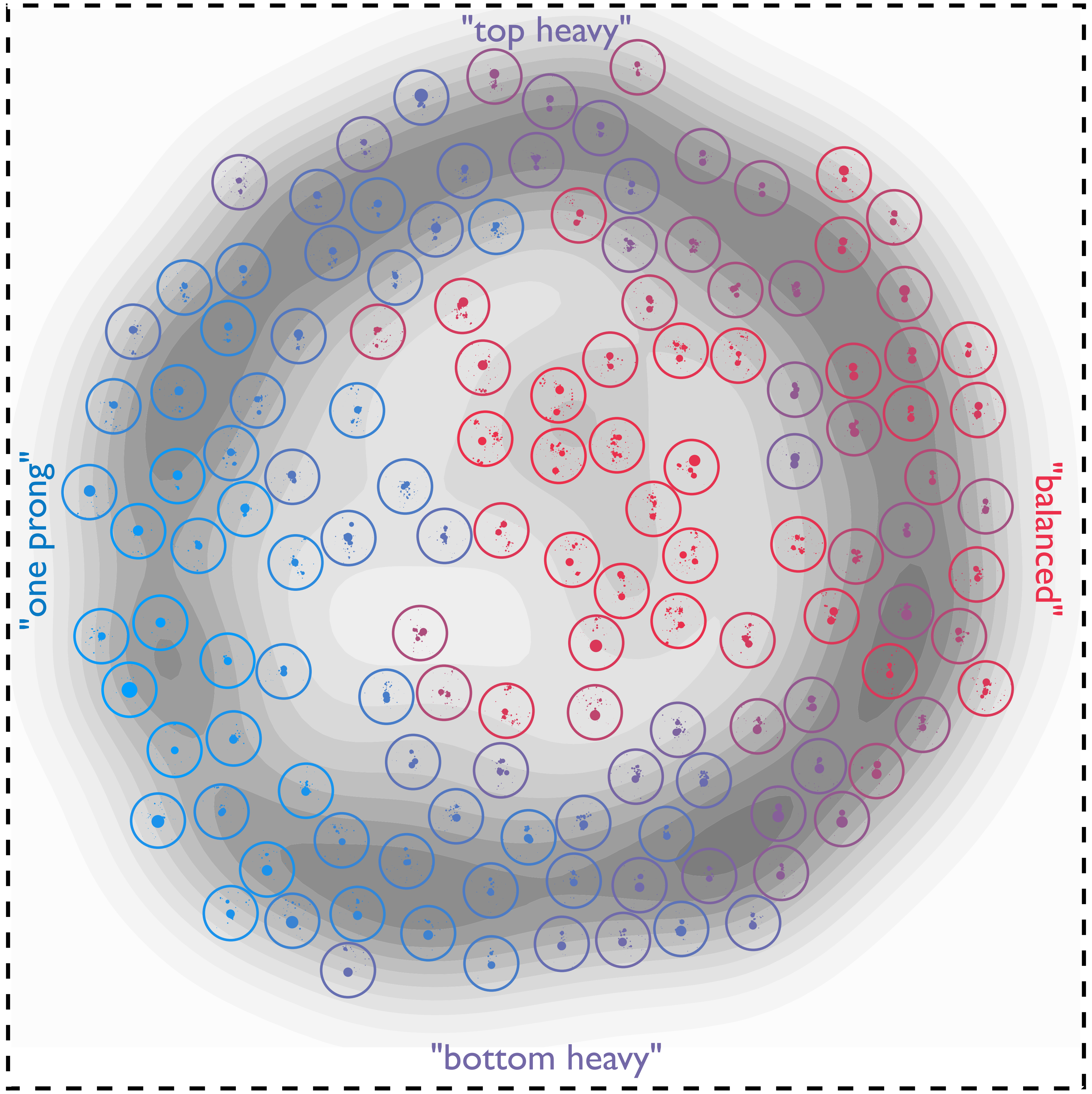
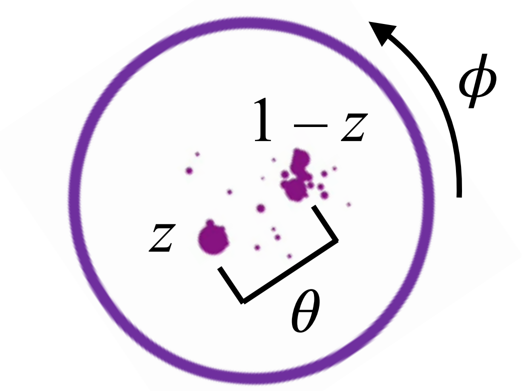
[PTK, Metodiev, Thaler, PRL 2019]

Geometric space of *W* jets



W Jet



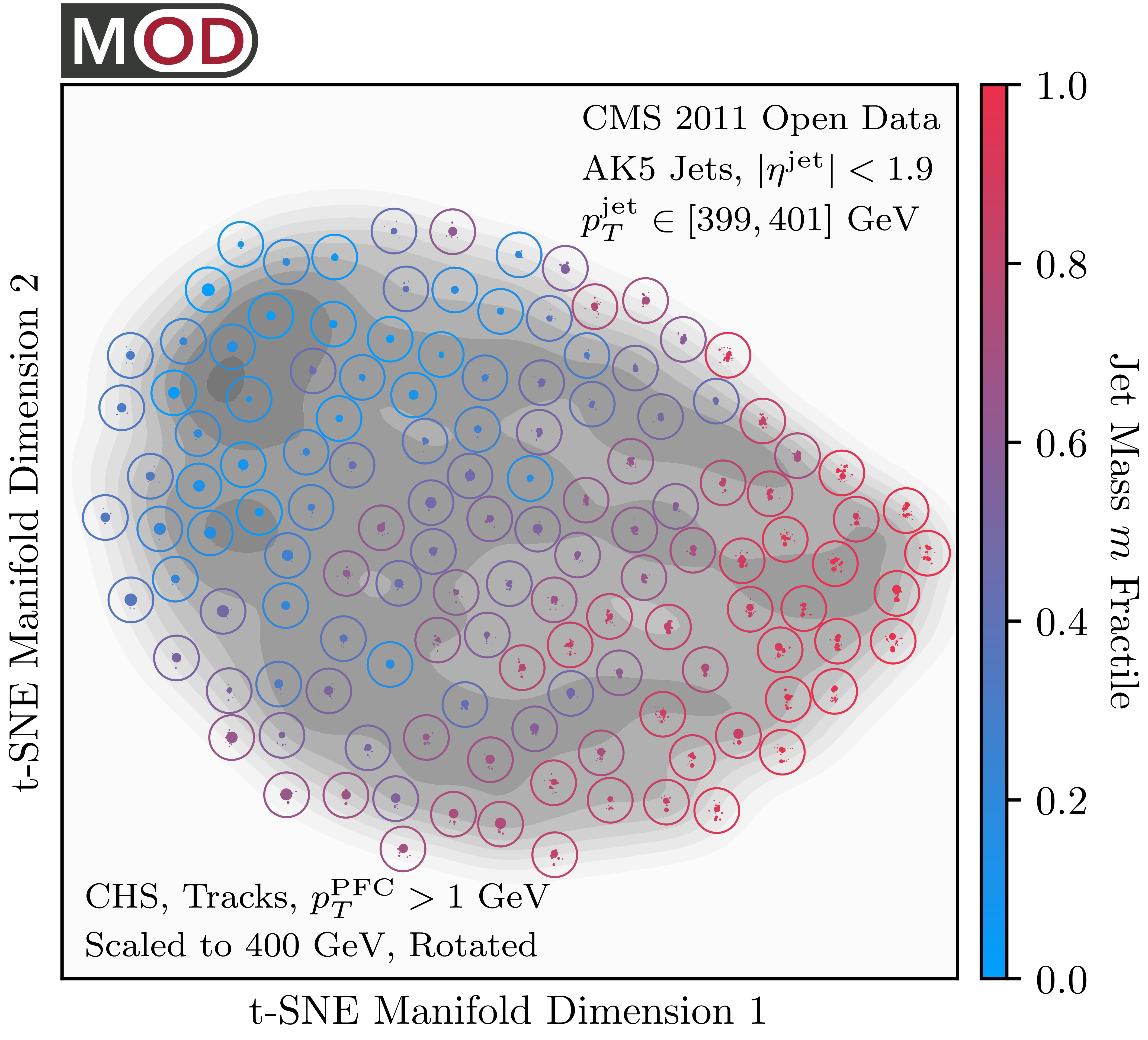
Constraints: *W* Mass and $\phi = 0$ preprocessing

Gray contours represent the density of jets

Each circle is a particular *W* jet

Visualizing Geometry in CMS Open Data

[PTK, Mastandrea, Metodiev, Naik, Thaler, PRD 2019; code and datasets at energyflow.network]



Example jets sprinkled throughout

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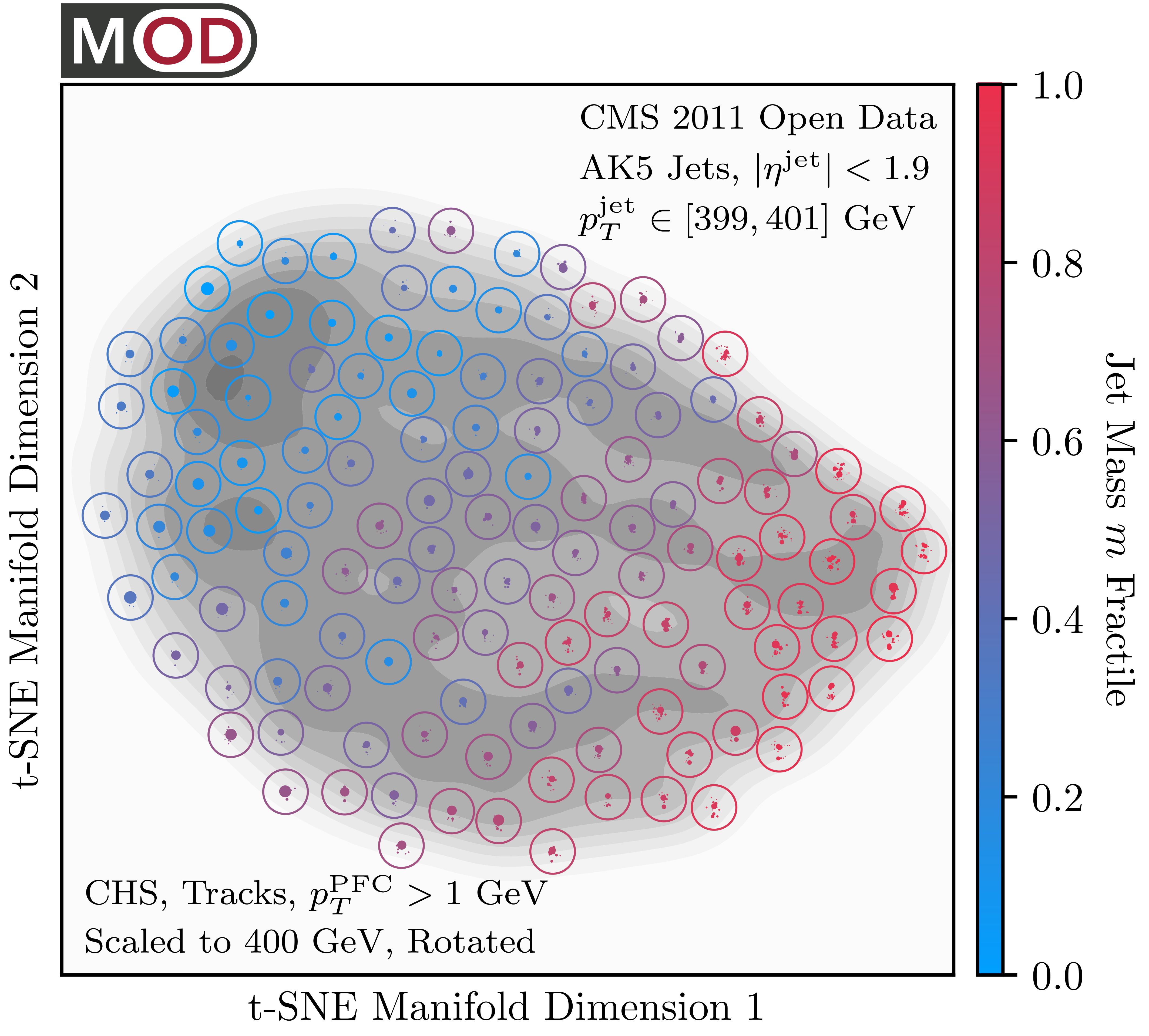

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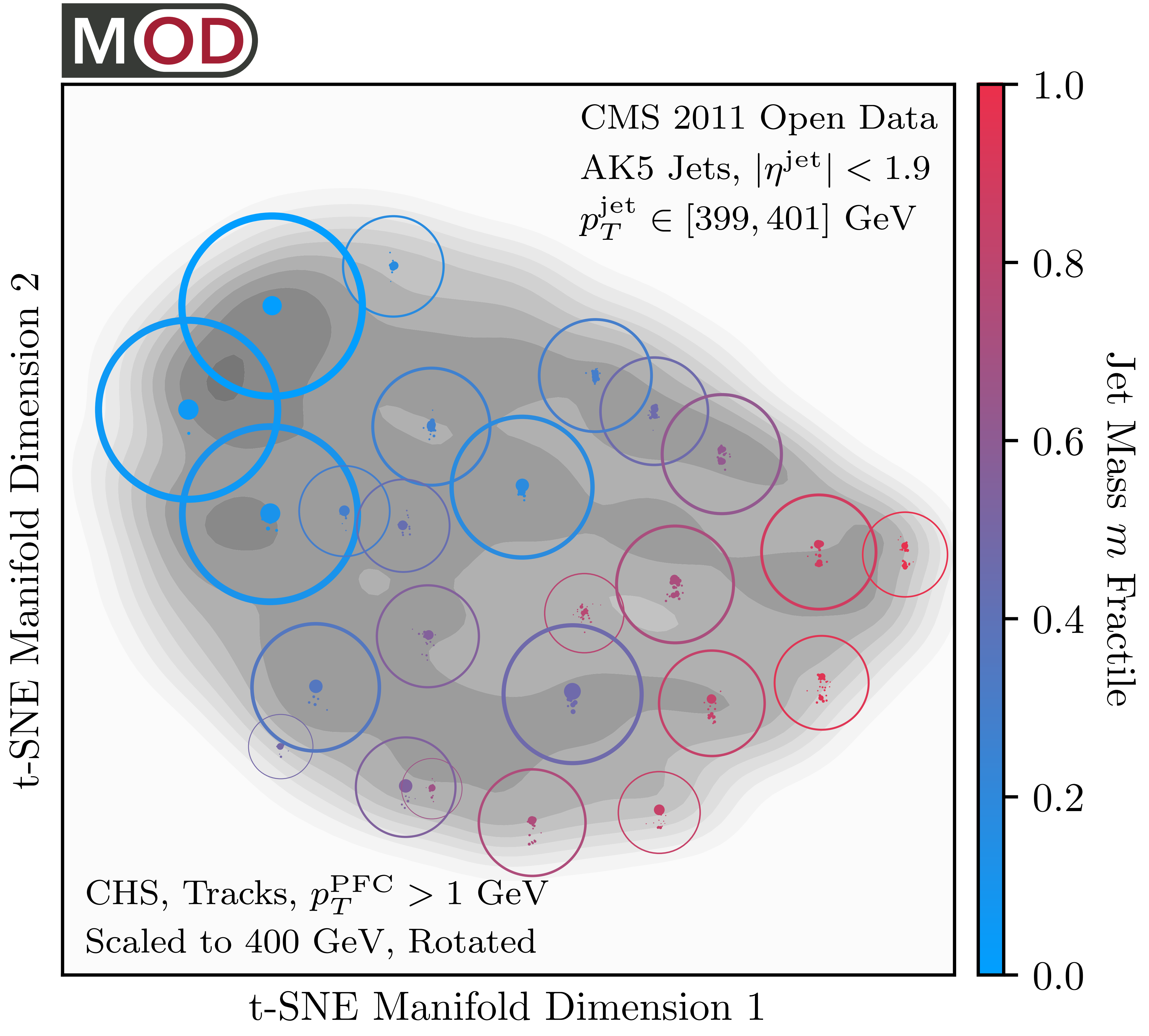
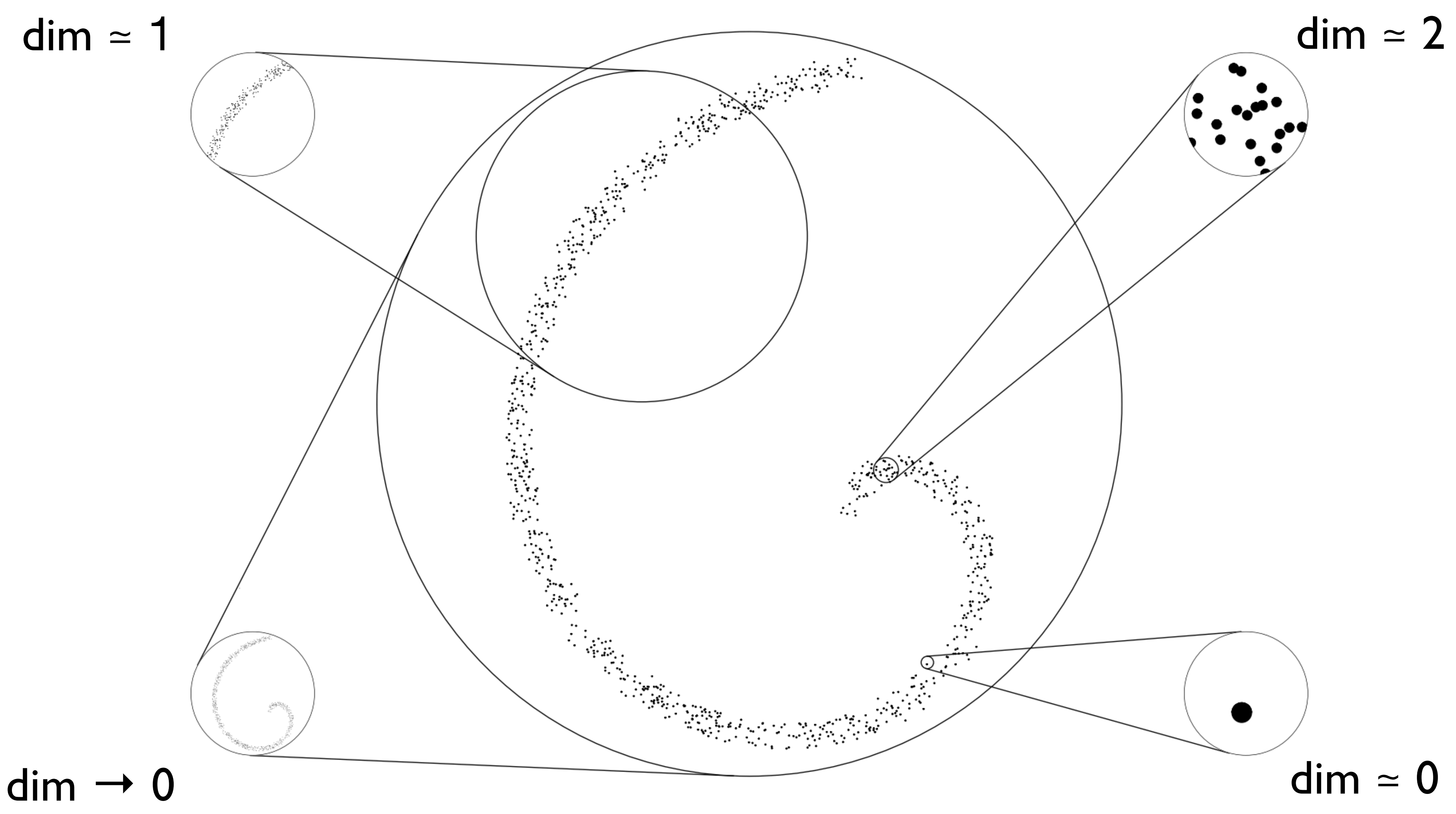


25 most representative jets ("medoids")
Size is proportional to number of jets associated to that medoid

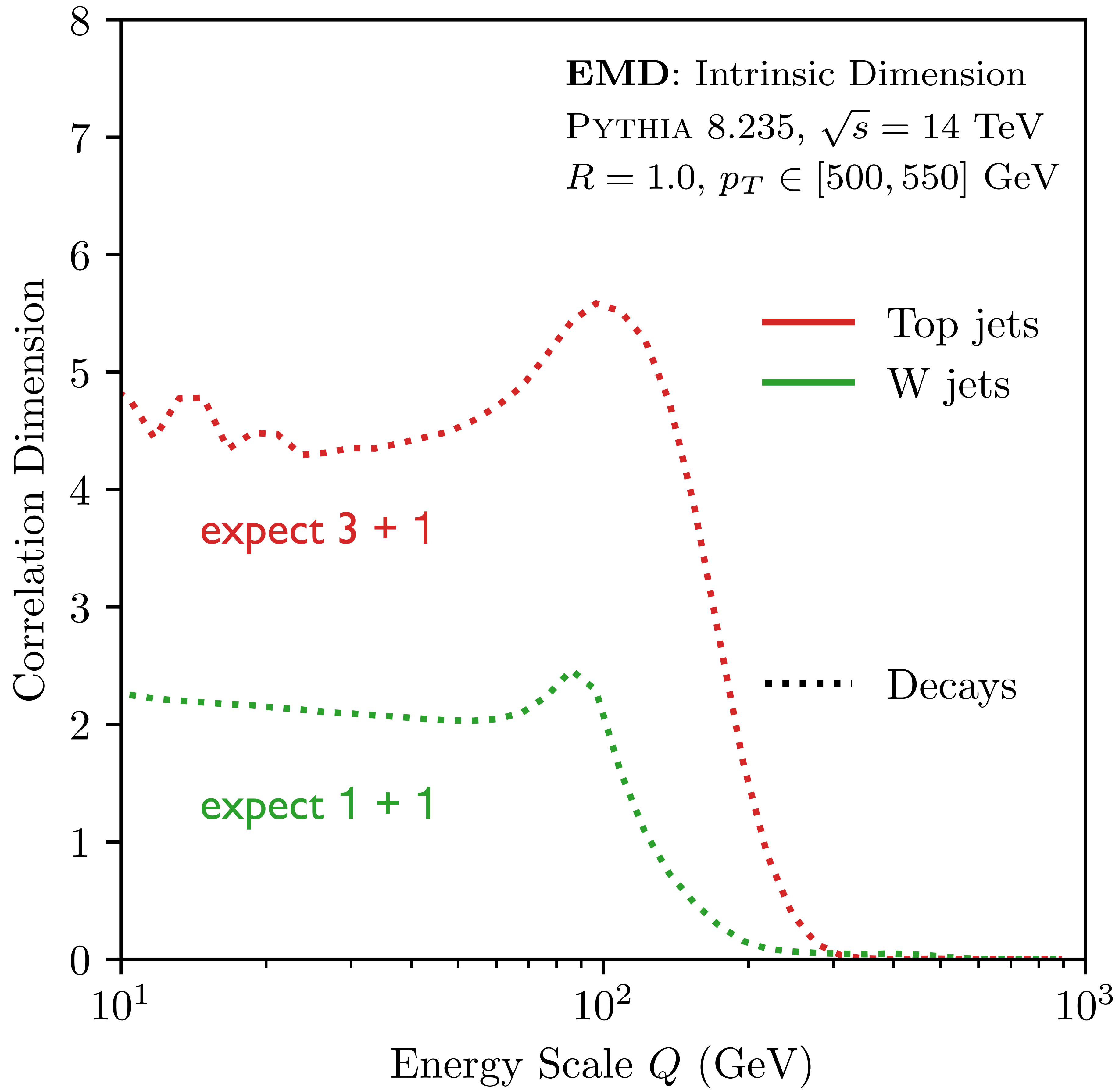
Quantifying Event-Space Manifolds

$$\dim(Q) = Q\frac{\partial}{\partial Q}\ln\sum_i\sum_j\Theta(\mathrm{EMD}(\mathcal{E}_i,\mathcal{E}'_j) < Q)$$

Correlation dimension: *how does the # of elements within a ball of size Q change?*



$$N_{\text{neigh.}}(Q) \propto Q^{\dim} \implies \dim(Q) = Q\frac{d}{dQ}\ln N_{\text{neigh.}}(Q)$$



Correlation dimension lessons:

Decays are "constant" dim. at low *Q*

[Grassberger, Procaccia, PRL 1983; PTK, Metodiev, Thaler, PRL 2019]

*Preliminary LL calculation for QG jets in backup

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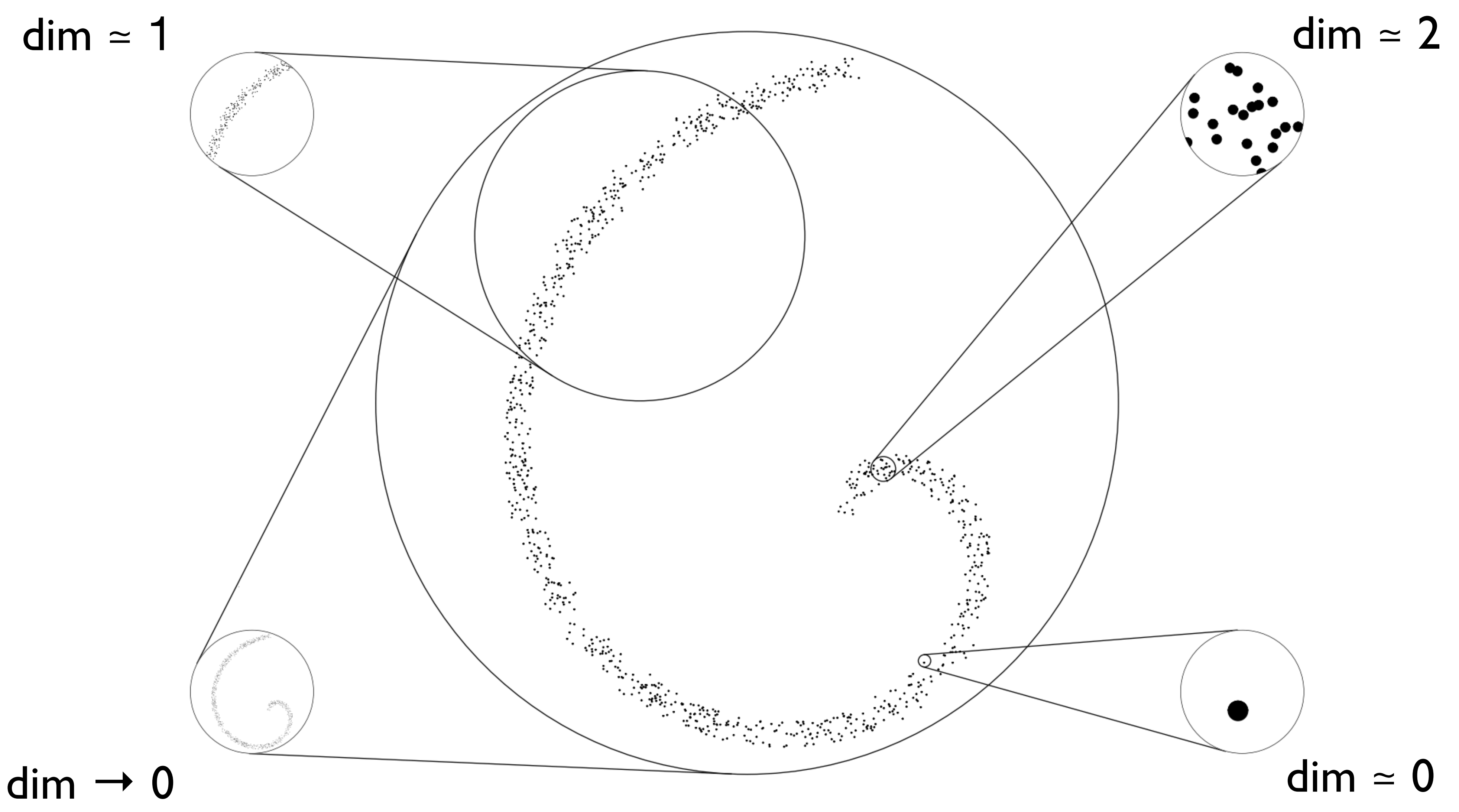

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Correlation dimension lessons:

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- Complexity hierarchy: QCD < W < Top
- Fragmentation increases dim. at smaller scales

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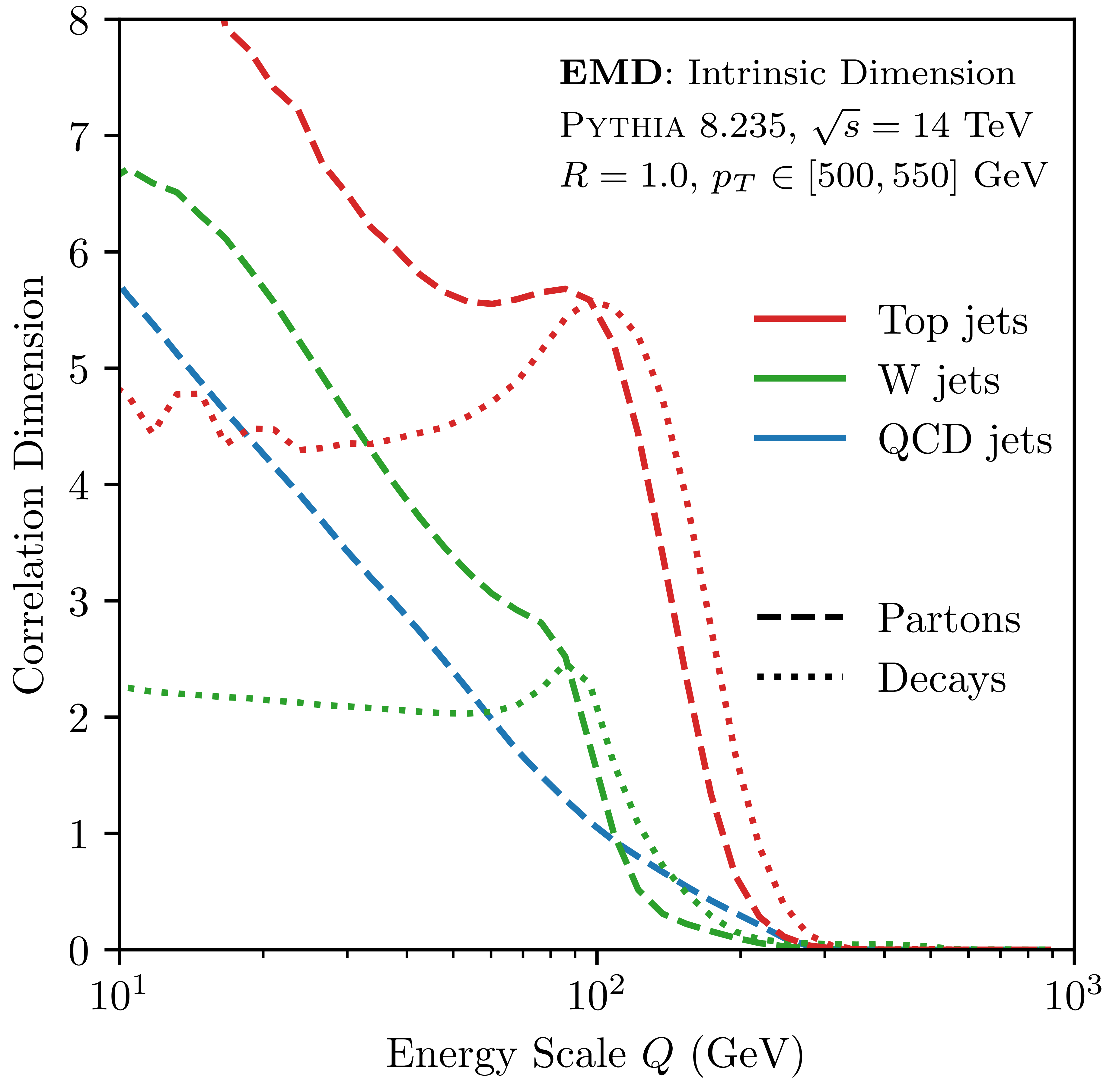

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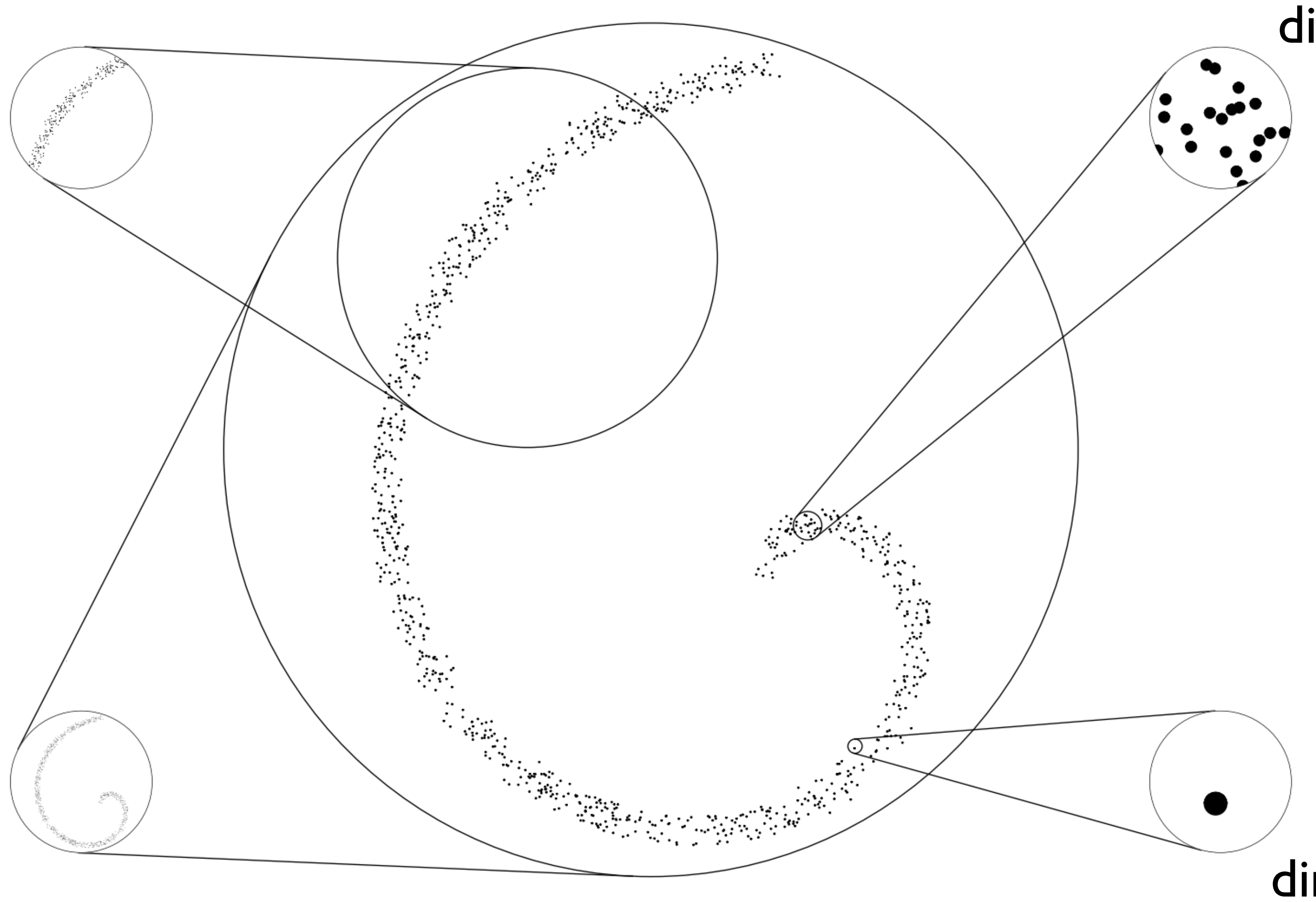

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Correlation dimension lessons:

- Decays are "constant" dim. at low Q
- Complexity hierarchy: QCD < W < Top
- Fragmentation increases dim. at smaller scales
- Hadronization important around 20-30 GeV

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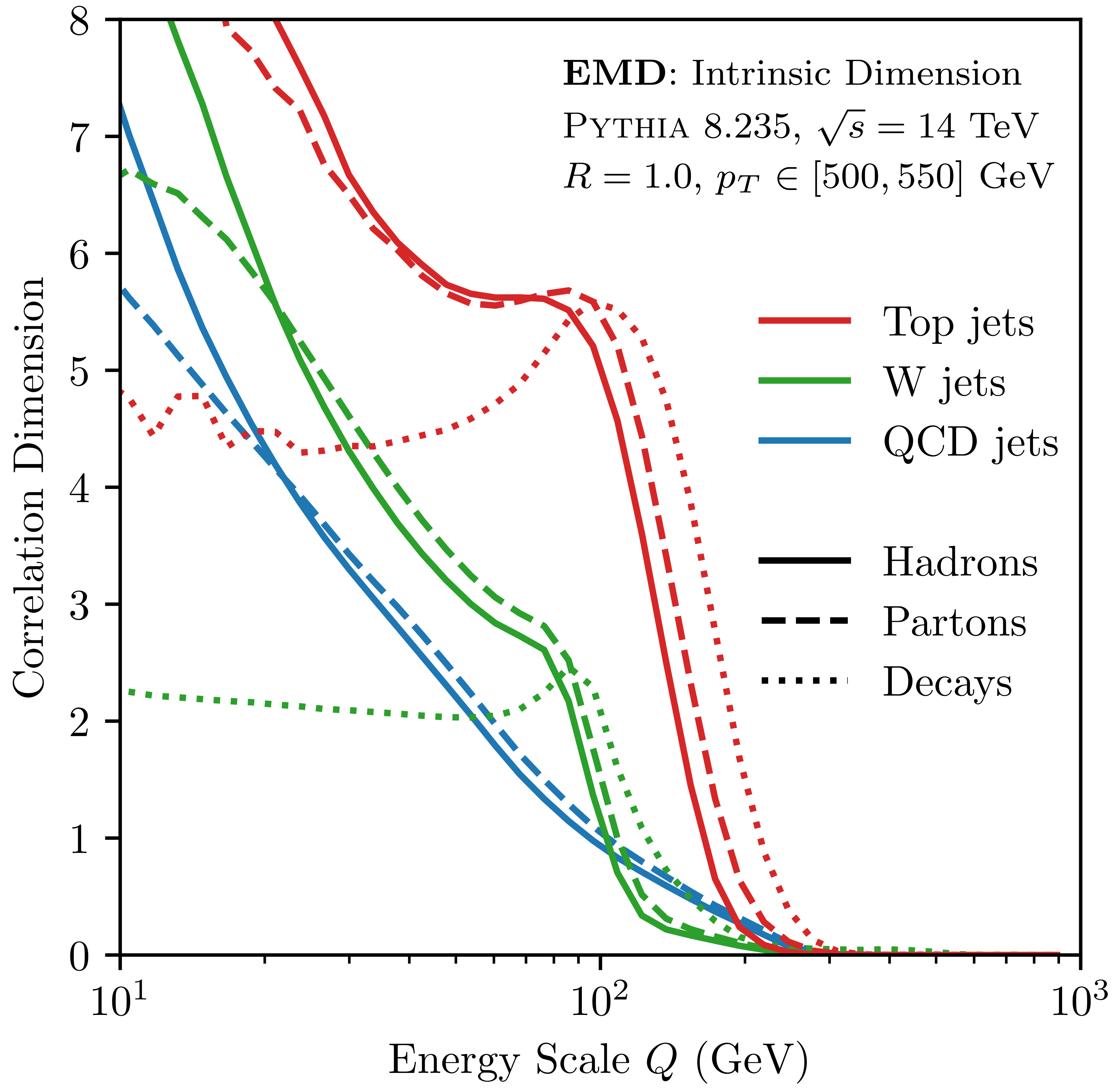

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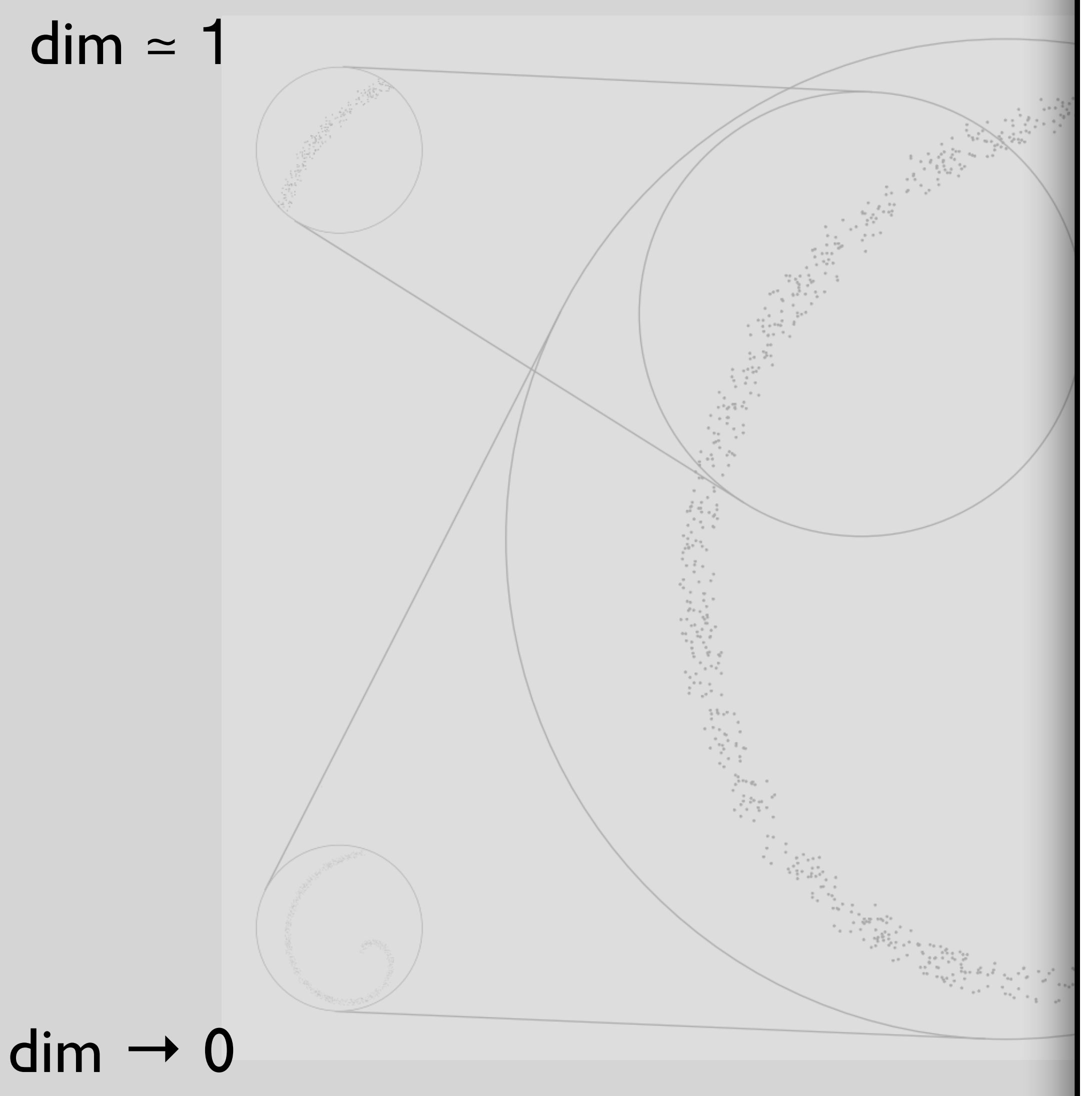
Quantifying Event-Space Manifolds

Correlation dimension
elements within a ball



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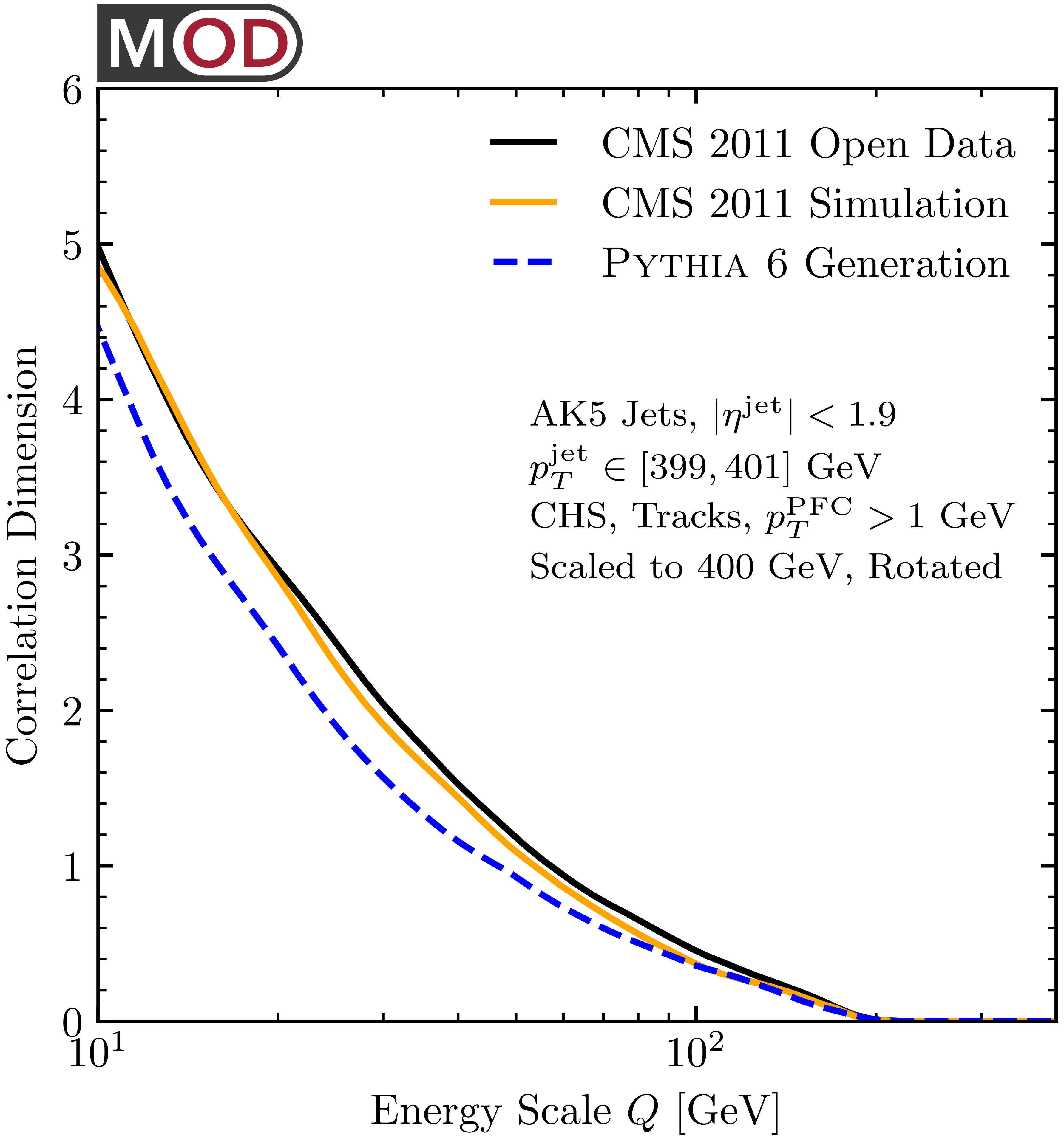
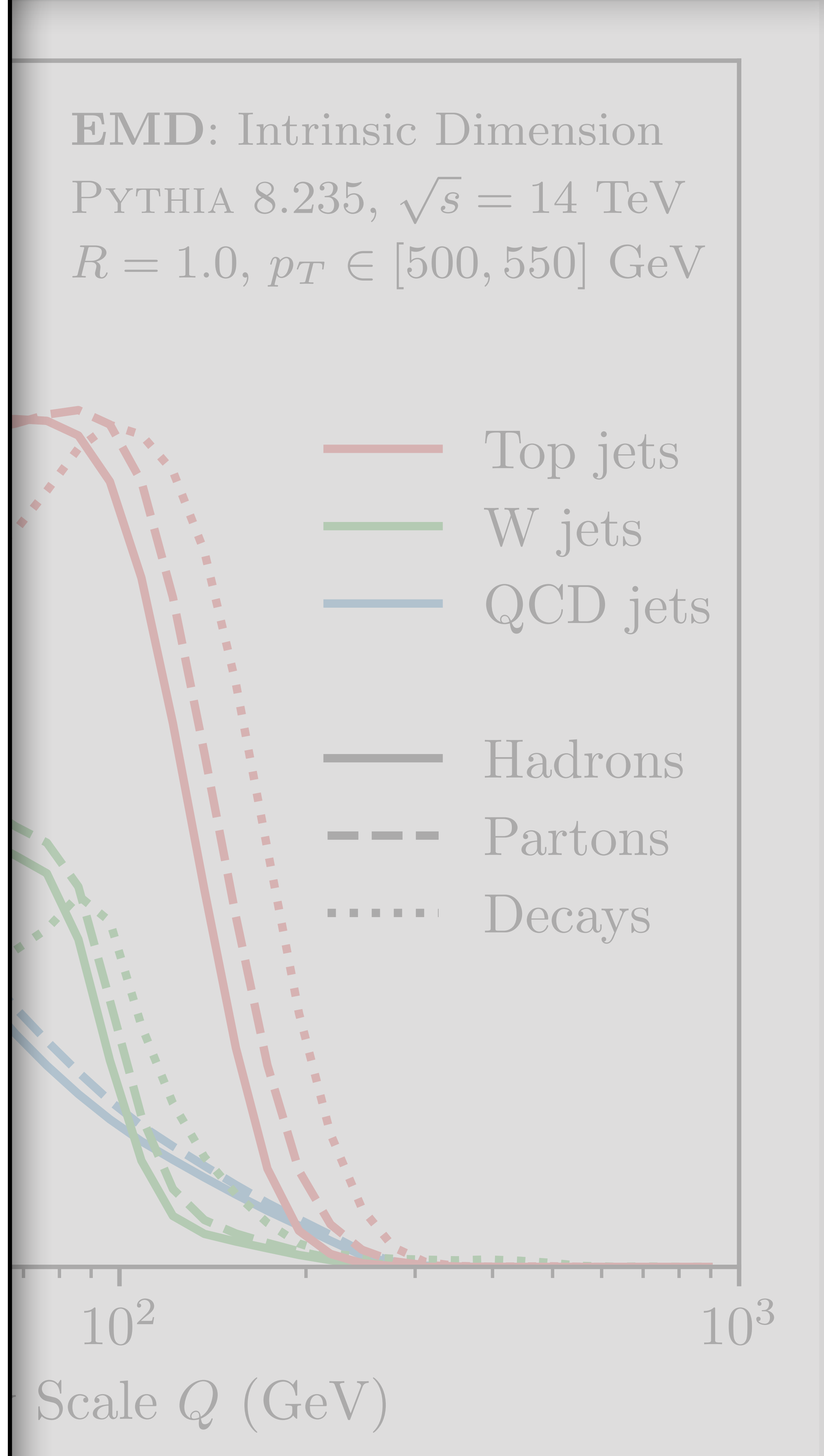
$\sum_j \Theta(\text{EMD}(\mathcal{E}_i, \mathcal{E}'_j) < Q)$

