Machine Learning
An Essential Toolkit for Particle Physics

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Snowmass Computational Frontier Workshop – ML Subgroup

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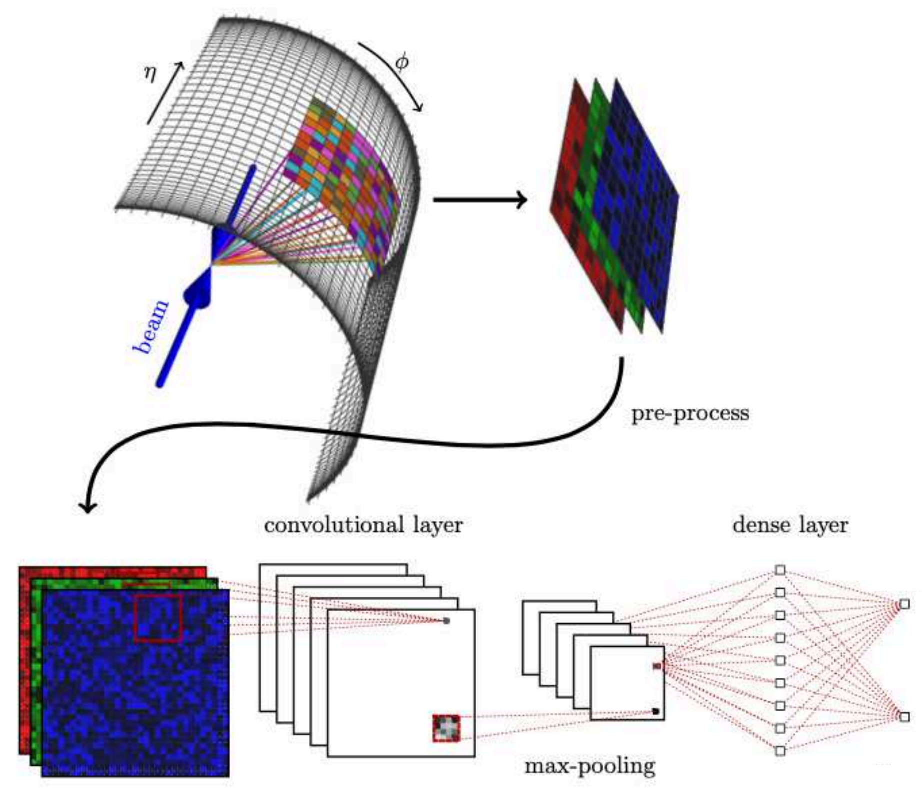
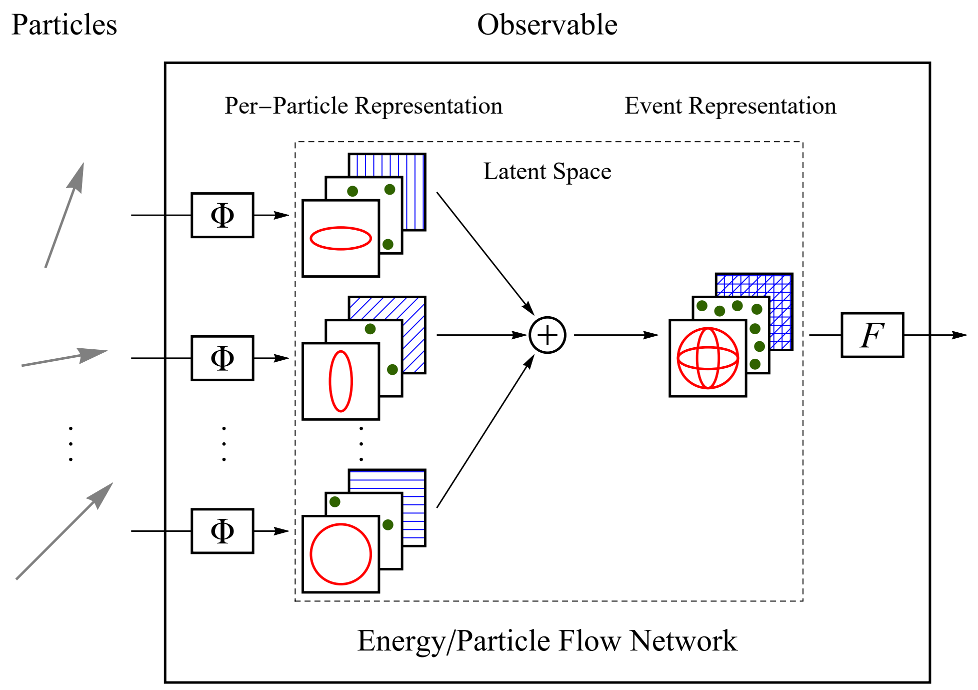
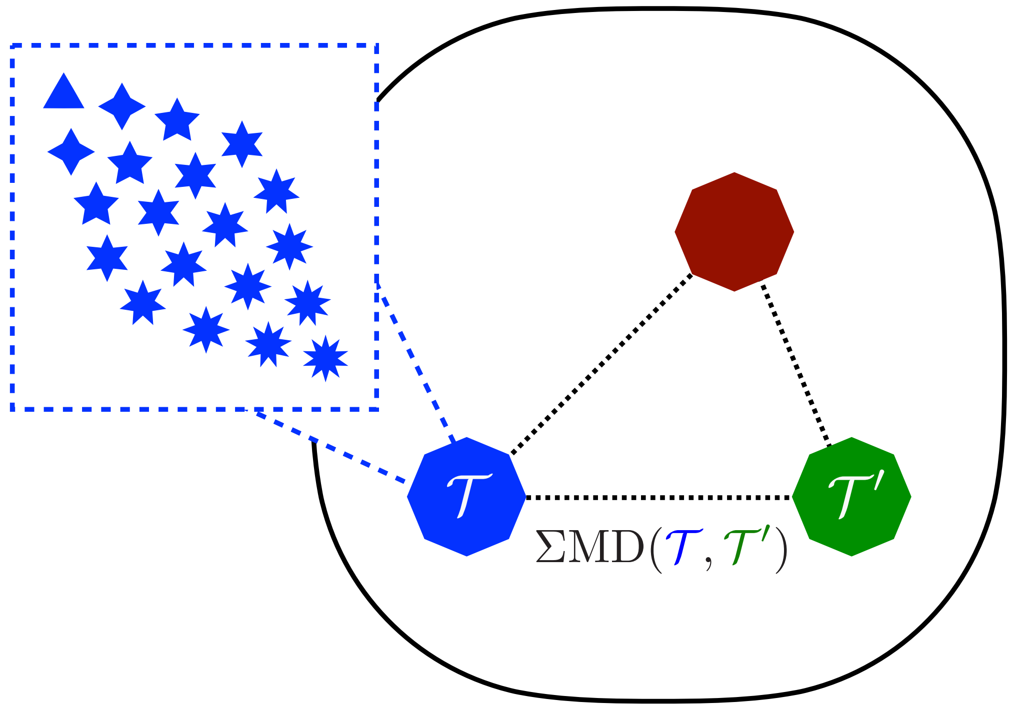


