Linear Jet Tagging with the Energy Flow Basis

Machine Learning for Jet Physics Workshop

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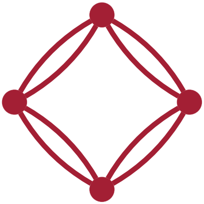
Based on work with Eric M. Metodiev and Jesse Thaler

Outline

Lightning Review of EFPs



Linear Classification with EFPs

Comparison with Modern Machine Learning

Fast Computation of EFPs

EFP Essentials

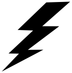
$$\mathrm{EFP}_G = \sum_{i_1=1}^{M} \cdots \sum_{i_N=1}^{M} z_{i_1} \cdots z_{i_N} \prod_{(k,\ell)\in G} \theta_{i_k i_\ell} \tag{1}$$

$$z_i = \frac{p_{T,i}}{\sum_i p_{T,i}}, \quad \theta_{ij} = (\Delta y_{ij}^2 + \Delta\phi_{ij}^2)^{\beta/2}$$

See Eric Metodiev's talk

- Jet substructure observables:
  - Fixed energy structure (necessary and sufficient for IRC safety)
  - Angular structure encoded by a multigraph G
  - Related to jet mass, ECFs, angularities, geometric moments, etc.
- Jet representation:
  - Complete, linear basis for IRC-safe information
  - Linearity of basis naturally suggests linear learning methods

Outline



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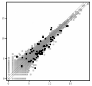
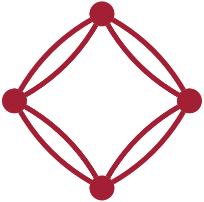


Linear Classification with EFPs



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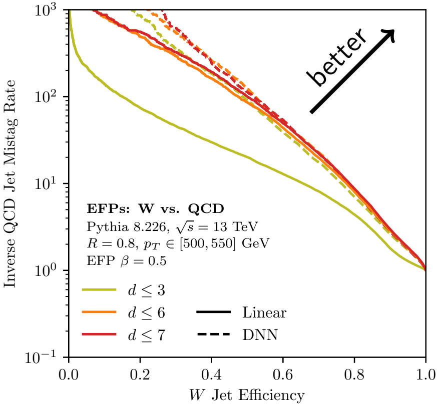
Linear Classification Overview

- Fit a decision plane, determined by a vector $\boldsymbol{w}$
  - Fisher's linear discriminant (LDA): closed-form solution
  - Logistic regression: Convex, iterative solution
- Decision threshold t is determined by distance from the plane
- $\mathcal{G}$ is finite set of graphs corresponding to the inputs
  - Organization by d is natural (equivalent to the order of the expansion)
  - Organization by N or χ also possible, (where is the information?)

$$\text{Classifier} = \left\{ \left(t + \sum_{G \in \mathcal{G}} w_G \text{EFP}_G \right) \begin{array}{ll} \geq 0, & \text{signal} \\ < 0, & \text{background} \end{array} \right.$$

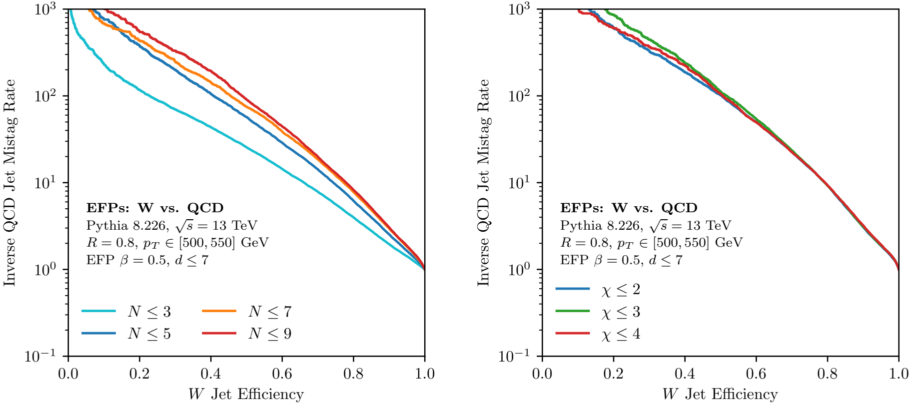
Linear Classification with EFPs

- W vs. QCD jet classification (quark/gluon and top tagging in backup)