… in CMS Open Data



*More in backup



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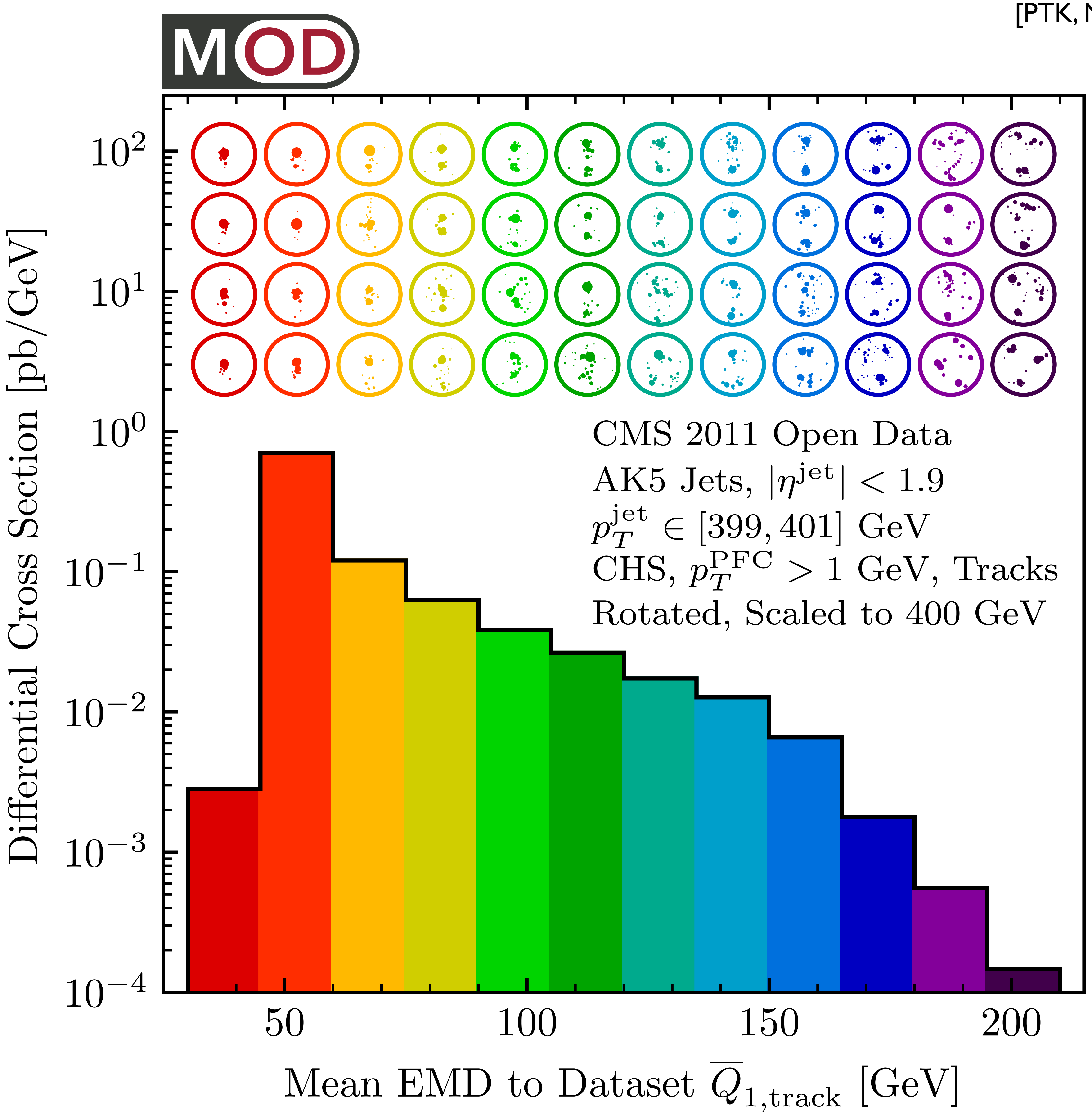

EMD for anomaly detection

← 4 medoids in each bin of anomaliness $\bar{Q}_1$

n^{th} moment of EMD distribution for a dataset

$$\bar{Q}_n(\mathcal{I}) = \sqrt[n]{\frac{1}{N}\sum_{k=1}^{N}(\text{EMD}(\mathcal{I},\mathcal{J}_k))^n}$$

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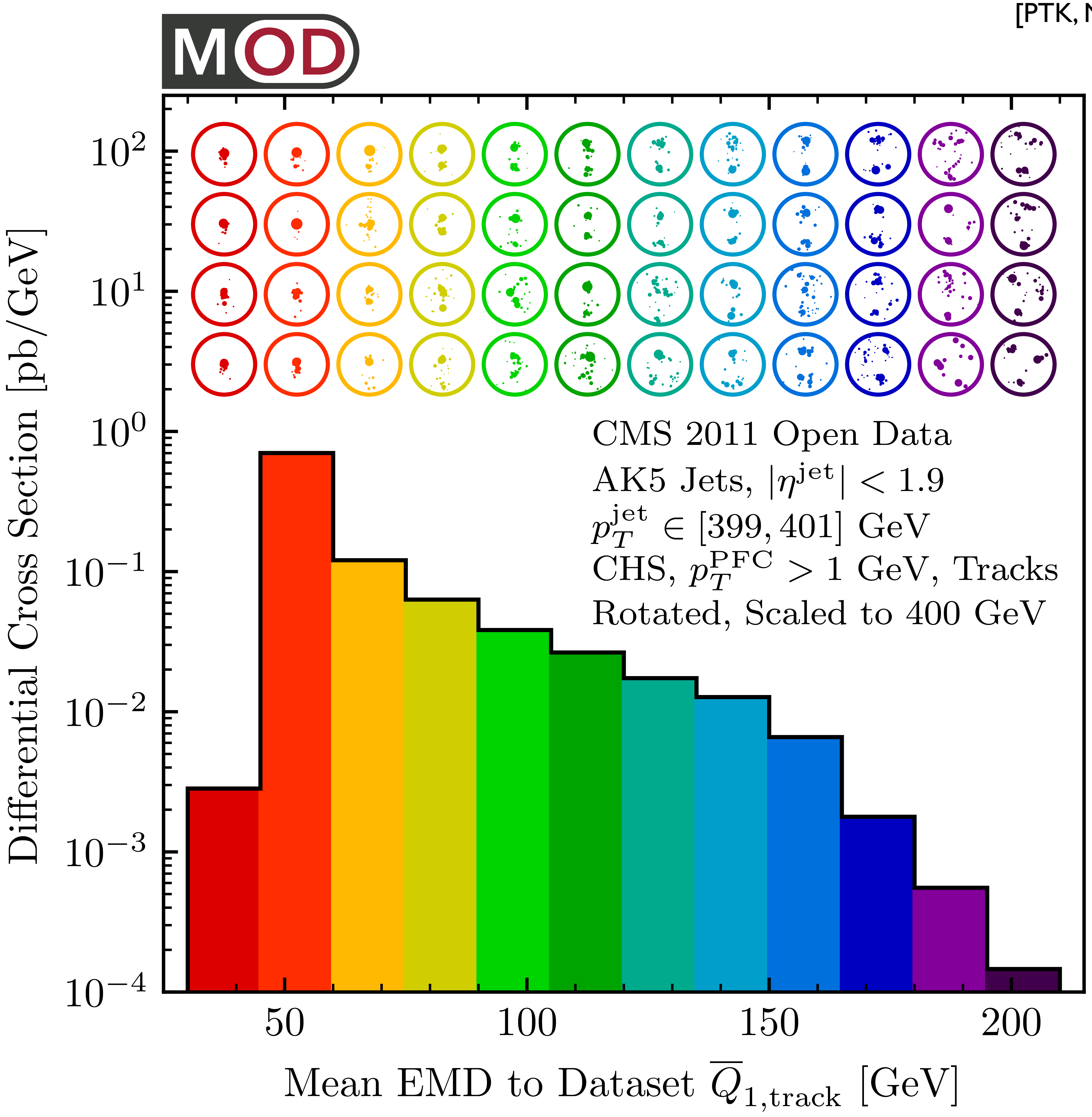

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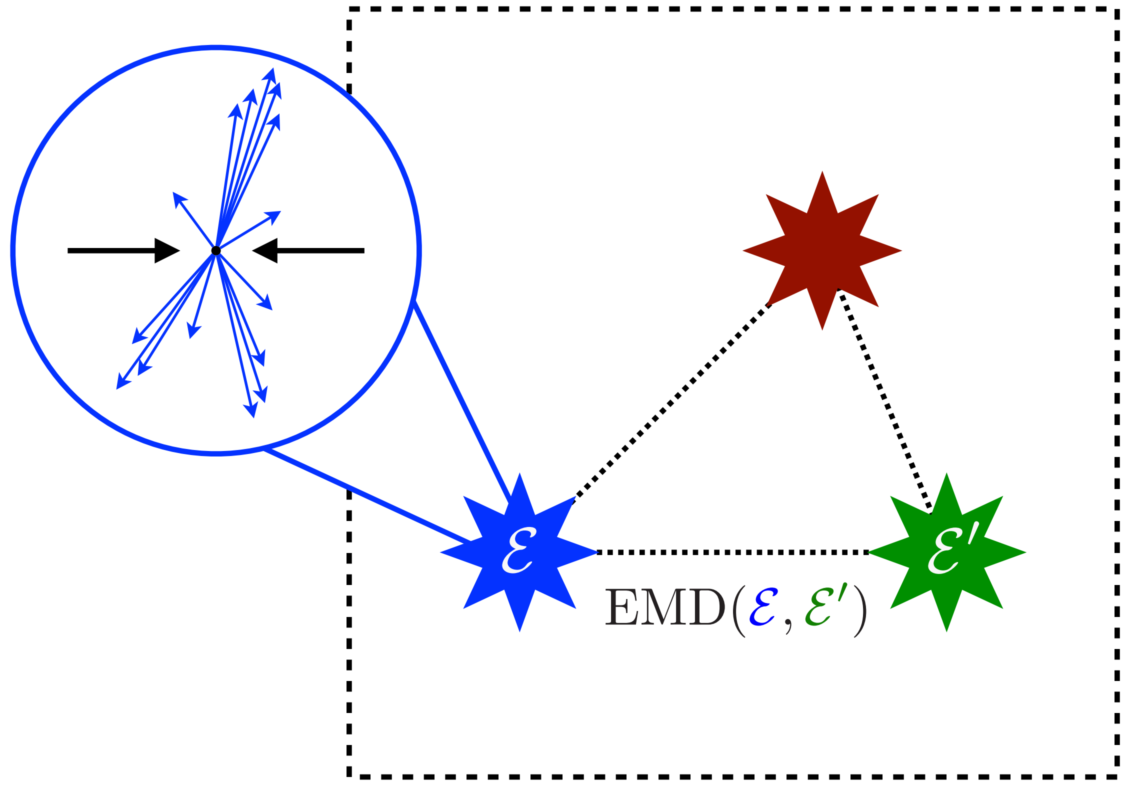
How far does this go?

$$\mathcal{V}_k = \frac{1}{N}\sum_{i=1}^{N}\min\left\{\mathrm{EMD}(\mathcal{J}_i,\mathcal{K}_1),\ldots,\mathrm{EMD}(\mathcal{J}_i,\mathcal{K}_k)\right\}$$

k-eventiness? jet from dataset medoids

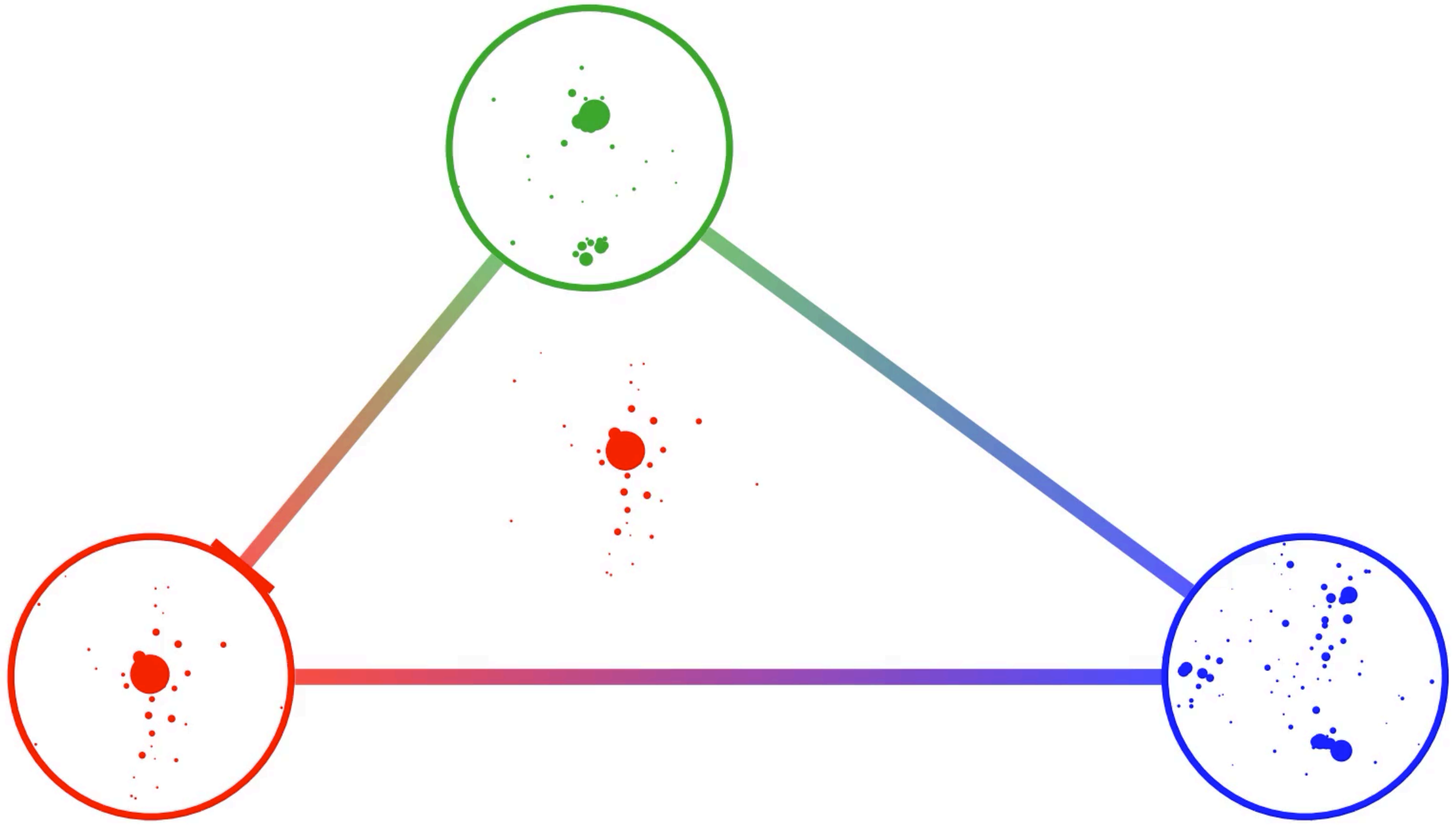


The (Metric) Space of Events

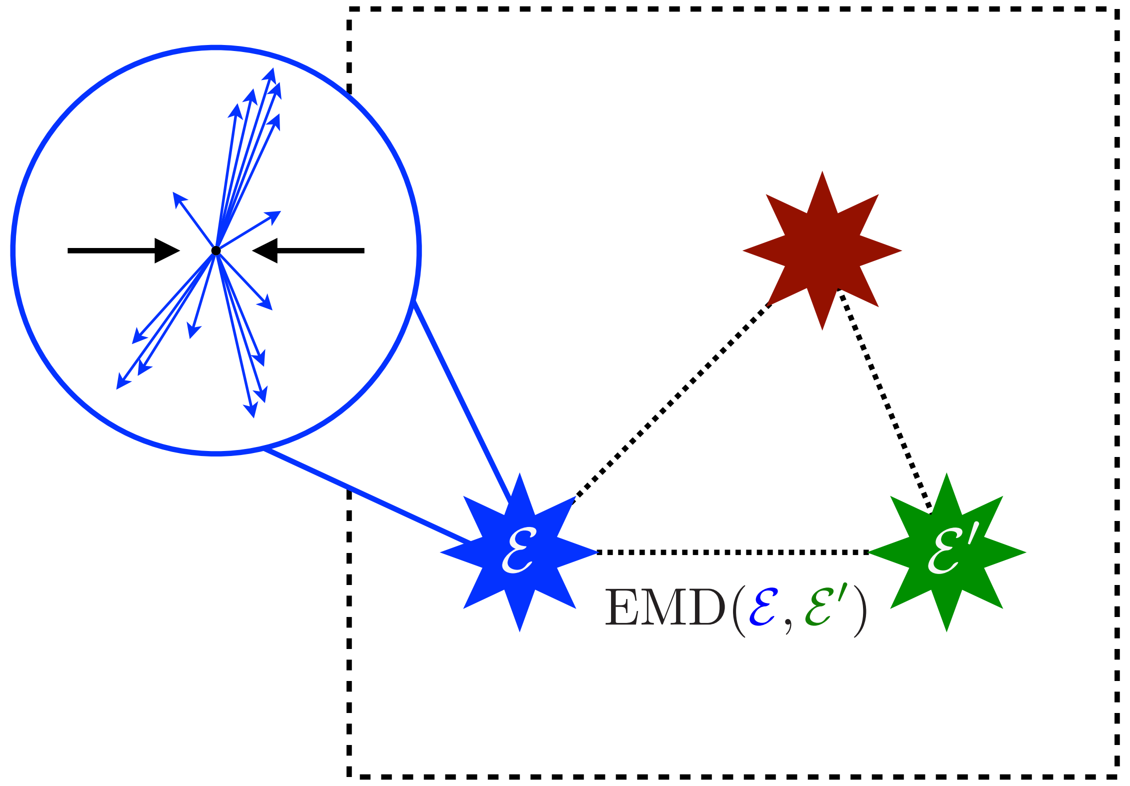
– Energy flow is theoretically and experimentally robust

– EMD metrizes the space of energy flows (events)

– Manifolds in the space of events can be visualized and quantified



What else can this geometry do for us?



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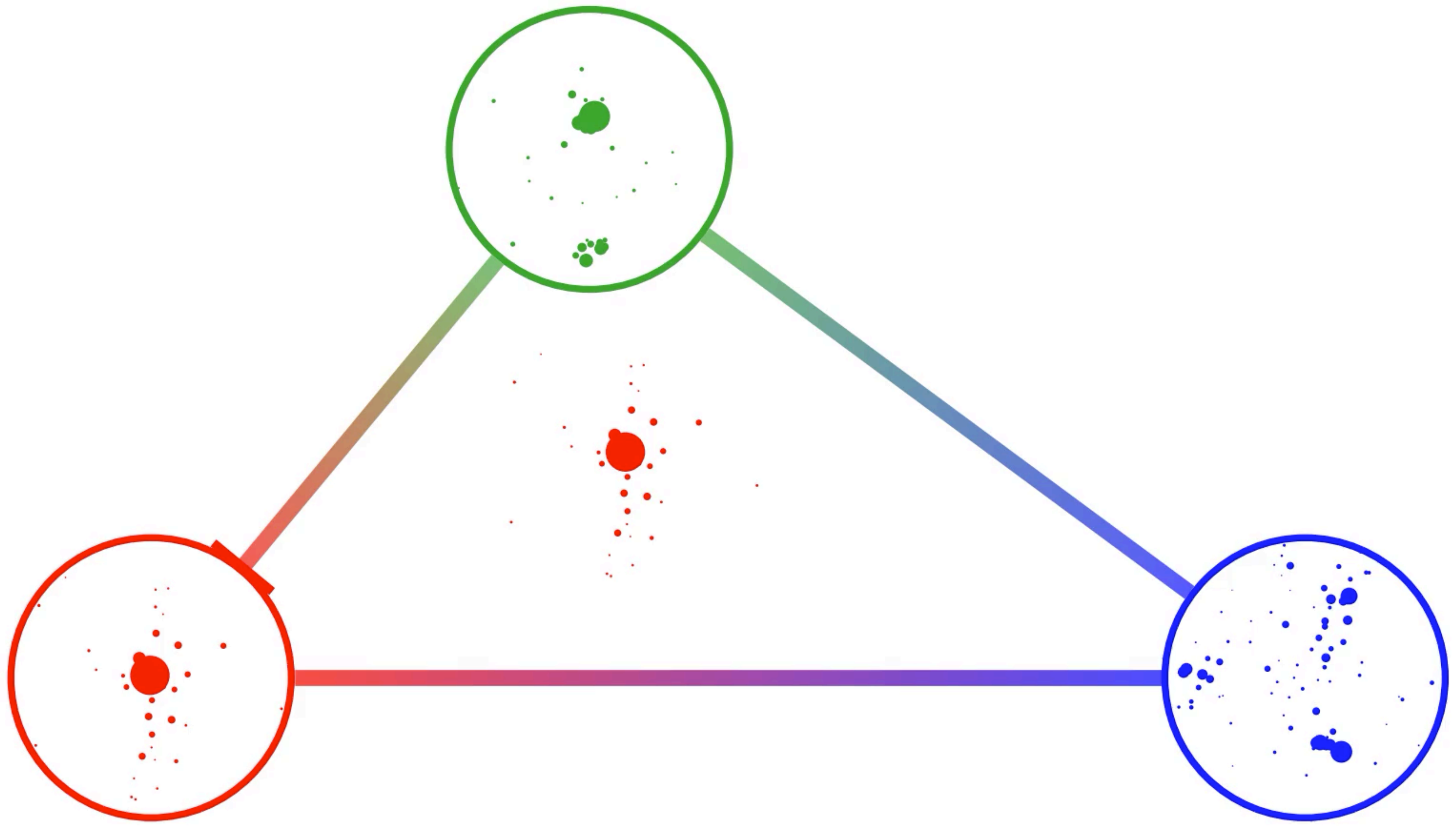

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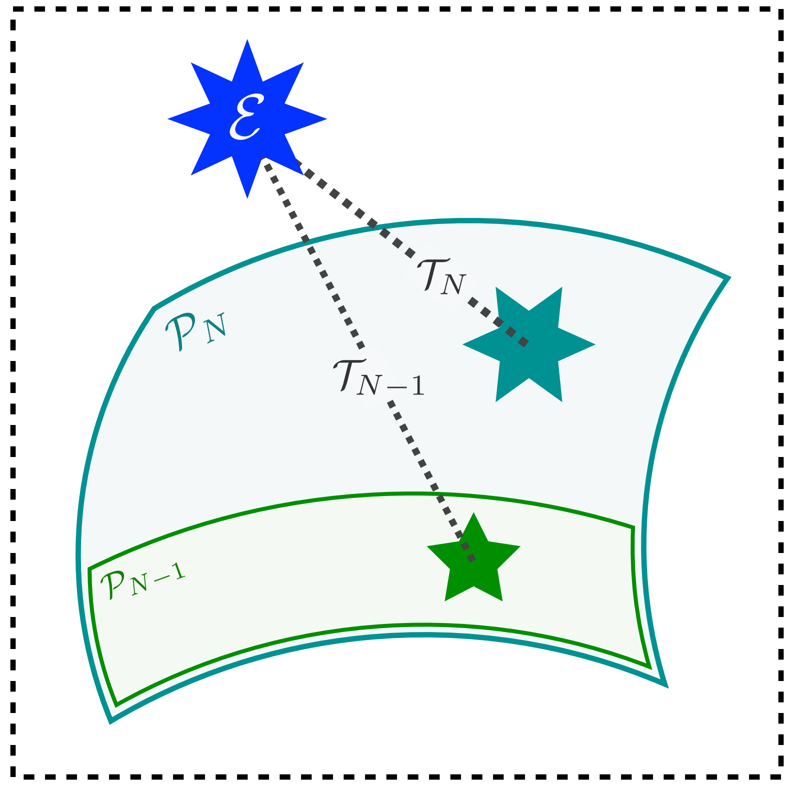


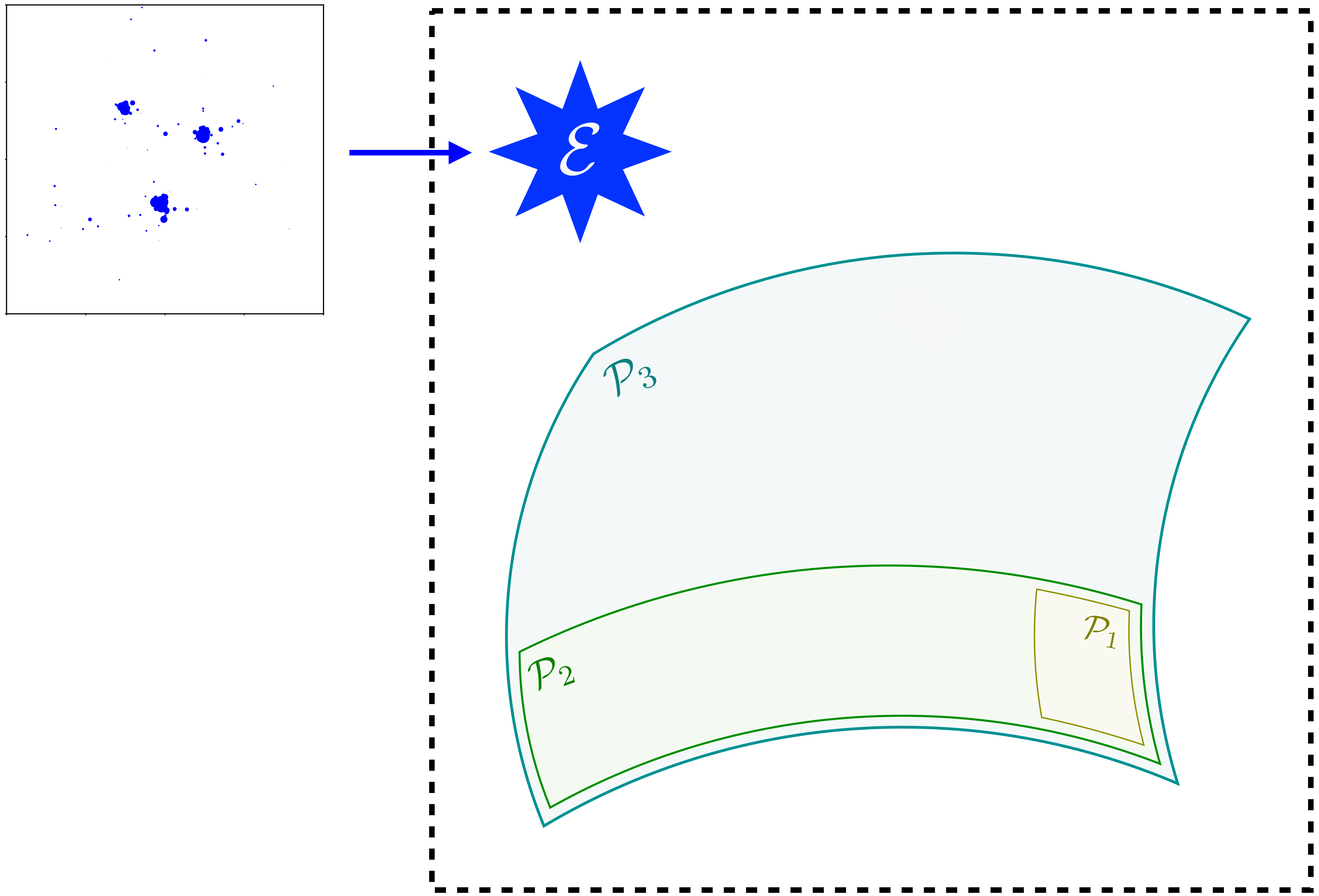
Revealing Hidden Geometry

Theory Space

N-particle Manifolds in the Space of Events

[PTK, Metodiev, Thaler, 2004.04159]

$$\mathcal{P}_N = \text{set of all N-particle configurations} = \left\{ \sum_{i=1}^{N} E_i\,\delta(\hat{n} - \hat{n}_i) \,\middle|\, E_i \geq 0 \right\}$$



$$\vdots$$

$$\mathcal{P}_N \supset \mathcal{P}_{N-1} \supset \cdots \supset \mathcal{P}_3 \supset \mathcal{P}_2 \supset \mathcal{P}_1$$

by soft and collinear limits

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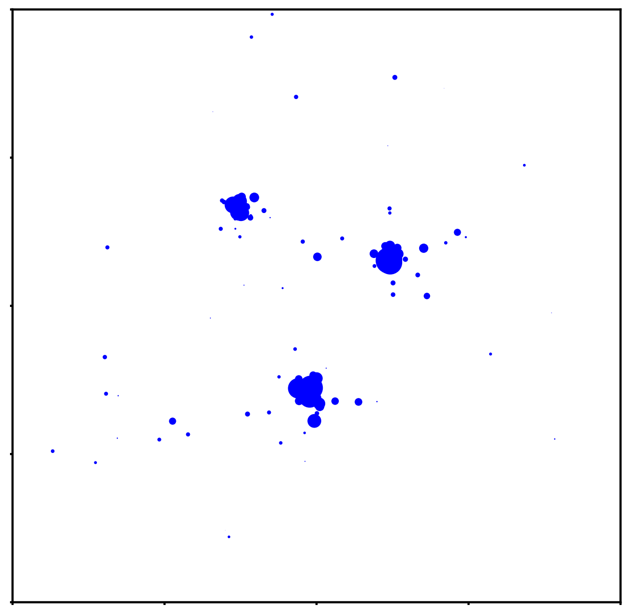

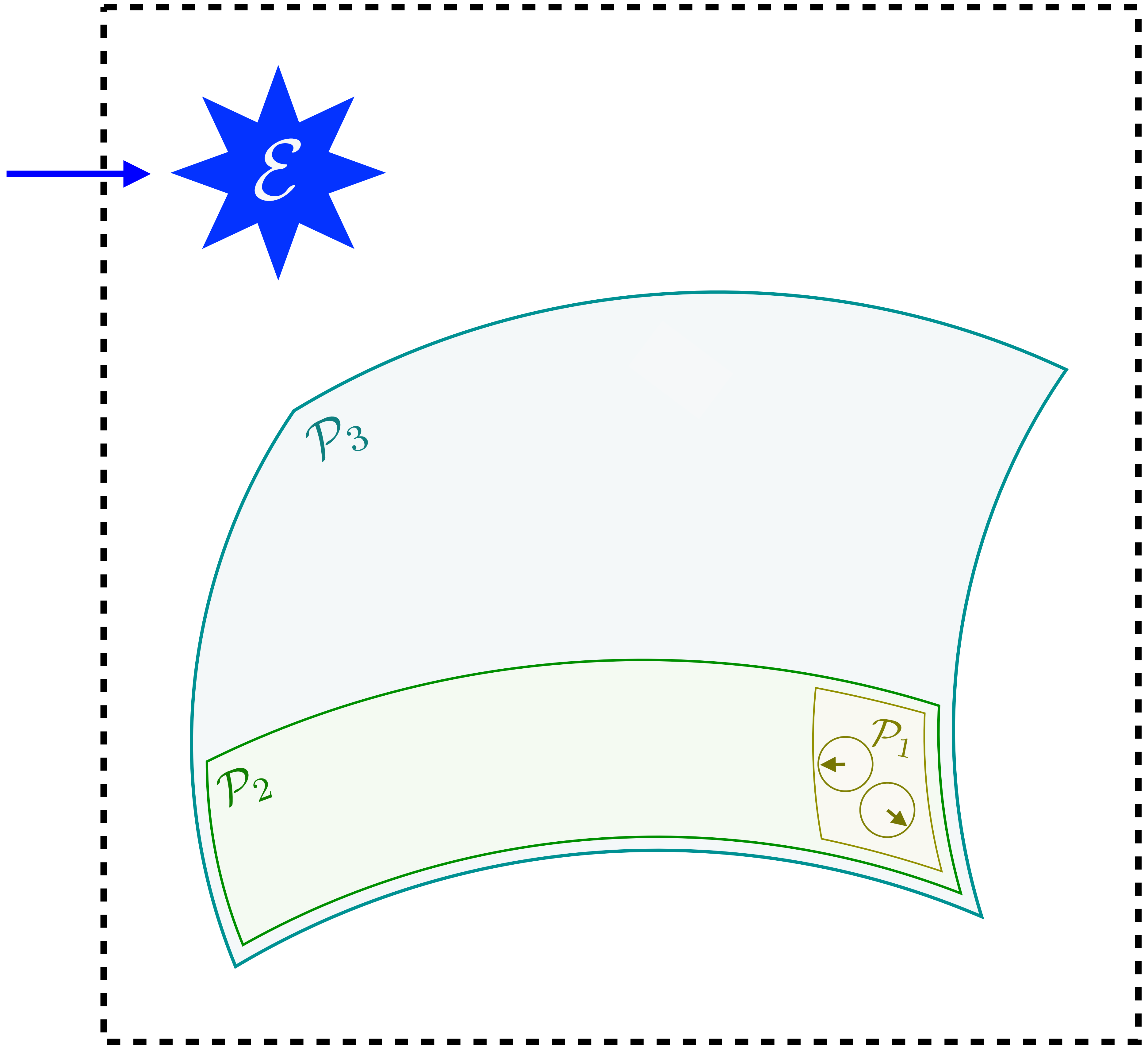

$\mathcal{P}_1$: manifold of events with one particle

⋮

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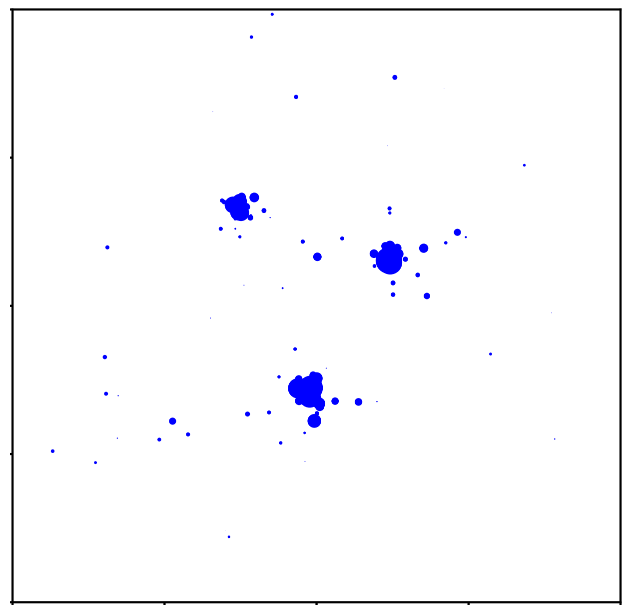

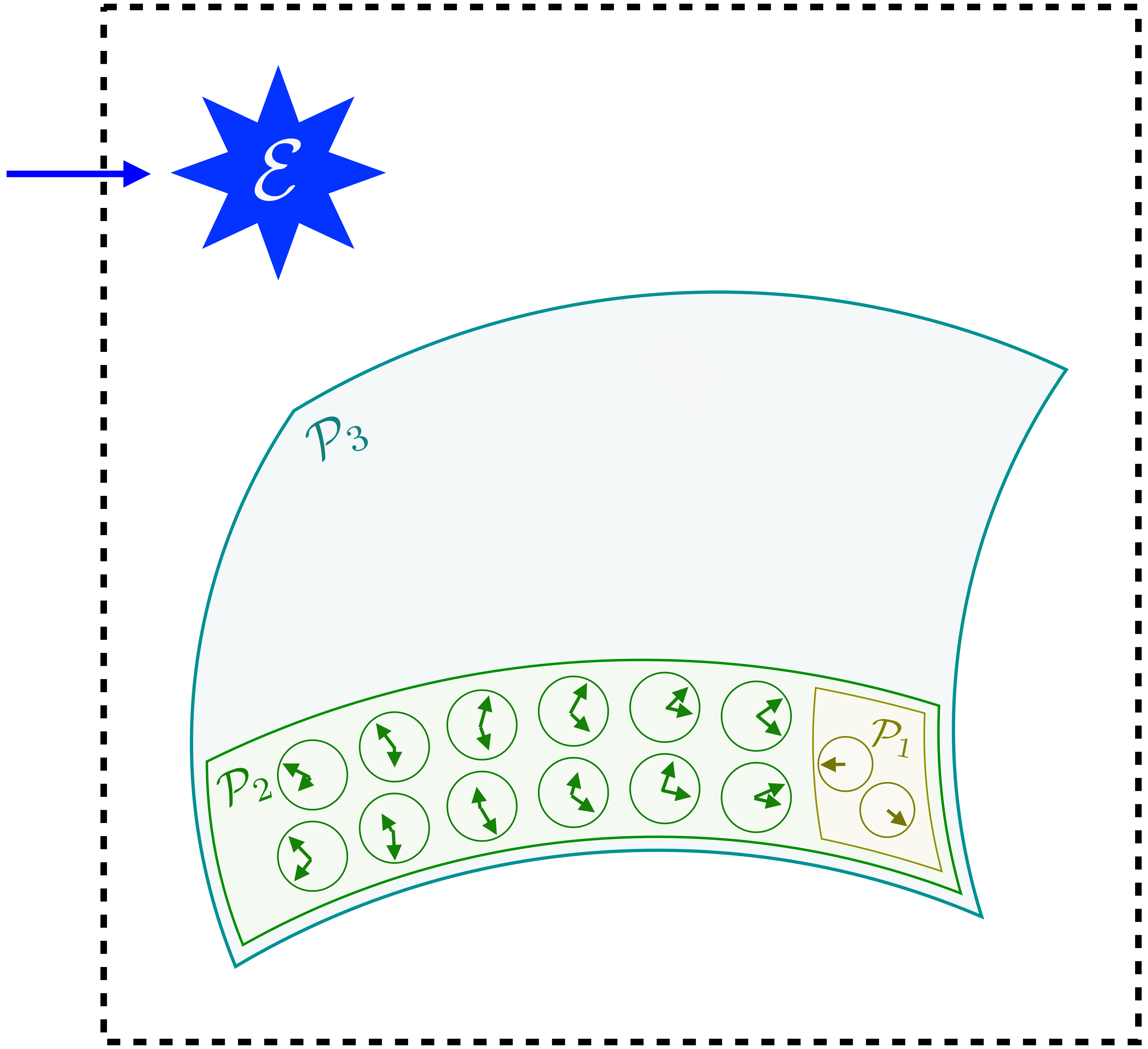

$\mathcal{P}_1$: manifold of events with one particle

$\mathcal{P}_2$: manifold of events with two particles

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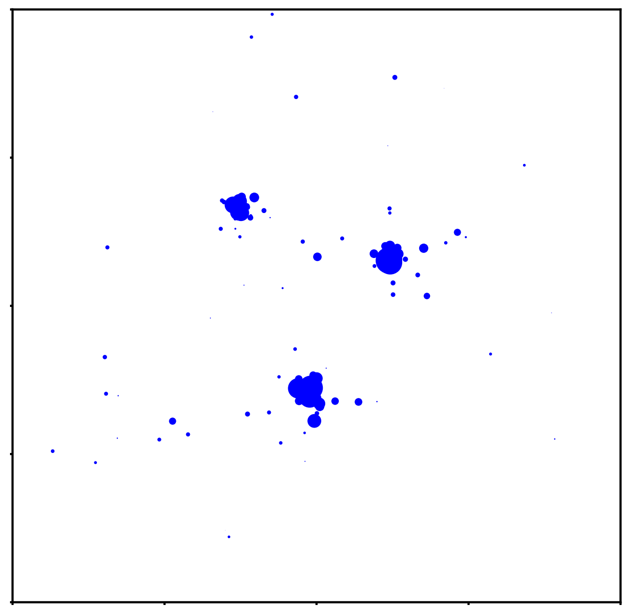

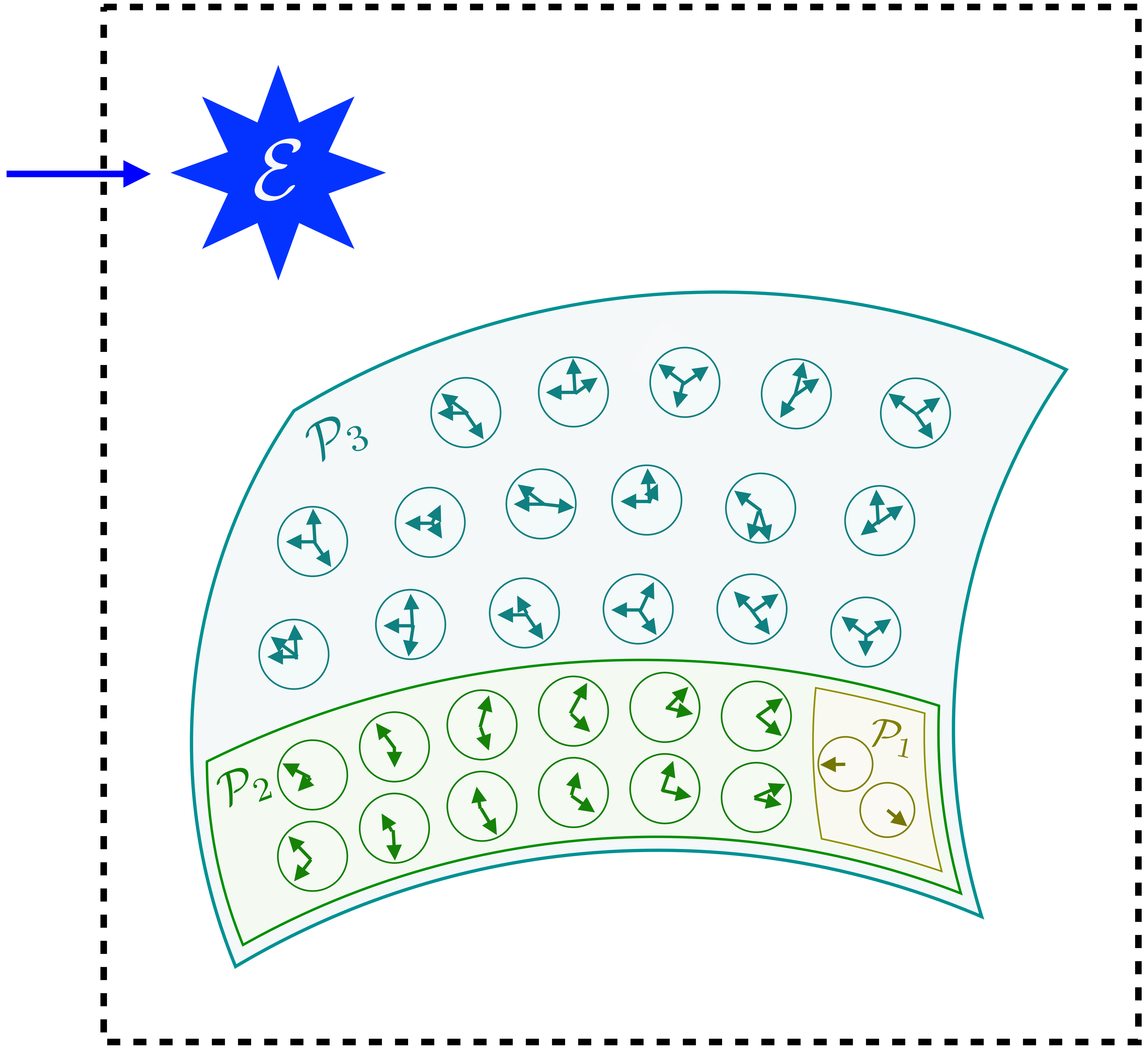

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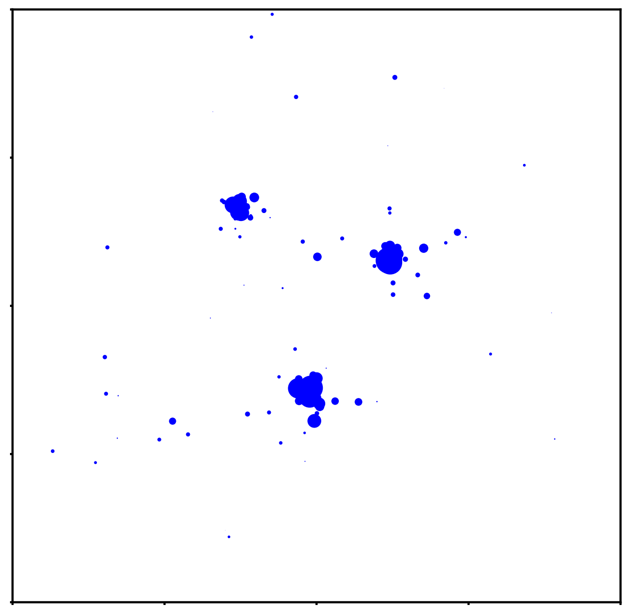

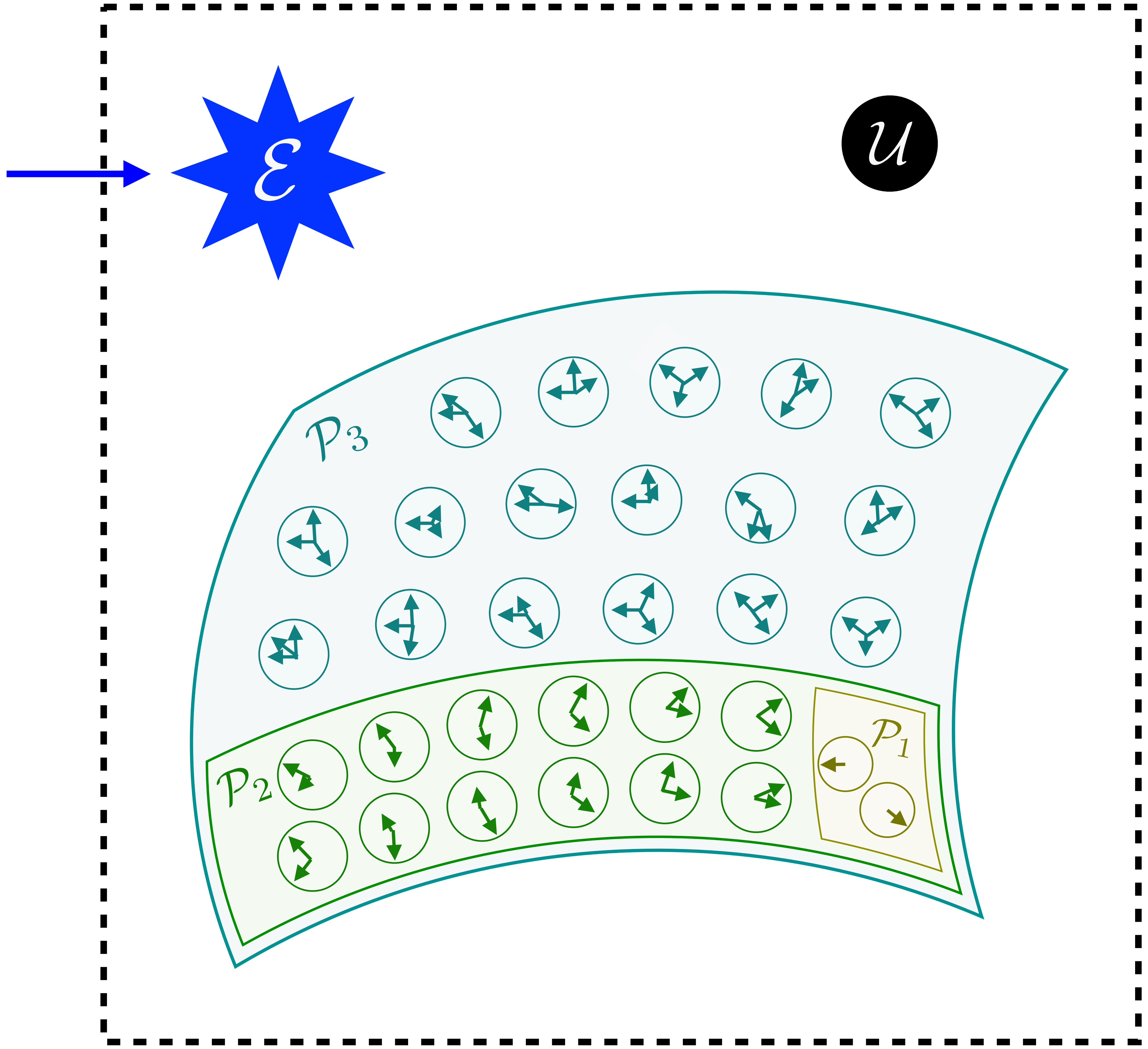

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by soft and collinear limits