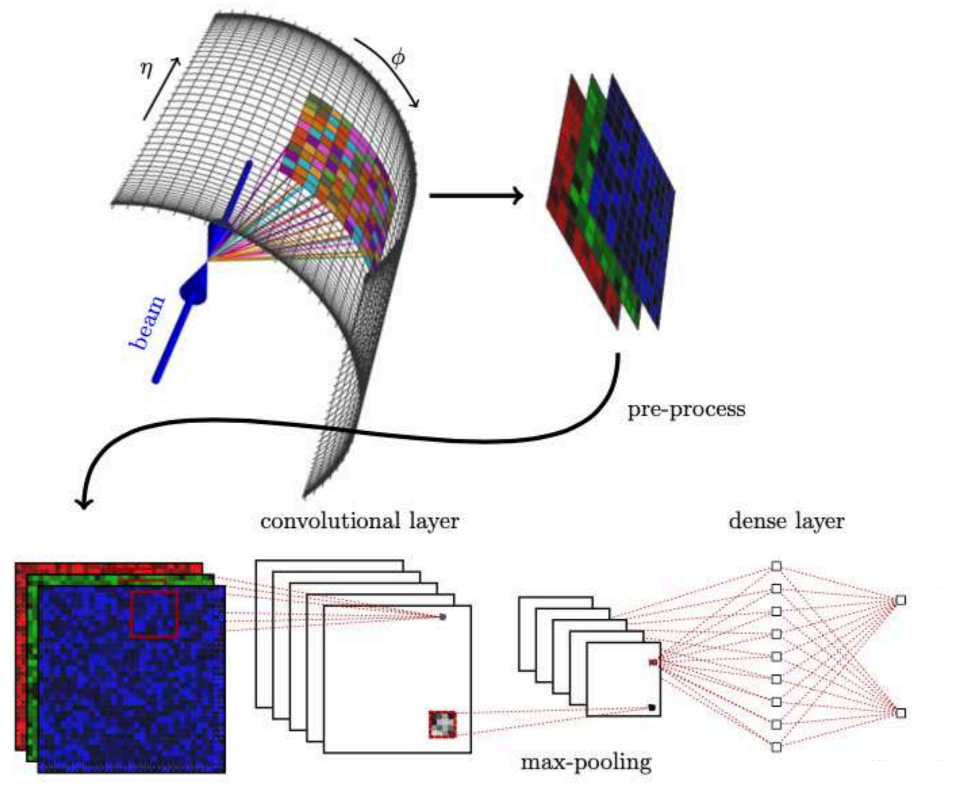




Ubiquity of ML in HEP

Lightning Review

Future Directions



Ubiquity of ML in HEP

Lightning Review

Future Directions

Machine Learning Permeates High-Energy Physics

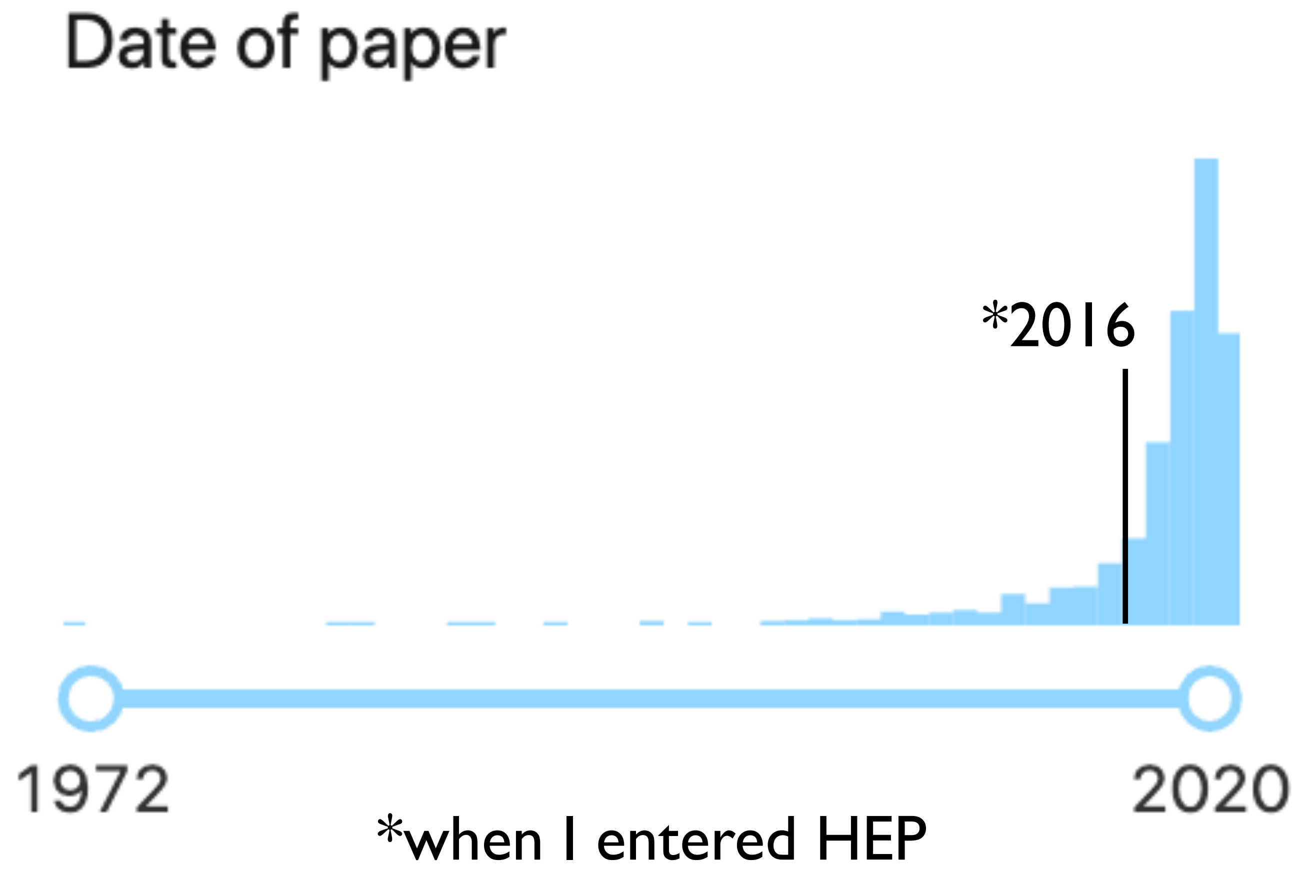
INSPIRE HEP | literature | machine learning or deep learning

arXiv Category

arXiv Category	
astro-ph.IM	316
astro-ph.CO	262
hep-ex	247
hep-ph	245
cs.LG	207
physics.data-an	183
astro-ph.HE	152
stat.ML	144
physics.ins-det	112
astro-ph.GA	110
physics.comp-ph	94
hep-th	87
gr-qc	85
cs.CV	44
nucl-th	44
cond-mat.dis-nn	41
cond-mat.stat-mech	41
hep-lat	40
quant-ph	39
nucl-ex	37