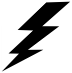
- 300k training samples
- Linear: Fisher's linear discriminant
  - num. params. = num. EFPs < 1000
  - 100k test samples
- DNN: Dense neural net
  - (100 node fully-connected layer) $\times$ 3
  - $\sim$ 120k parameters
  - 50k validation, 50k test samples

Which EFPs are Important?



- High-N EFPs are important for classification performance
- Great classification performance with just $\chi = 2$ EFPs!

Outline



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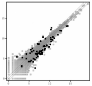

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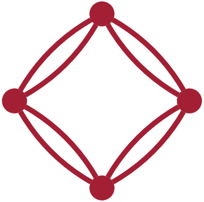

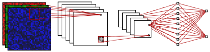

Comparison with Modern Machine Learning

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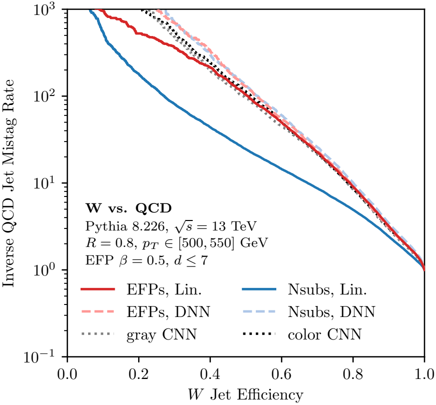
Modern Machine Learning Comparison

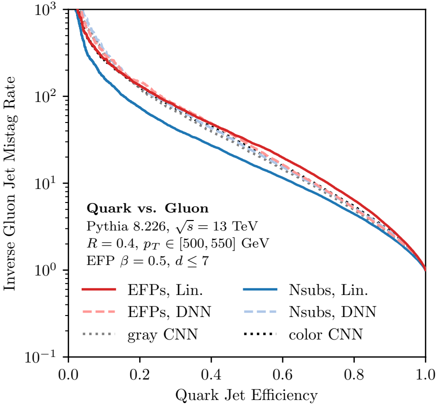


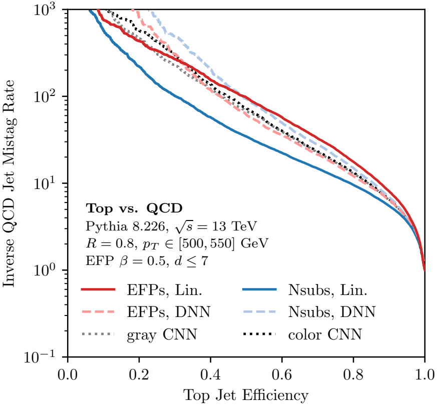
- Linear and DNN same as before
- CNN: Convolutional neural net
  - 33×33 jet images
  - (48 filters) $\times\, 3$
  - gray: p_T channel only
  - color: p_T and mult. channels
  - ~ 80k parameters

- (Linear classification with EFPs) $\sim$ (MML) for $\varepsilon_s \gtrsim 0.5$!

N-subjetiness: 1011.2268, N-subjetiness basis: 1704.08249, NN Review: 1709.04464

Modern Machine Learning Comparison





- (Linear classification with EFPs) $\gtrsim$ (MML) for $\varepsilon_s \gtrsim 0.5$

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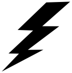

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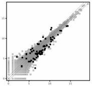
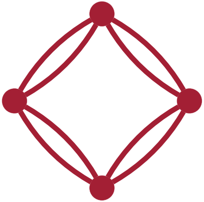
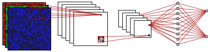