$\mathcal{U}$ Uniform event, not contained in any P_N

N-particle Manifolds in the Space of Events – Infrared Divergences

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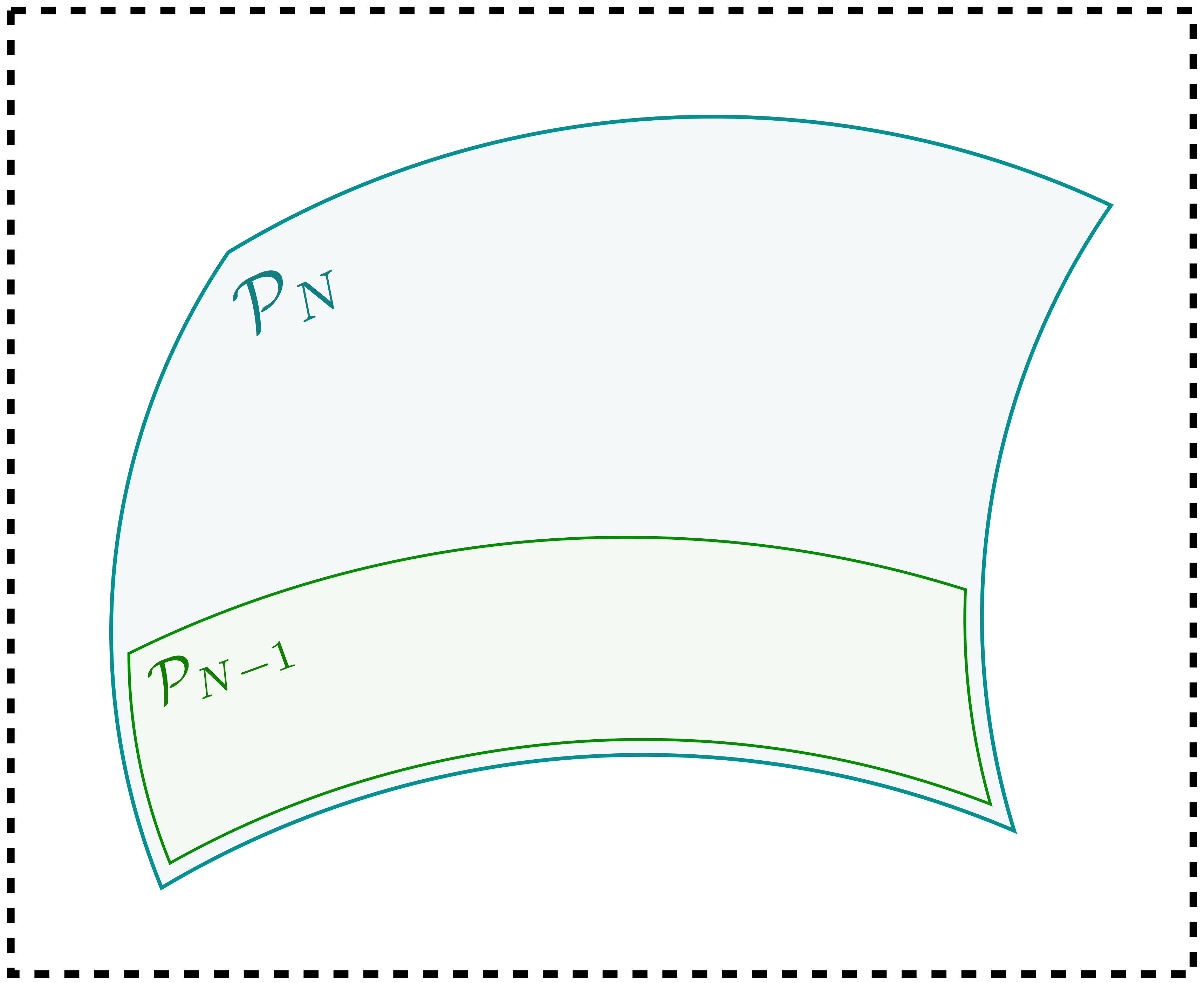

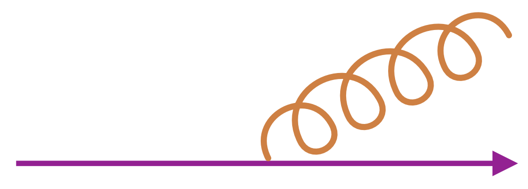

$$dP_{i\to ig} \simeq \frac{2\alpha_s}{\pi} C_a \frac{d\theta}{\theta} \frac{dz}{z}$$

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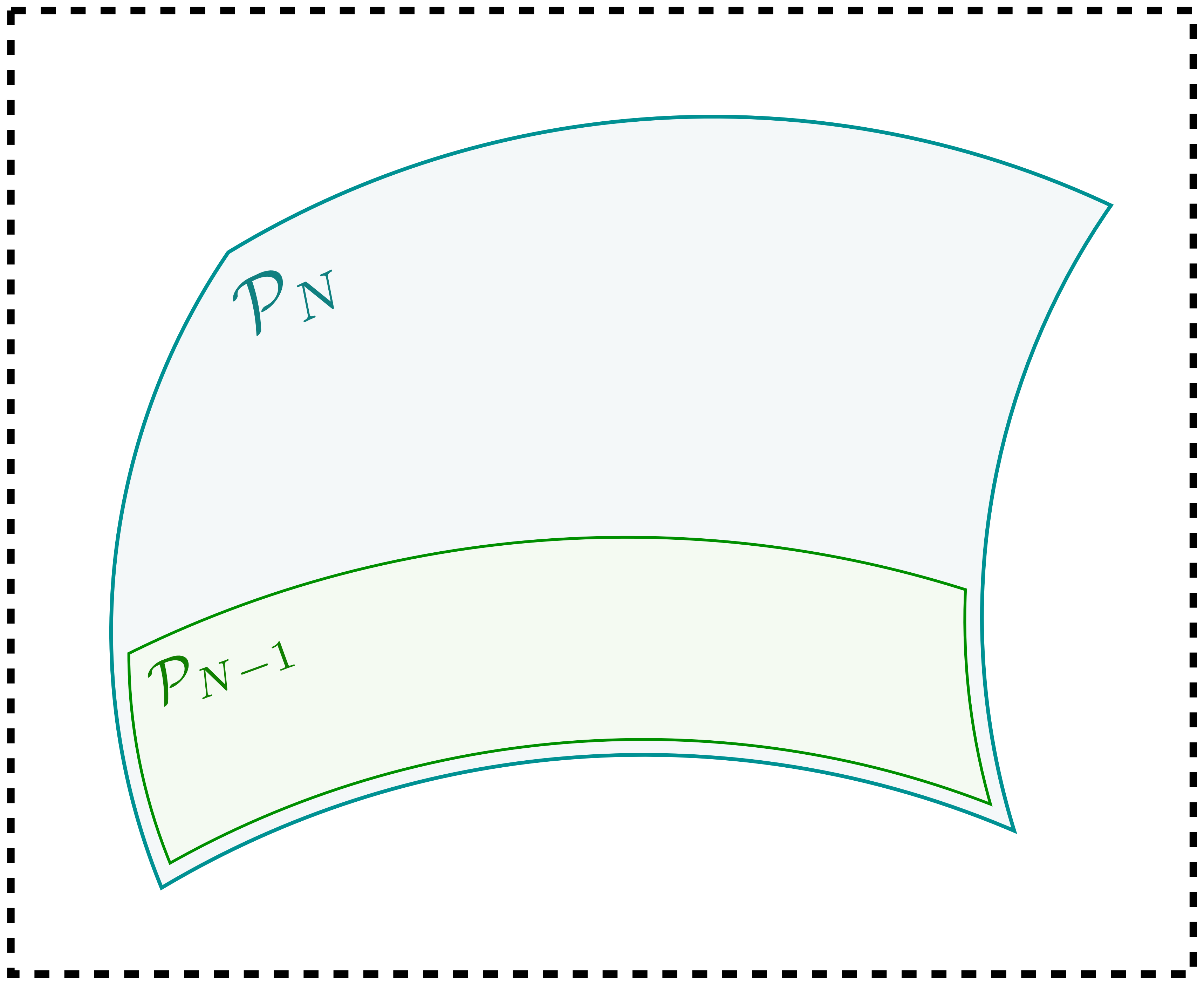

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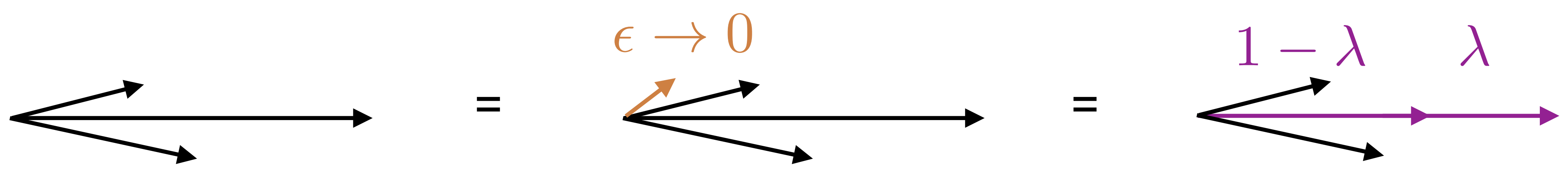
Energy flow is unchanged by exact soft/collinear emissions



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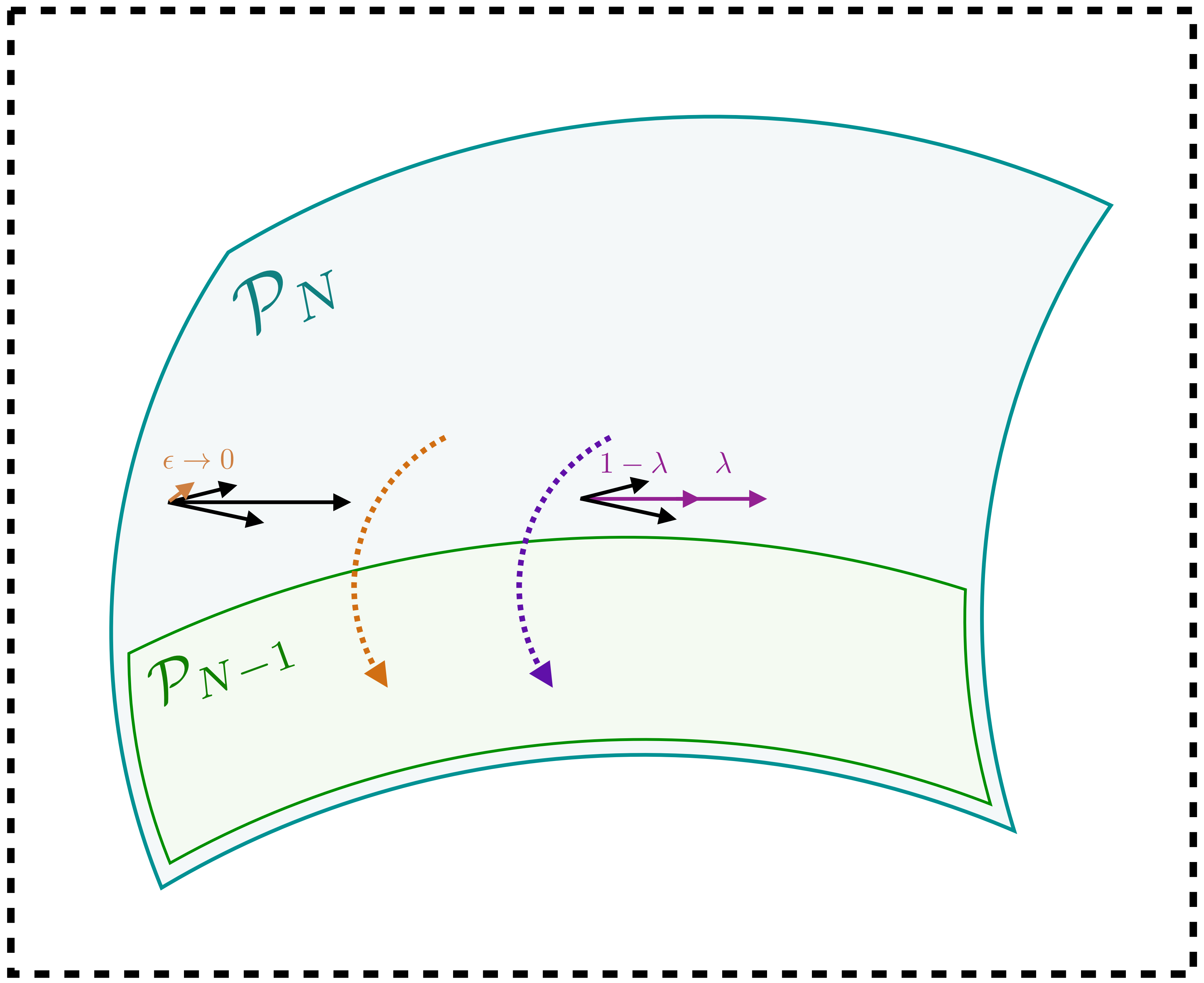

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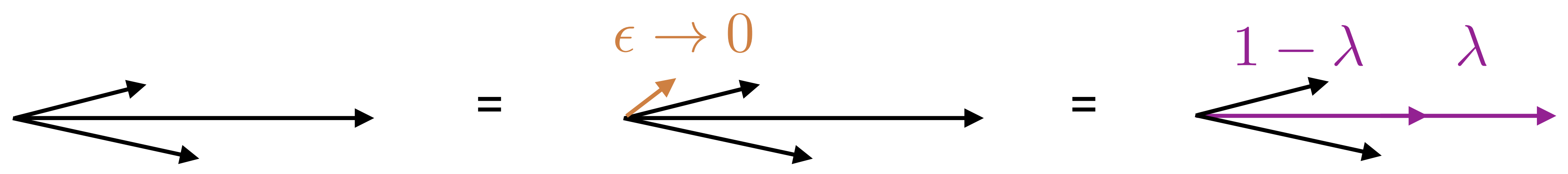

Functions of energy flow automatically satisfy exact IRC invariance!

Real and virtual divergences appear naturally together

$$\mathcal{P}_N \supset \mathcal{P}_{N-1} \supset \cdots \supset \mathcal{P}_3 \supset \mathcal{P}_2 \supset \mathcal{P}_1$$

by soft and collinear limits

Defining IRC Safety Precisely

[Sterman, Weinberg, PRL 1997; Sterman, PRD 1978; Banfi, Salam, Zanderighi, JHEP 2005]

Infrared and collinear safety is a proxy for perturbative calculability of an observable

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Exact IRC invariance

$$\mathcal{O}(p_1^\mu, \ldots, p_M^\mu) = \mathcal{O}(0p_0^\mu, p_1^\mu, \ldots, p_M^\mu)$$
$$\mathcal{O}(p_1^\mu, \ldots, p_M^\mu) = \mathcal{O}(\lambda p_1^\mu, (1-\lambda)p_1^\mu, \ldots, p_M^\mu)$$

Guarantees observable is well-defined on energy flows

Allows for pathological observables, e.g. pseudo-multiplicity

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Eliminates common observables with hard boundaries

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All Observables	Comments
Multiplicity ($\sum_i 1$)	IR unsafe and C unsafe
Momentum Dispersion [65] ($\sum_i E_i^2$)	IR safe but C unsafe
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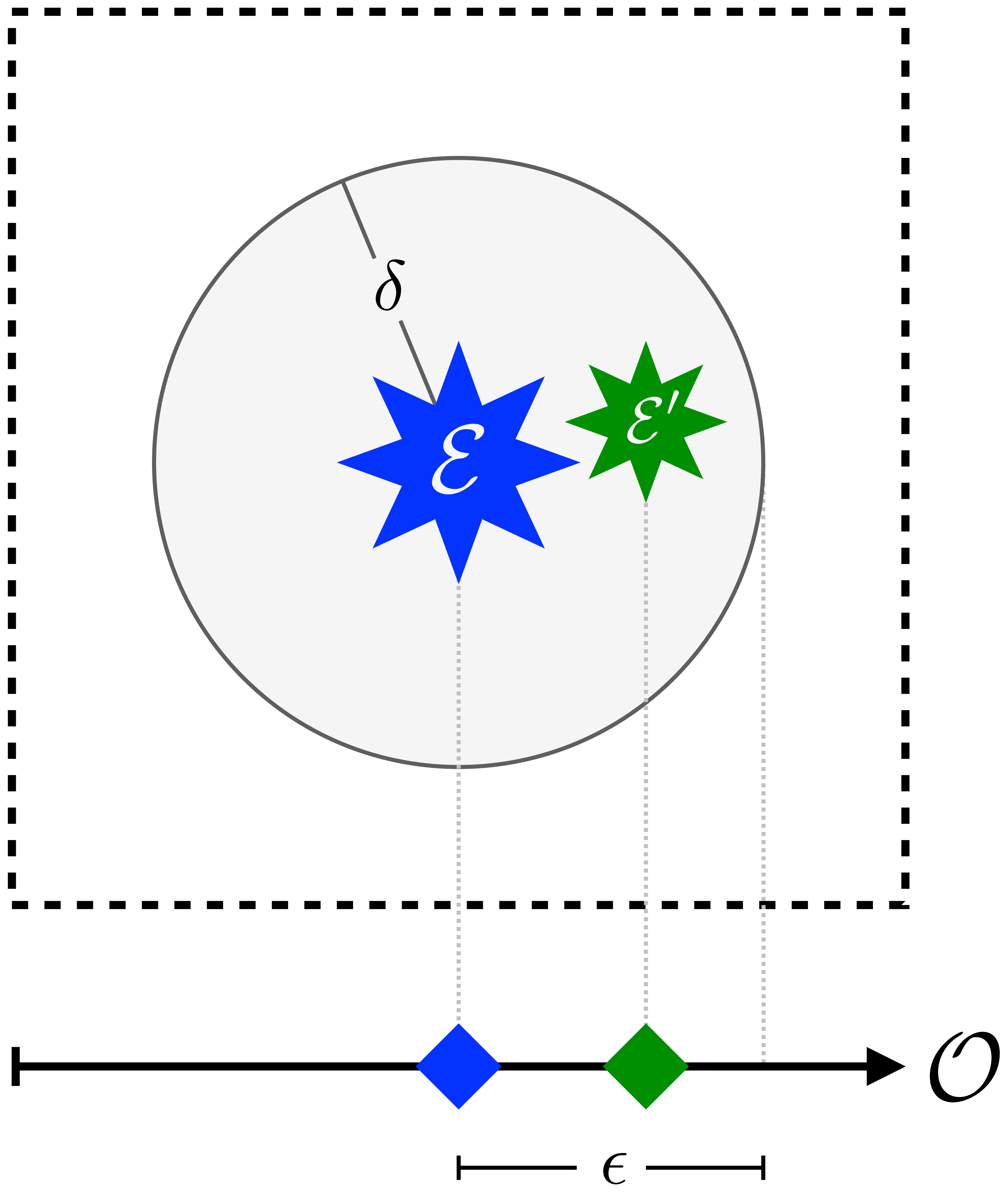
More EMD Geometry – Continuity in the Space of Events

[PTK, Metodiev, Thaler, 2004.04159]



Classic $\epsilon - \delta$ definition of continuity in a metric space

An observable $\mathcal{O}$ is EMD continuous at an event $\mathcal{E}$ if, for any $\epsilon > 0$, there exists a $\delta > 0$ such that for all events $\mathcal{E}'$:

$$\text{EMD}(\mathcal{E}, \mathcal{E}') < \delta \quad \Longrightarrow \quad |\mathcal{O}(\mathcal{E}) - \mathcal{O}(\mathcal{E}')| < \epsilon.$$

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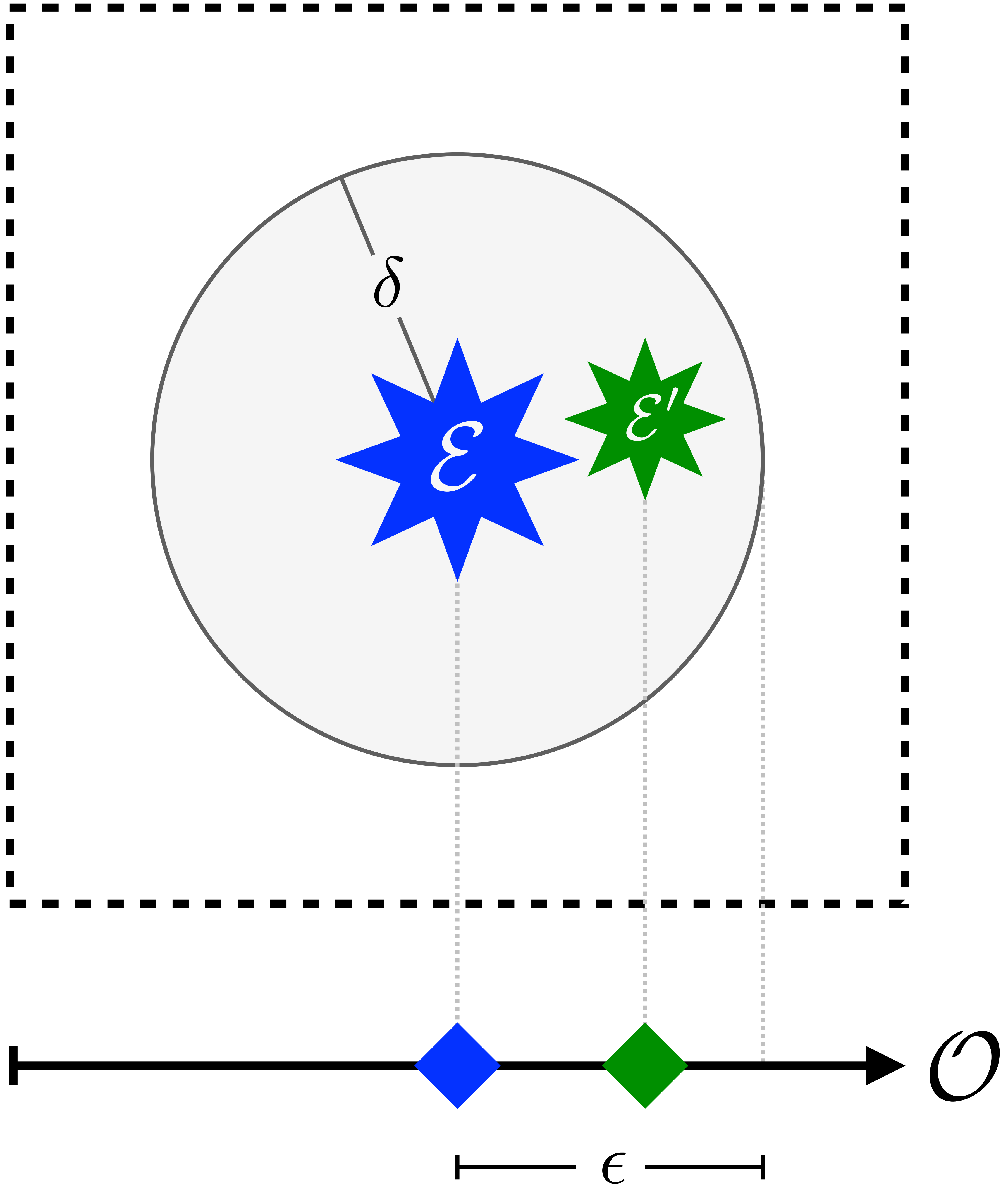

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Towards a geometric definition of IRC Safety

IRC Safety = EMD Continuity*

*on all but a negligible set‡ of events

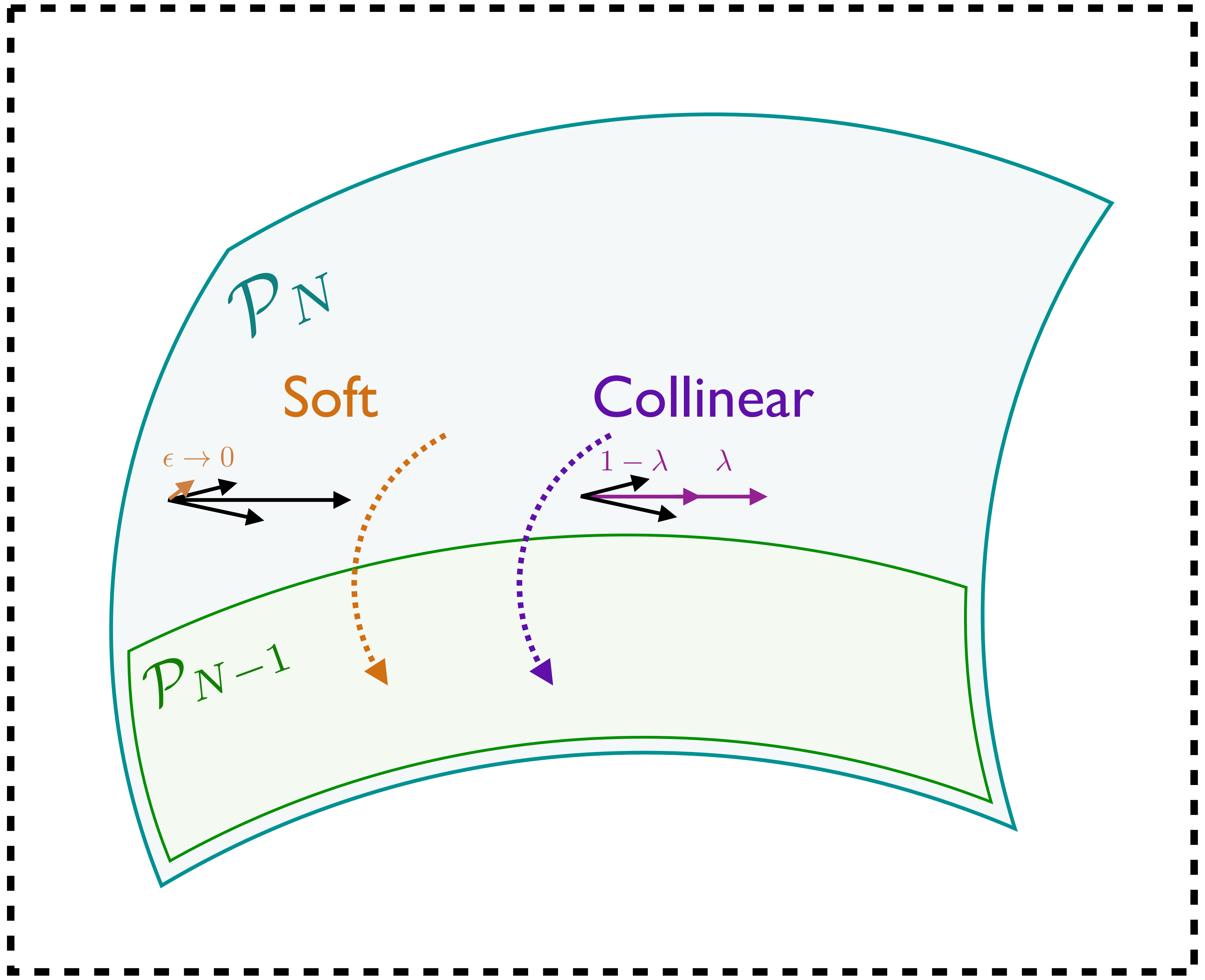
‡a negligible set is one that contains no positive-radius EMD-ball

⋮

Perturbation Theory in the Space of Events

[PTK, Metodiev, Thaler, 2004.04159]

Infrared singularities of massless gauge theories appear on each $\mathcal{P}_N$



Perturbation Theory in the Space of Events

[PTK, Metodiev, Thaler, 2004.04159]

Sudakov safety

[Larkoski, Thaler, JHEP 2014; Larkoski, Marzani, Thaler, PRD 2015]

Some observables have discontinuities on P_N for some N

A resummed IRC-safe companion can mitigate the divergences

$$p(\mathcal{O}_{\text{Sudakov}}) = \int d\mathcal{O}_{\text{Comp.}}\, p(\mathcal{O}_{\text{Sudakov}}|\mathcal{O}_{\text{Comp.}})\, p(O_{\text{Comp.}})$$

Event geometry suggests N-(sub)jettiness as universal companion

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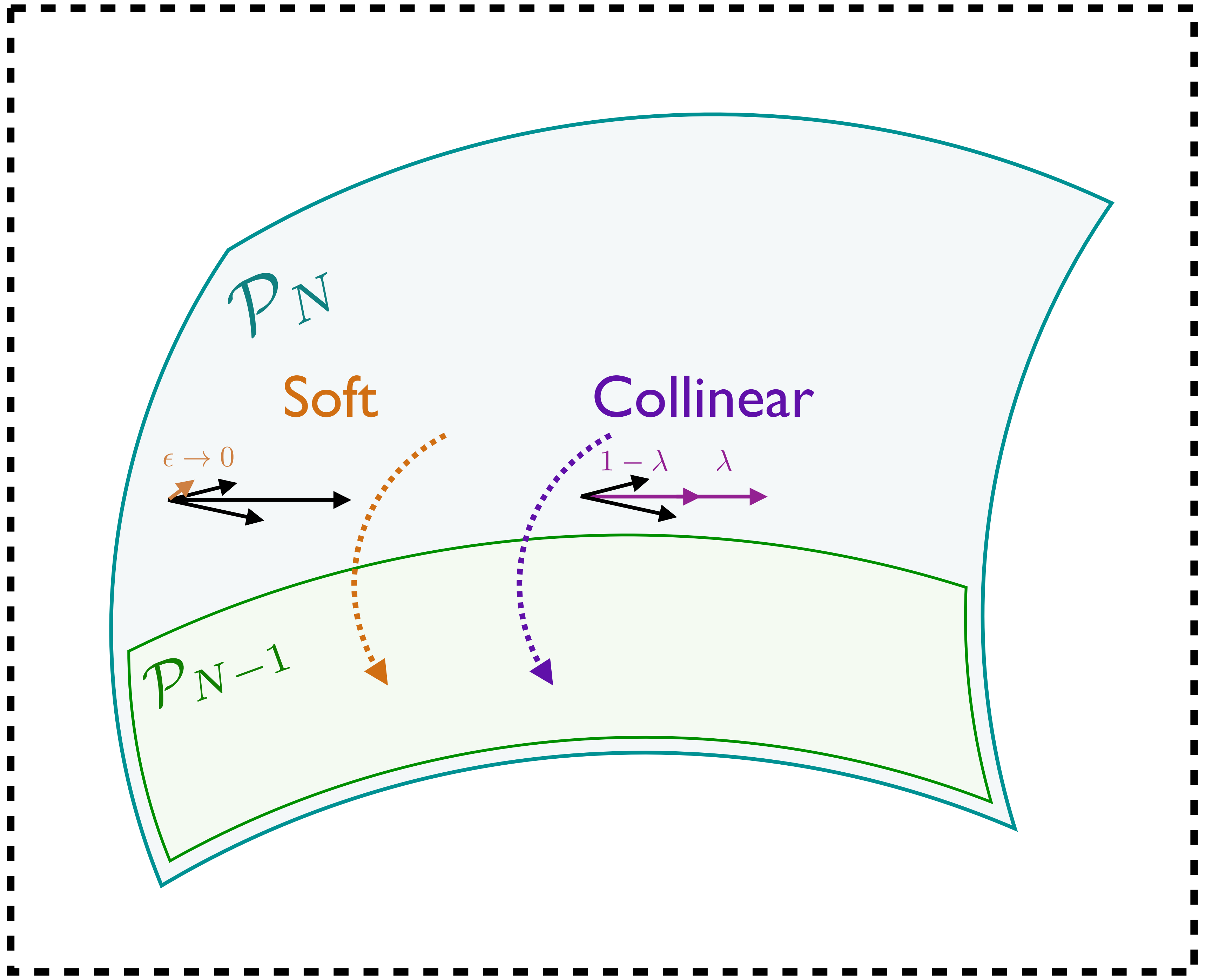

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Fixed-order calculability

[Sterman, PRD 1979; Banfi, Salam, Zanderighi, JHEP 2005]

Is a statement of integrability on each P_N

EMD continuity must be upgraded to EMD-Hölder continuity on each P_N

$$\lim_{\mathcal{E}\to\mathcal{E}'} \frac{\mathcal{O}(\mathcal{E}) - \mathcal{O}(\mathcal{E}')}{\text{EMD}(\mathcal{E},\mathcal{E}')^c} = 0, \quad c > 0$$

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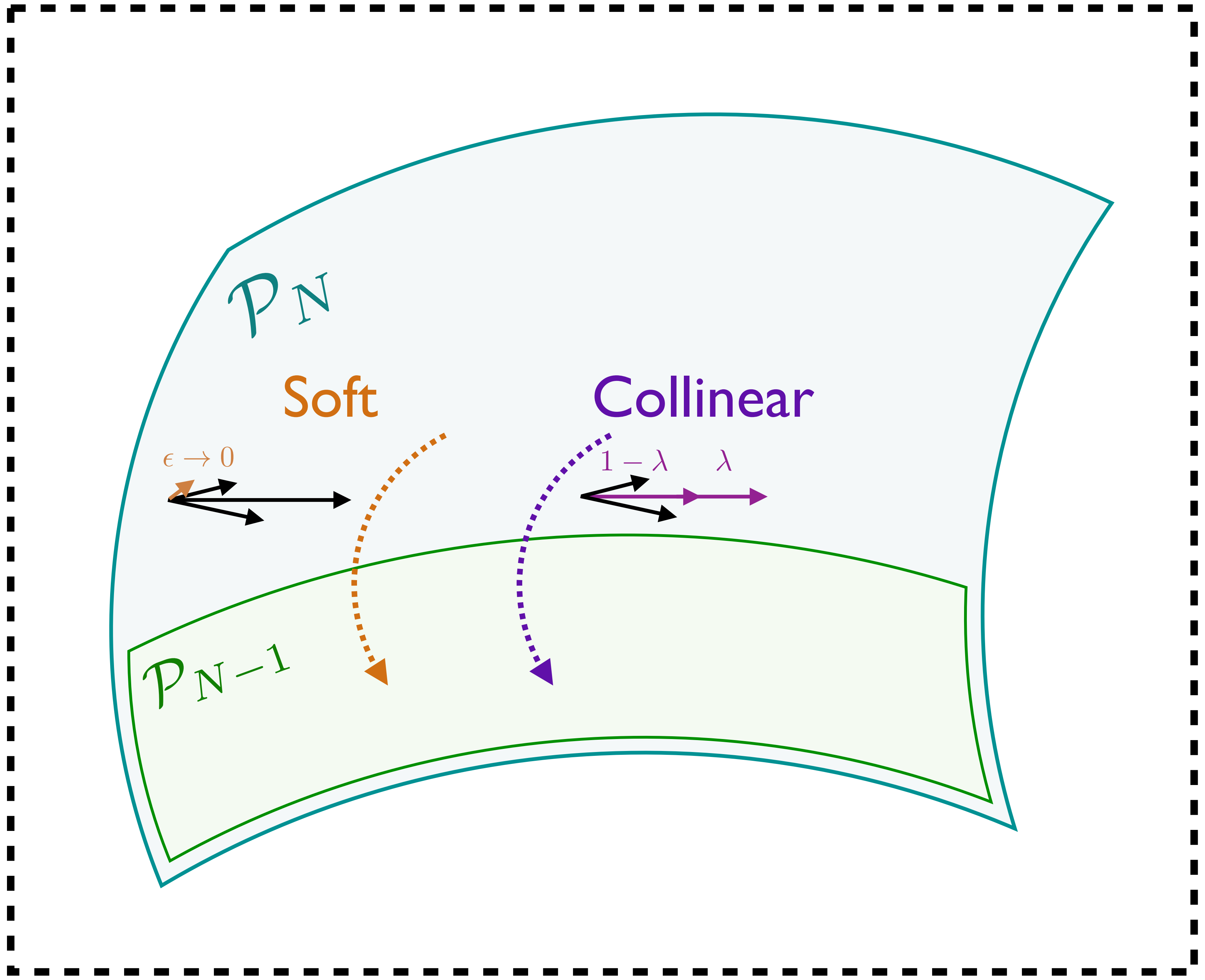
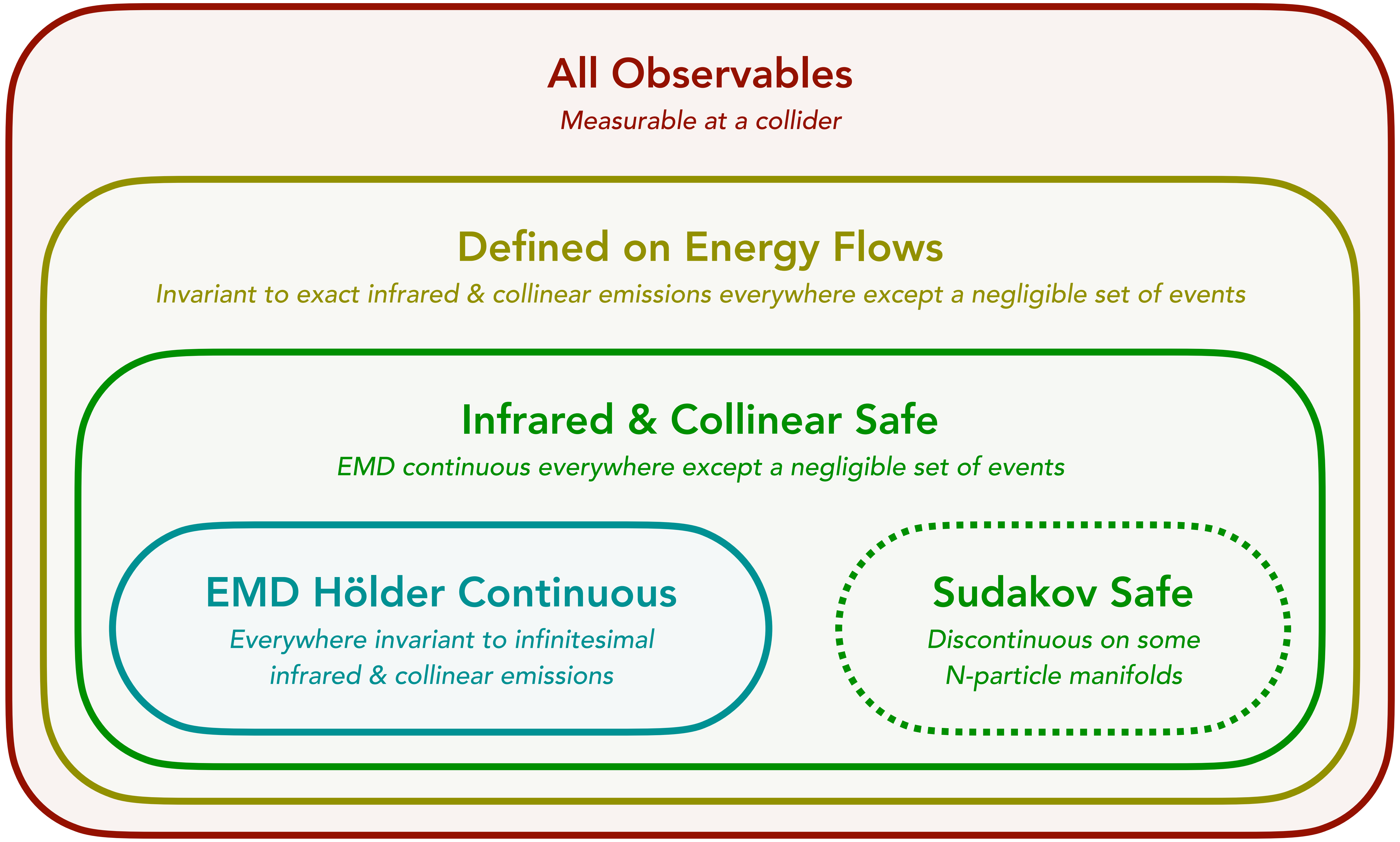


Example: $V(\mathcal{E}) = \mathcal{T}_2(\mathcal{E})\left(1 + \frac{1}{\ln E(\mathcal{E})/\mathcal{T}_3(\mathcal{E})}\right)$ is EMD continuous but not EMD Hölder continuous (it is Sudakov safe)

Hierarchy of IRC Safety Definitions

[PTK, Metodiev, Thaler, 2004.04159]



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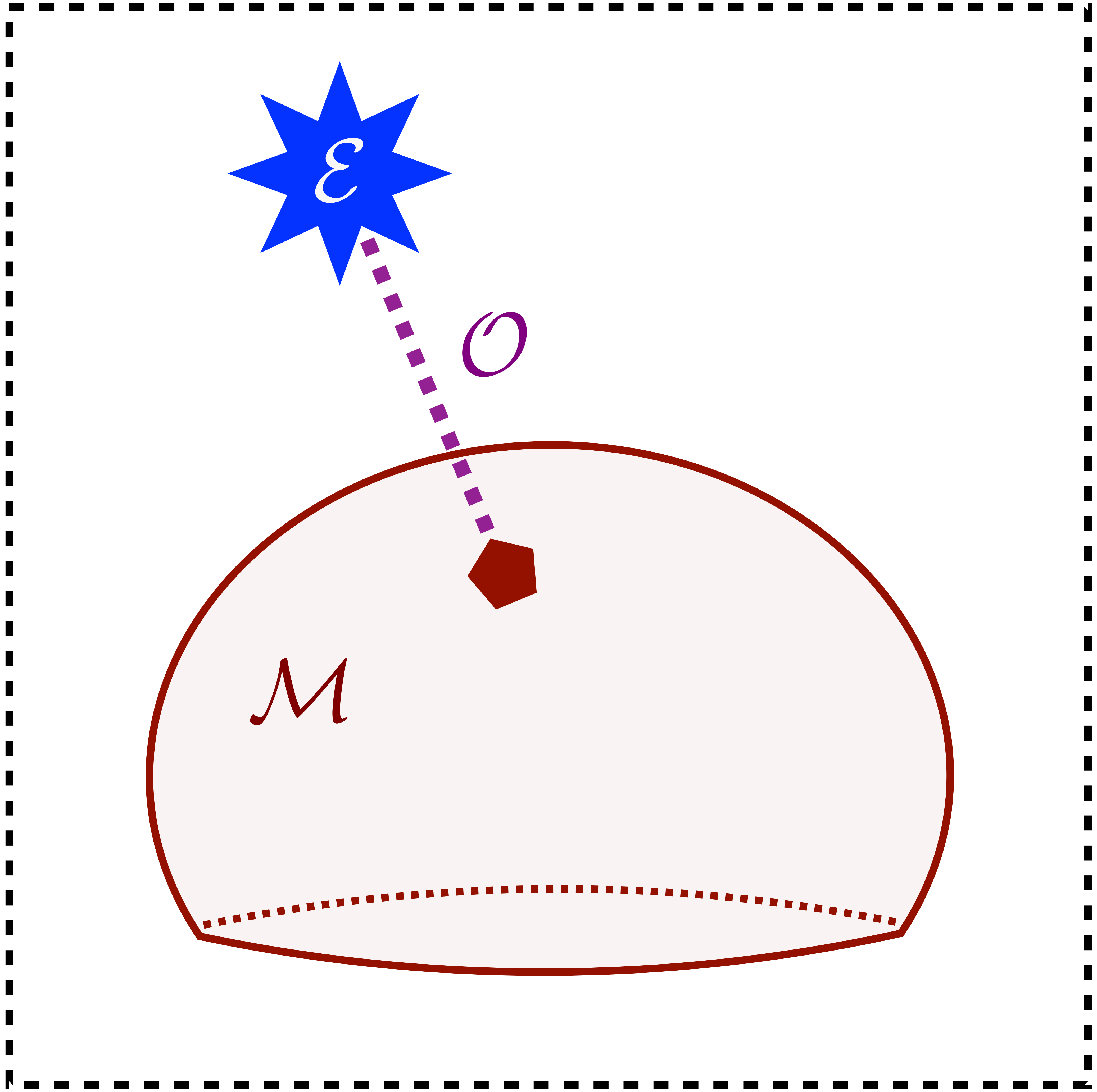
Sudakov Safe	
Groomed Momentum Fraction [39] (z_g)	Disc. on 1-particle manifold
Jet Angularity Ratios [37]	Disc. on 1-particle manifold
N-subjettiness Ratios [47, 48] (τ_{N+1}/τ_N)	Disc. on N-particle manifold
V parameter [36] (Eq. (2.11))	Hölder disc. on 3-particle manifold
EMD Hölder Continuous Everywhere	
Thrust [40, 41]	
Spherocity [42]	
Angularities [70]	
N-jettiness [44] ($\mathcal{T}_N$)	
C parameter [71–74]	Resummation beneficial at $C = \frac{3}{4}$
Linear Sphericity [72] ($\sum_i E_i n_i^\mu n_i^\nu$)	
Energy Correlators [36, 75–77]	
Energy Flow Polynomials [15, 17]	

Defining Observables via Event Space Geometry

[PTK, Metodiev, Thaler, 2004.04159]



Many common observables are distance of closest approach from event to a specific manifold

$$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \text{EMD}_{\beta,R}(\mathcal{E}, \mathcal{E}')$$

Defining Observables via Event Space Geometry

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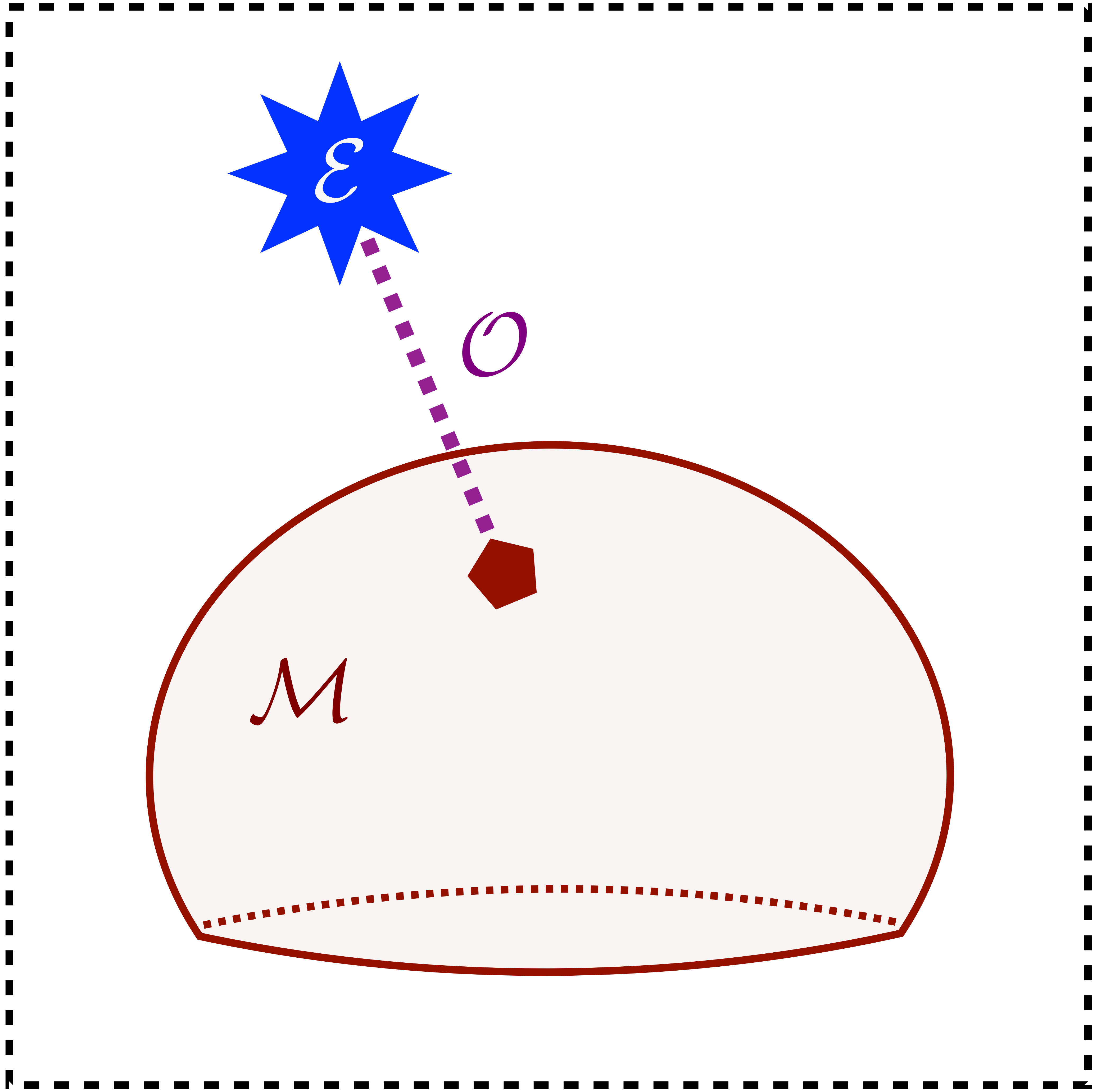

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EMD variant for equal-energy events

$$\text{EMD}_\beta(\mathcal{E}, \mathcal{E}') = \lim_{R\to\infty} R^\beta \text{EMD}_{\beta,R}(\mathcal{E}, \mathcal{E}') = \min_{\{f_{ij}\geq 0\}} \sum_{i=1}^{M} \sum_{j=1}^{M'} f_{ij}\theta_{ij}^\beta$$