Subject

Subject	
Astrophysics	556
Computing	473
Instrumentation	470
Experiment-HEP	377
Phenomenology-HEP	255
Data Analysis and Statistics	185
Other	172
General Physics	138
Gravitation and Cosmology	99
Theory-HEP	80



ML in HEP is a young, vibrant, growing field with exciting potential!

Machine Learning Fundamentals

The Power of ML

Comes at a cost

- *Interpolation in high-dimensional spaces*
  Combats the curse of dimensionality
  Loses analytic understandability/tractability

- *Automatic feature extraction*
  Ensures relevant features are not missed
  Cannot easily convey what features are used

- *Asymptotically optimizes performance*
  Provides useful/practical statistical power
  Training is difficult with few global guarantees

Responsible ML Considerations

- *Available data*
  Data source, number of samples, labels, reliability

- *Learning paradigm*
  Fully/weakly/un-supervised, classification/regression/generation

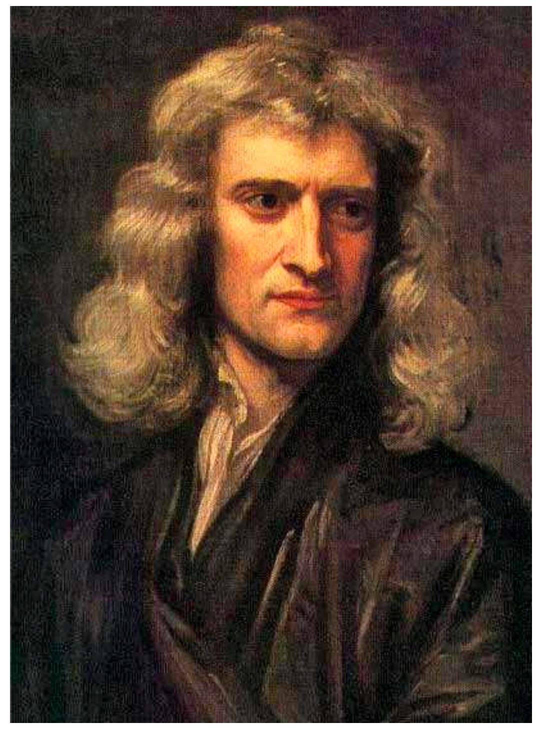
- *Inputs and outputs*
  Size/shape, symmetries, dimensionality

- *Model architecture*
  Expressibility, loss function, hyperparameters, validation/testing

- *Deployment strategy*
  Model implementation, training/evaluation speed, uncertainties

My Perspective on ML in HEP

Machine learning is here to stay in high-energy physics

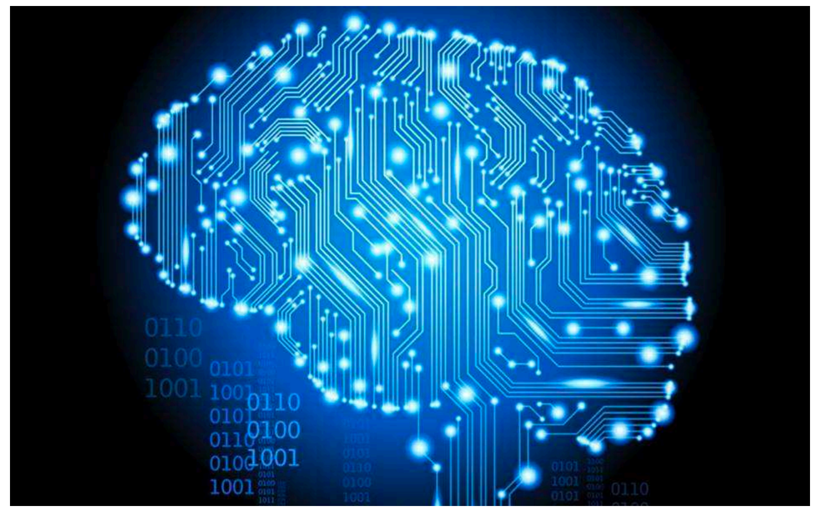


Calculus

- Fundamental to quantitative study of functions
- Taught to all undergraduate physics students
- Essential for understanding modern physics