Linear Classification with EFPs





Comparison with Modern Machine Learning

Fast Computation of EFPs

Computational Complexity of ECF(G)s

$$\sum_{i_1=1}^{M} \cdots \sum_{i_N=1}^{M} E_{i_1} \cdots E_{i_N} \begin{cases} \prod_{i<j\in\{i_1,\ldots,i_N\}} \theta_{ij}^{\beta}, & \mathsf{ECF}_N^{\beta} \quad \text{1305.0007} \\ \prod_{m=1}^{v} \min_{(m)}\{\theta_{ij}^{\beta}\}_{i<j\in\{i_1,\ldots,i_N\}}, & {}_v\mathsf{ECFG}_N^{\beta} \quad \text{1609.07483} \end{cases}$$

- Implementation of ECF(G) formula runs in time $\mathcal{O}(M^N)$
- With $M \sim 100$, ECF(G)$_{N=4}$ $\sim$ one hundred million operations
- $N = 4$ is barely tractable, $N \geq 5$ is essentially inaccessible

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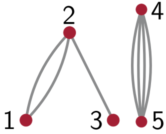
```
// if N > 5, then throw error
if (_N > 5) {
    throw Error("EnergyCorrelator is only hard coded for N = 0,1,2,3,4,5");
}
```

fjcontrib – EnergyCorrelator 1.2.0

Computational Complexity of EFPs

- Like other energy correlators, EFPs are naively $\mathcal{O}(M^N)$
- *Factorability* of summand in EFP formula can speed up computation



$$= \left(\sum_{i_1=1}^{M}\sum_{i_1=1}^{M}\sum_{i_3=1}^{M} z_{i_1} z_{i_2} z_{i_3} \theta_{i_1 i_2}^2 \theta_{i_2 i_3}\right)\left(\sum_{i_4=1}^{M}\sum_{i_5=1}^{M} z_{i_4} z_{i_5} \theta_{i_4 i_5}^4\right)$$

Composite EFPs are products of prime EFPs

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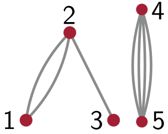


$$= \left(\sum_{i_1=1}^{M}\sum_{i_1=1}^{M}\sum_{i_3=1}^{M} z_{i_1} z_{i_2} z_{i_3} \theta_{i_1 i_2}^2 \theta_{i_2 i_3}\right)\left(\sum_{i_4=1}^{M}\sum_{i_5=1}^{M} z_{i_4} z_{i_5} \theta_{i_4 i_5}^4\right)$$

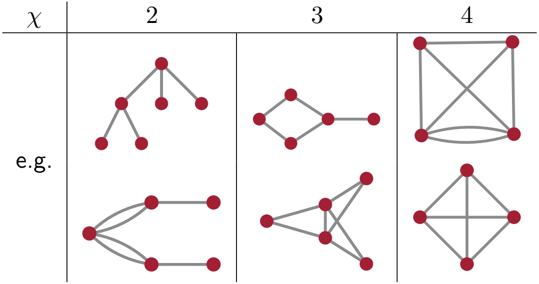
Composite EFPs are products of prime EFPs

$$= \underbrace{\sum_{i_1=1}^{M}\sum_{i_2=1}^{M}\sum_{i_3=1}^{M}\sum_{i_4=1}^{M}\sum_{i_5=1}^{M}\sum_{i_6=1}^{M}\sum_{i_7=1}^{M}\sum_{i_8=1}^{M} z_{i_1} z_{i_2} z_{i_3} z_{i_4} z_{i_5} z_{i_6} z_{i_7} z_{i_8} \theta_{i_1 i_2}\theta_{i_1 i_3}\theta_{i_1 i_4}\theta_{i_1 i_5}\theta_{i_1 i_6}\theta_{i_1 i_7}\theta_{i_1 i_8}}_{\mathcal{O}(M^8)}$$

$$= \underbrace{\sum_{i_1=1}^{M} z_{i_1}\left(\sum_{i_2=1}^{M} z_{i_2}\theta_{i_1 i_2}\right)^7}_{\mathcal{O}(M^2)}$$