Enforces equal energy (else infinity)

on equal-energy events

Defining Observables via Event Space Geometry

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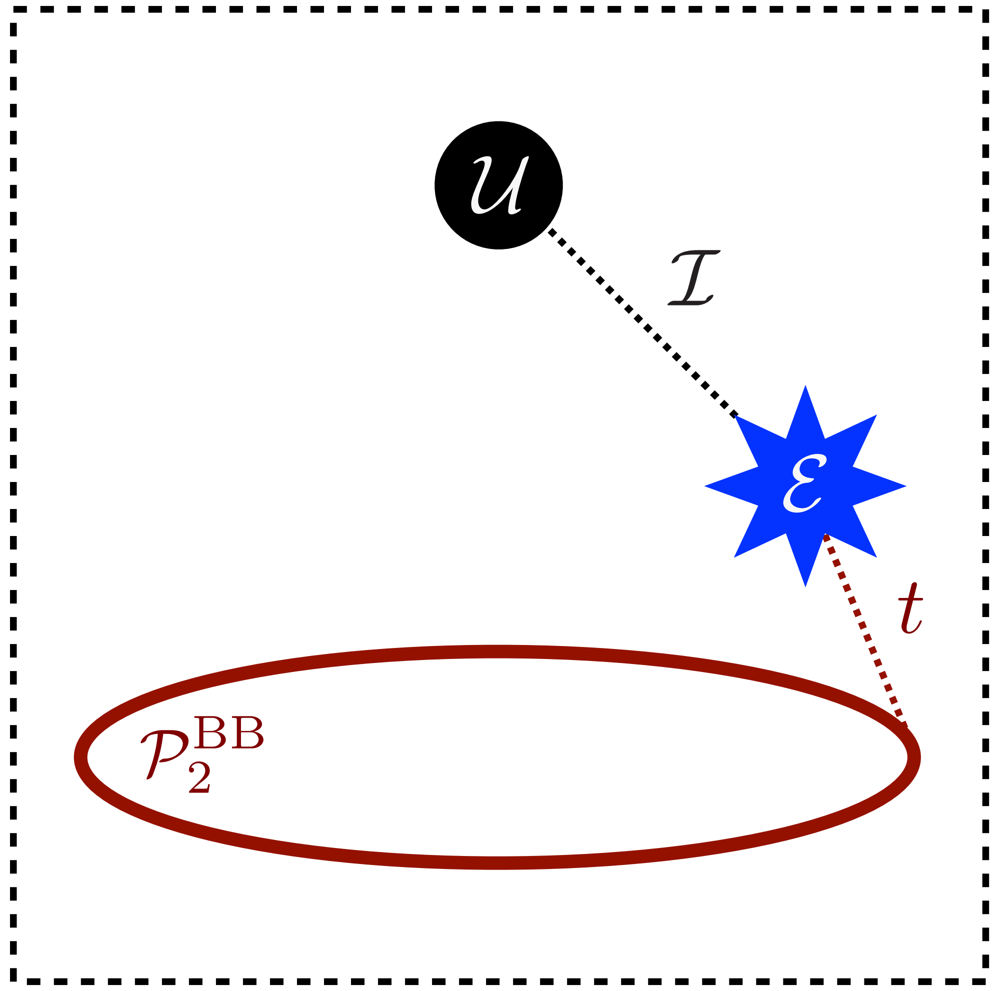
Thrust, spherocity, isotropy*

Distance of closest approach to a specific manifold



$$t(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{P}_2^{\text{BB}}} \text{EMD}_2(\mathcal{E}, \mathcal{E}')$$

$$\sqrt{s(\mathcal{E})} = \min_{\mathcal{E}' \in \mathcal{P}_2^{\text{BB}}} \text{EMD}_1(\mathcal{E}, \mathcal{E}')$$

$$\mathcal{I}^{(\beta)}(\mathcal{E}) = \min_{\mathcal{E}' \in M_{\mathcal{U}}} \text{EMD}_\beta(\mathcal{E}, \mathcal{E}')$$

[Farhi, PRL 1977; Georgi, Machacek, PRL 1977]
*New! [Cesarotti, Thaler, 2004.06125]

Defining Observables via Event Space Geometry

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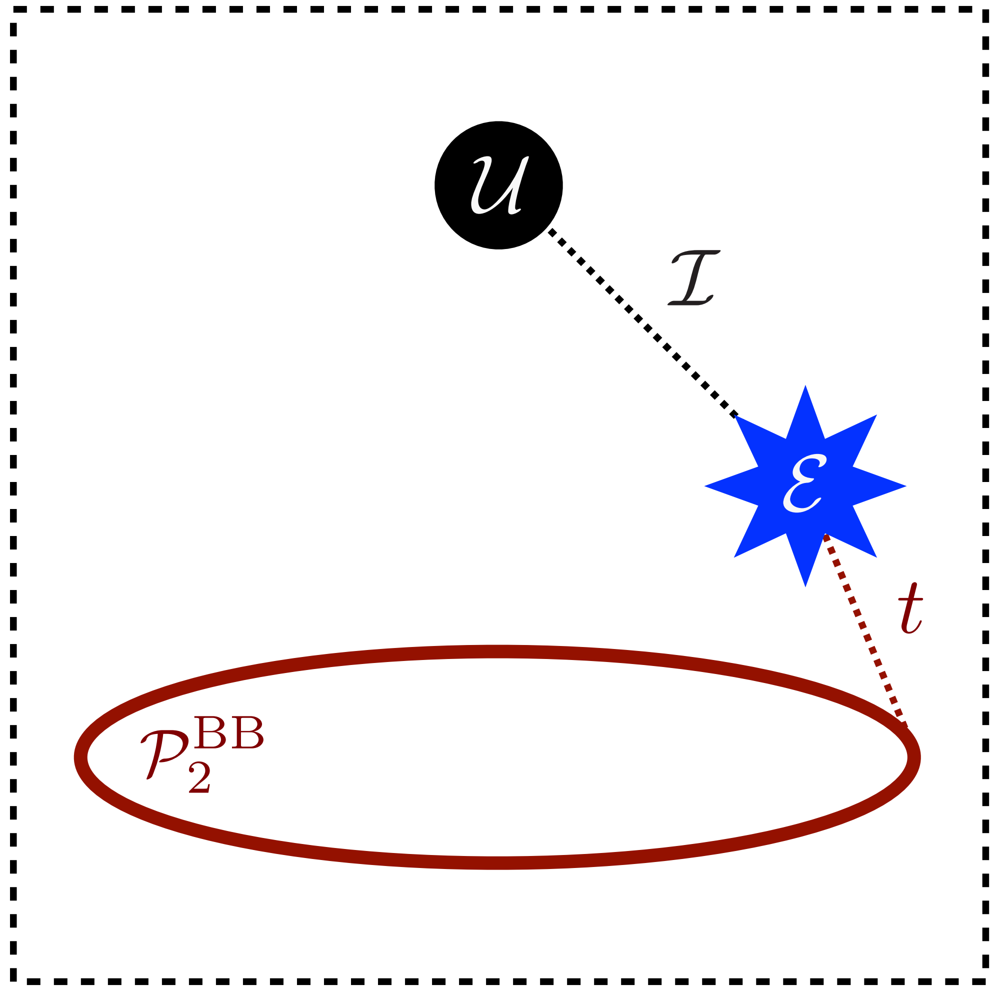

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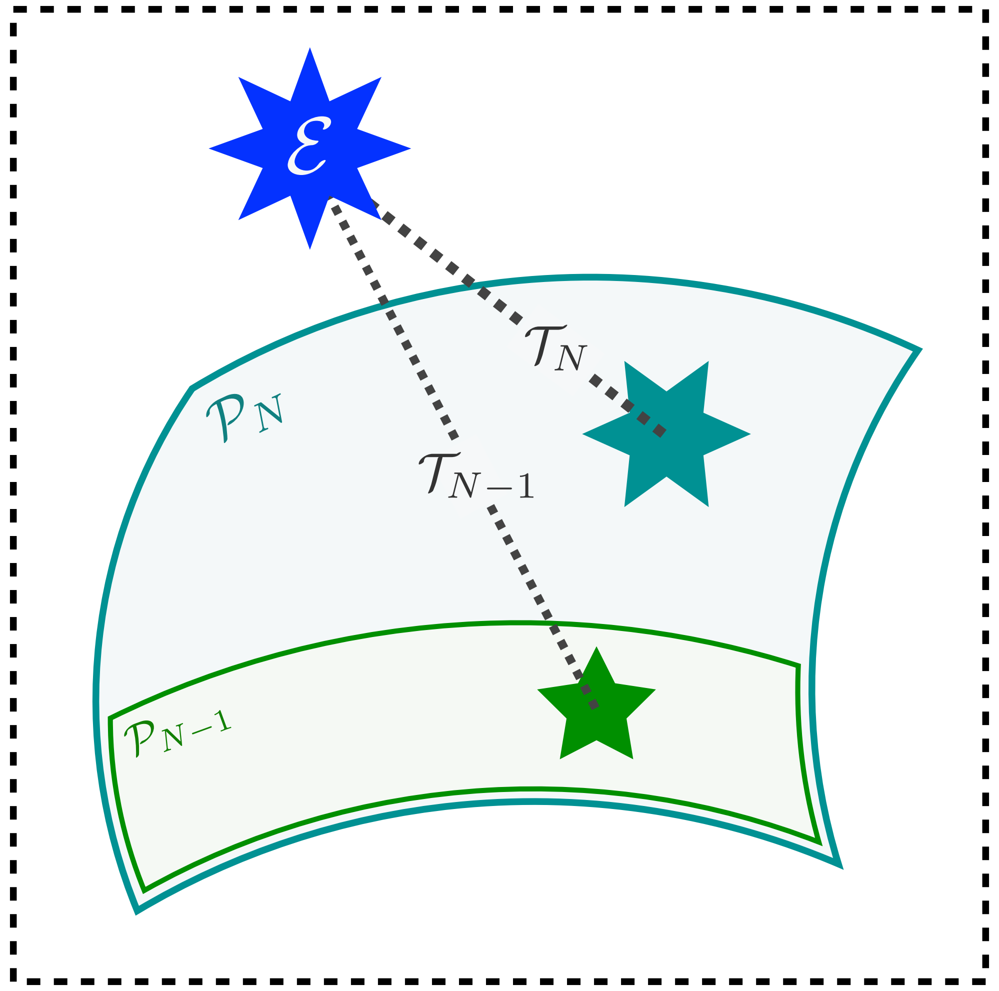
[Farhi, PRL 1977; Georgi, Machacek, PRL 1977]
*New! [Cesarotti, Thaler, 2004.06125]

N-jettiness

Minimum distance from event to N-particle manifold



without beam region

$$\mathcal{T}_N^{(\beta)}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{P}_N} \text{EMD}_\beta(\mathcal{E},\mathcal{E}')$$

with constant beam distance R^β

$$\mathcal{T}_N^{(\beta,R)}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{P}_N} \text{EMD}_{\beta,R}(\mathcal{E},\mathcal{E}')$$

[Brandt, Dahmen, Z. Phys 1979;
Stewart, Tackmann, Waalewijn, PRL 2010]

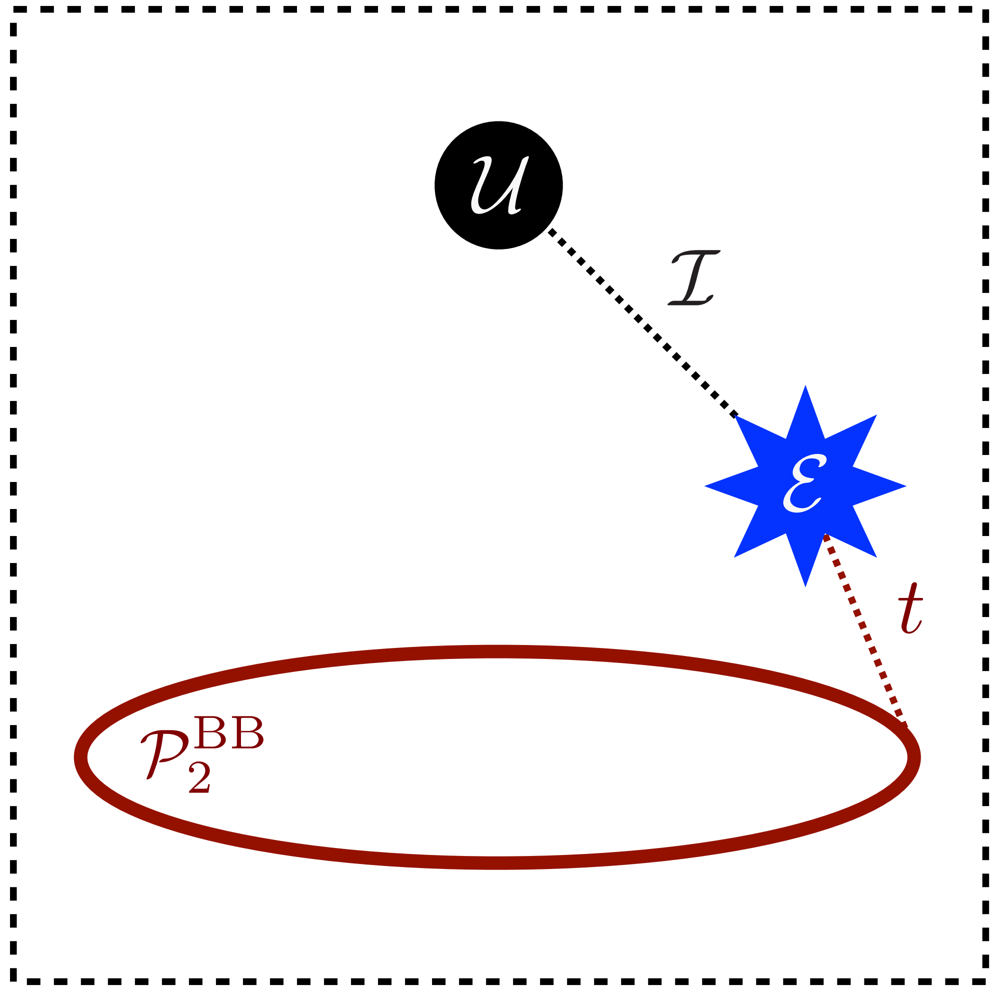
Defining Observables via Event Space Geometry

$$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \mathrm{EMD}_{\beta,R}(\mathcal{E},\mathcal{E}')$$

[PTK, Metodiev, Thaler, 2004.04159]

Thrust, spherocity, isotropy*

Distance of closest approach to a specific manifold



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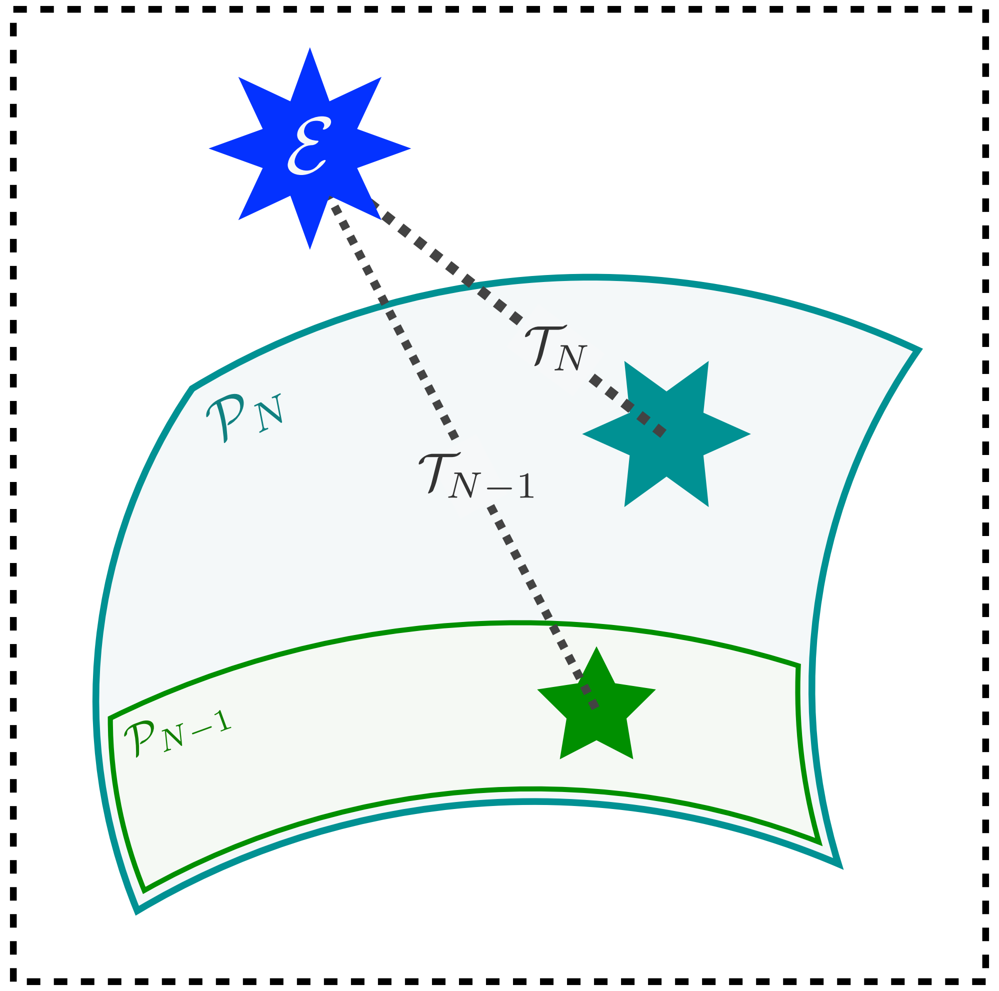

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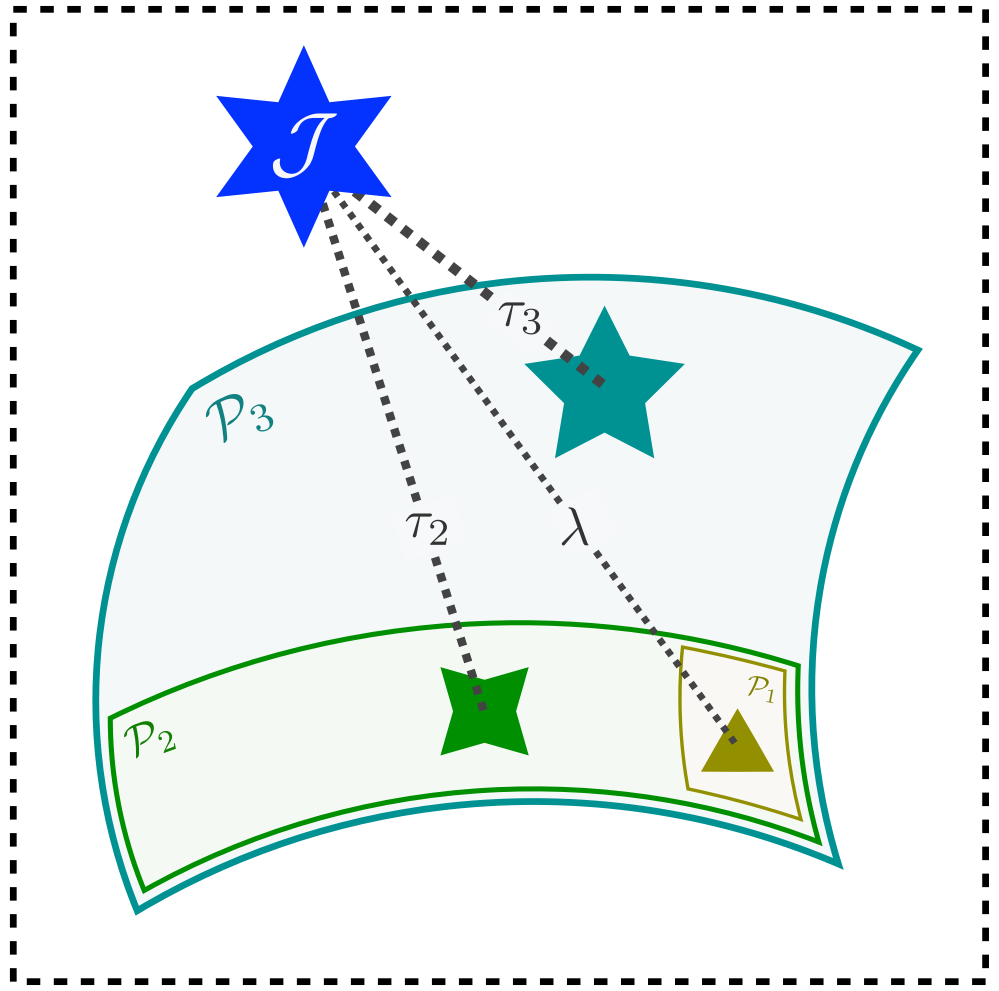
N-subjettiness, angularities

Smallest distance from jet to N-particle manifold



for recoil-free angularity

$$\lambda_\beta(\mathcal{J}) = \min_{\mathcal{J}' \in \mathcal{P}_1} \mathrm{EMD}_\beta(\mathcal{J},\mathcal{J}')$$

$$\tau_N^{(\beta)}(\mathcal{J}) = \min_{\mathcal{J}' \in \mathcal{P}_N} \mathrm{EMD}_\beta(\mathcal{J},\mathcal{J}')$$

[Ellis, Vermilion, Walsh, Hornig, Lee, JHEP 2010;
Thaler, Van Tilburg, JHEP 2011, JHEP 2012]

Jets in the Space of Events – The Closest *N*-particle Description of an *M*-particle Event

[PTK, Metodiev, Thaler, 2004.04159]

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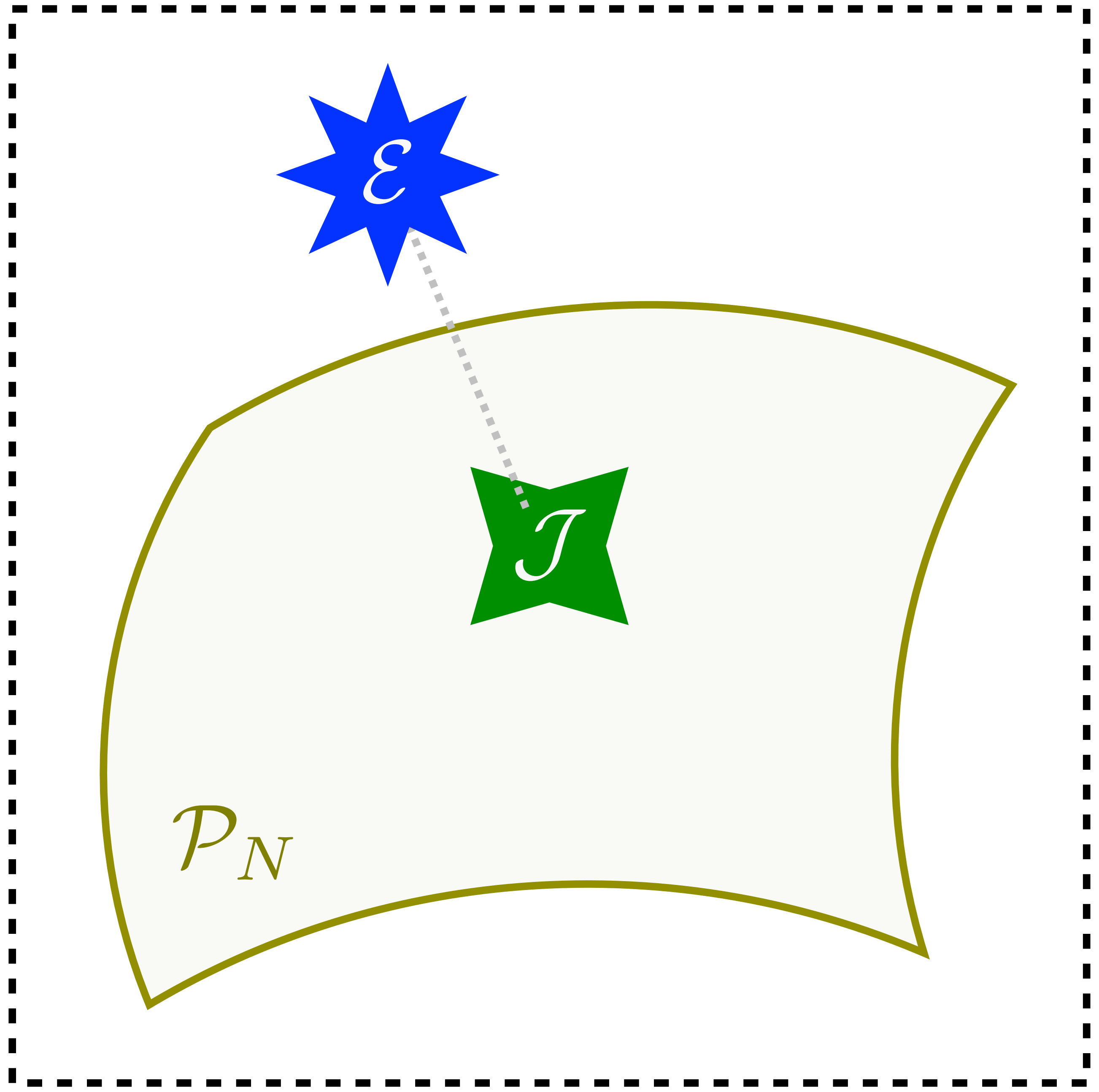
Exclusive cone finding

XCone finds N jets by minimizing N-jettiness



$$\mathcal{J}^{\text{XCone}}_{N,\beta,R}(\mathcal{E}) = \underset{\mathcal{J}\in\mathcal{P}_N}{\arg\min}\, \text{EMD}_{\beta,R}(\mathcal{E},\mathcal{J})$$

[Stewart, Tackmann, Thaler, Vermilion, Wilkason, JHEP 2015;
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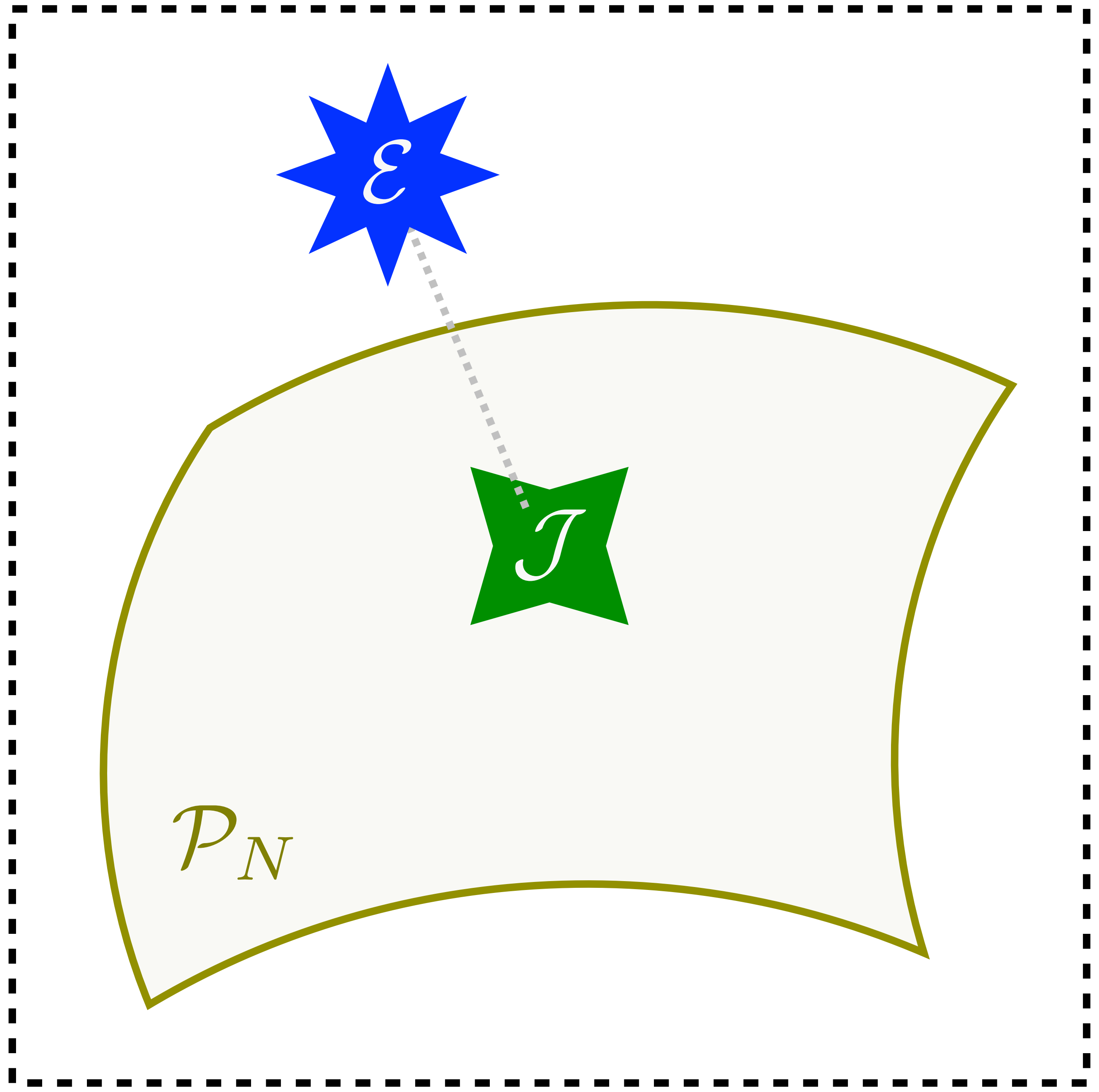

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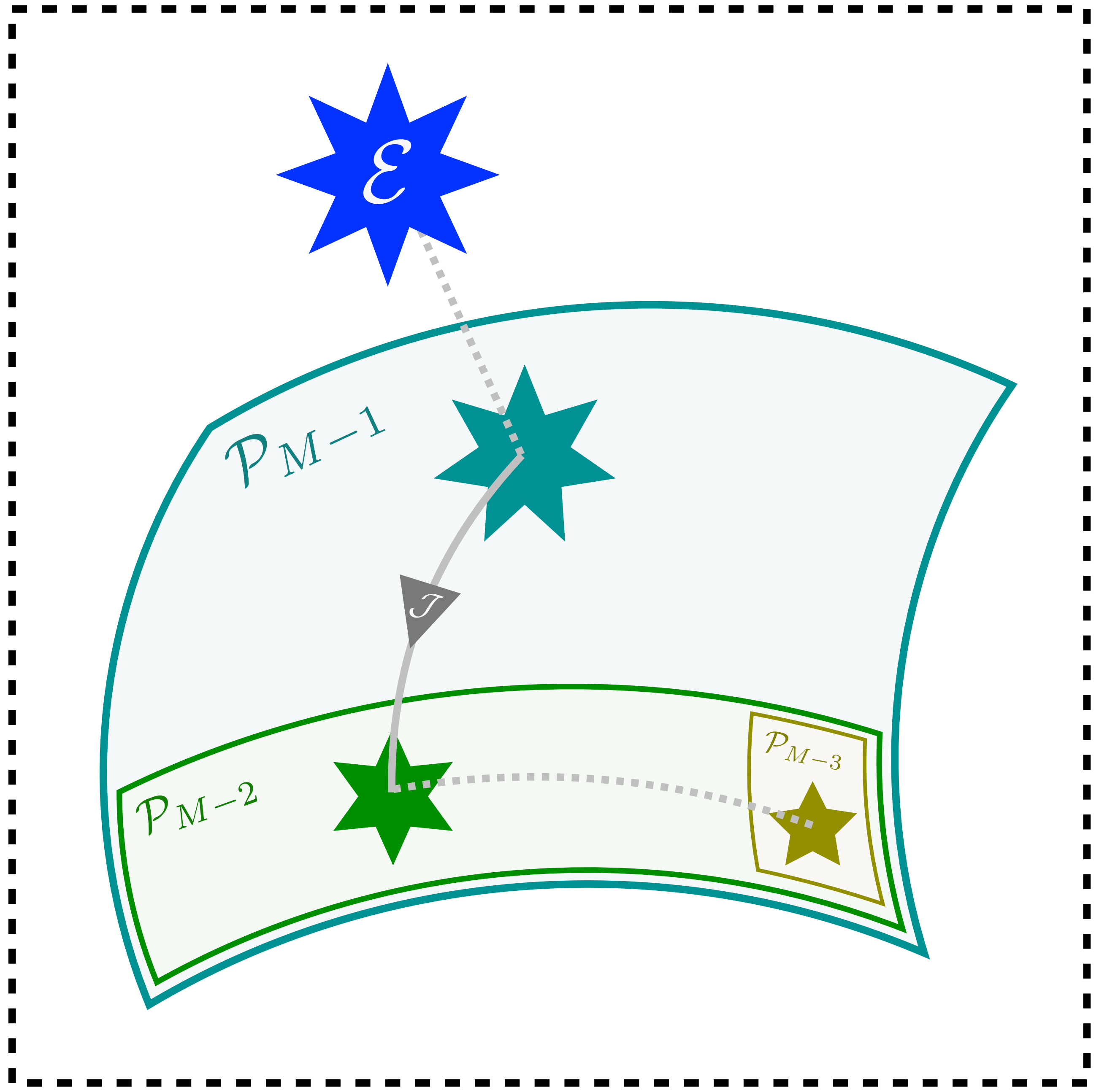
[Stewart, Tackmann, Thaler, Vermilion, Wilkason, JHEP 2015; Thaler, Wilkason, JHEP 2015]

Sequential recombination

Iteratively merges particles or identifies a jet



"destroying" energy corresponds to identifying a jet

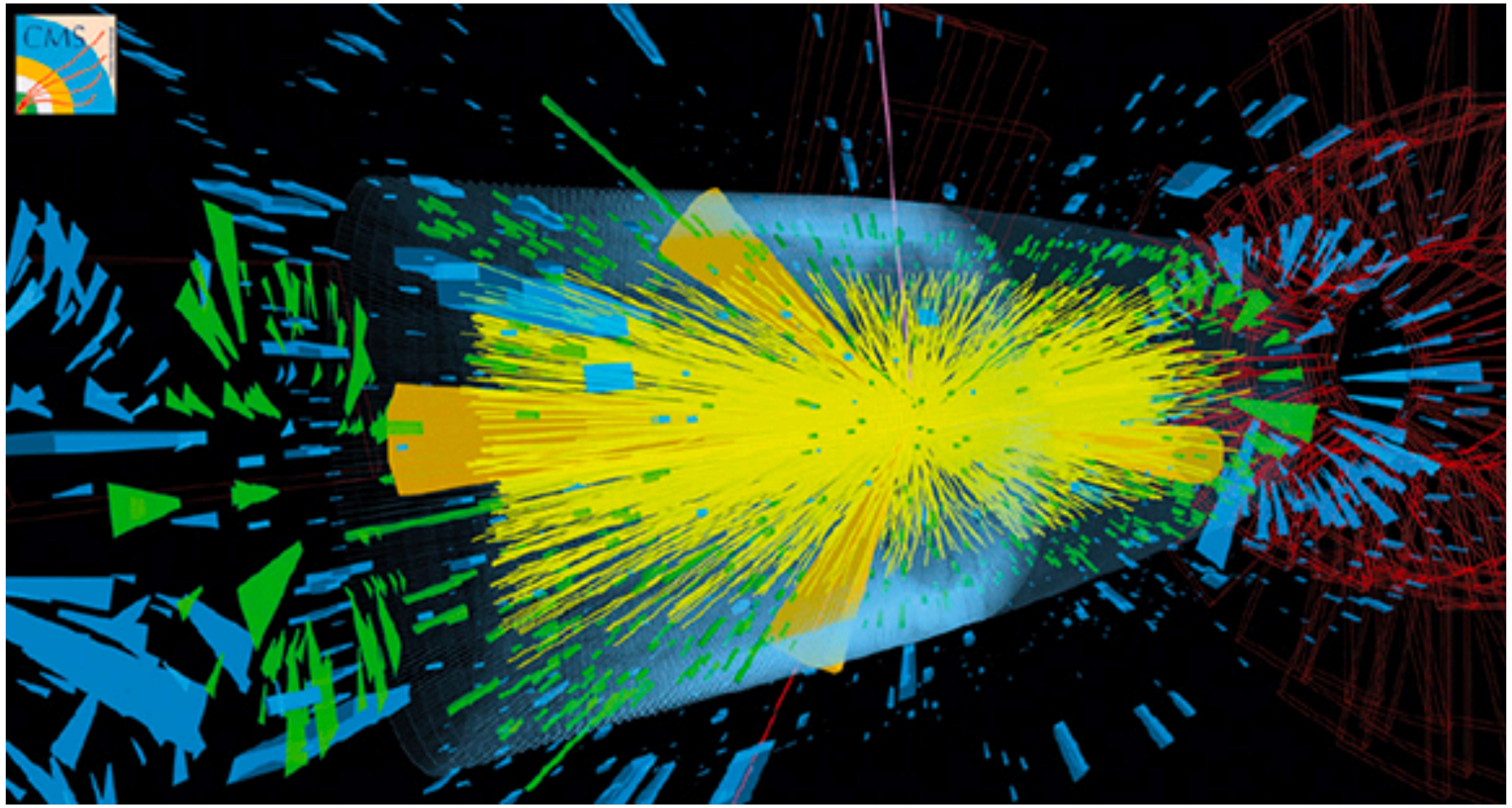
event with one fewer particle after one step

$$\mathcal{E}^{(\beta,R)}_{M-1}(\mathcal{E}_M) = \underset{\mathcal{E}'_{M-1}\in\mathcal{P}_{M-1}}{\arg\min}\, \text{EMD}_{\beta,R}(\mathcal{E}_M,\mathcal{E}'_{M-1})$$

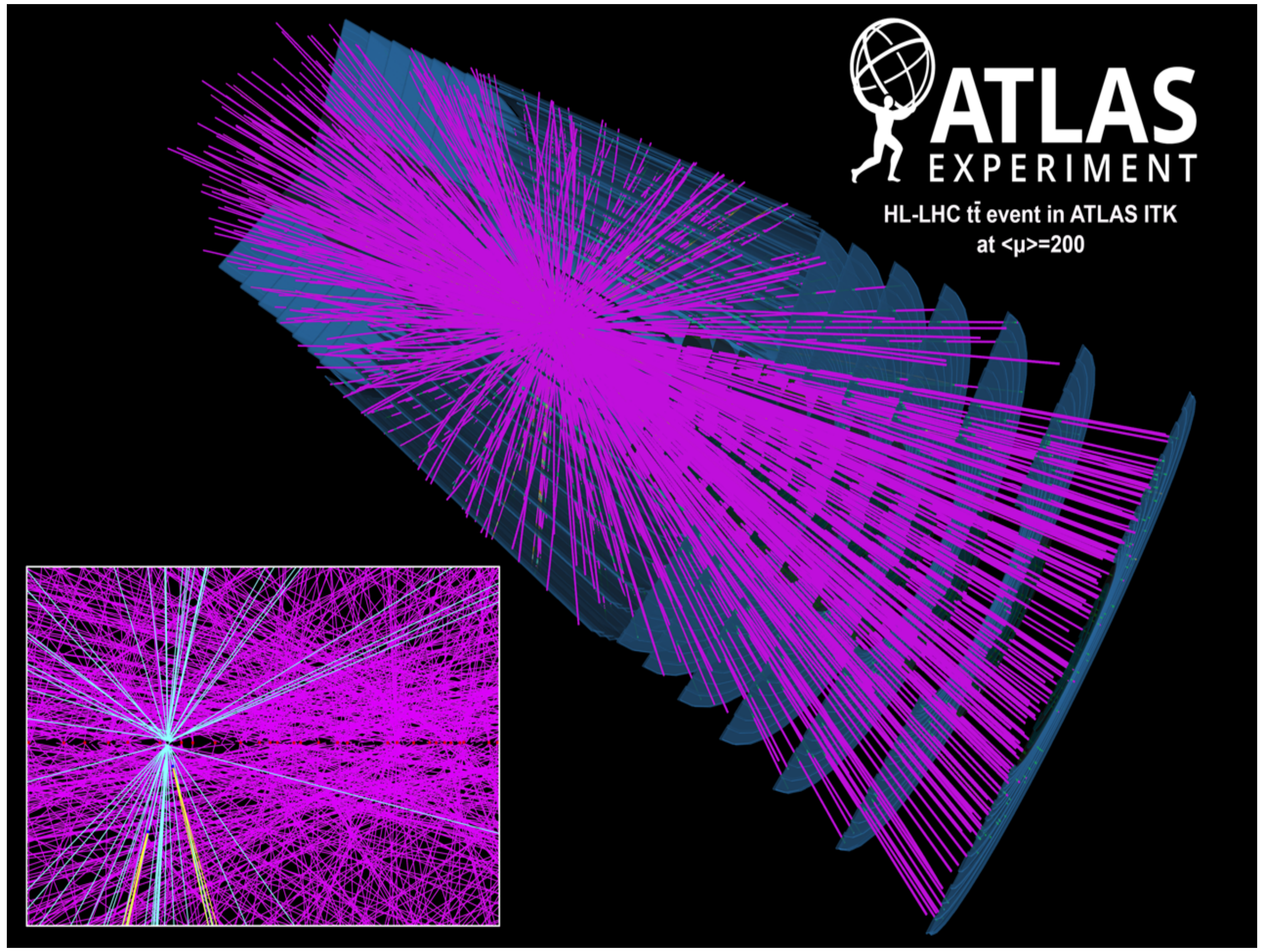
[Catani, Dokshitzer, Seymour, Webber, Nucl. Phys. B 1993; Ellis, Soper, PRD 1993; Dokshitzer, Leder, Moretti, Webber, JHEP 1997; Cacciari, Salam, Soyez, JHEP 2008]

Pileup at the (HL-)LHC

Pileup is uniform (on average) radiation from additional proton-proton collisions



VBF Higgs + 200 pileup vertices

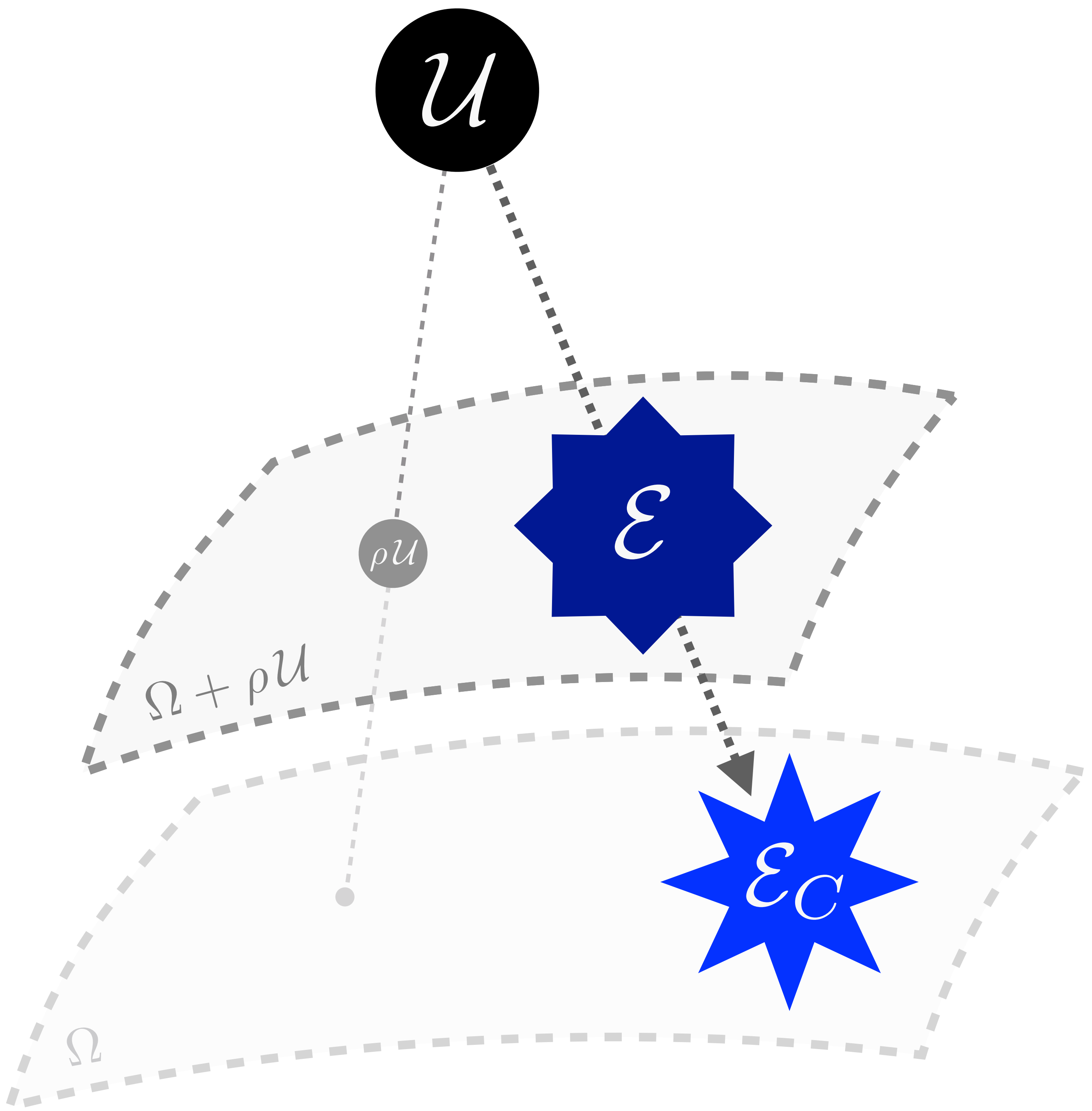


$t\bar{t}$ + 200 pileup vertices

Pileup Mitigation in Event Space

Pileup: uniform (on average) radiation from additional proton-proton collisions

Pileup mitigation: "moving away" from the uniform event



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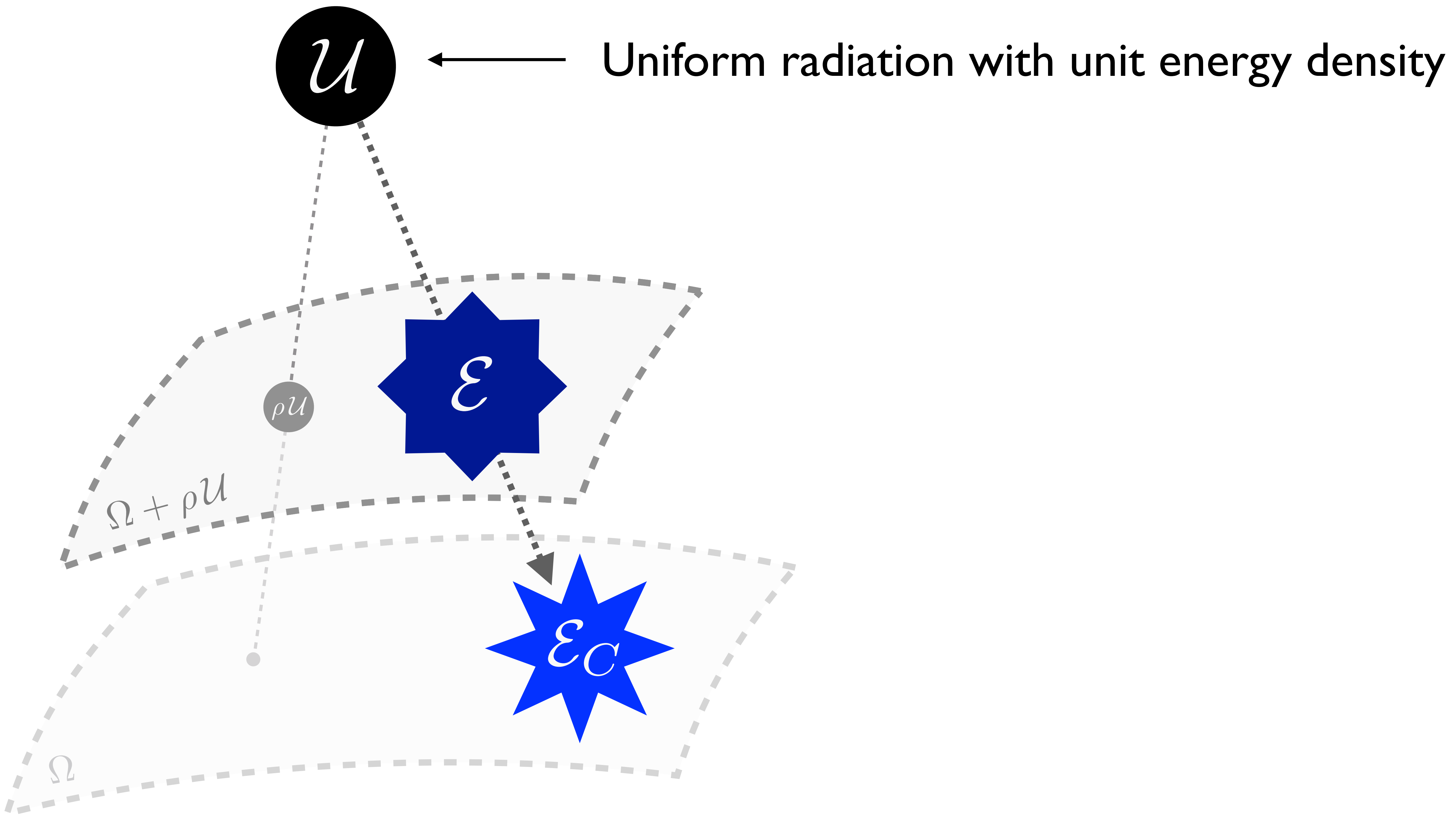

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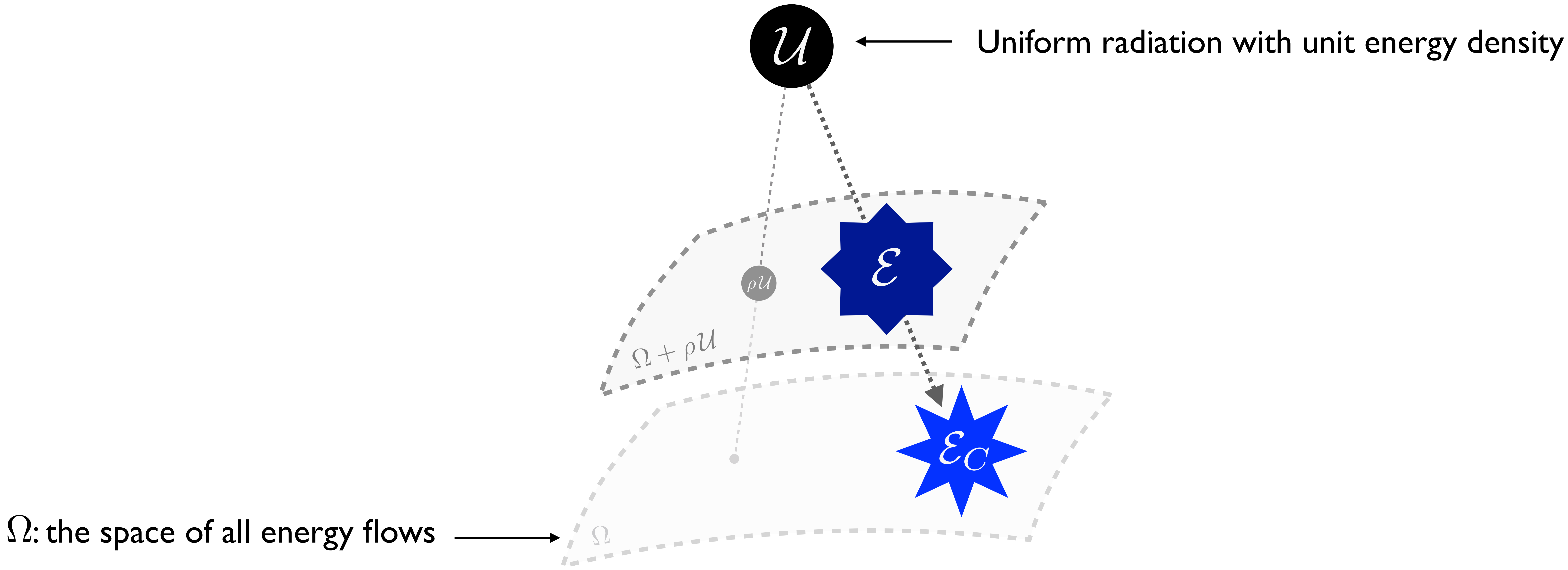