Machine Learning

- Fundamental to statistical data analysis
- Increasingly taught in undergrad science programs
- Increasingly essential for modern physics/science



Machine learning straddles theory/experiment/computation divide

Close coordination between theory and experiment is essential
in the current era of uncertainty in particle physics

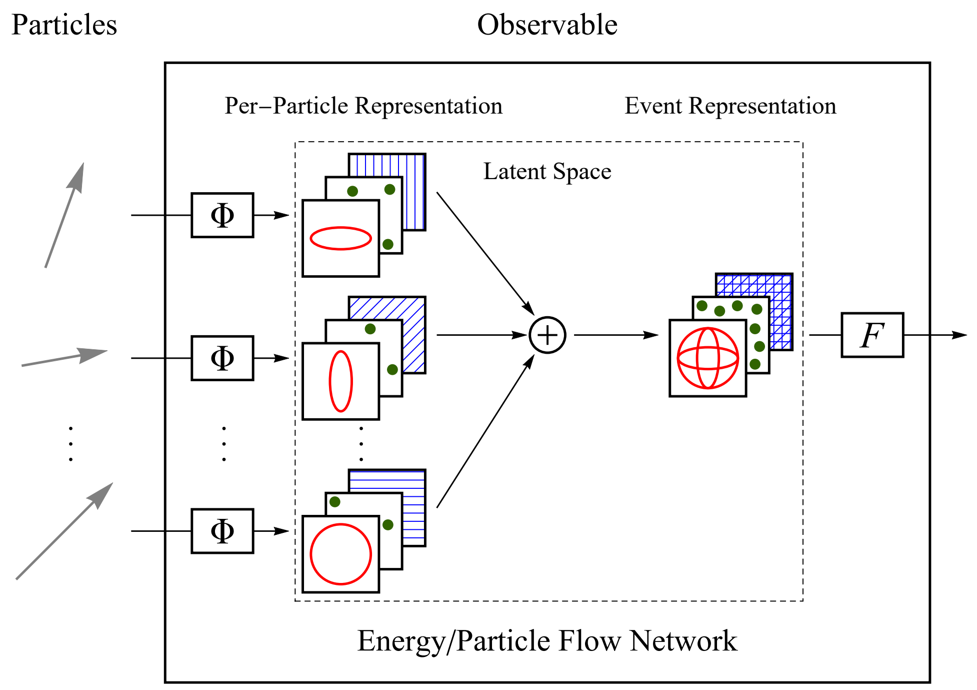
- *Where has ML already had an impact in high-energy physics?*

Part II of this talk

Key Questions During the Snowmass Process

- *What is the role of ML in particle physics (and vice-versa) in the future?*

Part III of this talk



Ubiquity of ML in HEP

Lightning Review

Disclaimers

- Examples biased towards collider physics
- References are not exhaustive

Future Directions

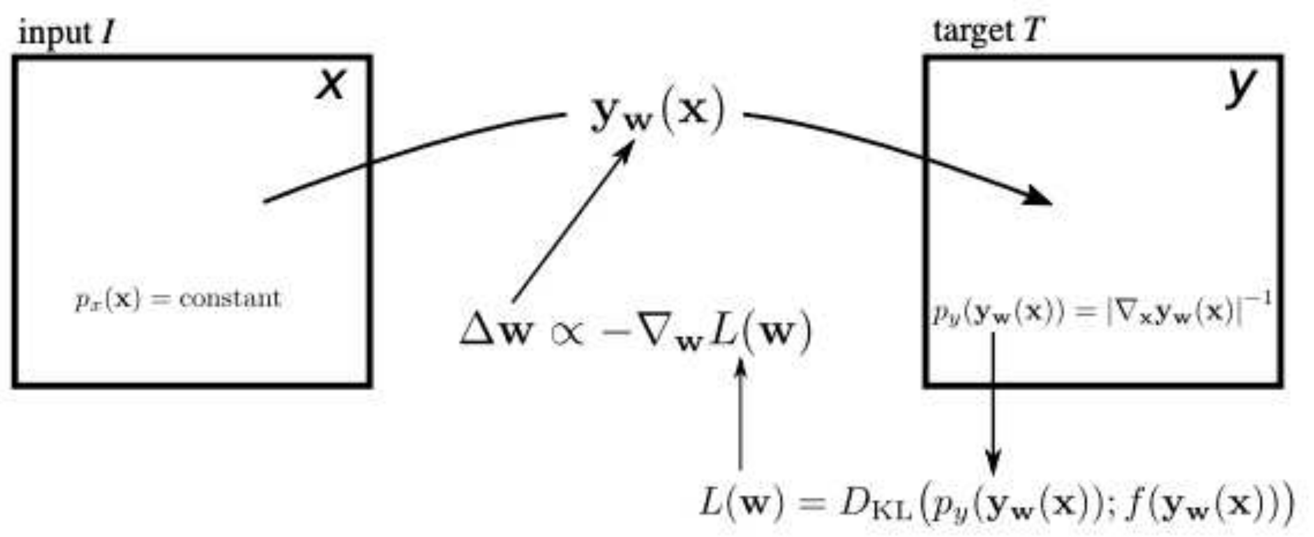
“Uncontroversial” Applications of Machine Learning in HEP