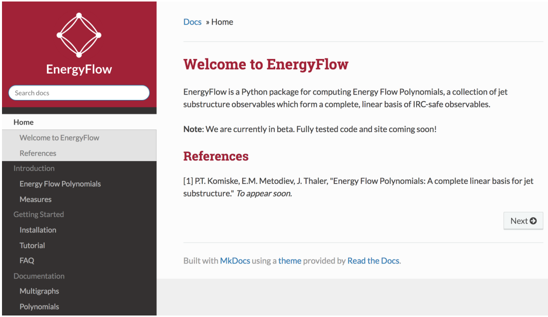
Variable Elimination (VE)

- Algorithm for finding optimal parentheses placement in EFP formula
- Reduces EFP computational complexity to $\mathcal{O}(M^{\chi})$:
  - Best case (NP-hard): $\chi = \texttt{treewidth}(G) + 1$
  - Heuristics can be used which work excellently for our small graphs
  - $\chi = 2$ for all tree graphs, $\chi = 3$ for single-cycle graphs, $\chi = N$ for K_N



EnergyFlow Python Package

- A convenient and simple package for efficient implementation of EFPs
- Currently written in pure Python using the NumPy library
  - Need a fast, arbitrary dimension multi-array
  - We've considered a C++ implementation (possible, but not simple)




```
pip install energyflow
```

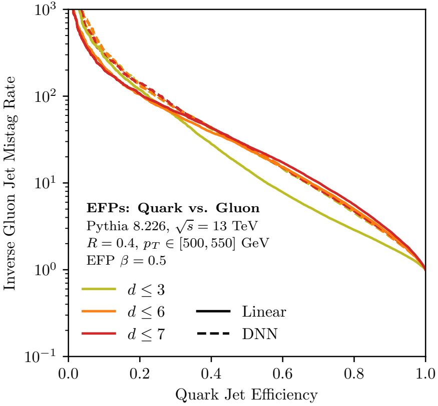
```python
import energyflow as ef
efps = ef.EFPSet('d<=7')
results = efps.compute(event)
```

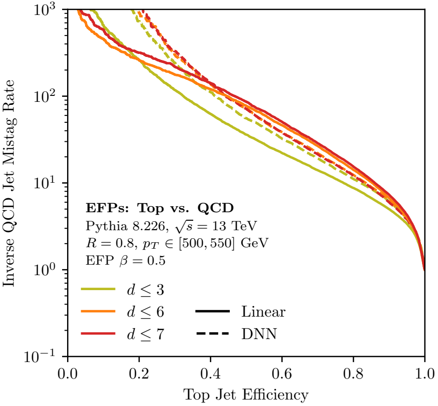
Conclusions

- Linear classification with EFPs very comparable to MML methods
- Linear methods $\implies$ very nice both theoretically and experimentally
  - EFP linear structure potentially allows for theoretical calculation
  - Fully differentiable model, uncertainty/error propagation simple
  - Convex, global minimum is guaranteed
  - No hyperparameters
  - Interesting methods made possible by linearity
    - Lasso regression for automatic feature selection
    - PCA, orthogonal subspaces, etc.
- Efficient computation of EFPs has been achieved
  - `EnergyFlow` Python package here, stay tuned for more
- EFPs potentially bridge MML performance & theory understanding