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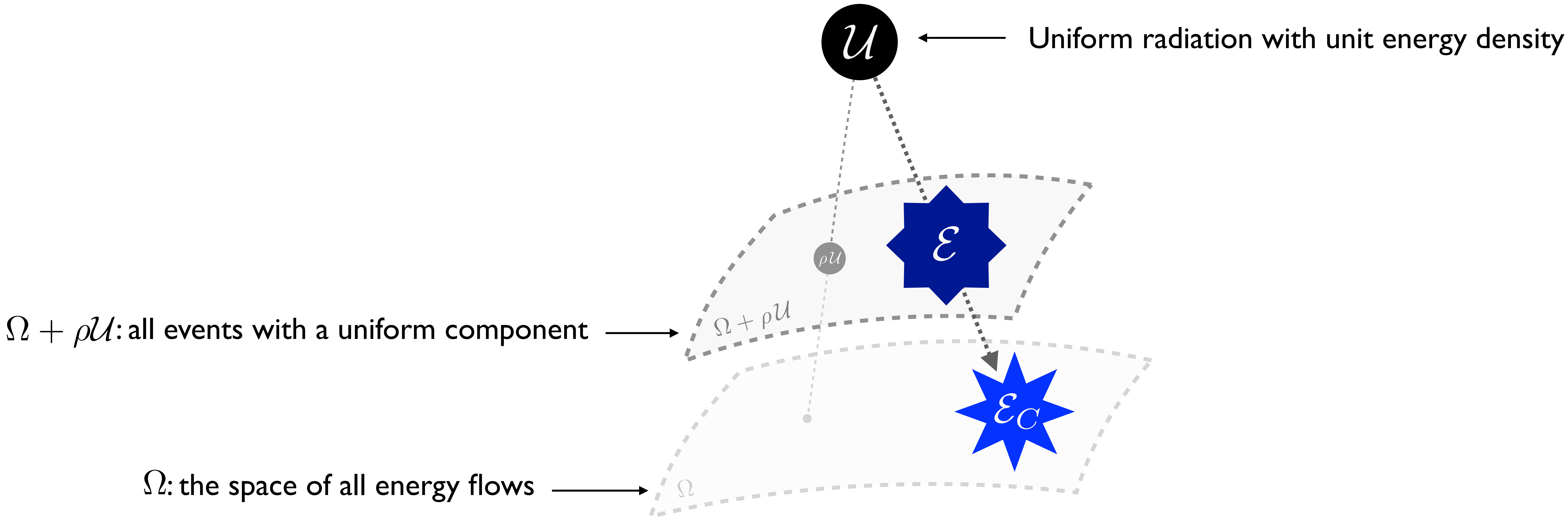

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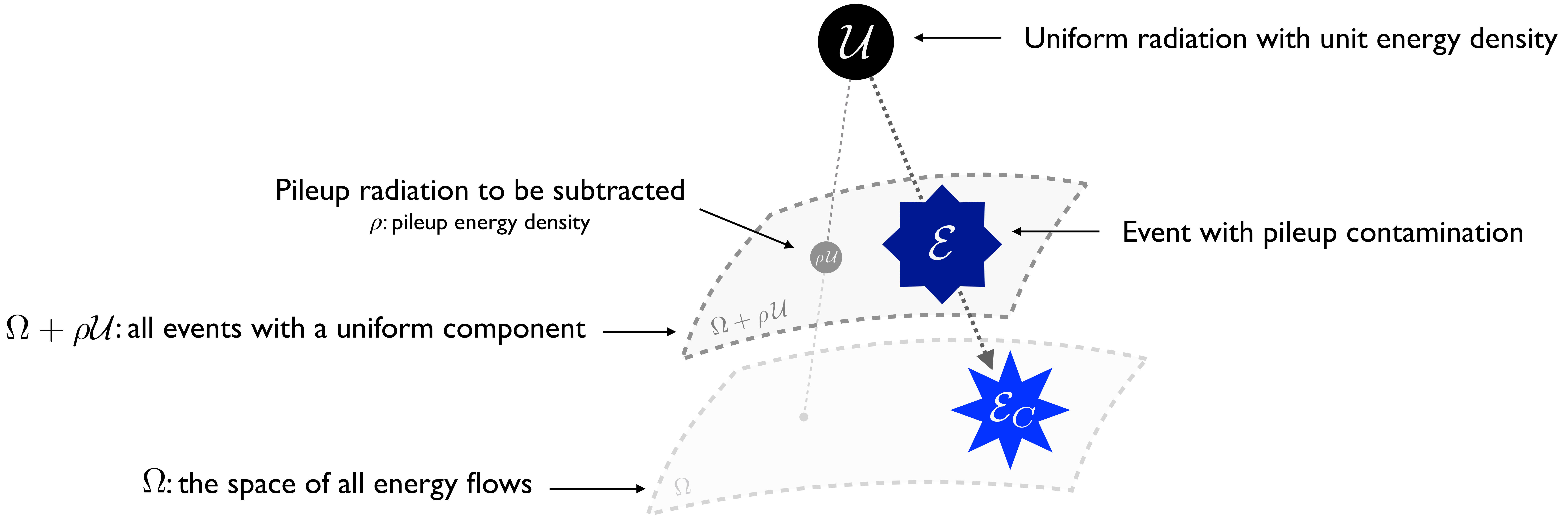

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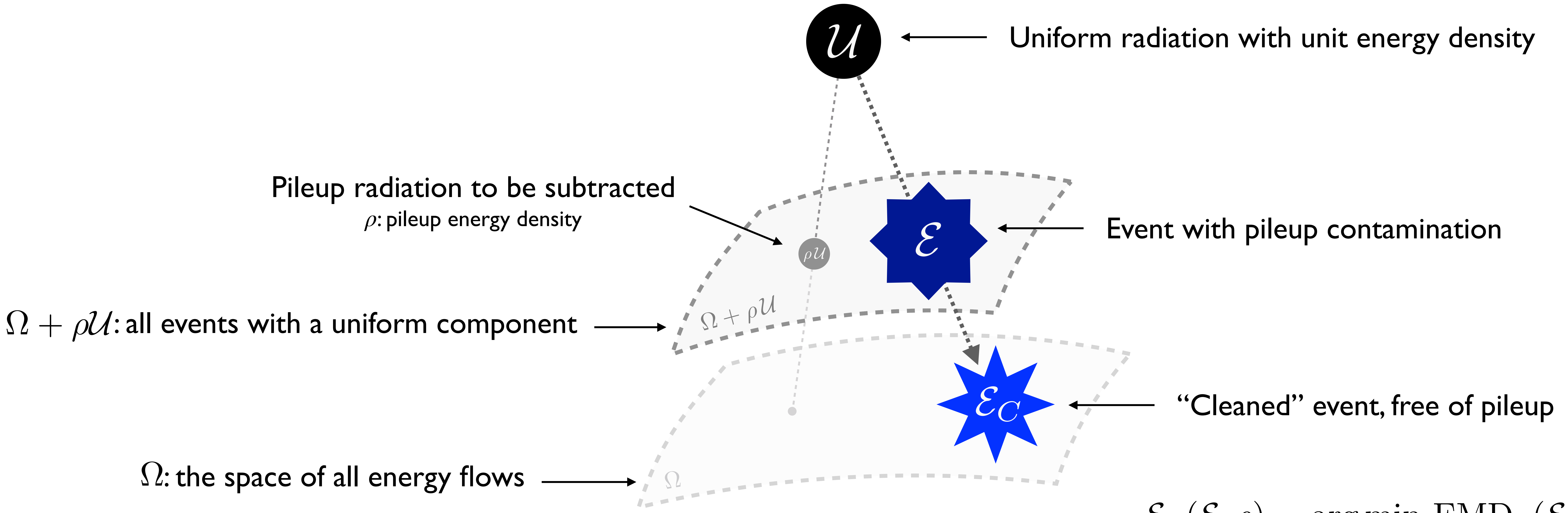

$$\mathcal{E}_C(\mathcal{E}, \rho) = \underset{\mathcal{E}' \in \Omega}{\arg\min}\ \mathrm{EMD}_\beta(\mathcal{E}, \mathcal{E}' + \rho\mathcal{U})$$

Pileup Mitigation in Event Space

Area subtraction

Particle energies corrected proportional to area of associated region

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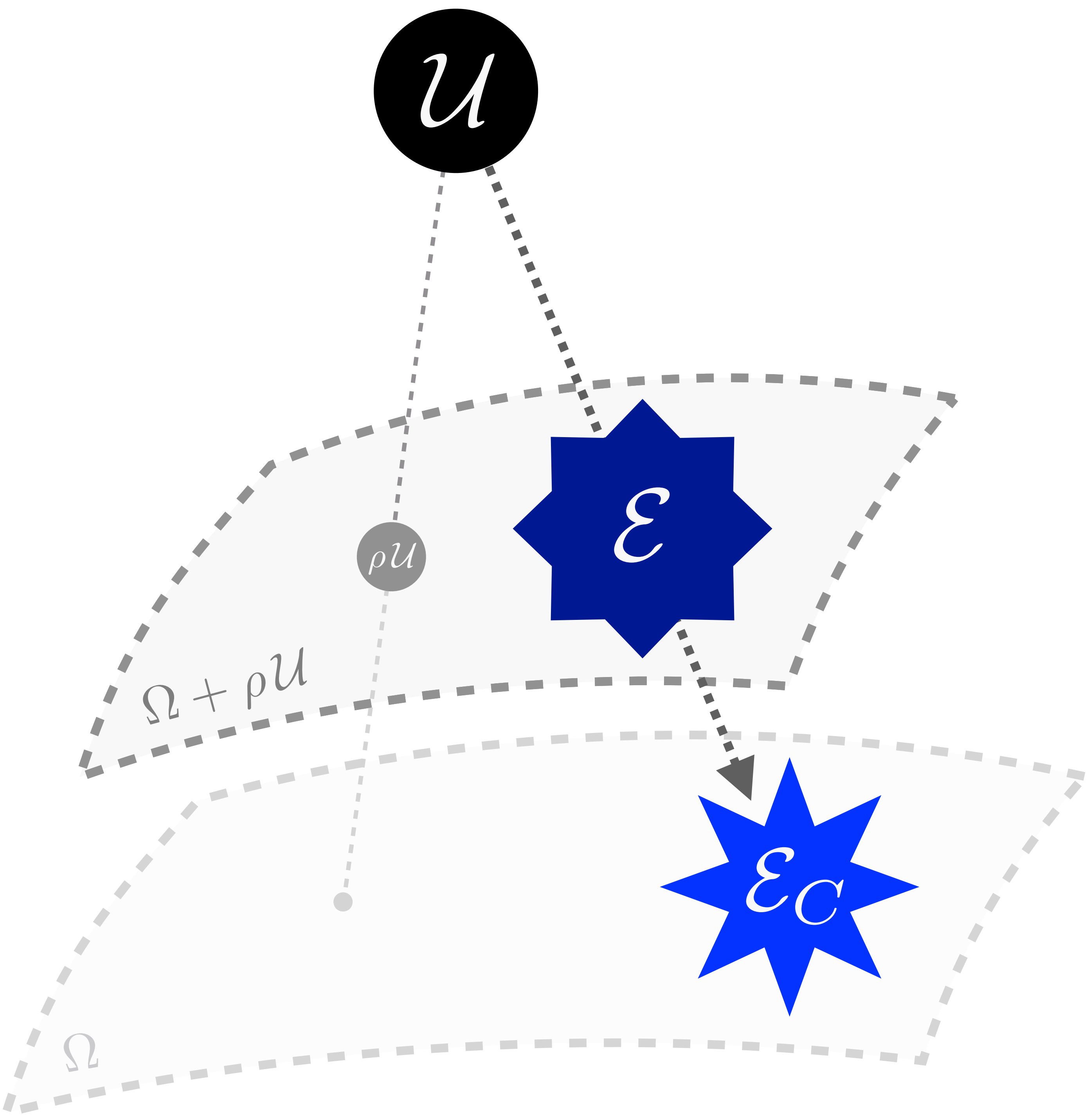

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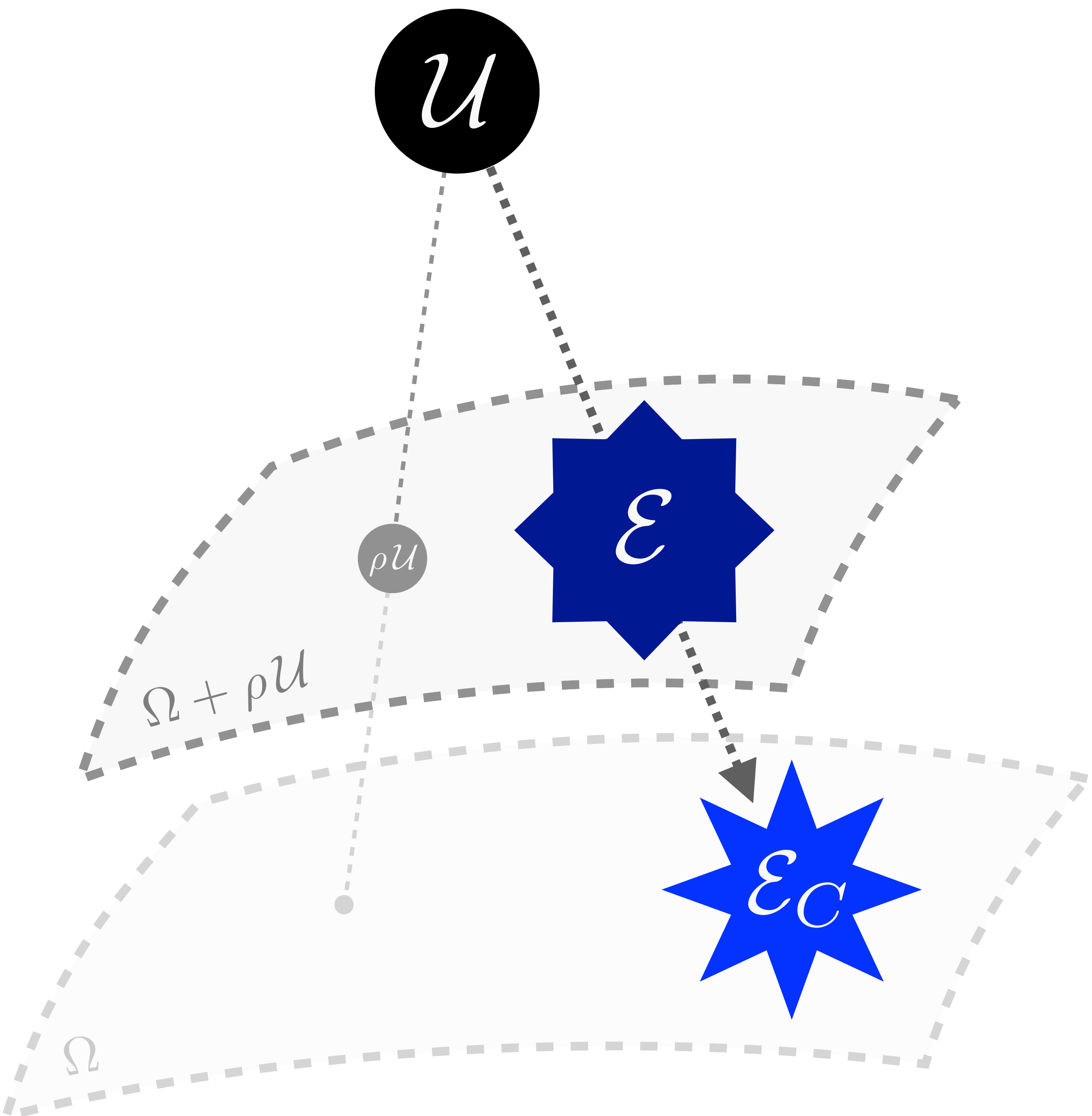

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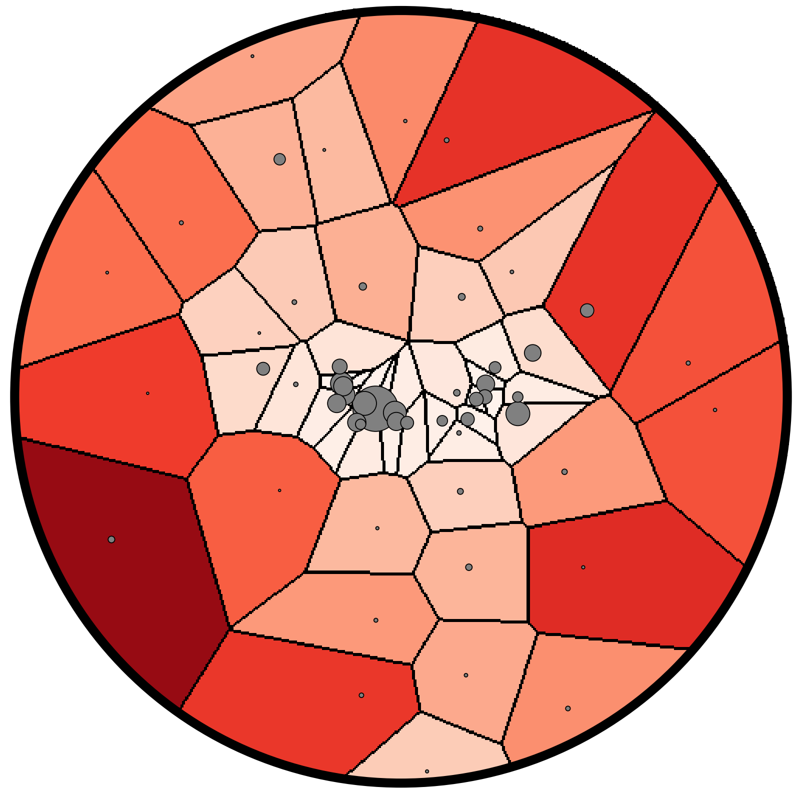
Voronoi

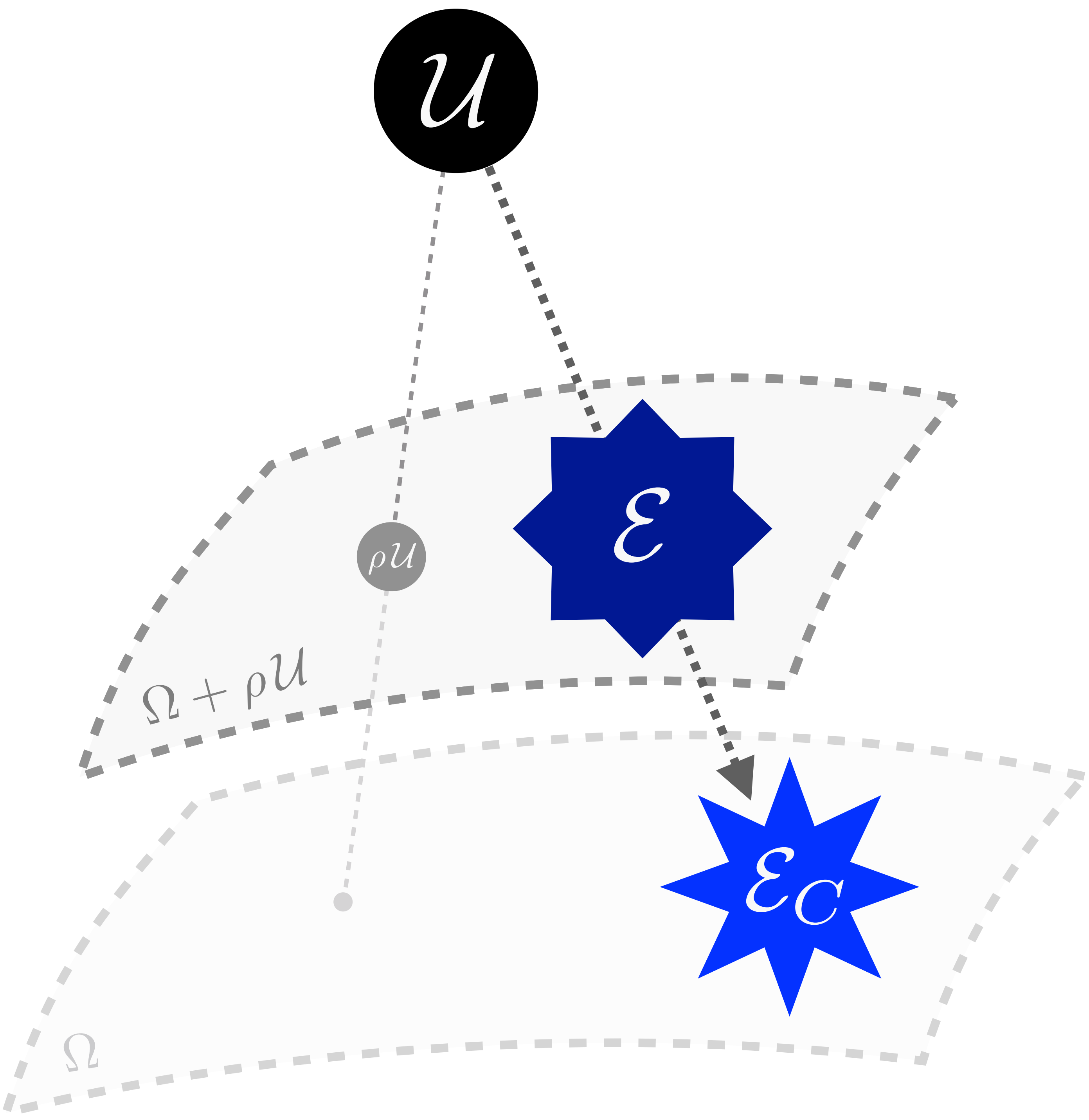


[Cacciari, Salam, Soyez, JHEP 2008]

Voronoi regions IRC unsafe

Sensitive to small modifications

Pileup Mitigation in Event Space



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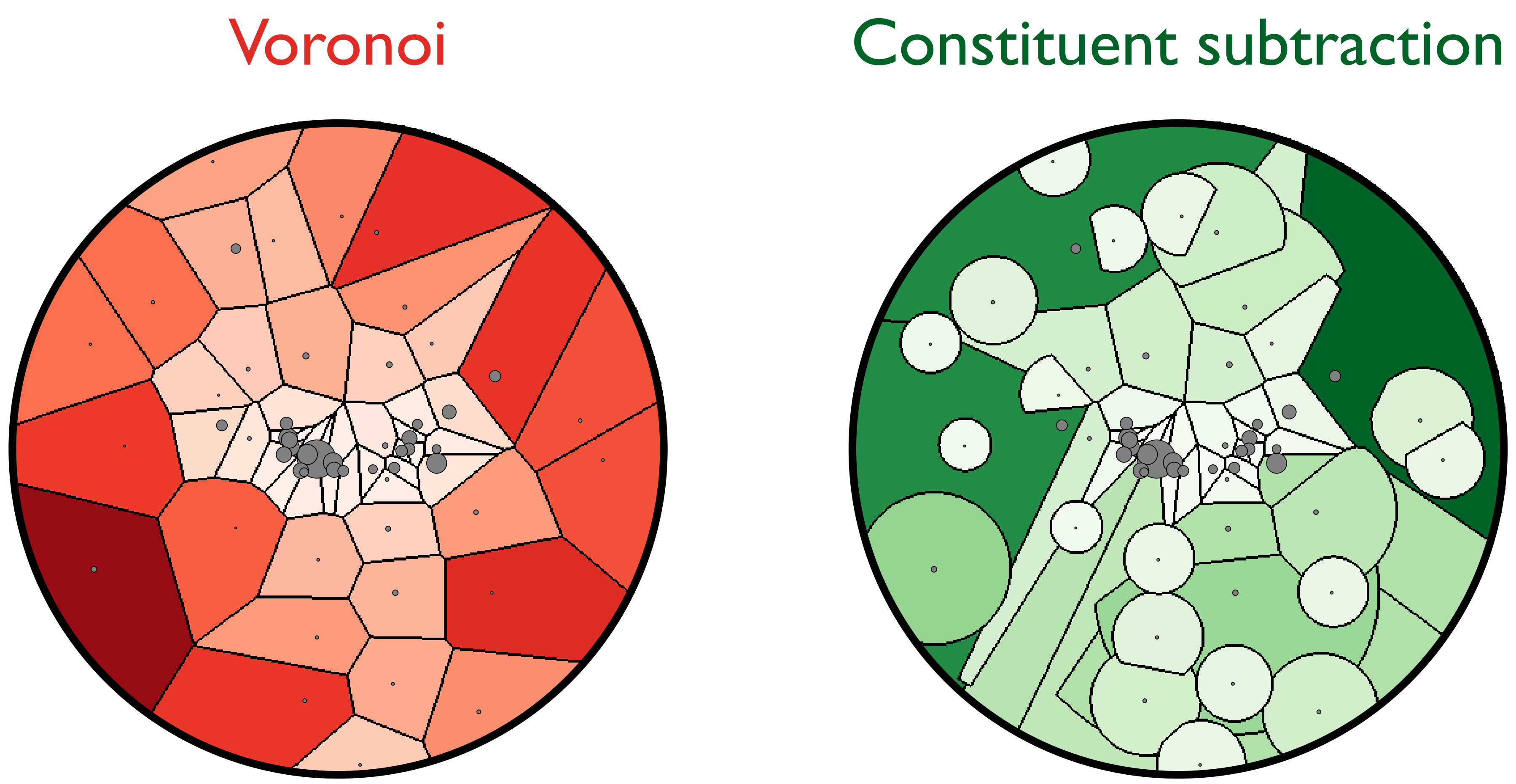

Voronoi

[Cacciari, Salam, Soyez, JHEP 2008]

- Voronoi regions IRC unsafe
- Sensitive to small modifications

Constituent subtraction

[Berta, Spousta, Miller, Leitner, JHEP 2008]

- Lays down grid of "ghost" particles
- Ghosts associate to nearest particle
- Vanished particles don't attract ghosts

Pileup Mitigation in Event Space

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Particle energies corrected proportional to area of associated region

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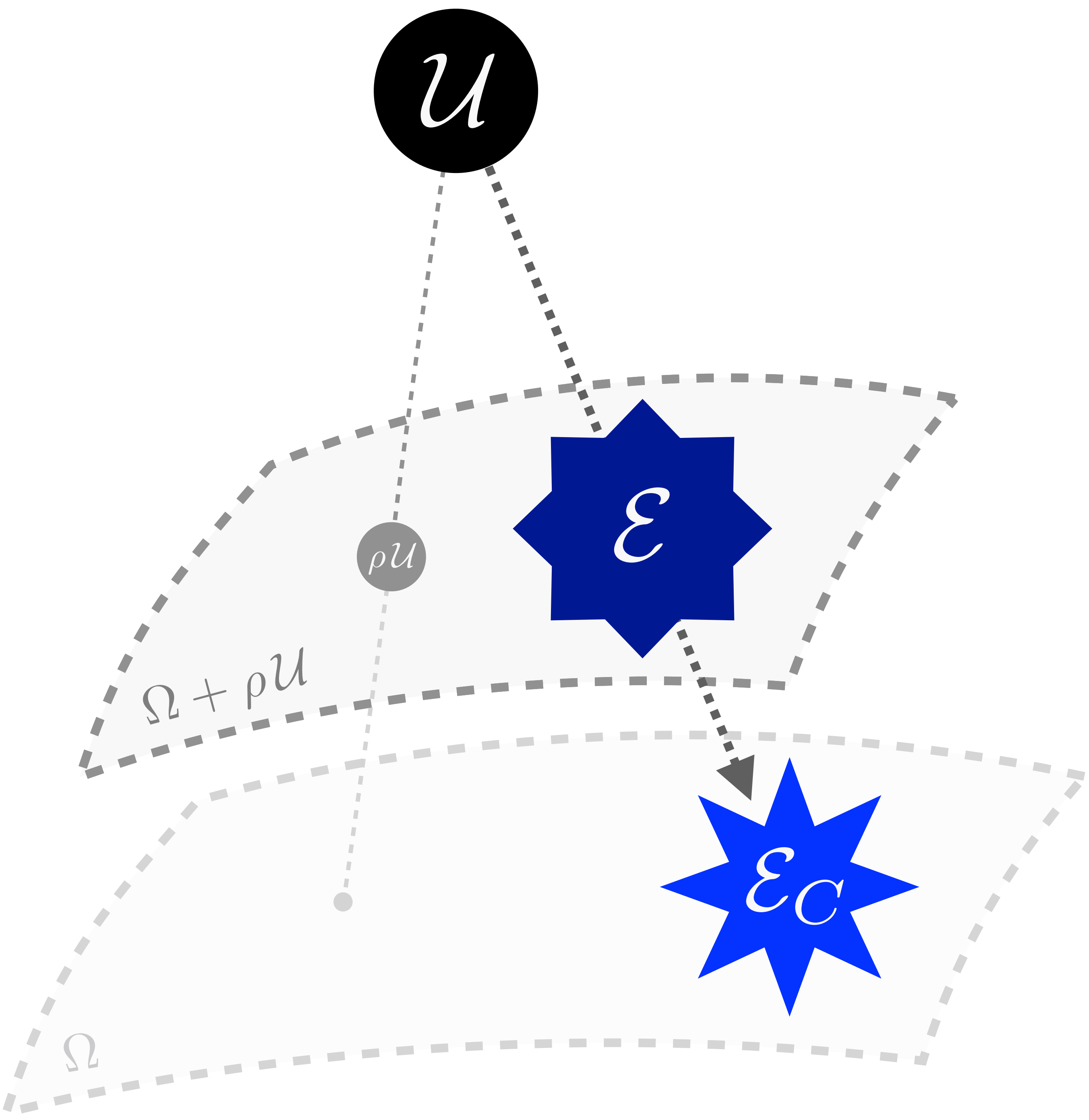

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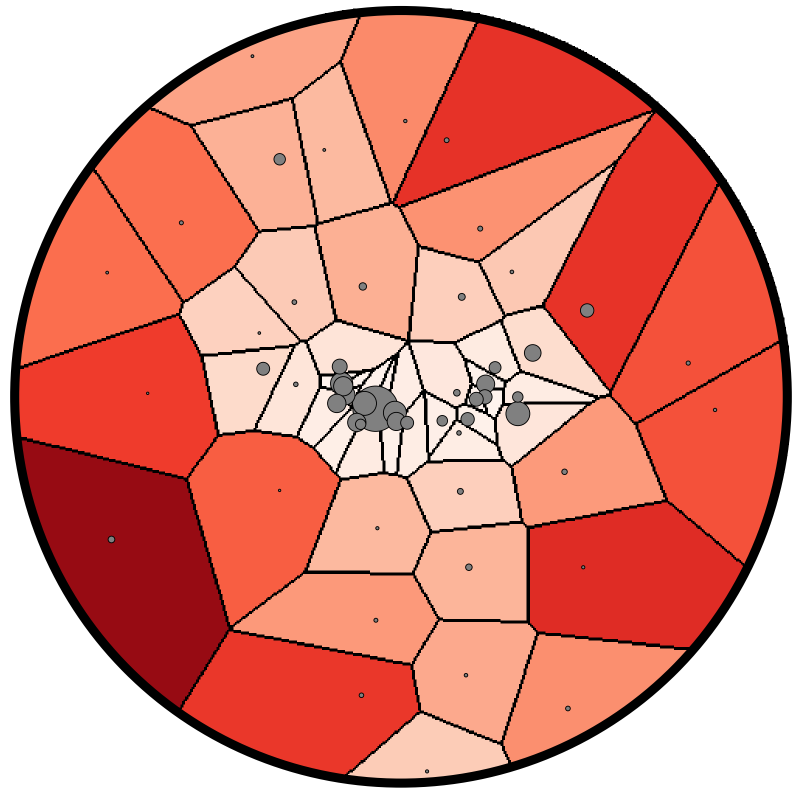

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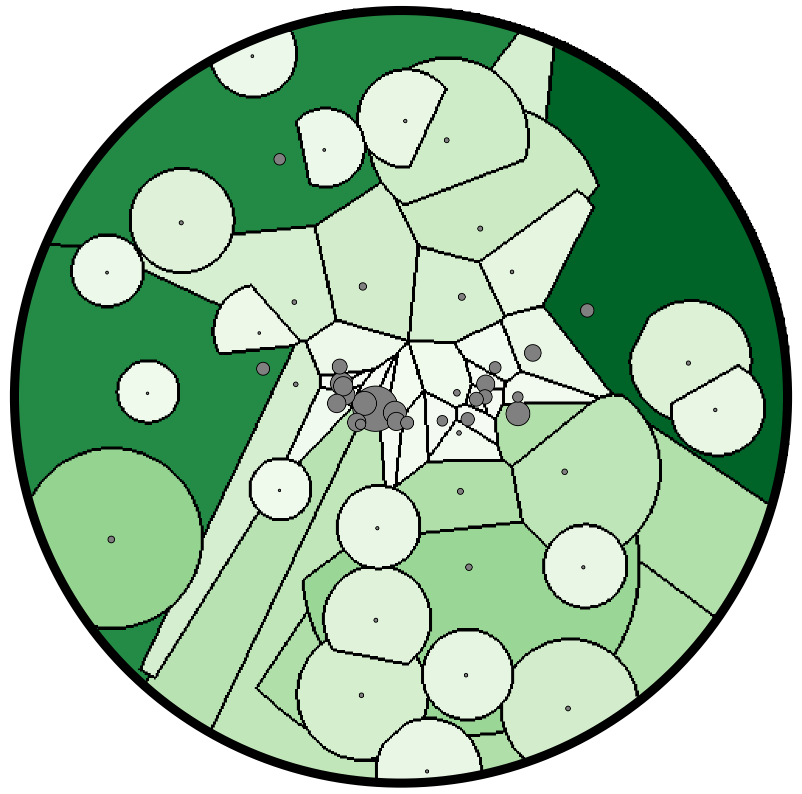


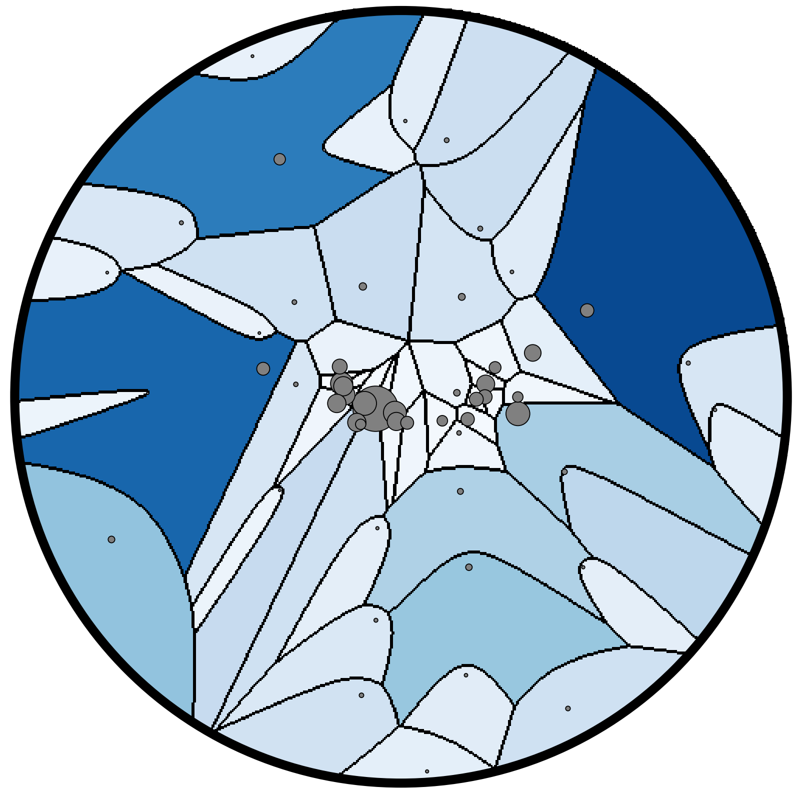
[Berta, Spousta, Miller, Leitner, JHEP 2008]

Lays down grid of "ghost" particles

Ghosts associate to nearest particle

Vanished particles don't attract ghosts

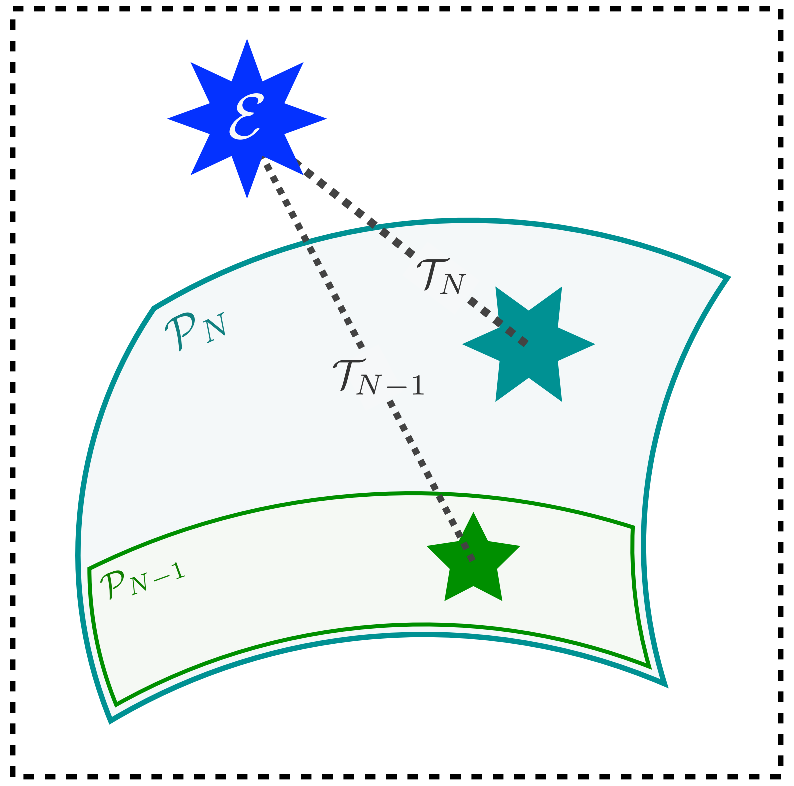
Apollonius



[PTK, Metodiev, Thaler, 2004.04159]

Ghosts are optimally assigned to particles by minimizing EMD

Apollonius regions have an understood continuum limit

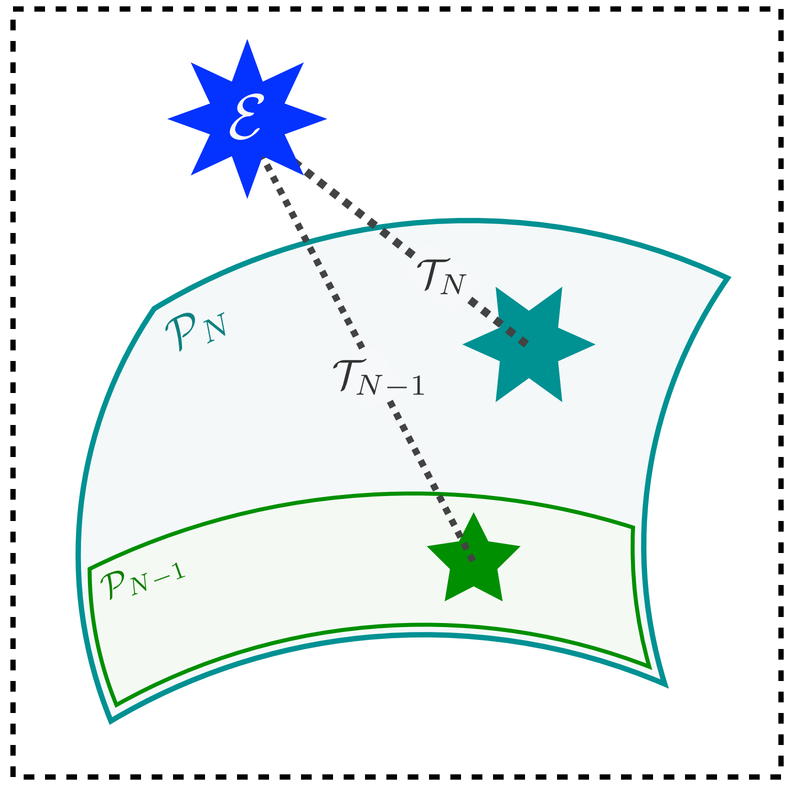


Revealing Hidden Geometry

– Event space exhibits a rich geometry that can be probed using the EMD

– Decades worth of collider techniques are naturally described in this geometry

– Many new techniques are suggested, and new light is shed on old ones



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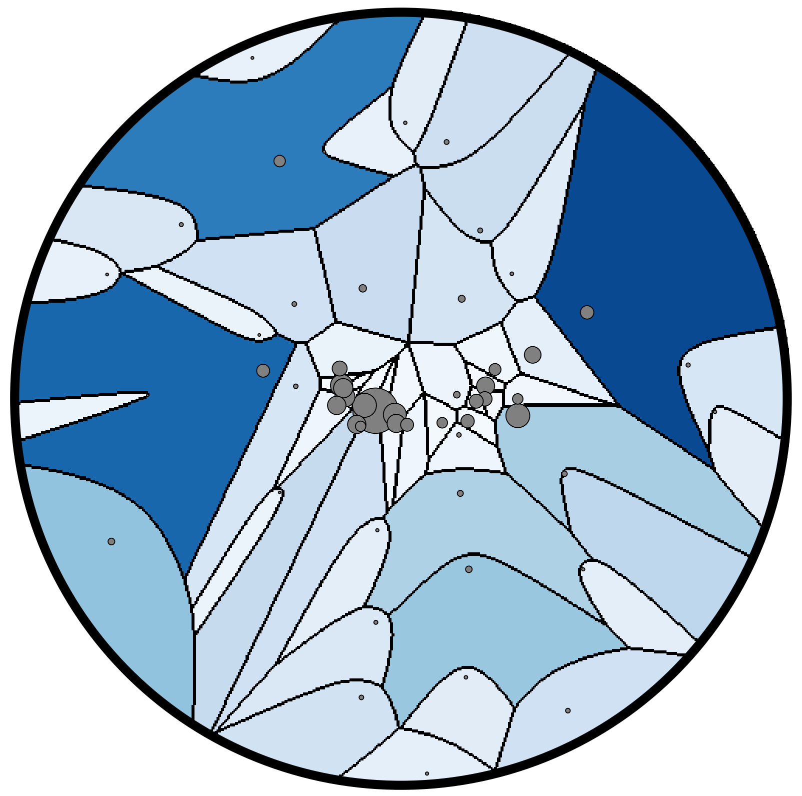
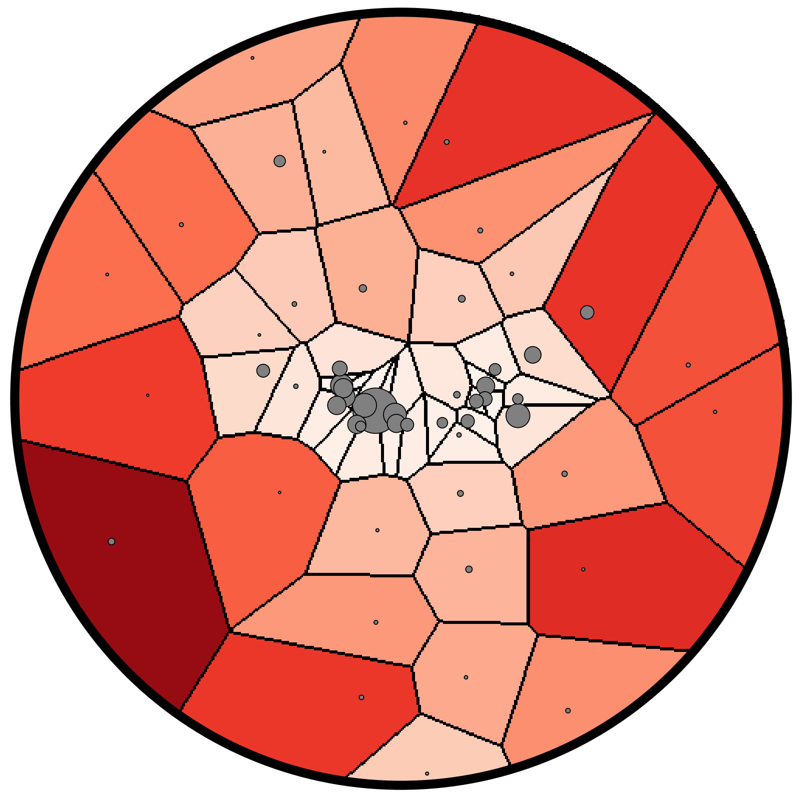
Can pileup mitigators be effective jet groomers?