ML can provide a uniformly improved, drop-in replacement for some mathematical tasks

Efficient Monte Carlo Integration

[Bendavid, 1707.00028; Klimek, Perelstein, 1810.11509]

“A continuum implementation of the VEGAS algorithm”

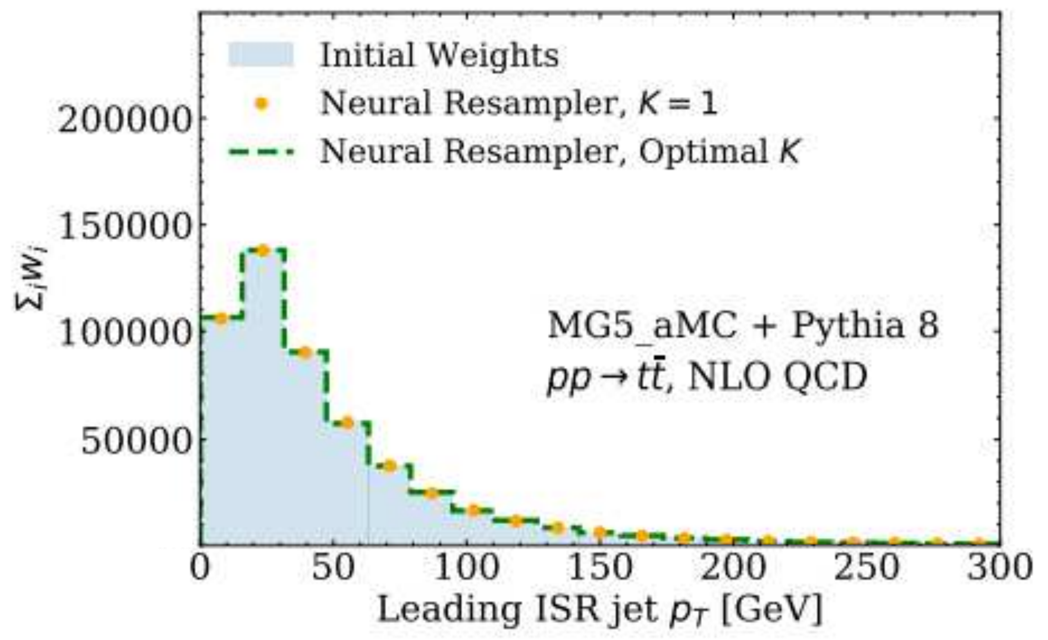


Algorithm	# of Func. Evals	$\sigma_w / < w >$	σ_I / I (2e6 add. evts)
VEGAS	300,000	2.820	$\pm 2.0 \times 10^{-3}$
Foam	3,855,289	0.319	$\pm 2.3 \times 10^{-4}$
Generative BDT	300,000	0.082	$\pm 5.8 \times 10^{-5}$
Generative BDT (staged)	300,000	0.077	$\pm 5.4 \times 10^{-5}$
Generative DNN	294,912	0.083	$\pm 5.9 \times 10^{-5}$
Generative DNN (staged)	294,912	0.030	$\pm 2.1 \times 10^{-5}$