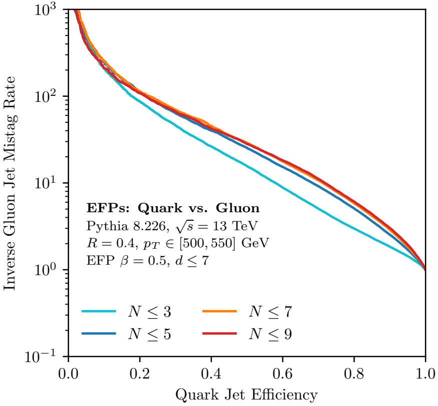
Additional Slides

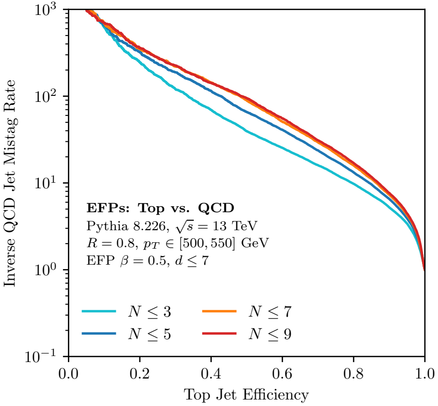
Quark/Gluon, Top Linear Classification with EFPs



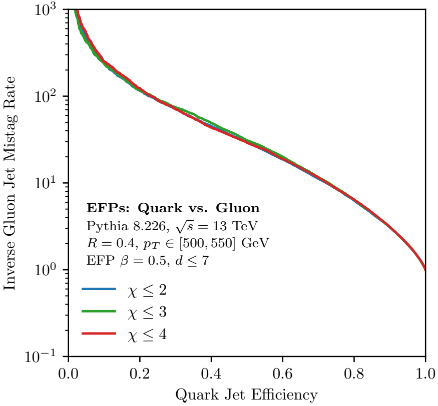


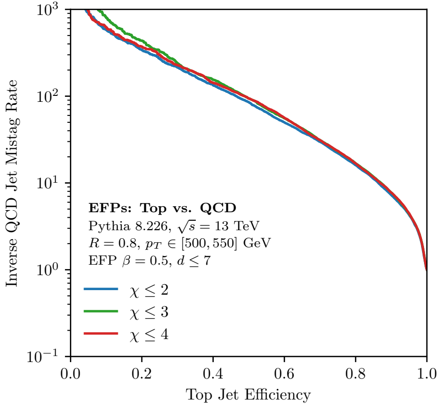
Quark/Gluon and Top Tagging N Sweep



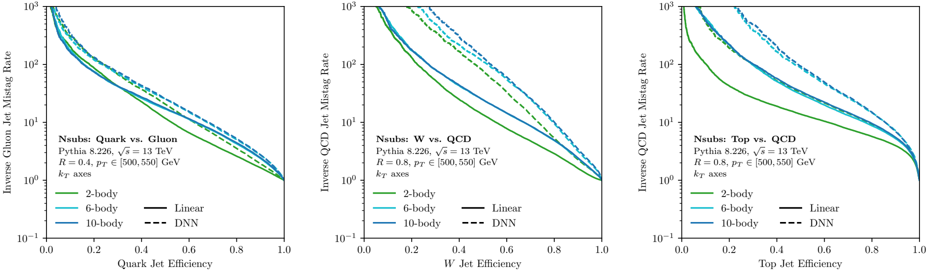


Quark/Gluon and Top Tagging χ Sweep



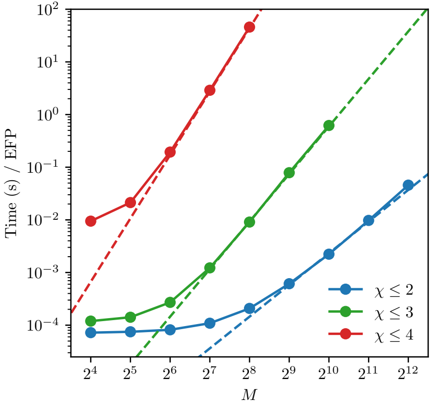


N-Subjettiness Linear/DNN Comparison



VE Timing

- Test our implementation of VE averaged over all EFPs with $d \leq 7$
- This includes prime EFPs up to $N = 8$! (Imagine $N = 8$ ECF, OMG)



N		2	3	4	5	6	7	8
χ	2	7	12	33	50	65	48	23
	3		11	42	82	80	33	
	4			2	1			

Prime EFPs with $d \leq 7$