Apollonius

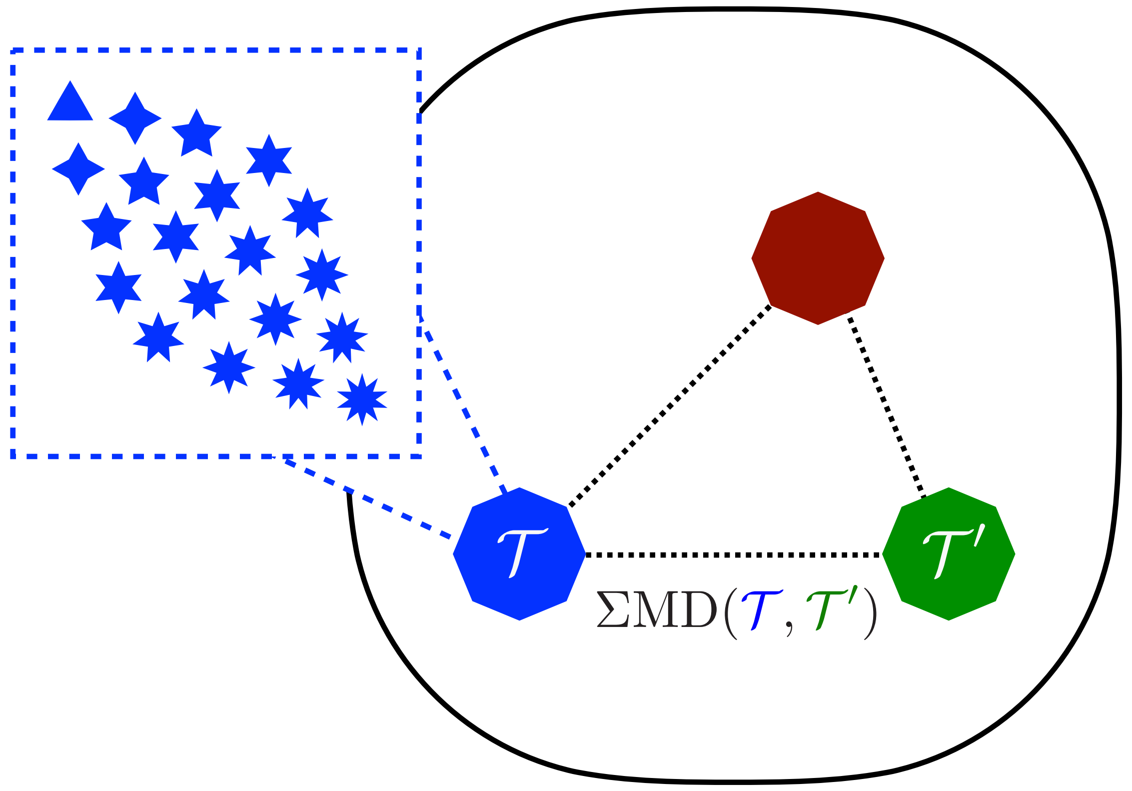


can be approximated by iterating

Voronoi



[Alipour-fard, PTK, Metodiev, Thaler, *to appear soon*]



The (Metric) Space of Events

Revealing Hidden Geometry

Theory Space

Templated Metric Construction

Inputs

– "Points" that live in the ground metric space
– "Ground metric" that measures point distances
– "Weights" associated to each point

Output

– A new metric for collections of weighted points
– A metric space where these distributions live

p-Wasserstein metric from optimal transport theory

$$W_p(\mu,\nu) = \left(\inf_{J\in\mathcal{J}(\mu,\nu)} \int_{M\times M} d(x,y)^p \, \mathrm{d}J(x,y) \right)^{1/p}$$

$(M,\ d)$, metric space

$\mathcal{J}(\mu,\nu)$, space of joint distributions with marginals $\mu,\ \nu$

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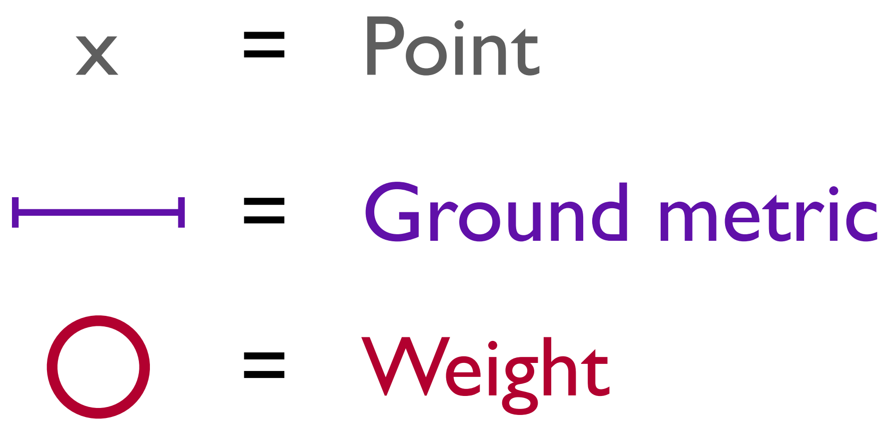
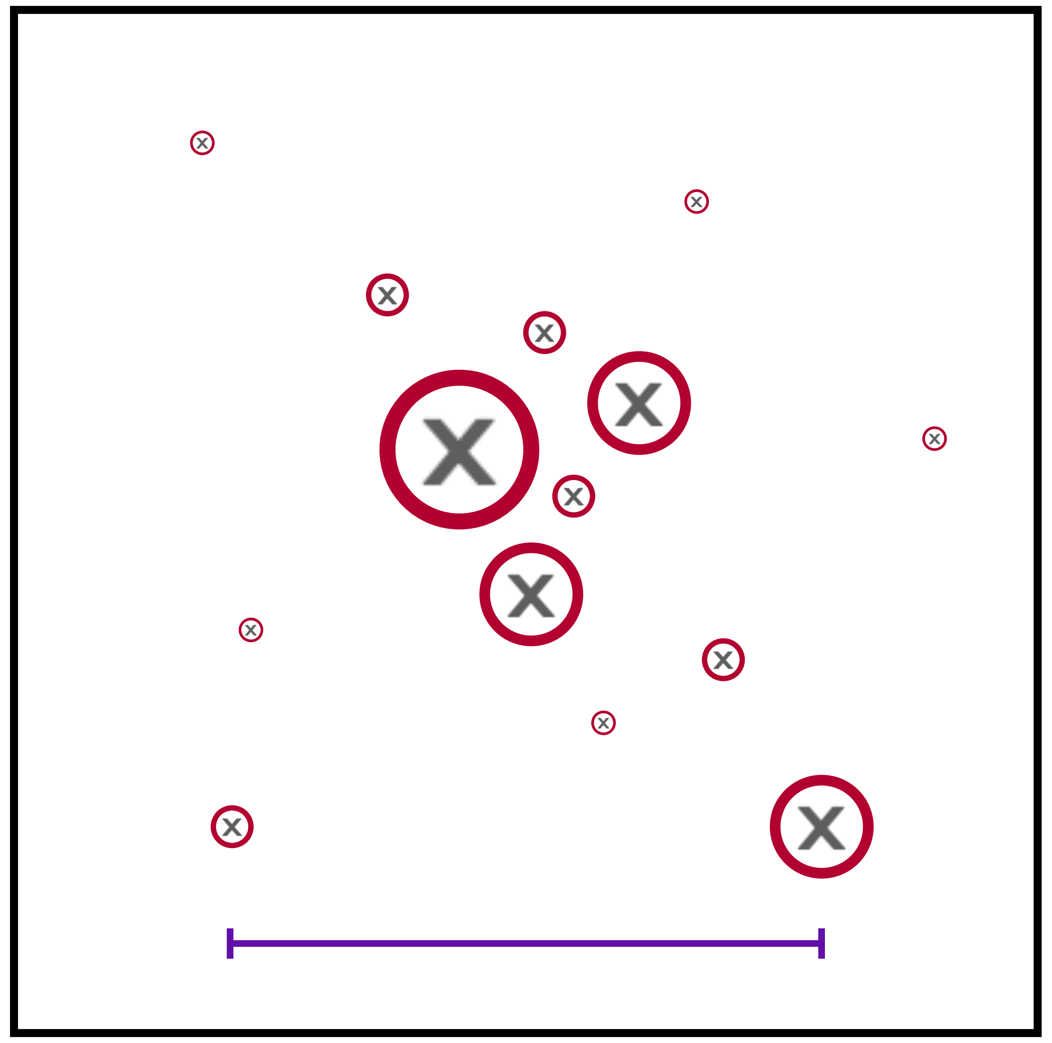


Ground space



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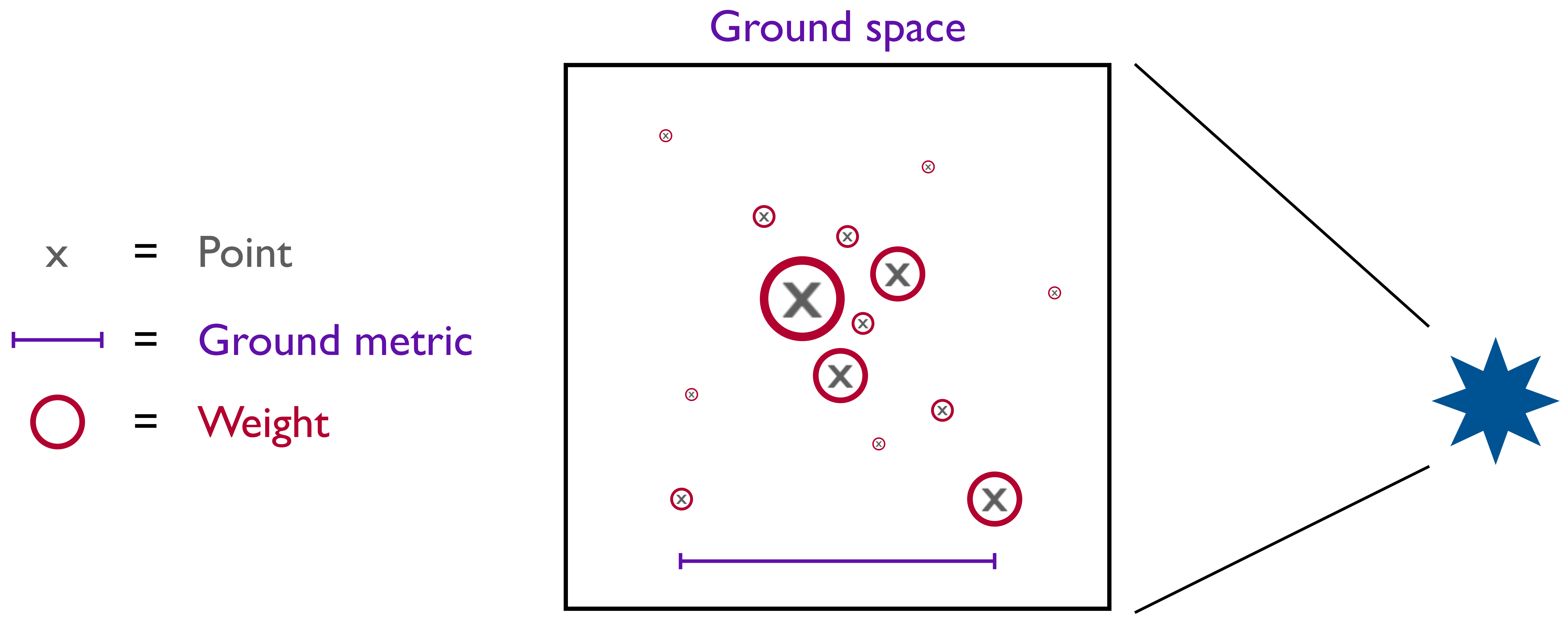

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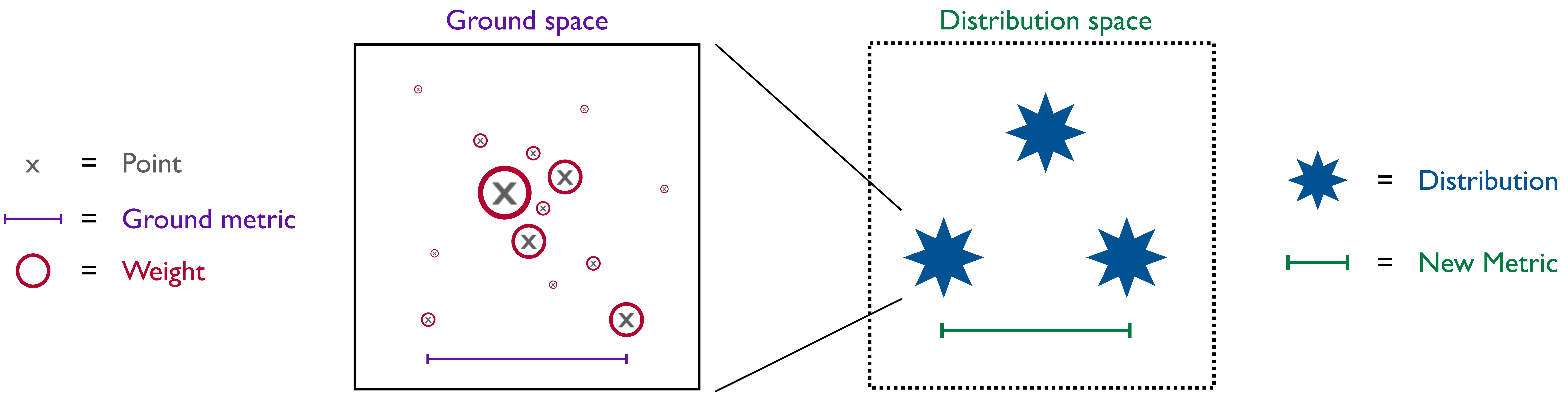

Templated Metric Construction – Energy Mover's Distance

[PTK, Metodiev, Thaler, PRL 2019]

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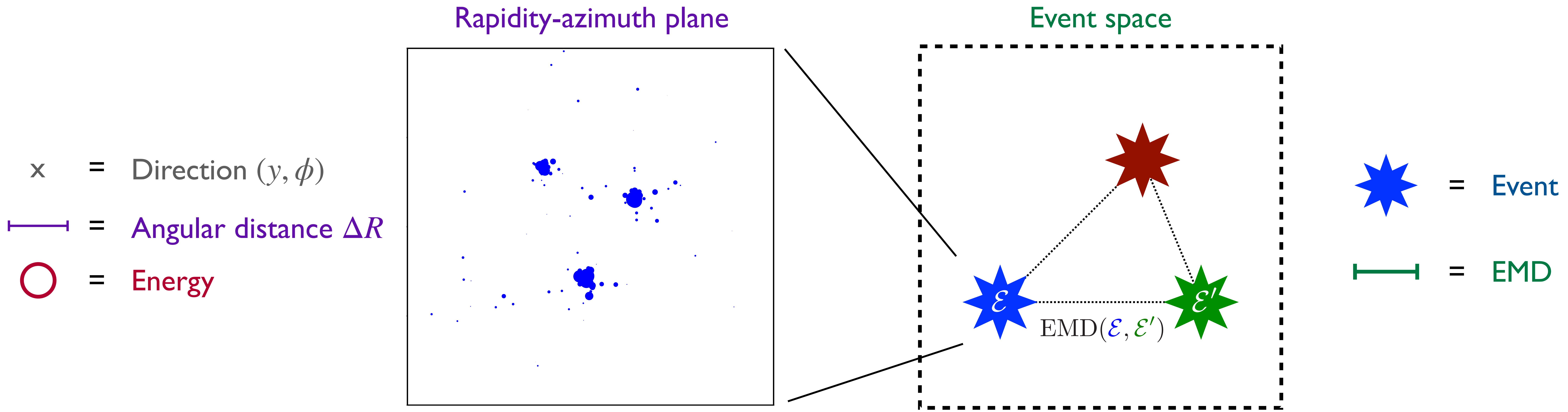


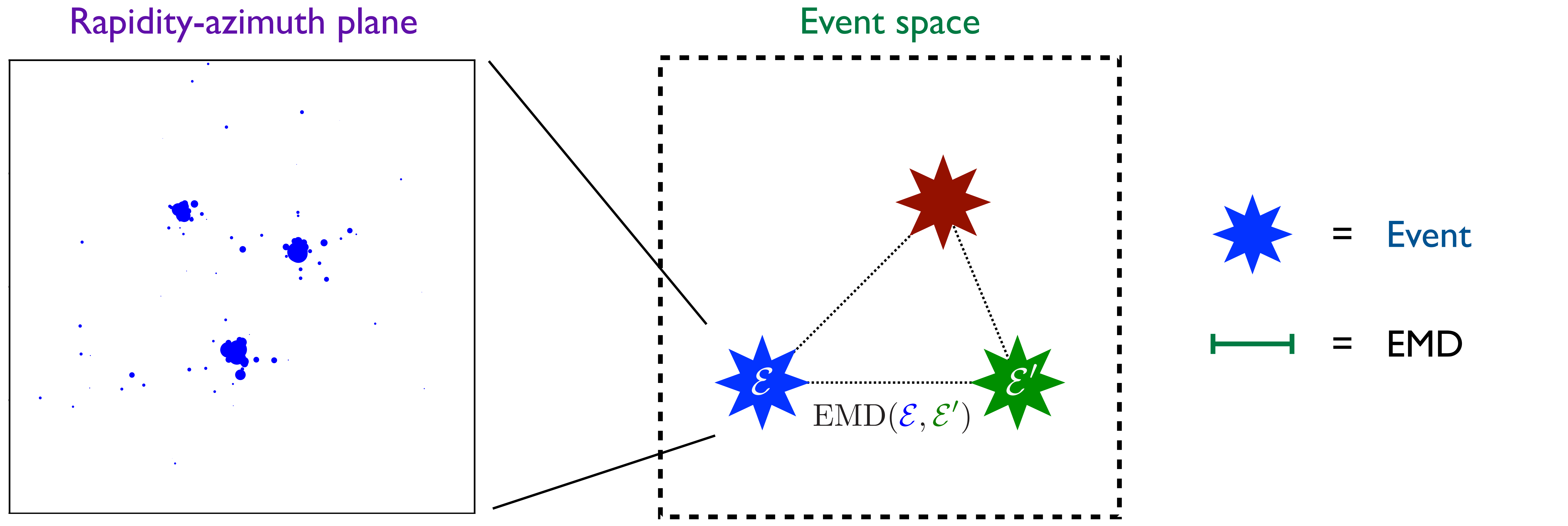
Bootstrapping to the Cross-Section Mover's Distance (ΣMD)

[PTK, Metodiev, Thaler, 2004.04159]

x = Direction (y, ϕ)

= Angular distance ΔR

= Energy



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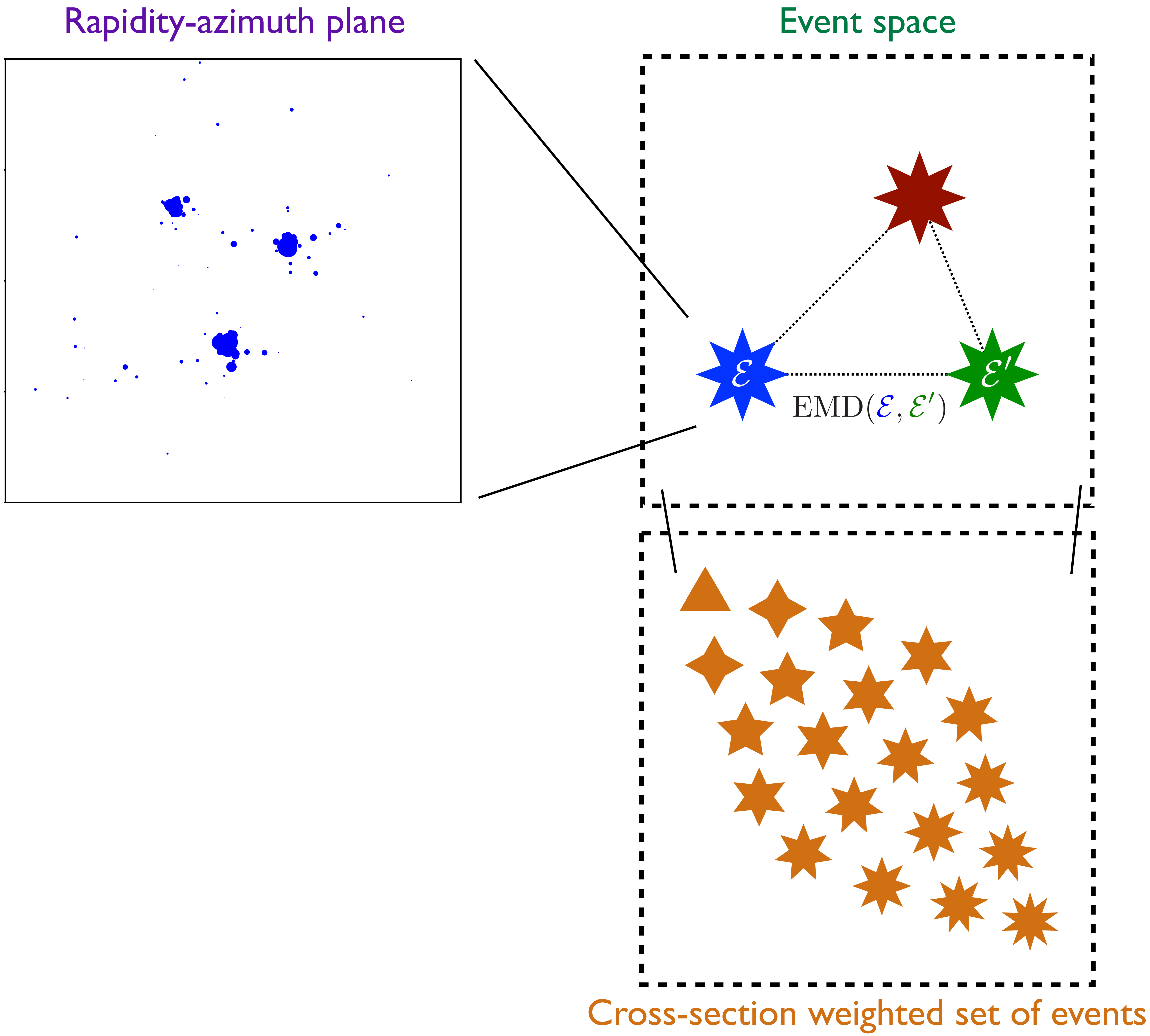

= Event

= EMD

event weights = *cross section*

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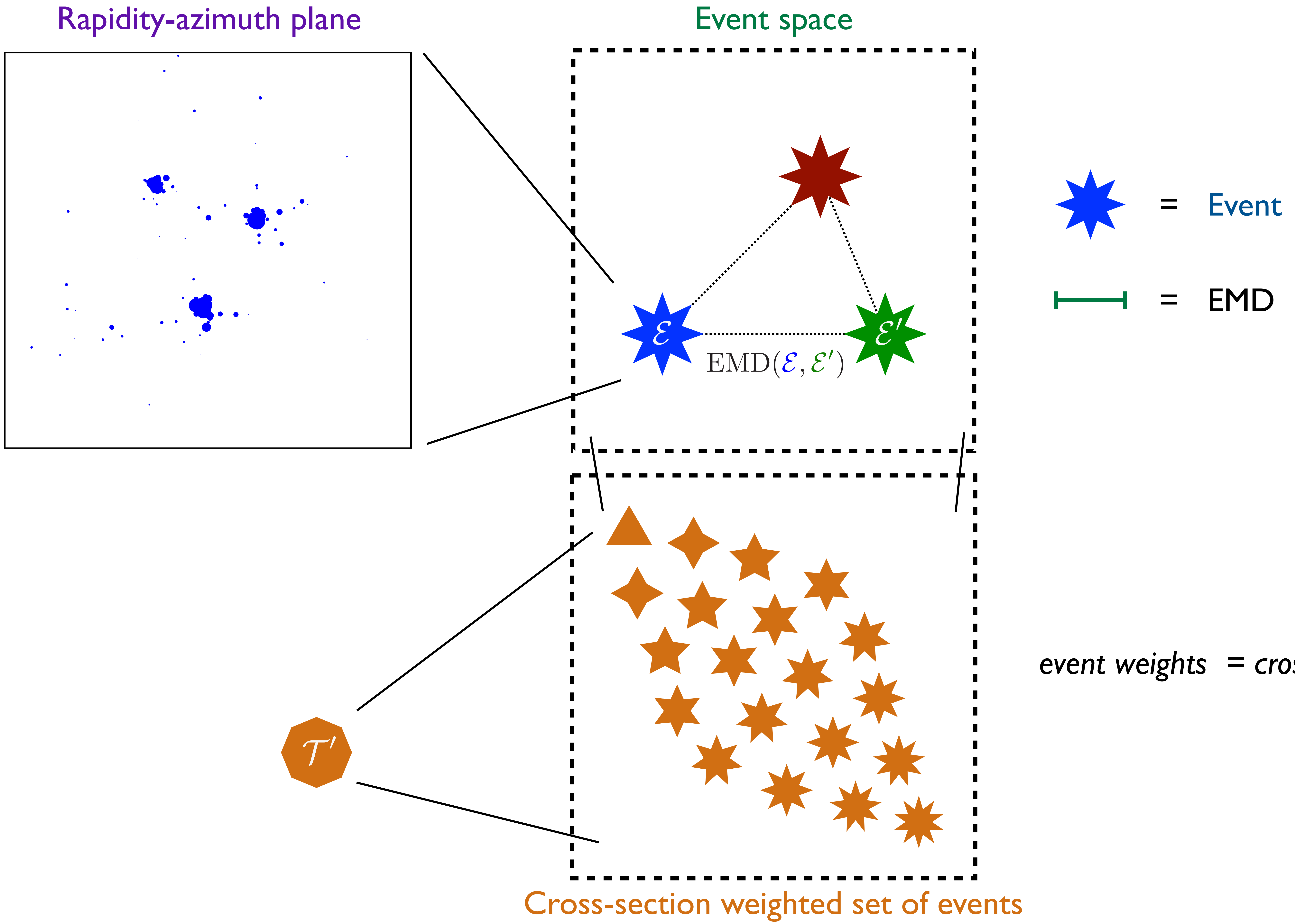

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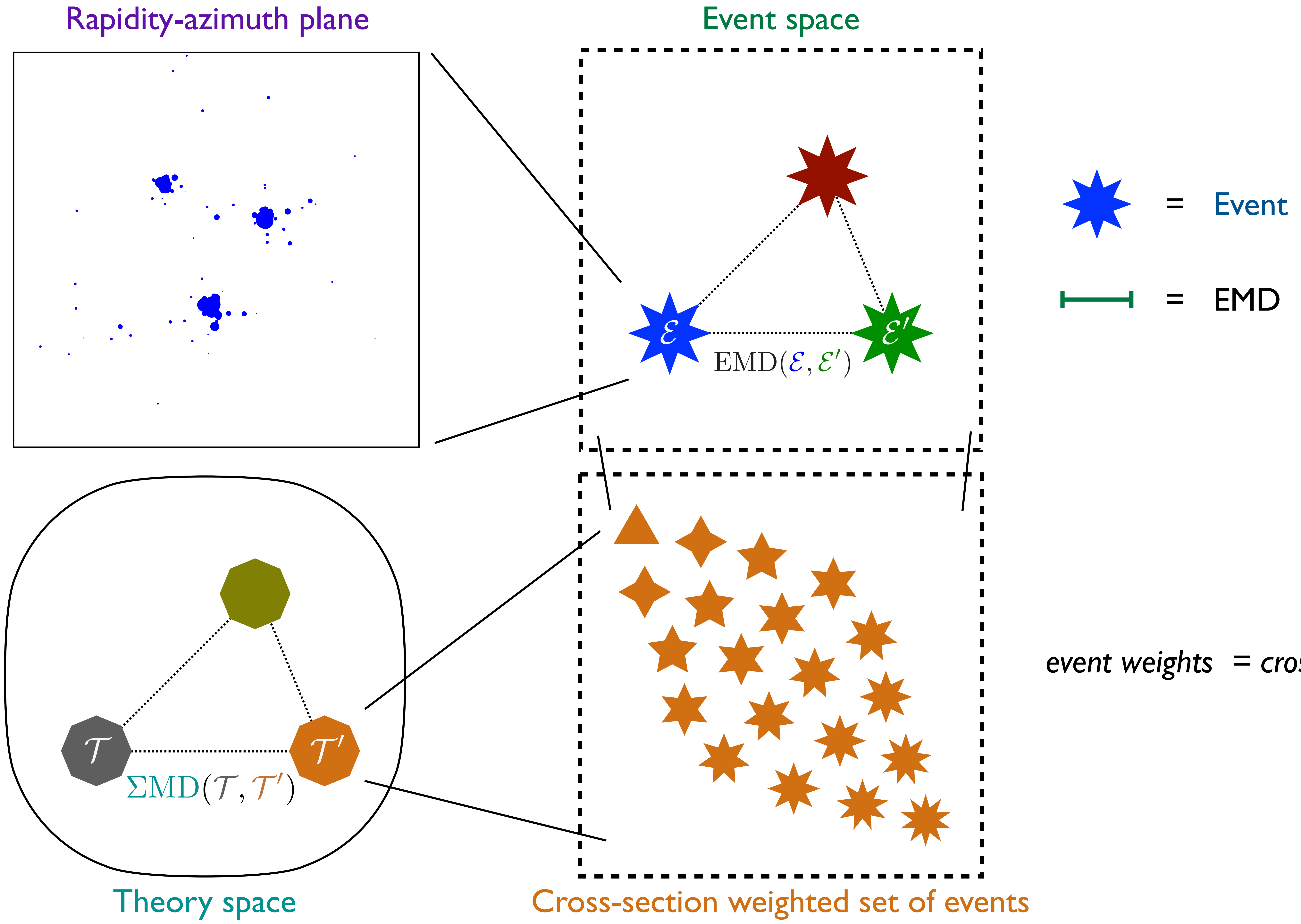
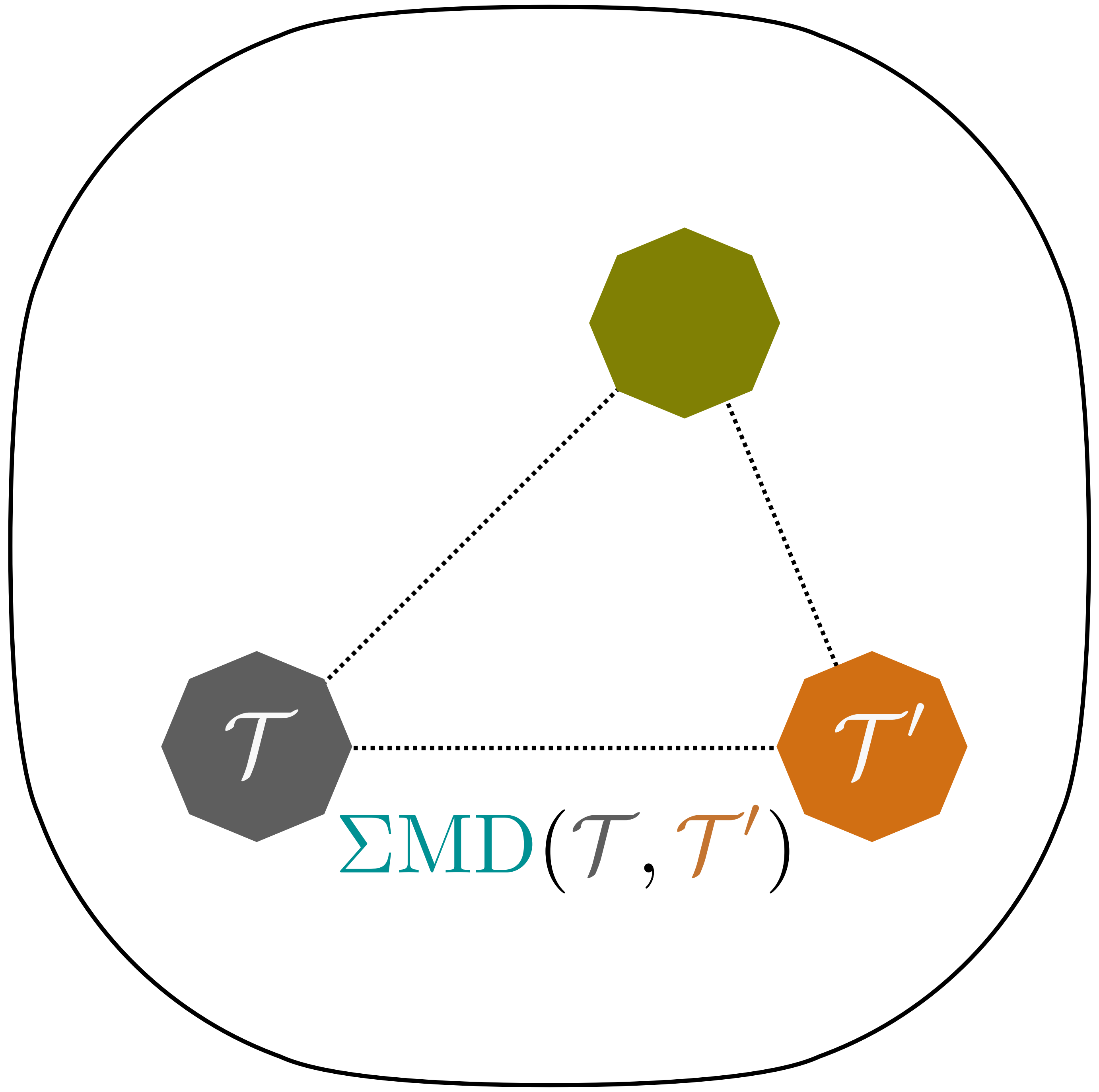


event weights = cross section

ΣMD is a metric between theories!

The Cross-Section Mover's Distance (ΣMD)

[PTK, Metodiev, Thaler, 2004.04159]



ΣMD uses EMD as the ground metric and event cross sections as weights

$$\Sigma\text{MD}_{\gamma,S;\beta,R}(\mathcal{T},\mathcal{T}') = \min_{\mathcal{F}_{ij}\ge 0} \sum_{i=1}^{N}\sum_{j=1}^{N'} \mathcal{F}_{ij} \left(\frac{\text{EMD}_{\beta,R}(\mathcal{E}_i,\mathcal{E}'_j)}{S}\right)^{\gamma} + \left|\sum_{i=1}^{N}\sigma_i - \sum_{j=1}^{N'}\sigma'_j\right|$$

$$\sum_{i=1}^{N}\mathcal{F}_{ij} \le \sigma'_j, \quad \sum_{j=1}^{N'}\mathcal{F}_{ij} \le \sigma_i, \quad \sum_{i=1}^{N}\sum_{j=1}^{N'}\mathcal{F}_{ij} = \min\left(\sum_{i=1}^{N}\sigma_i, \sum_{j=1}^{N'}\sigma'_j\right)$$

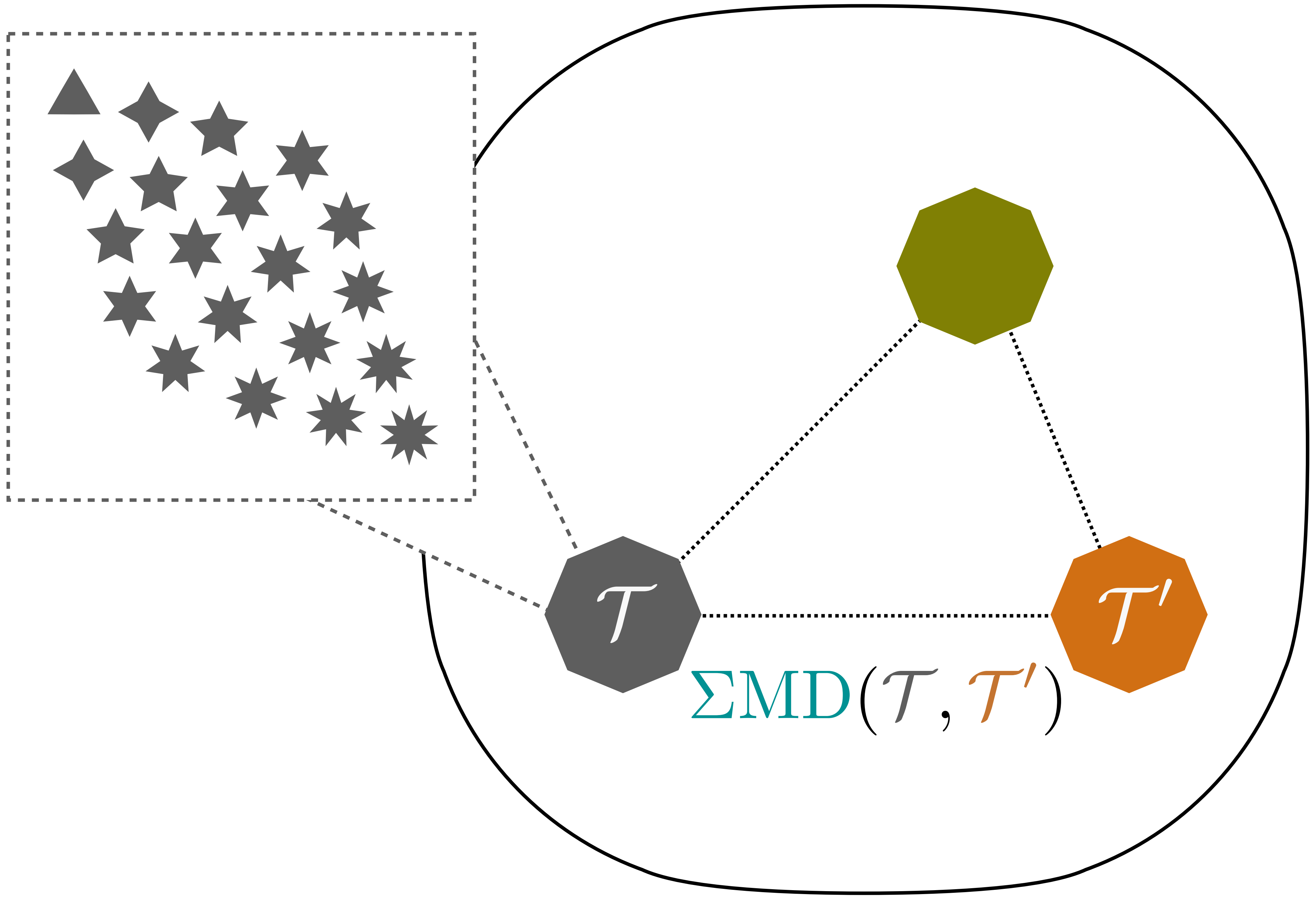
Usual constraints to ensure proper transport

	Energy Mover's Distance	Cross Section Mover's Distance
Symbol	EMD	ΣMD
Description	Distance between events	Distance between theories
Weight	Particle energies E_i	Event cross sections σ_i
Ground Metric	Particle distances θ_{ij}	Event distances $\text{EMD}(\mathcal{E}_i,\mathcal{E}_j)$

The Space of Theories

[PTK, Metodiev, Thaler, 2004.04159]

ΣMD provides a rigorous construction of theory space



*Theories are distinguished by their energy flows only

The Space of Theories

[PTK, Metodiev, Thaler, 2004.04159]

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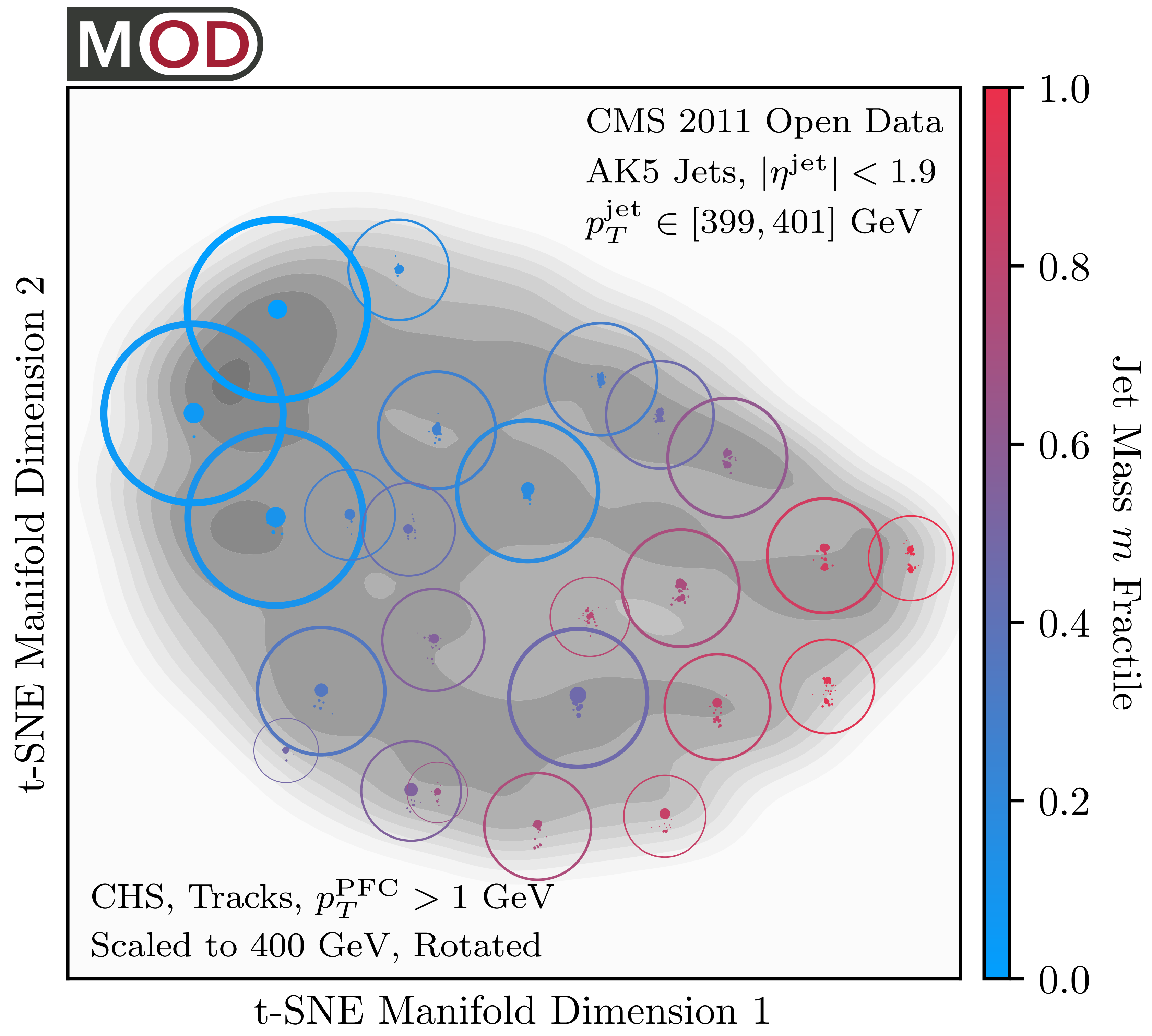

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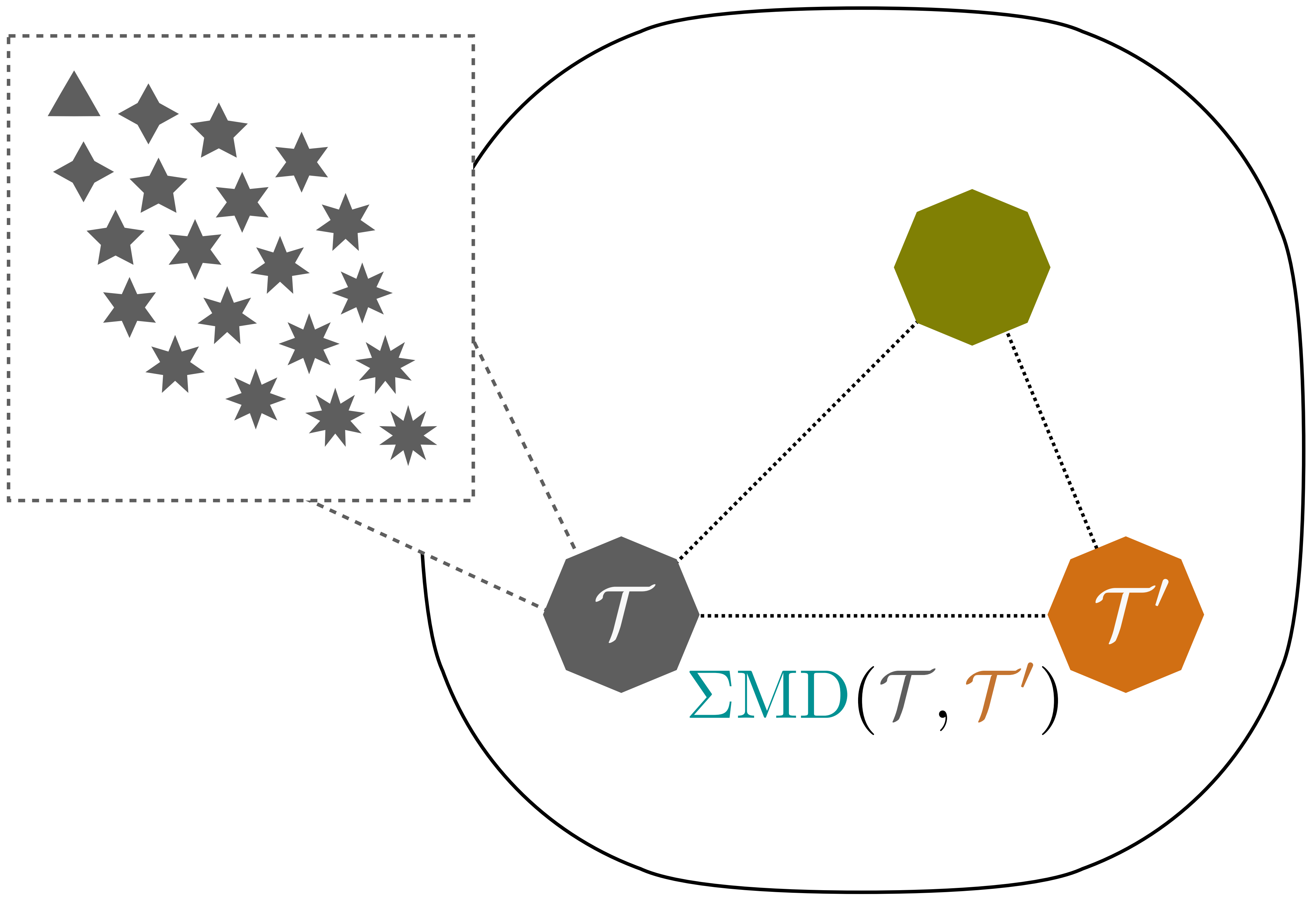

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Applications of ΣMD and the Space of Theories

Applications of ΣMD and the Space of Theories

N-(sub)jettiness ⇨

_k_-eventiness defined

$$\mathcal{V}_k^{(\gamma)}(\{\sigma_i, \mathcal{E}_i\}) = \min_{\mathcal{K}_1,\ldots,\mathcal{K}_k} \sum_{i=1}^{N} \sigma_i \min\{\mathrm{EMD}(\mathcal{E}_i, \mathcal{K}_1), \ldots, \mathrm{EMD}(\mathcal{E}_i, \mathcal{K}_k)\}^\gamma$$

$$\mathcal{V}_k^{(\gamma)}(\mathcal{T}) = \min_{|\mathcal{T}'|=k} \Sigma\mathrm{MD}_\gamma(\mathcal{T}, \mathcal{T}')$$

Applications of ΣMD and the Space of Theories

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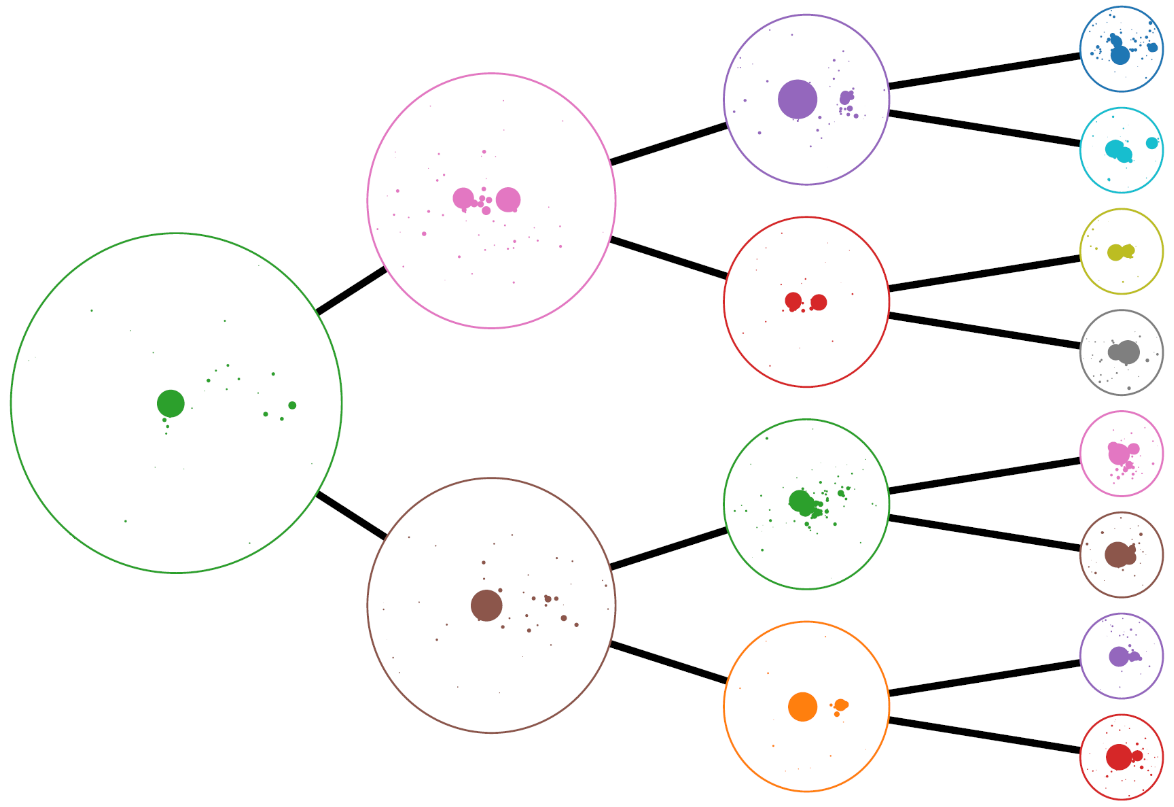
Jet clustering ⇒

Event clustering enabled
- Exclusive cone finding
- Sequential recombination



Applications of ΣMD and the Space of Theories

N-(sub)jettiness ⇨

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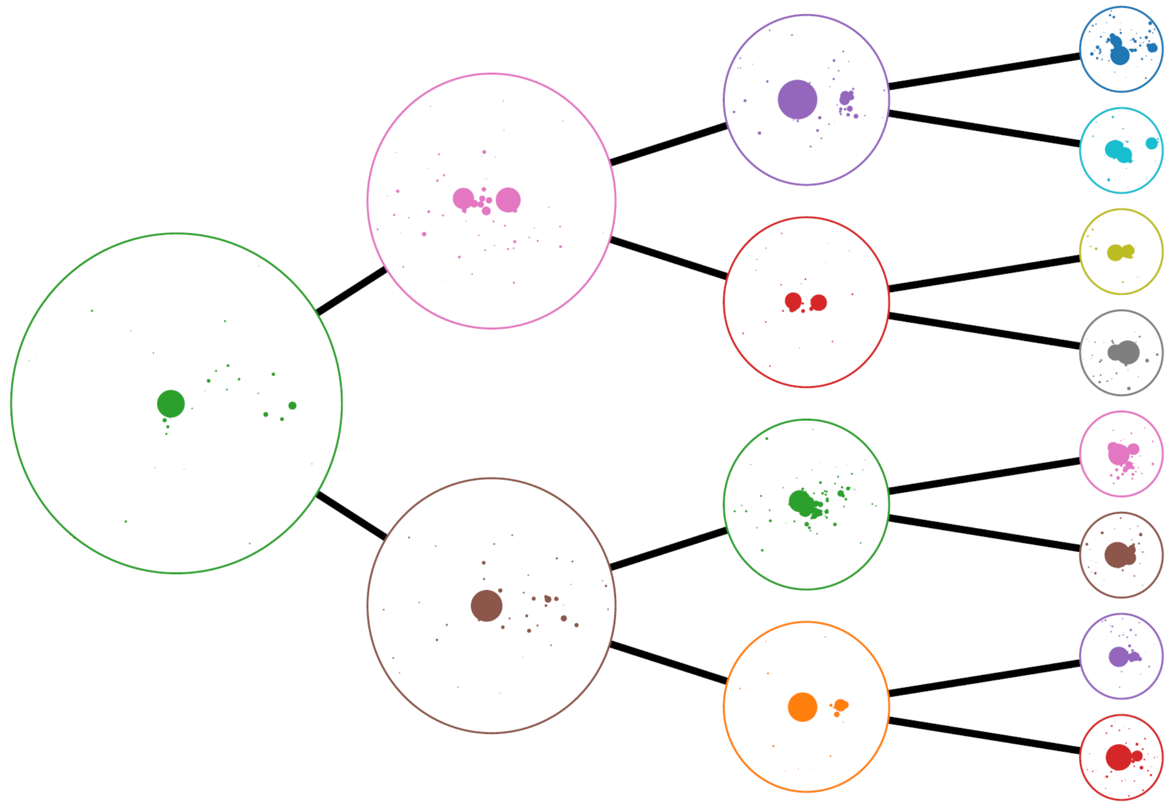
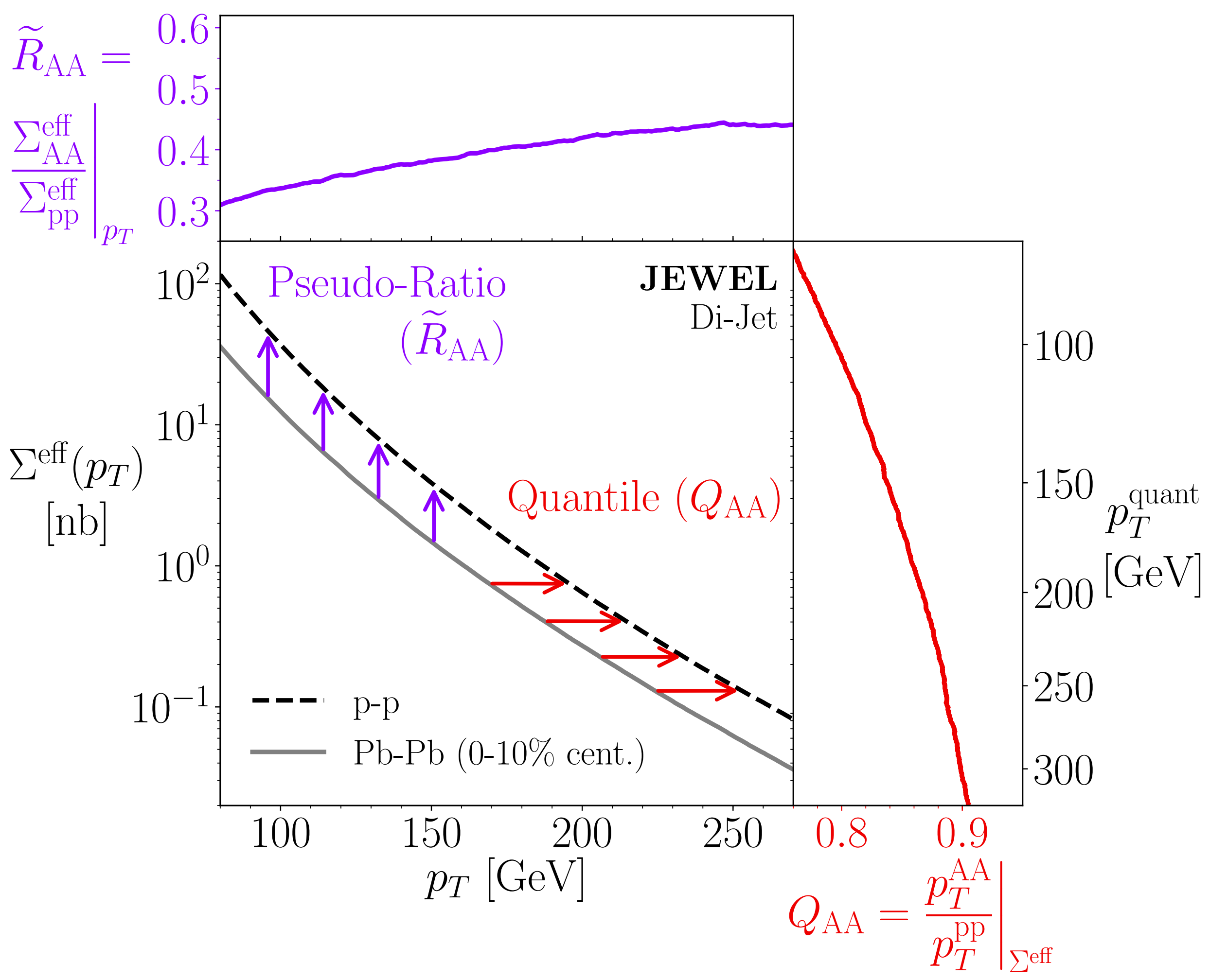