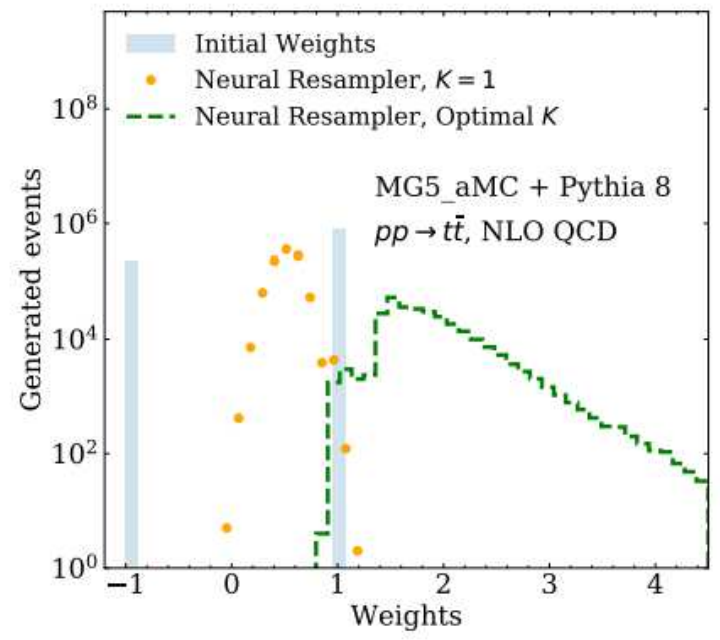
DNN has 10-100x smaller error with the same or fewer samples!

Positive MC-Weight Resampling

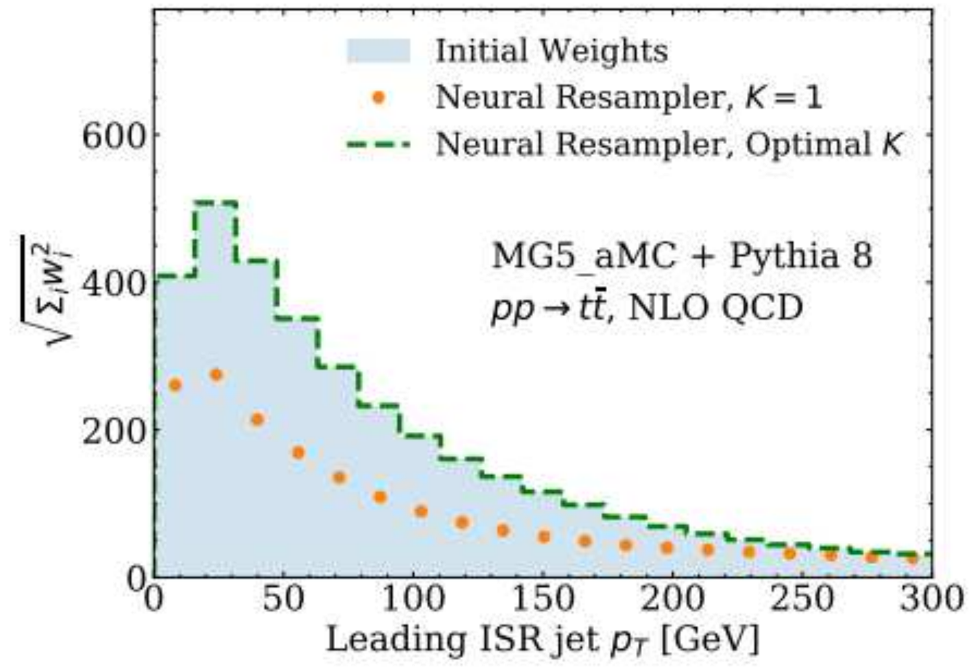
[Andersen, Gutschow, Maier, Prestel, 2005.09375;
Nachman, Thaler, 2007.11586]