Jet quenching in HI collisions ⇨

[Brewer, Milhano, Thaler, PRL 2019]



Quantile matching:

$$\Sigma^{\text{eff}}_{pp}(p_T^{\text{quant}}) \equiv \Sigma^{\text{eff}}_{AA}(p_T^{AA})$$

Applications of ΣMD and the Space of Theories

N-(sub)jettiness ⇨

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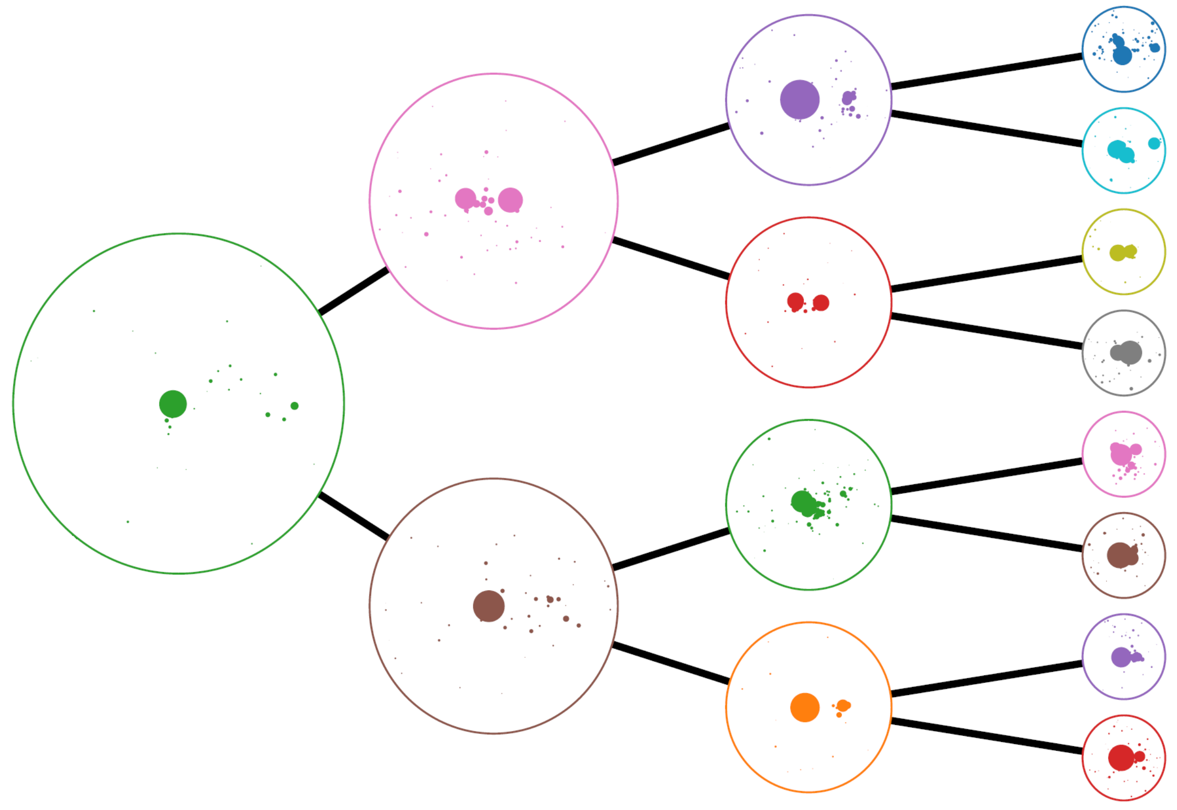

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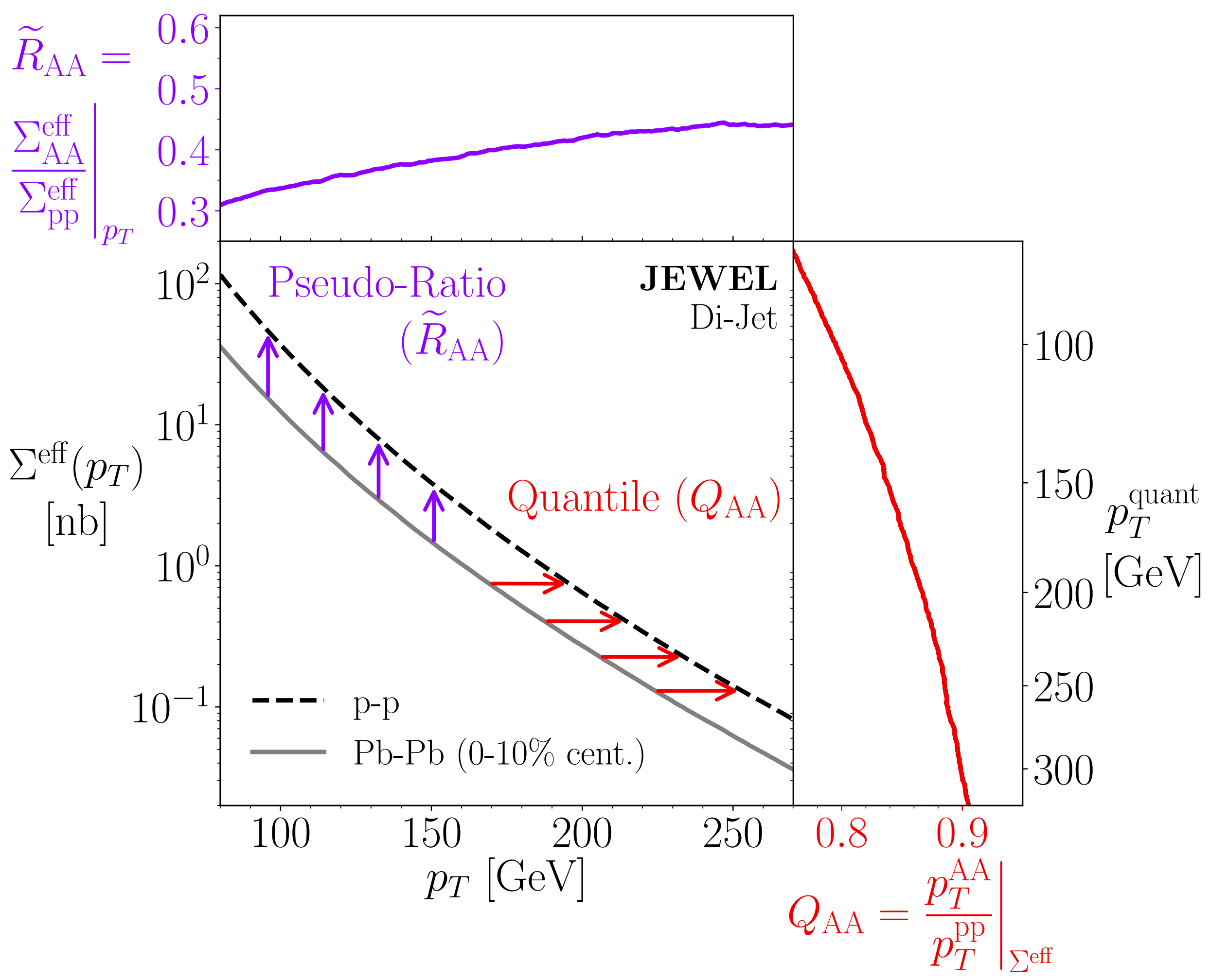


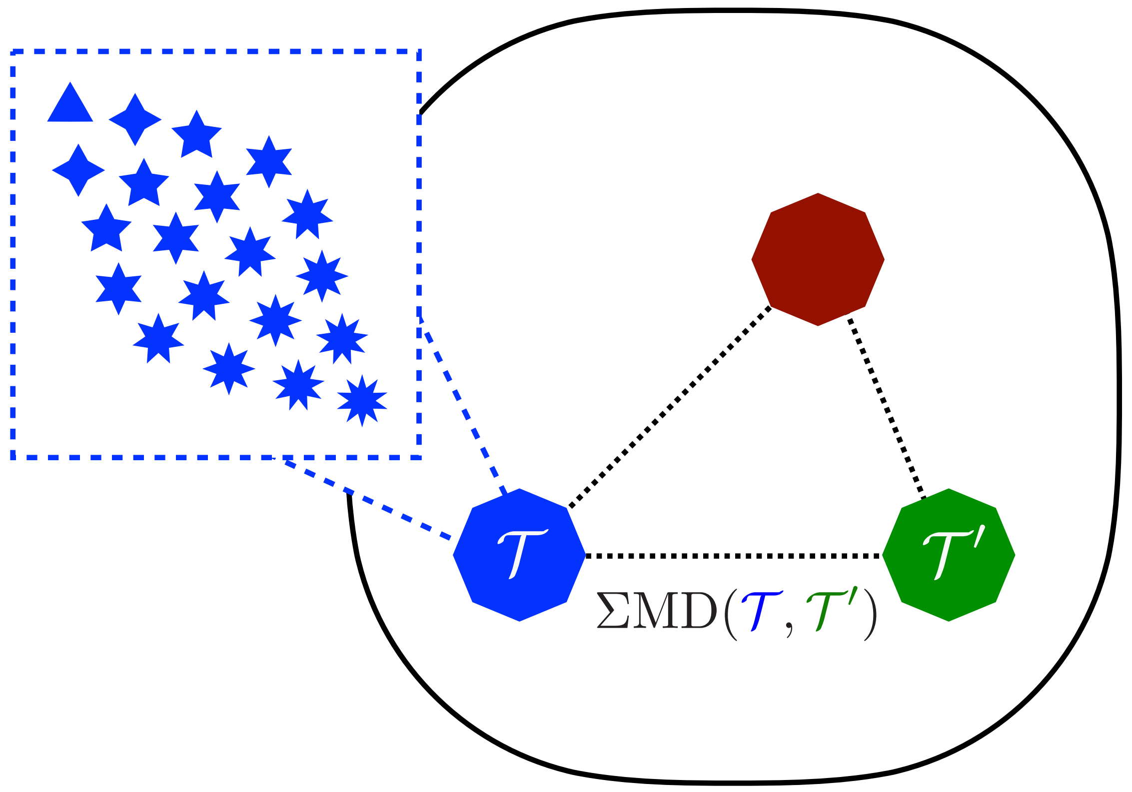
Quantile matching:

$$\Sigma^{\rm eff}_{\rm pp}(p_T^{\rm quant}) \equiv \Sigma^{\rm eff}_{\rm AA}(p_T^{\rm AA})$$

…is exactly a theory moving problem!

$$p_T^{\rm quant} = \mathrm{TM}(\mathcal{T}_{\rm AA}, \mathcal{T}_{\rm pp})[p_T^{\rm AA}]$$

↑ optimal p_T-only theory movement



Theory Space

– Is rigorously constructed using the cross-section mover's distance ΣMD

– ΣMD uses the EMD as ground metric and cross sections as weights

– Allows for theories to be explored with tools developed for events

	Energy Mover's Distance	Cross Section Mover's Distance
Symbol	EMD	ΣMD
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Weight	Particle energies E_i	Event cross sections σ_i
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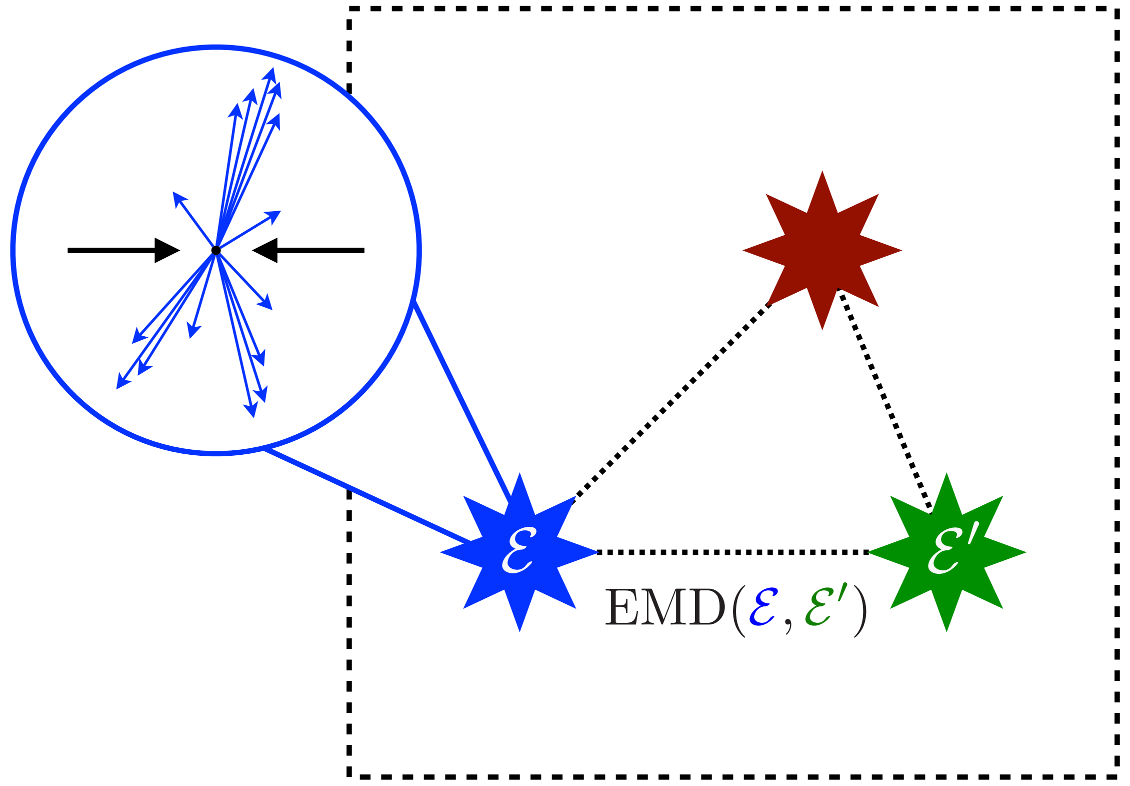
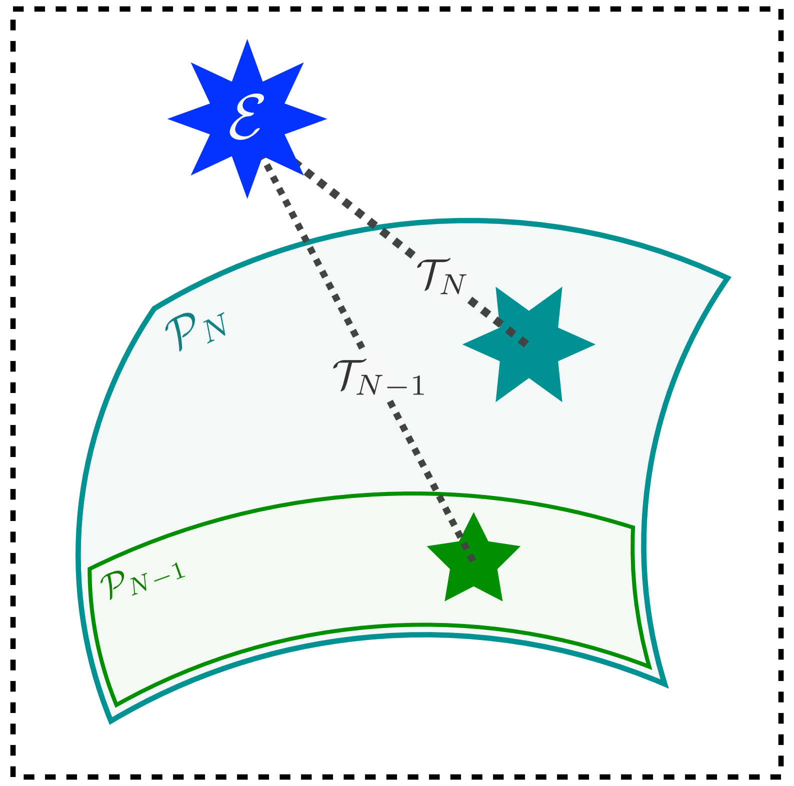
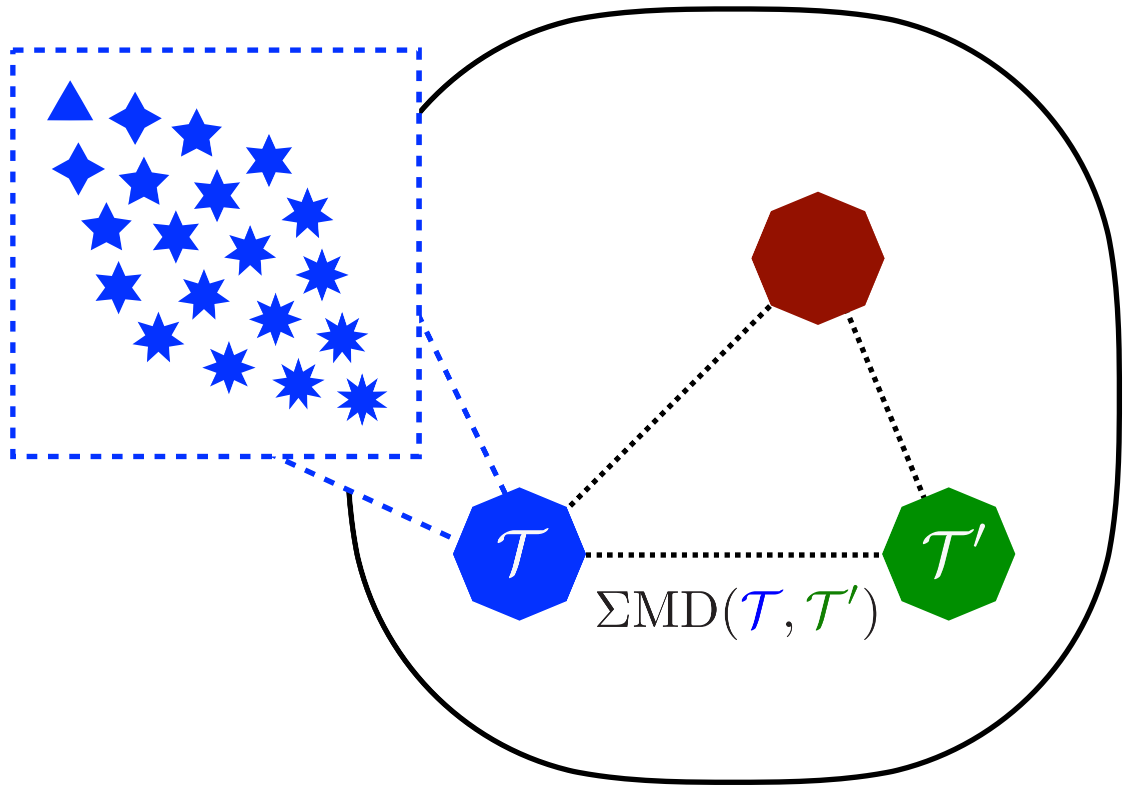
How else can ΣMD and theory space be utilized?

Perhaps Monte Carlo tuning/benchmarking…







The (Metric) Space of Events

– Energy flow is theoretically and experimentally robust

– EMD metrizes the space of energy flows (events)

– Manifolds in the space of events can be visualized and quantified

Revealing Hidden Geometry

– Event space exhibits a rich geometry that can be probed using the EMD

– Decades worth of collider techniques are naturally described in this geometry

– Many new techniques are suggested, and new light is shed on old ones

Theory Space

– Is rigorously constructed using the cross-section mover's distance ΣMD

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– Allows for theories to be explored with tools developed for events

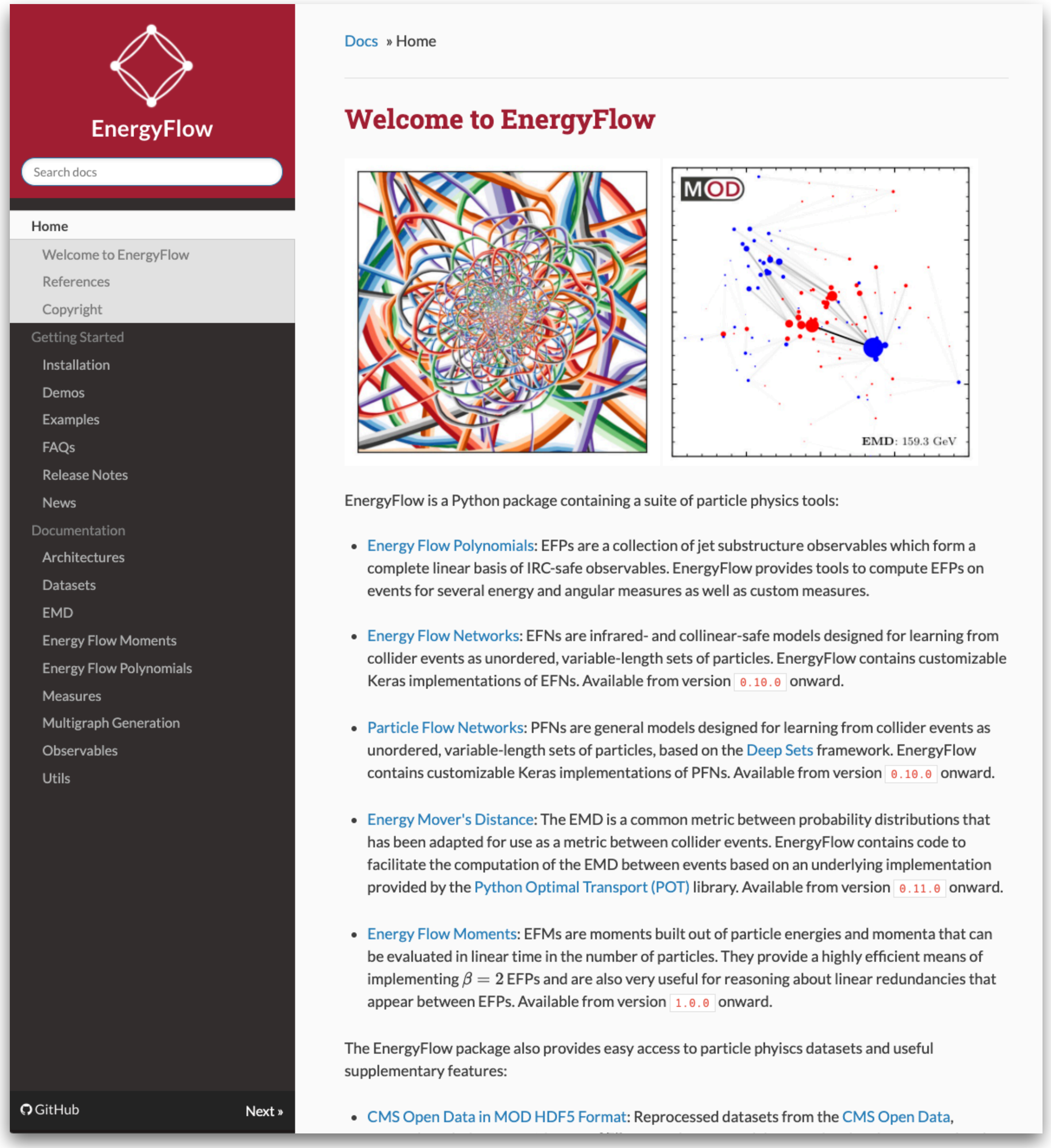
EnergyFlow Python Package

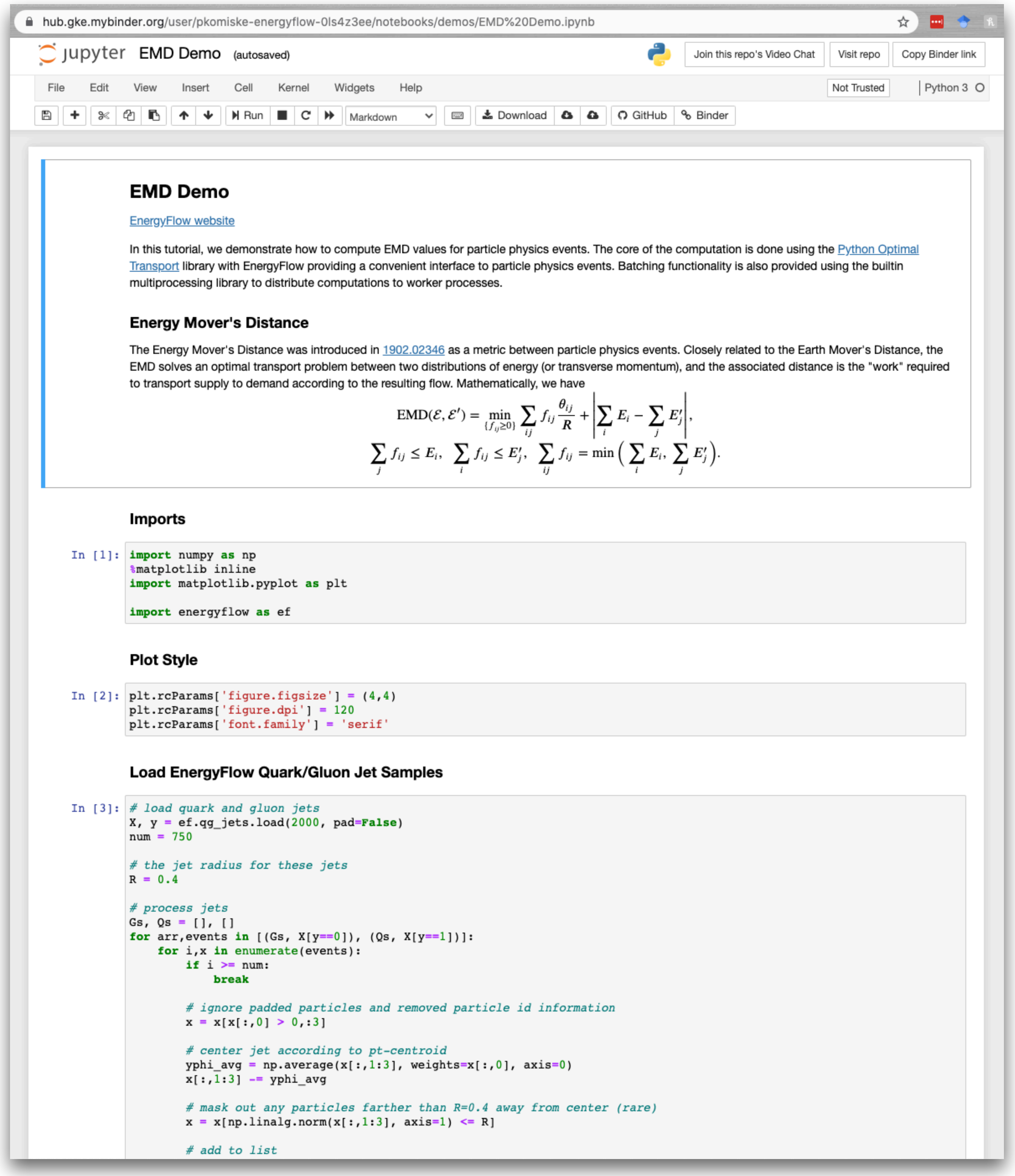
pip3 install energyflow

Parallelized EMD calculations via the Python Optimal Transport library

Detailed examples, demos, and documentation

Interfaces with CMS 2011A Jet Primary Dataset hosted on Zenodo





Additional Slides

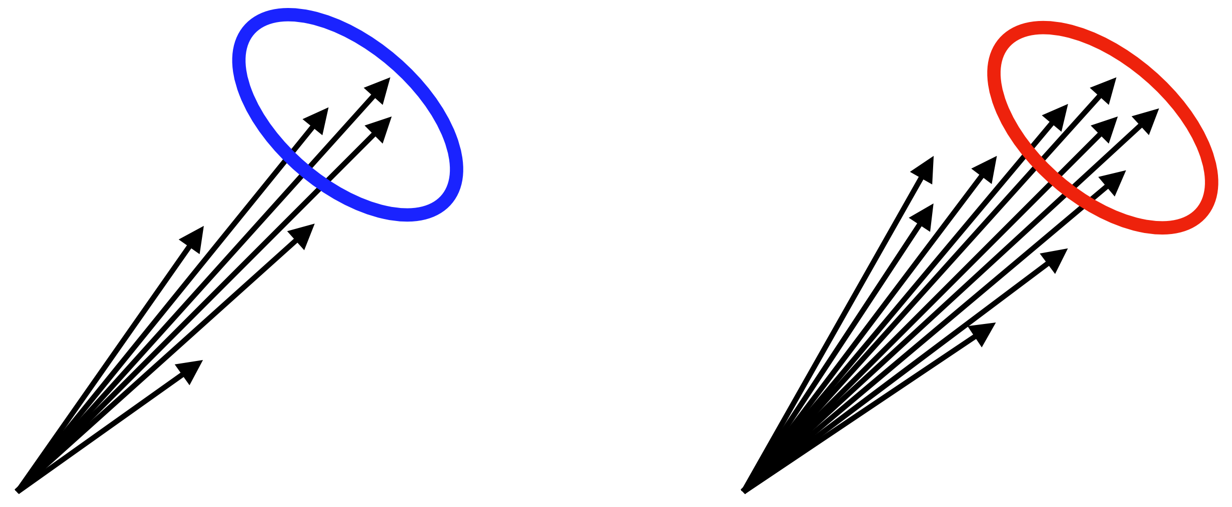
Quark and Gluon Correlation Dimensions

$$\dim(Q) = Q\frac{\partial}{\partial Q}\ln\sum_i\sum_j\Theta(\mathrm{EMD}(\mathcal{E}_i,\mathcal{E}'_j) < Q)$$

Leading log (single emission) calculation:

$$\dim_i(Q) \simeq -\frac{8\alpha_s}{\pi}C_i\ln\frac{Q}{p_T/2}$$

color factor

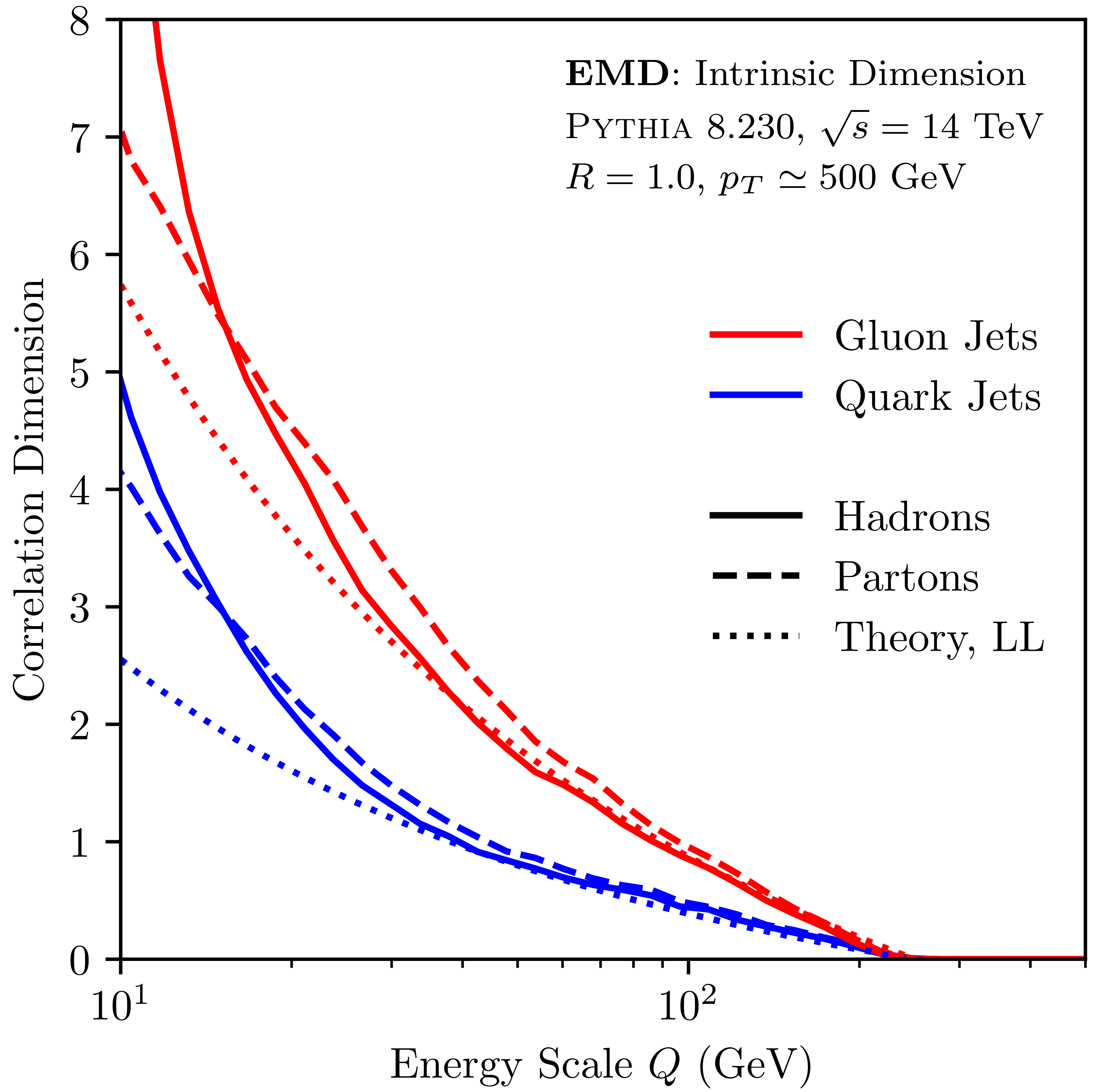


Quark

Gluon

$C_F = 4/3$

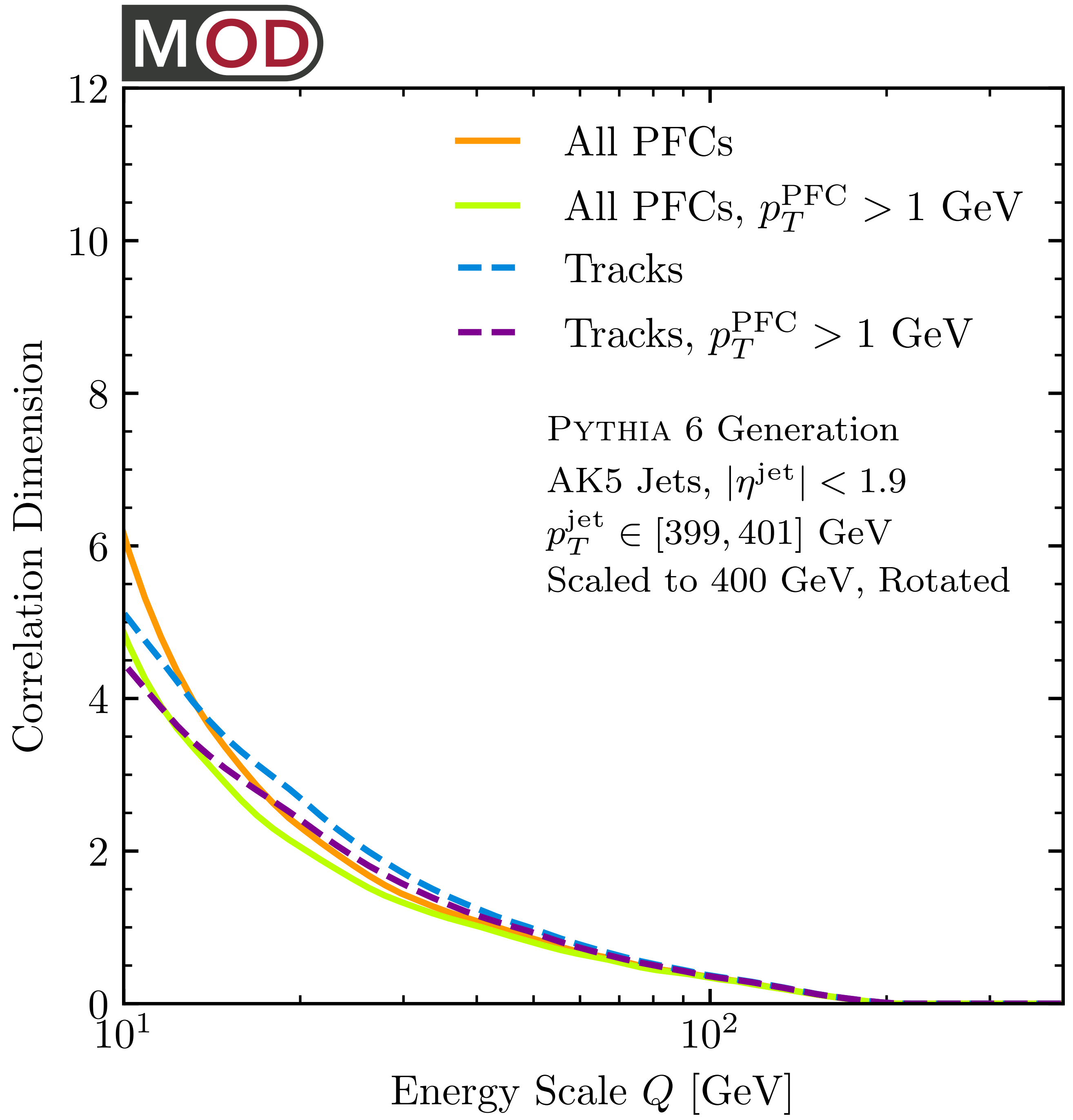
$C_A = 3$



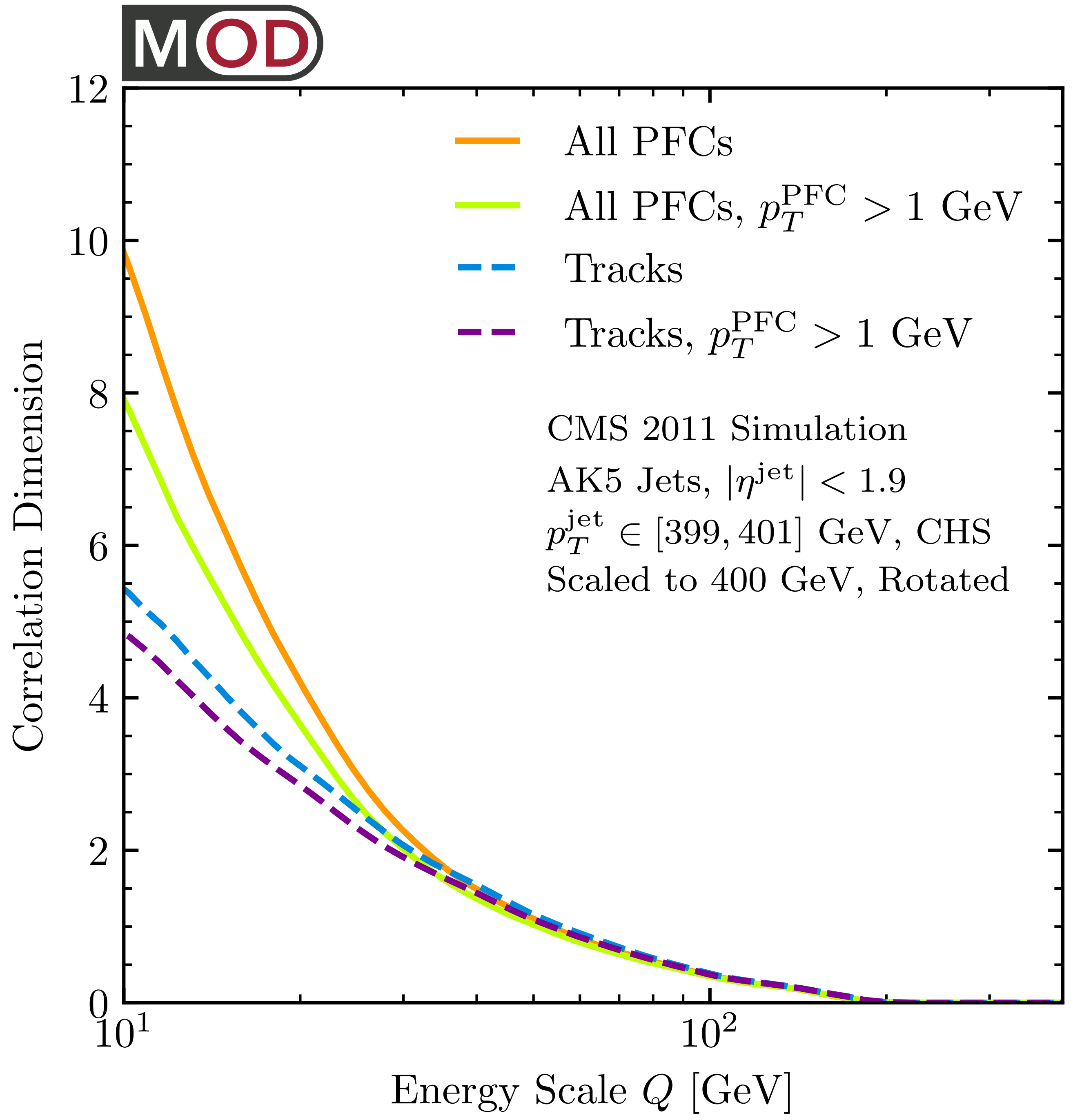
[PTK, Metodiev, Thaler, *to appear soon*]

Correlation Dimension at Particle and Detector Levels

Particle-level (PYTHIA)



Detector-level (PYTHIA + GEANT 4)



CMS Open Data

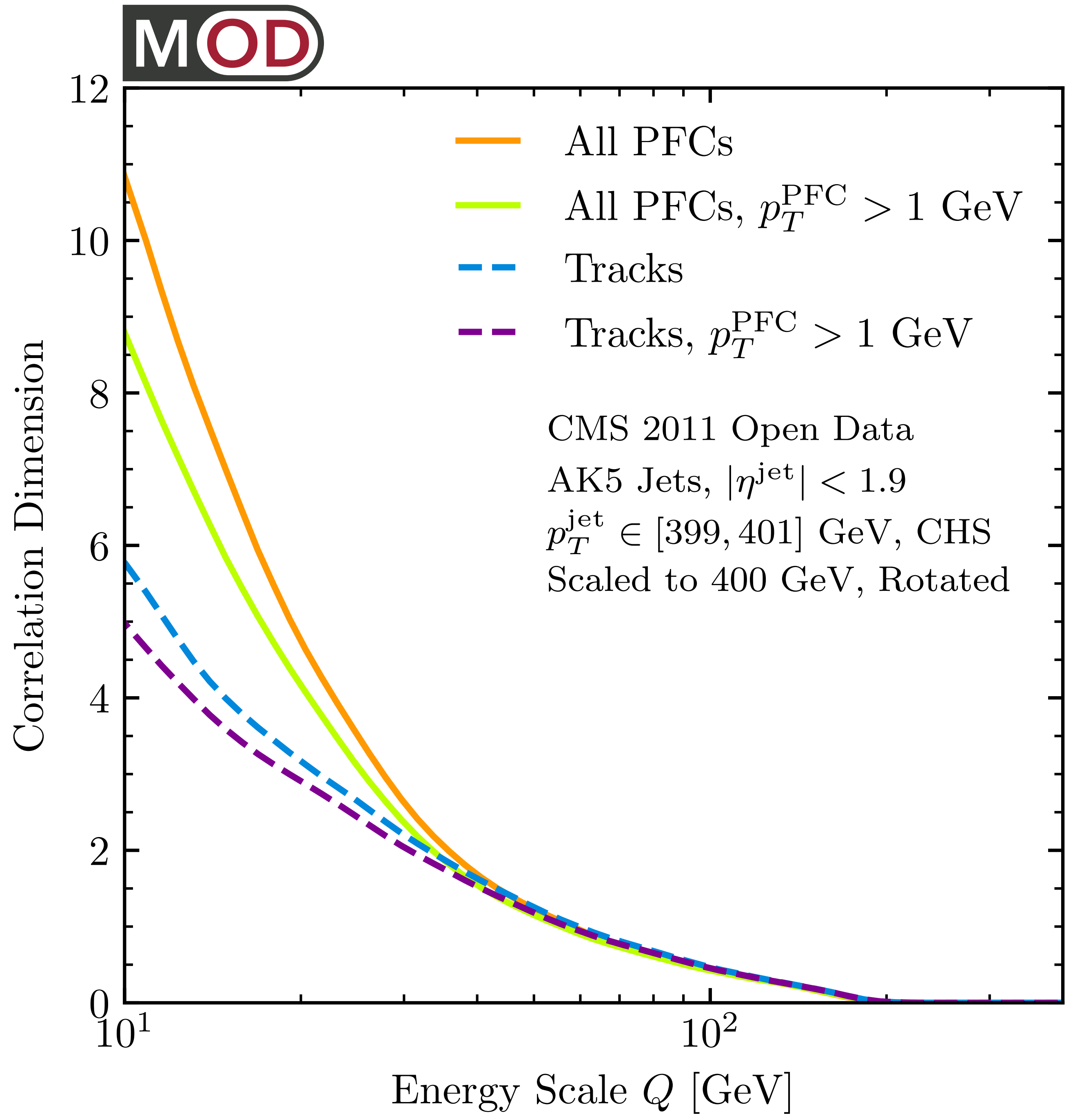
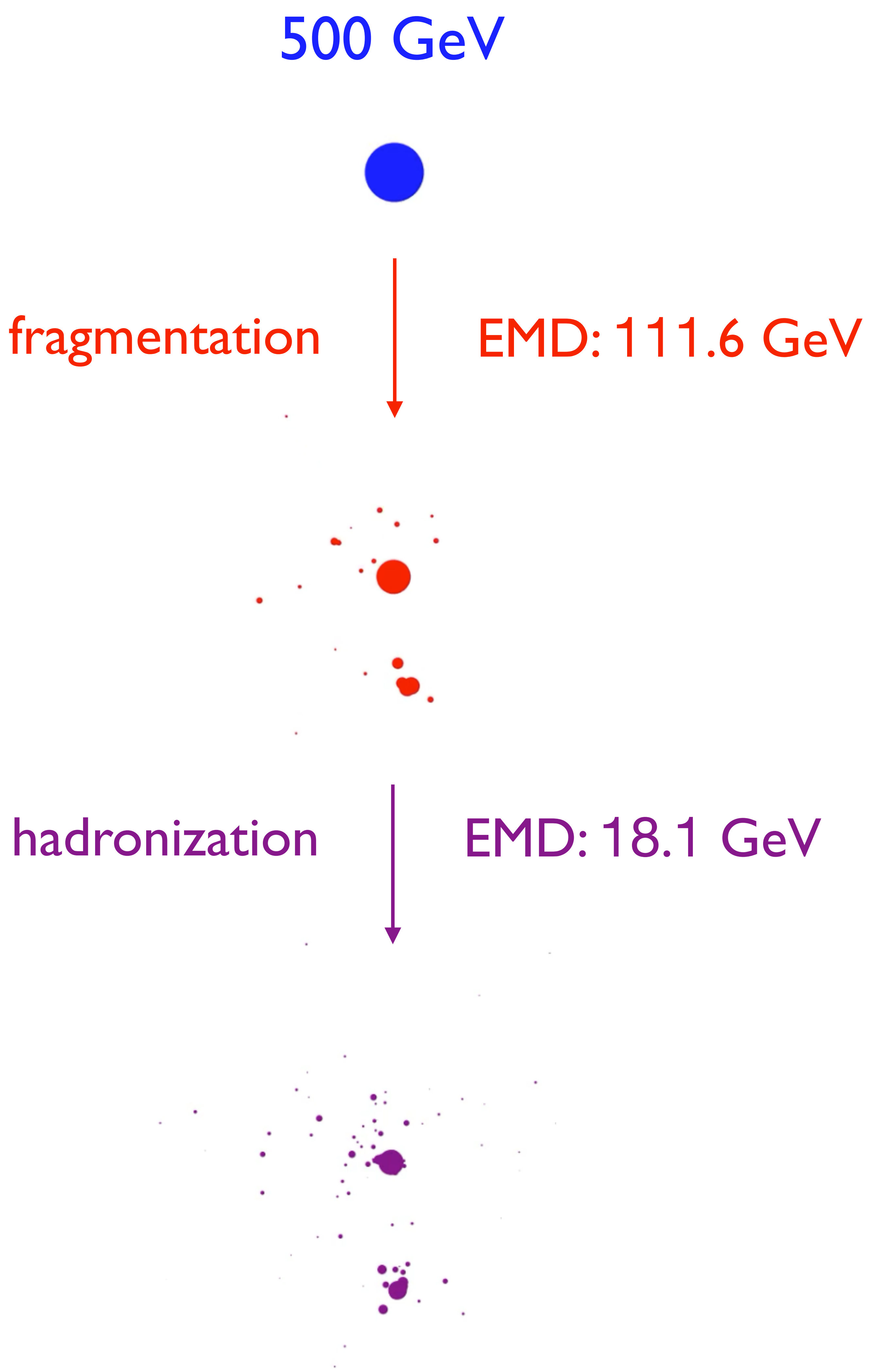


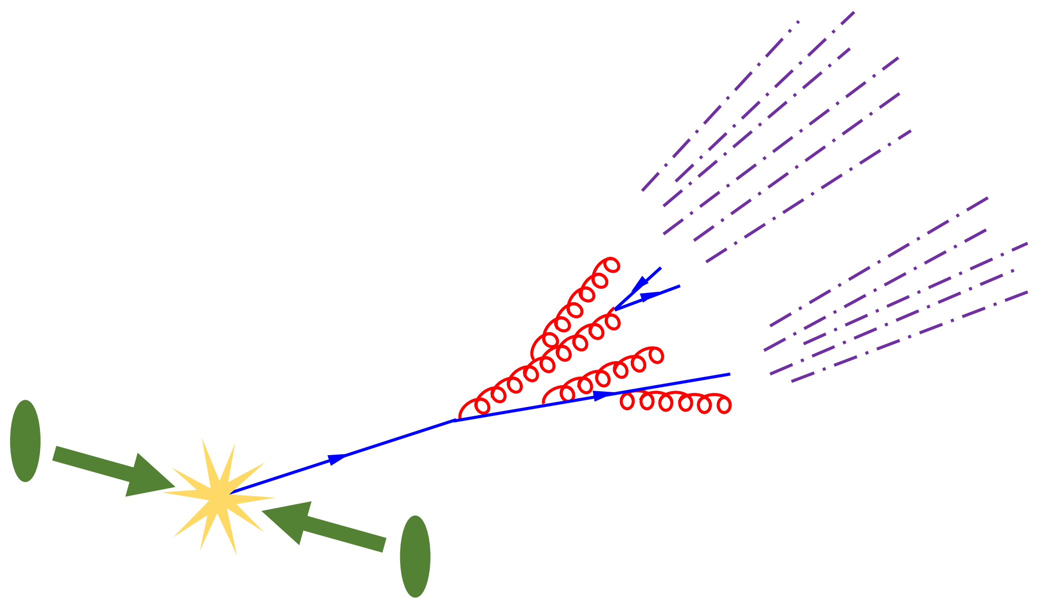
Table of Observables Defined via Event Space Geometry

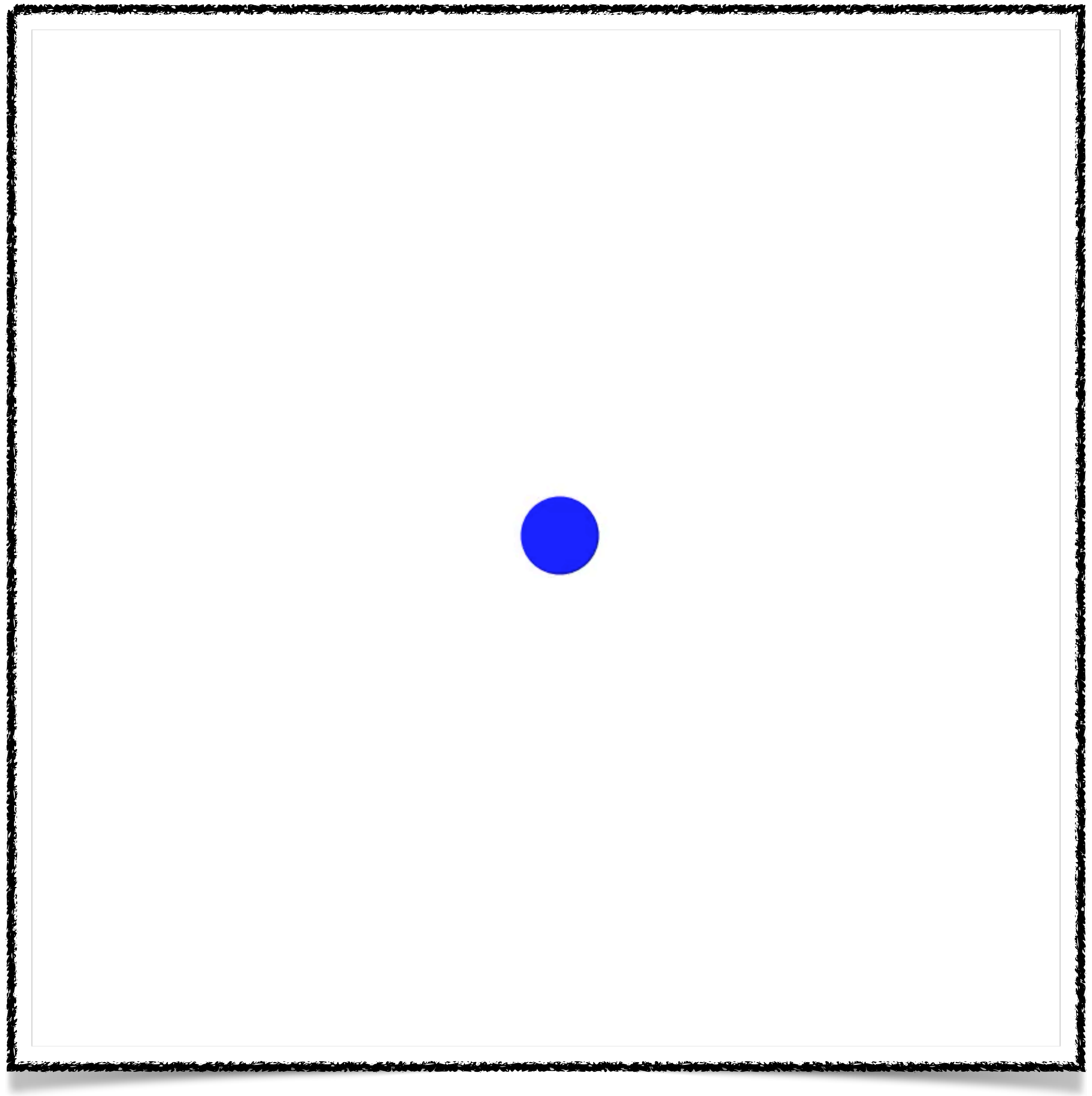
$$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \text{EMD}_{\beta,R}(\mathcal{E}, \mathcal{E}')$$

Name	$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \text{EMD}_{\beta}(\mathcal{E}, \mathcal{E}')$	β	Manifold $\mathcal{M}$
Thrust	$t(\mathcal{E})$	2	$\mathcal{P}_2^{\text{BB}}$: 2-particle events, back to back
Spherocity	$\sqrt{s(\mathcal{E})}$	1	$\mathcal{P}_2^{\text{BB}}$: 2-particle events, back to back
Broadening	$b(\mathcal{E})$	1	$\mathcal{P}_2$: 2-particle events
N-jettiness	$\mathcal{T}_N^{(\beta)}(\mathcal{E})$	β	$\mathcal{P}_N$: N-particle events
Isotropy	$\mathcal{I}^{(\beta)}(\mathcal{E})$	β	$\mathcal{M}_{\mathcal{U}}$: Uniform events
Jet Angularities	$\lambda_\beta(\mathcal{J})$	β	$\mathcal{P}_1$: 1-particle jets
N-subjettiness	$\tau_N^{(\beta)}(\mathcal{J})$	β	$\mathcal{P}_N$: N-particle jets

Visualizing Jet Formation – QCD Jets







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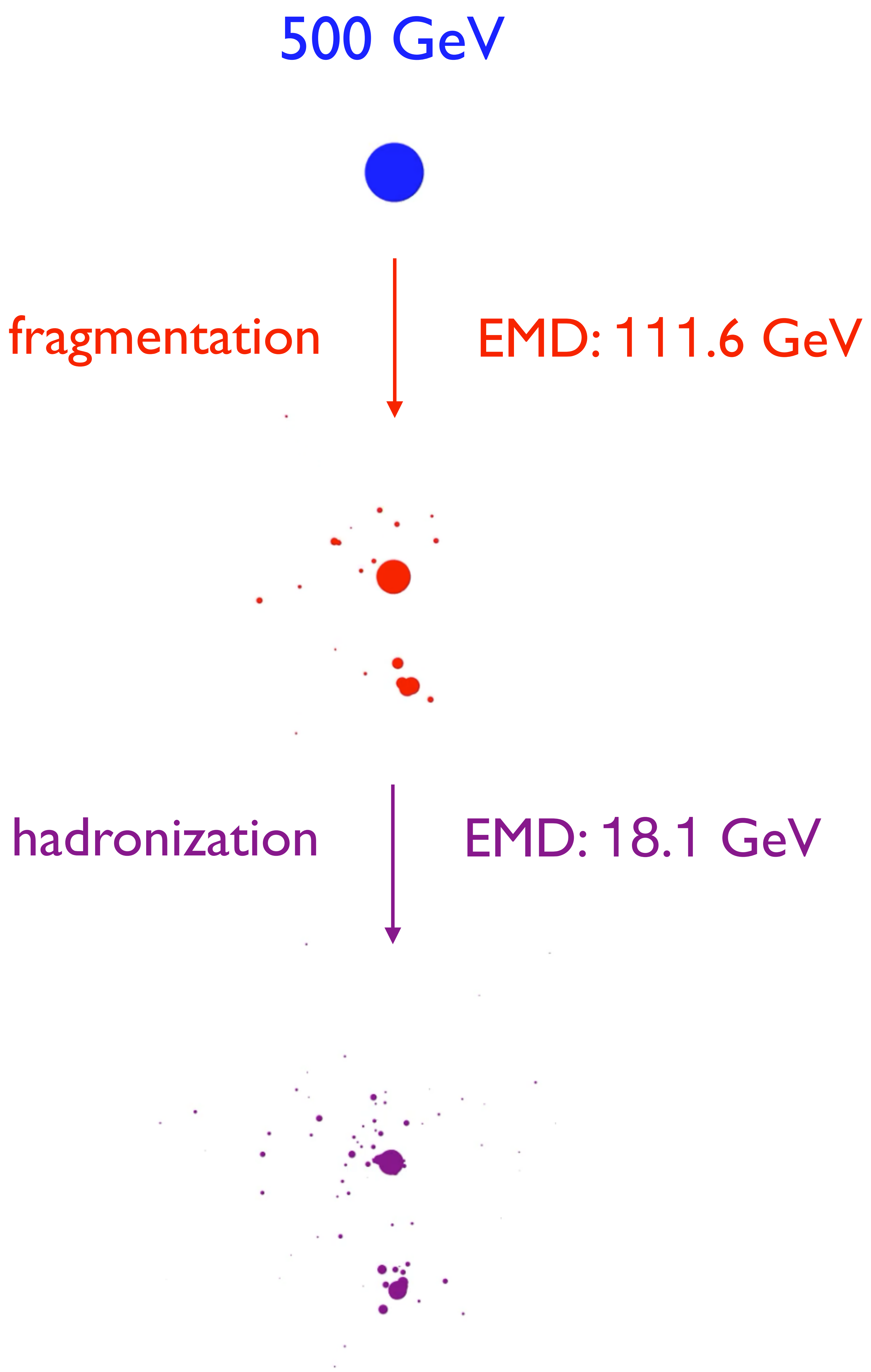
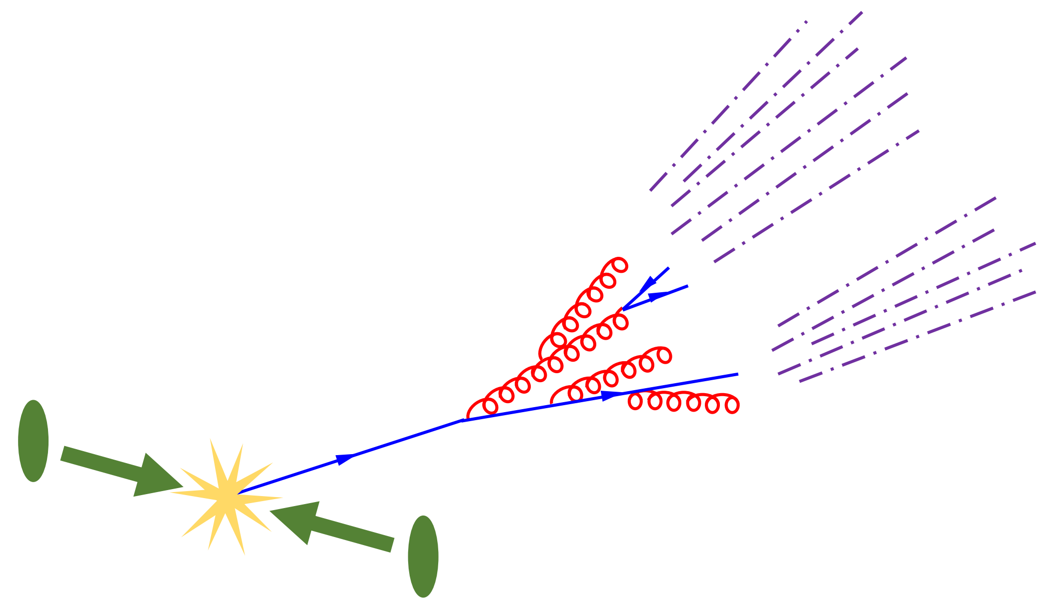
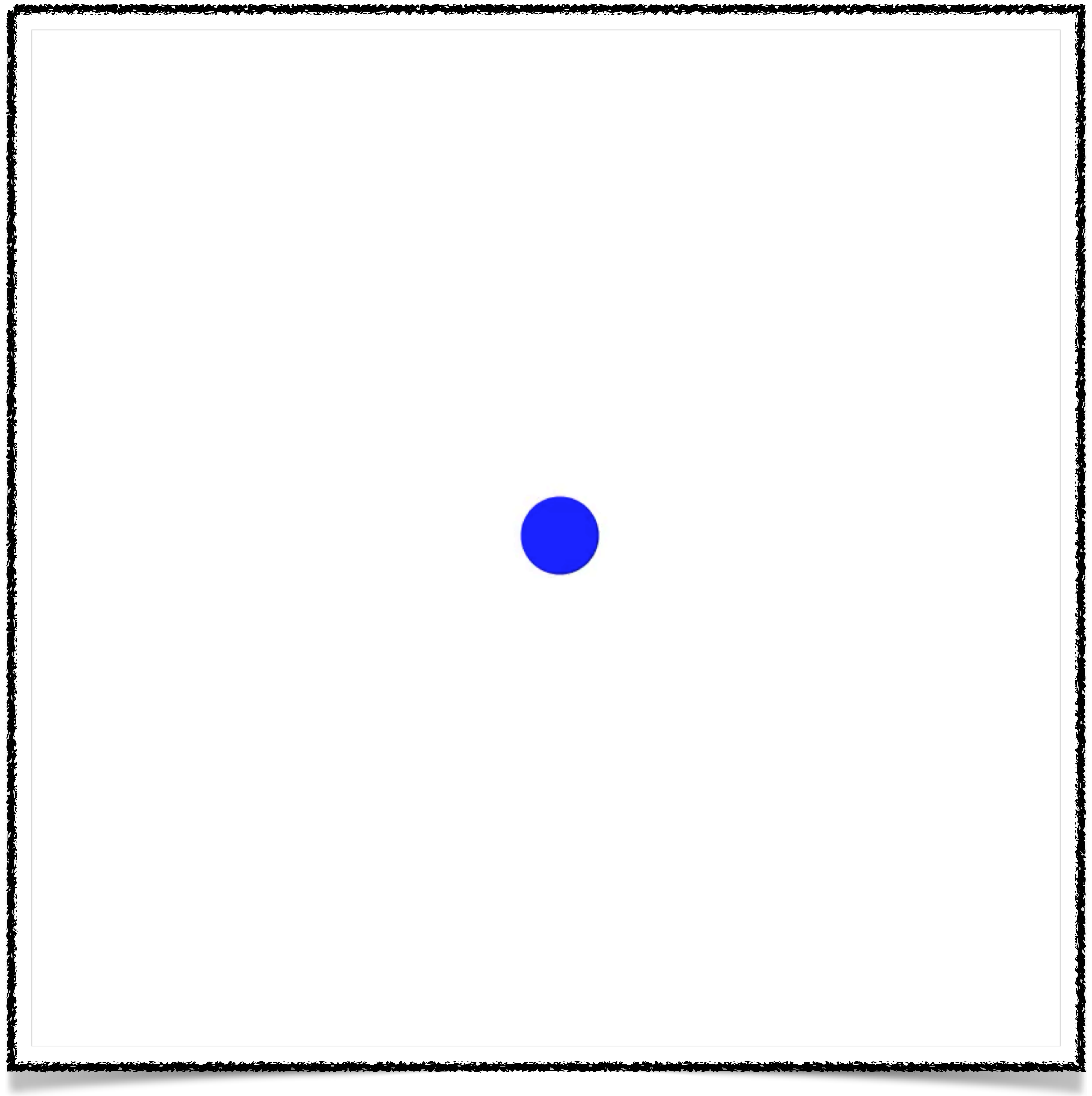
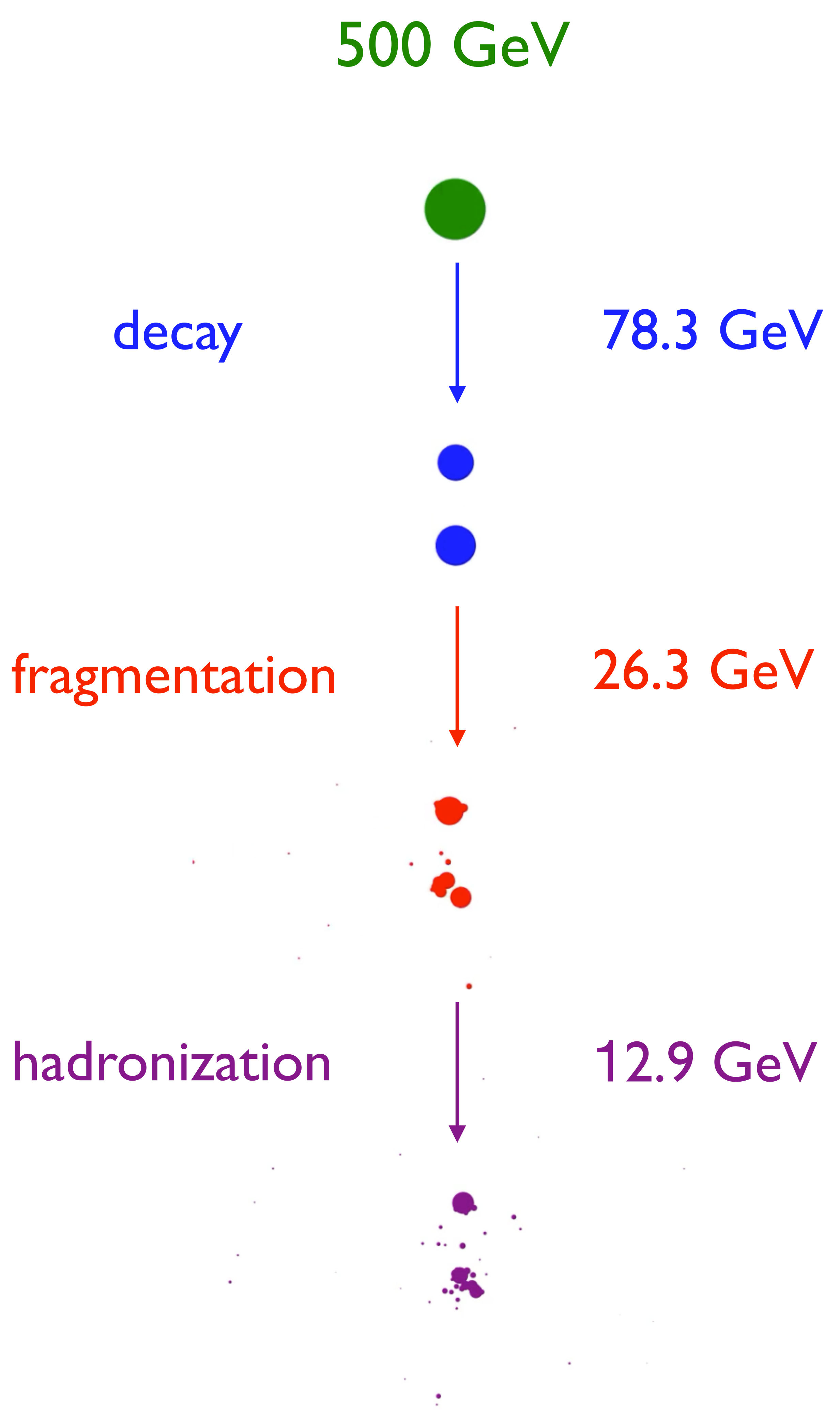
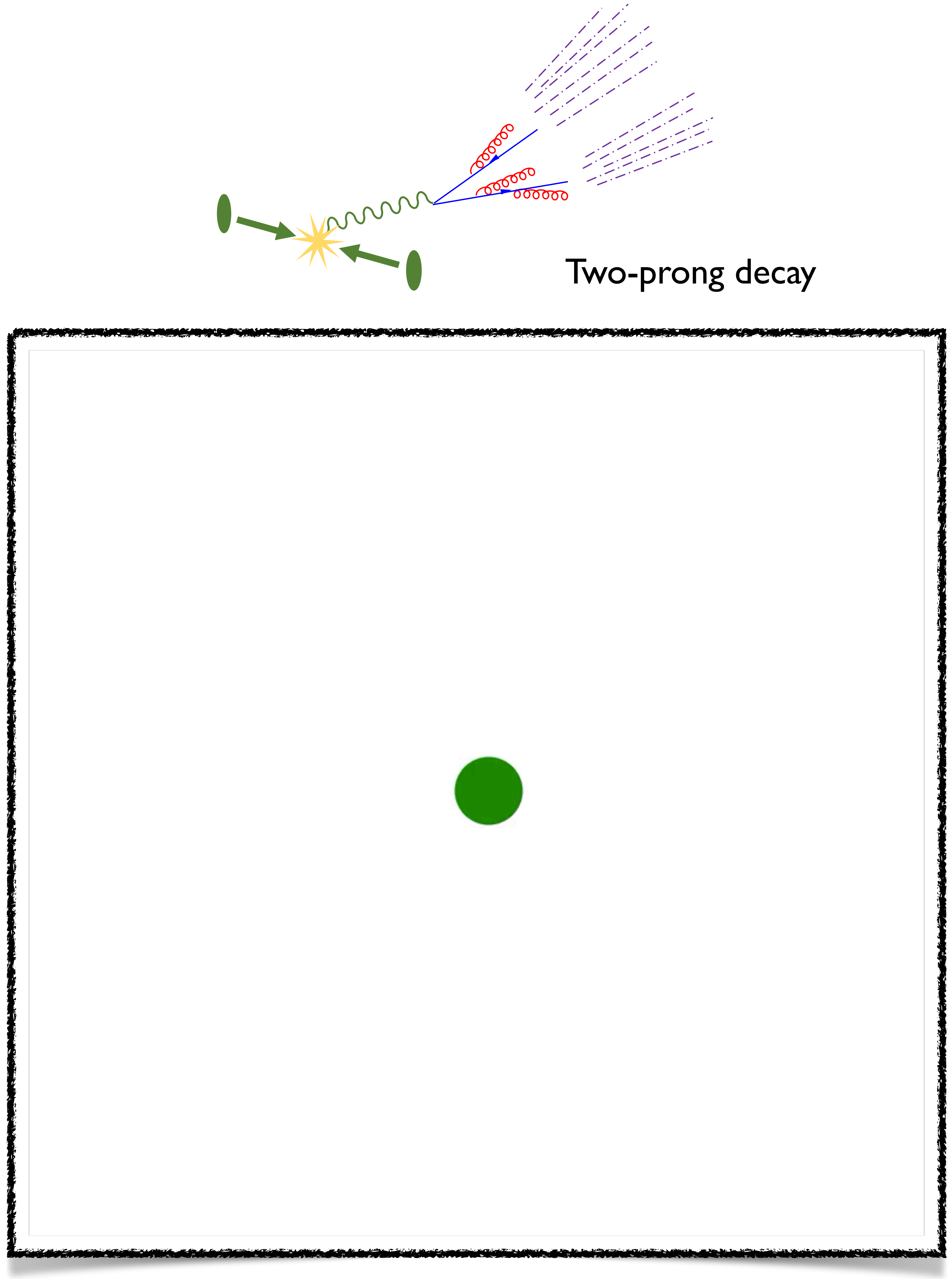






Visualizing Jet Formation – W Jets





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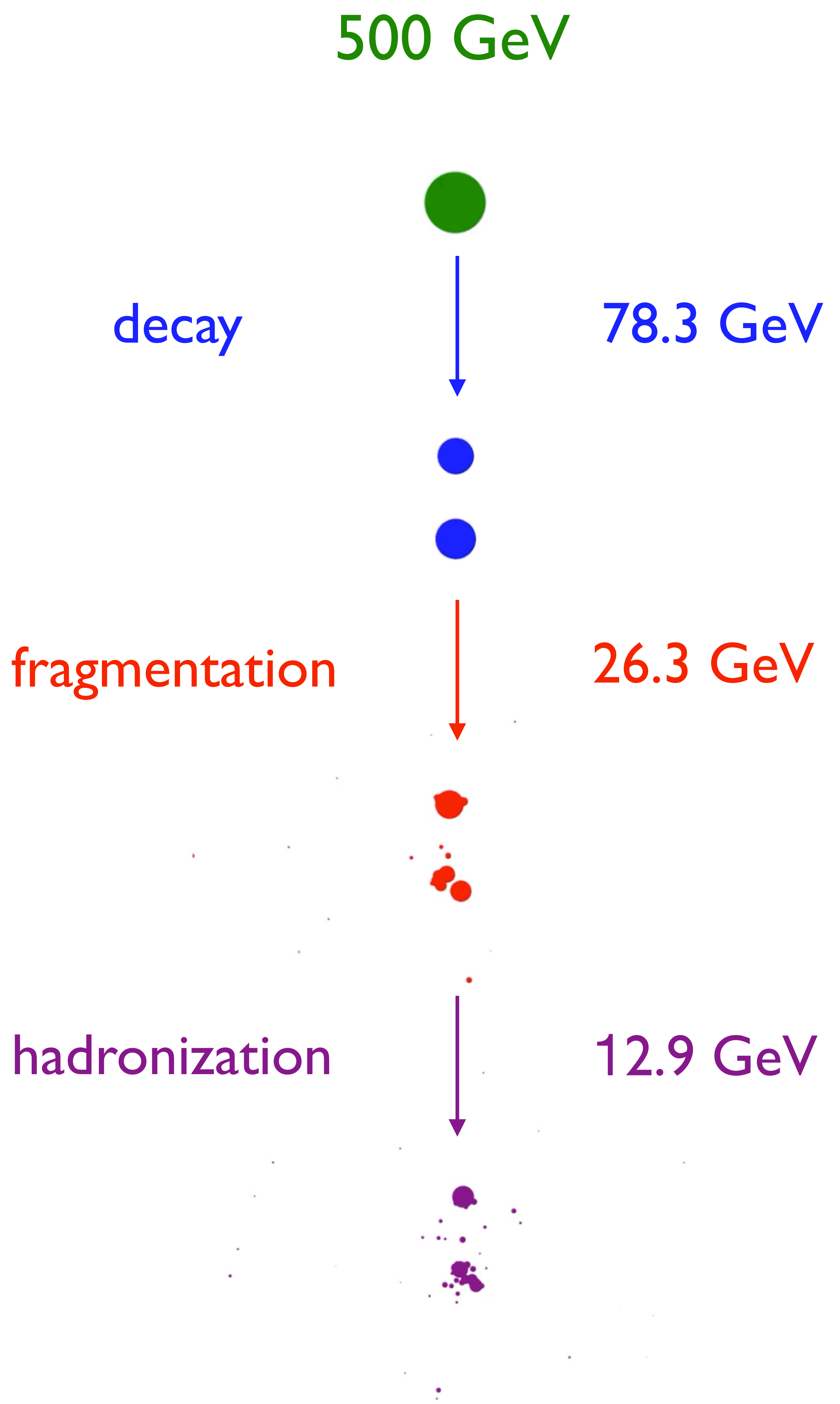
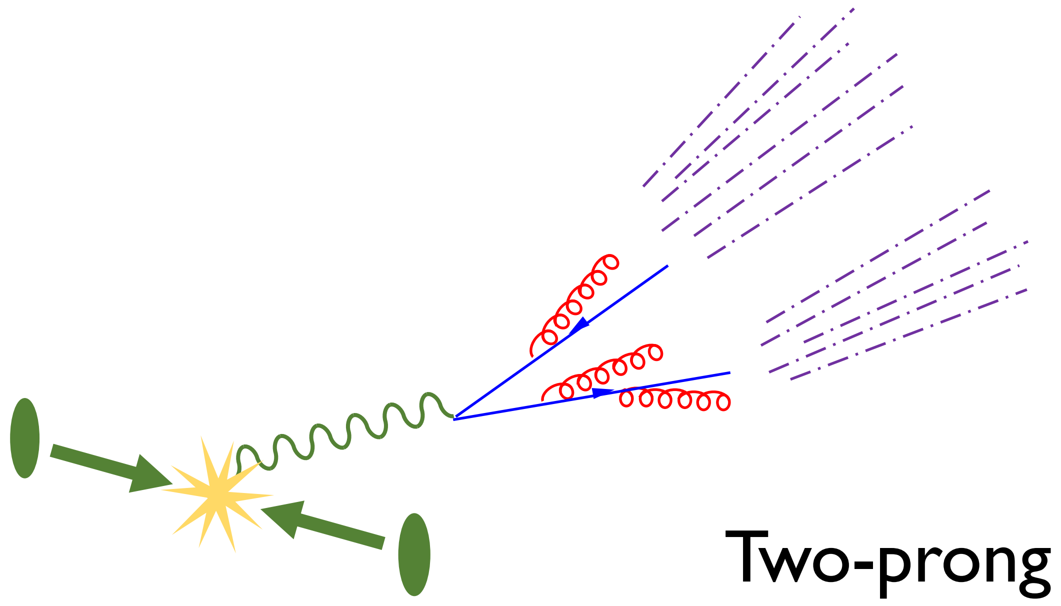
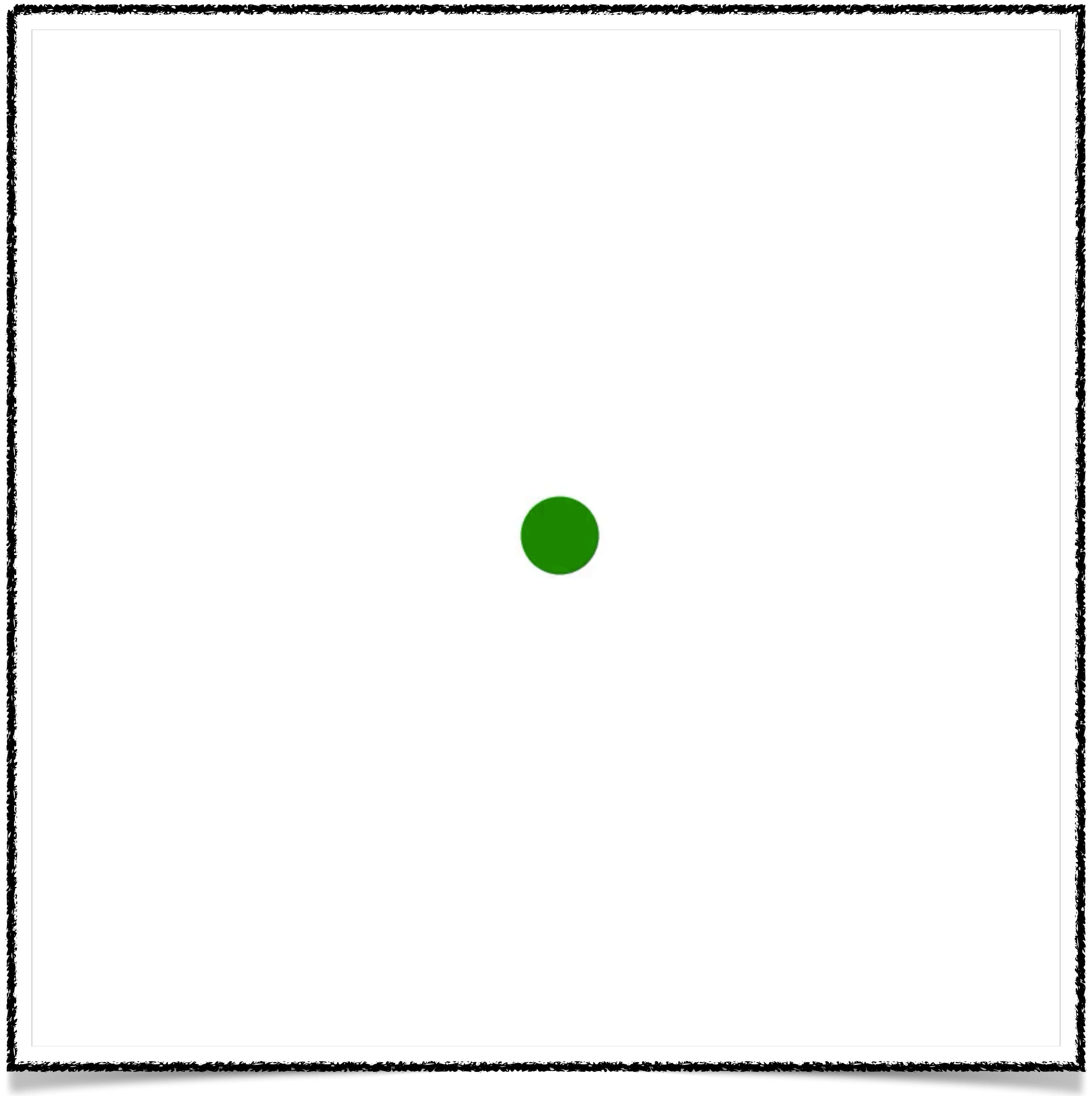
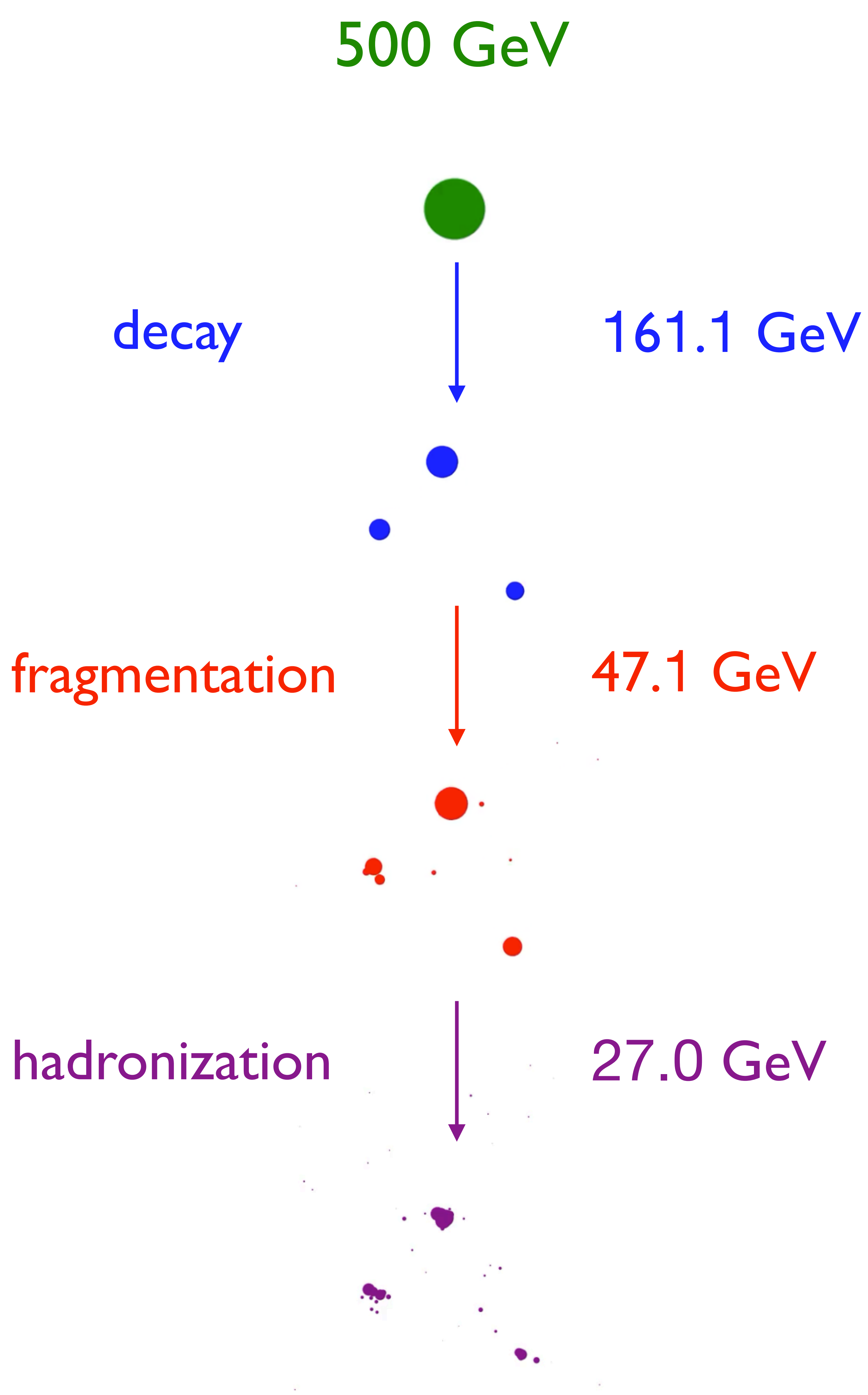


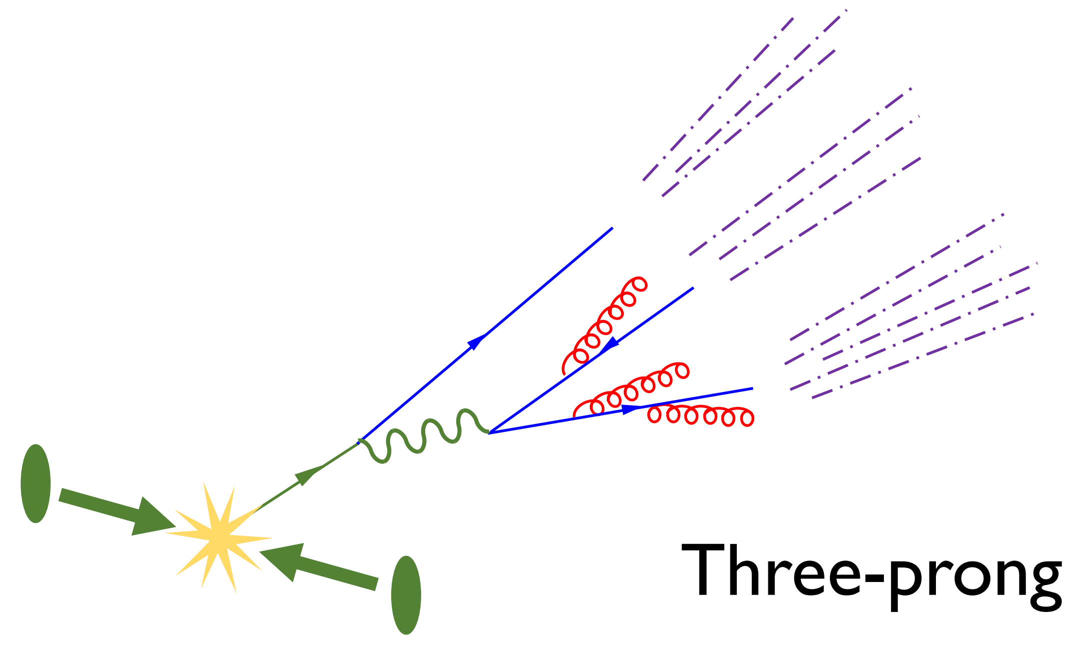


Two-prong decay

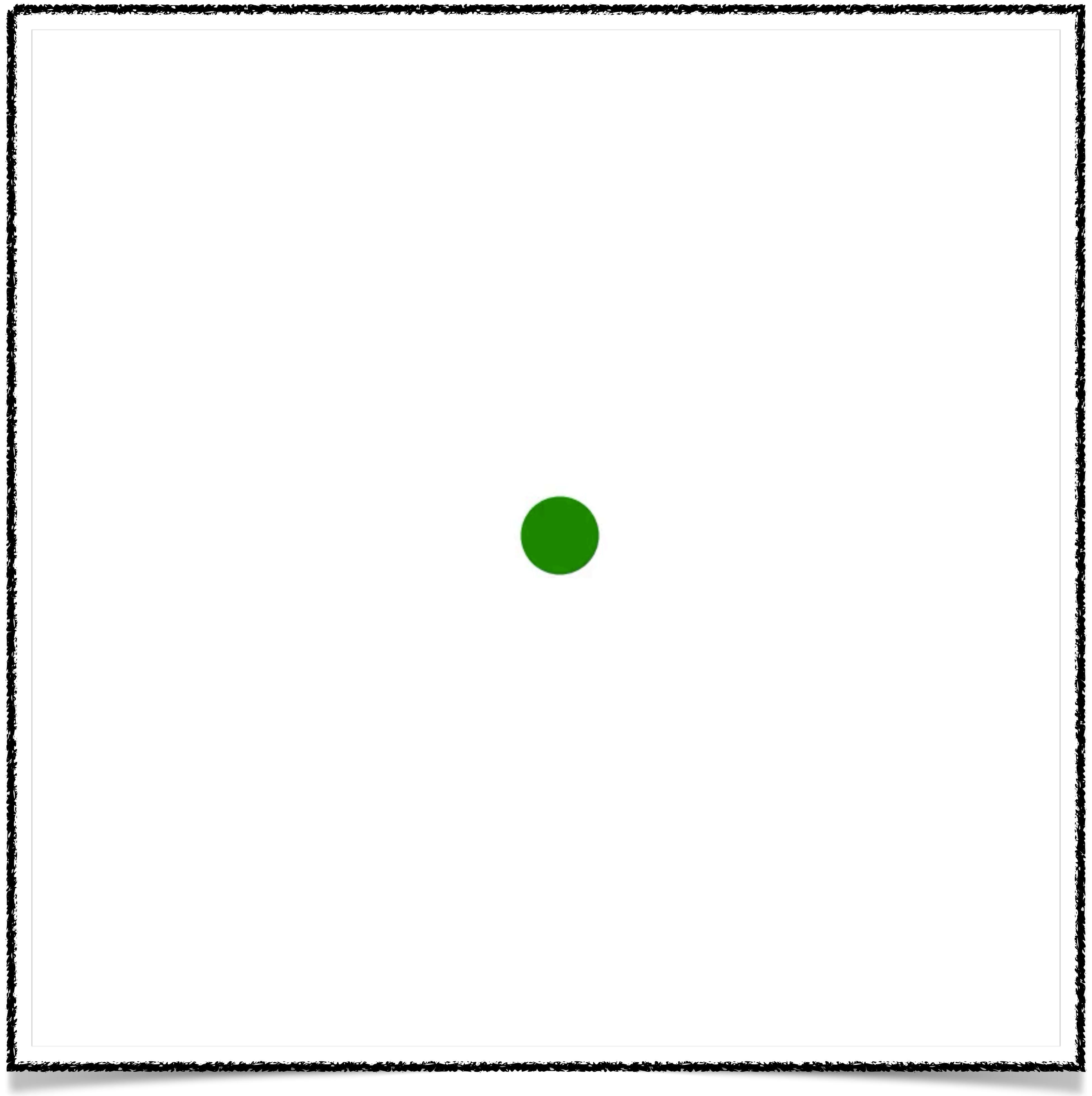


Visualizing Jet Formation – Top Jets

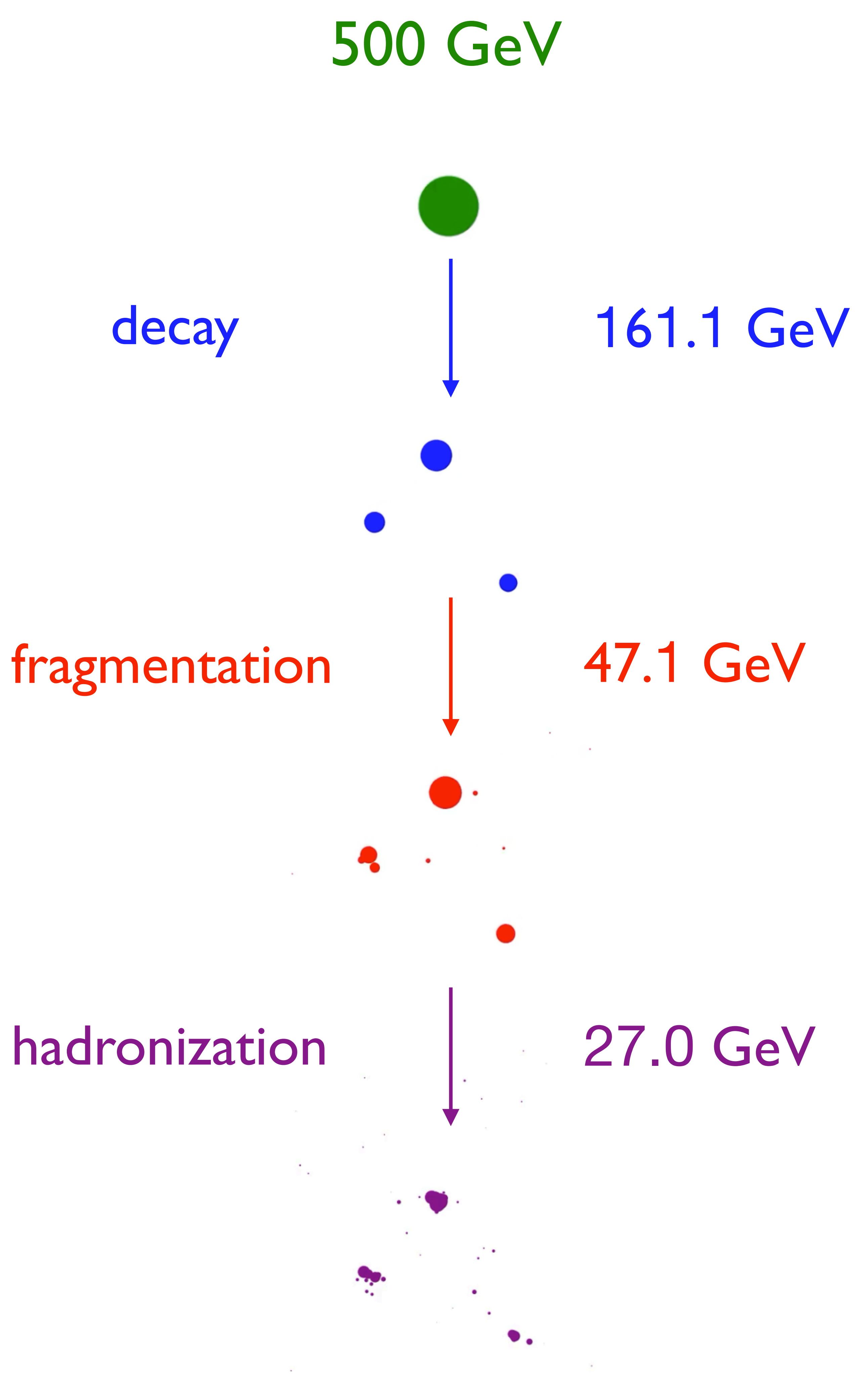


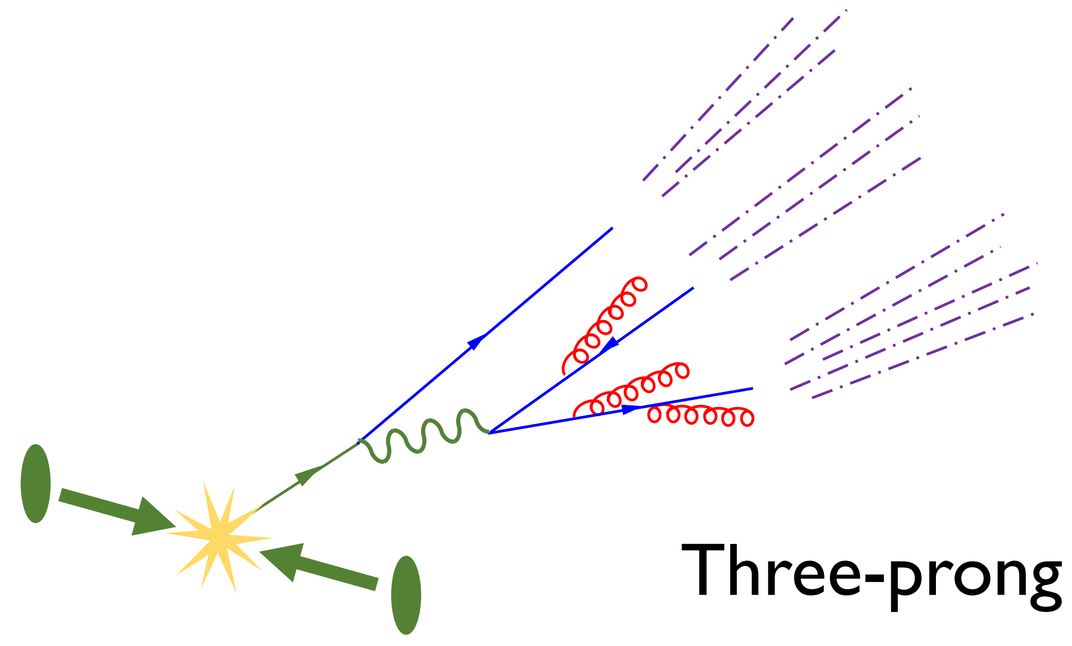


Three-prong decay



Visualizing Jet Formation – Top Jets





Three-prong decay