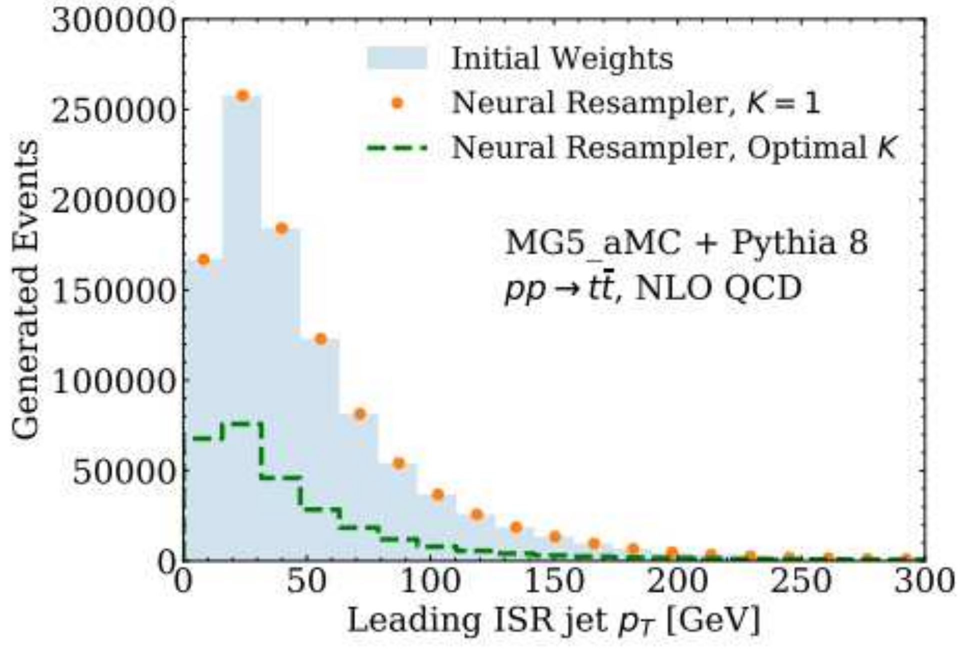
Central value preserved …



using only positive weights …



while preserving uncertainty …



and requiring fewer events!

Improvement to Computational Speed/Efficiency with ML

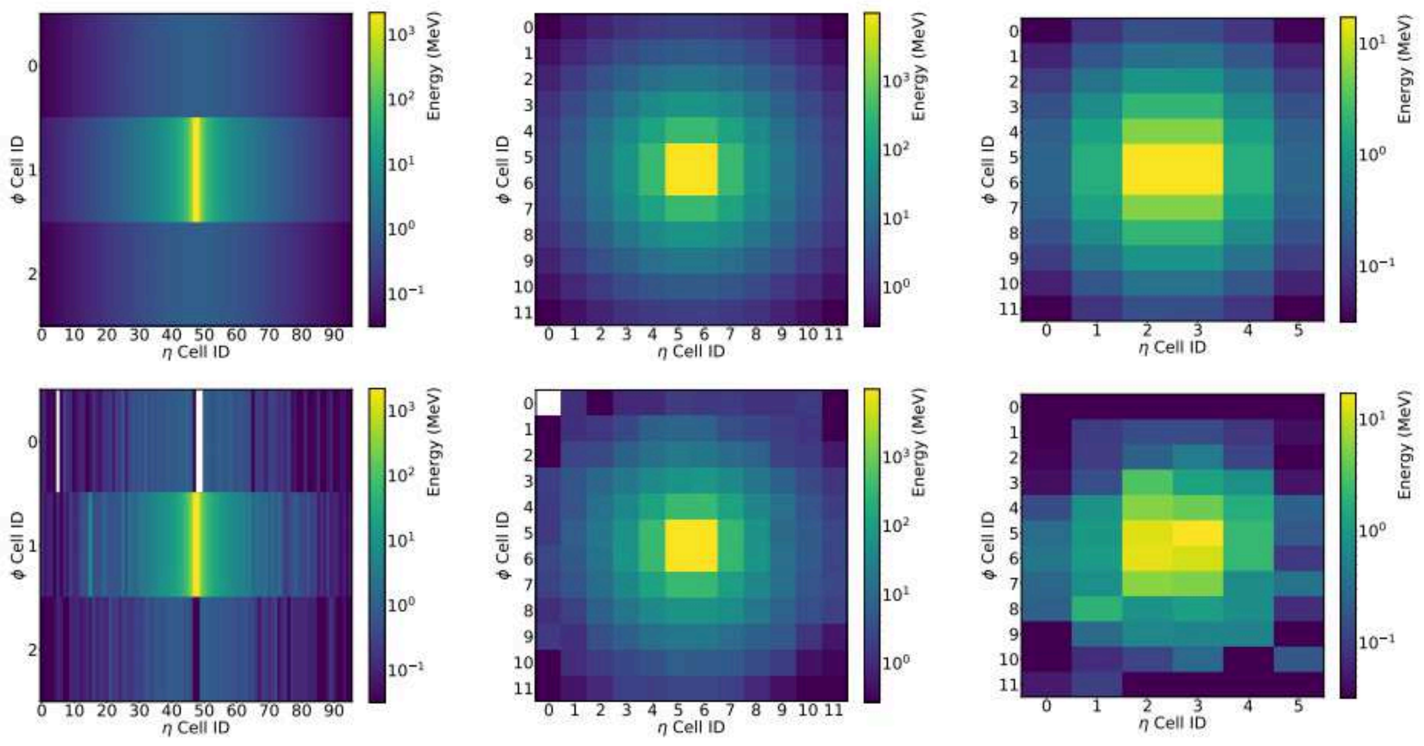
Fast Electromagnetic Calorimeter Simulation

[Paganini, de Oliviera, Nachman, 1705.02355 1712.10321;
Erdmann, Glombitza, Quast, 1807.01954]

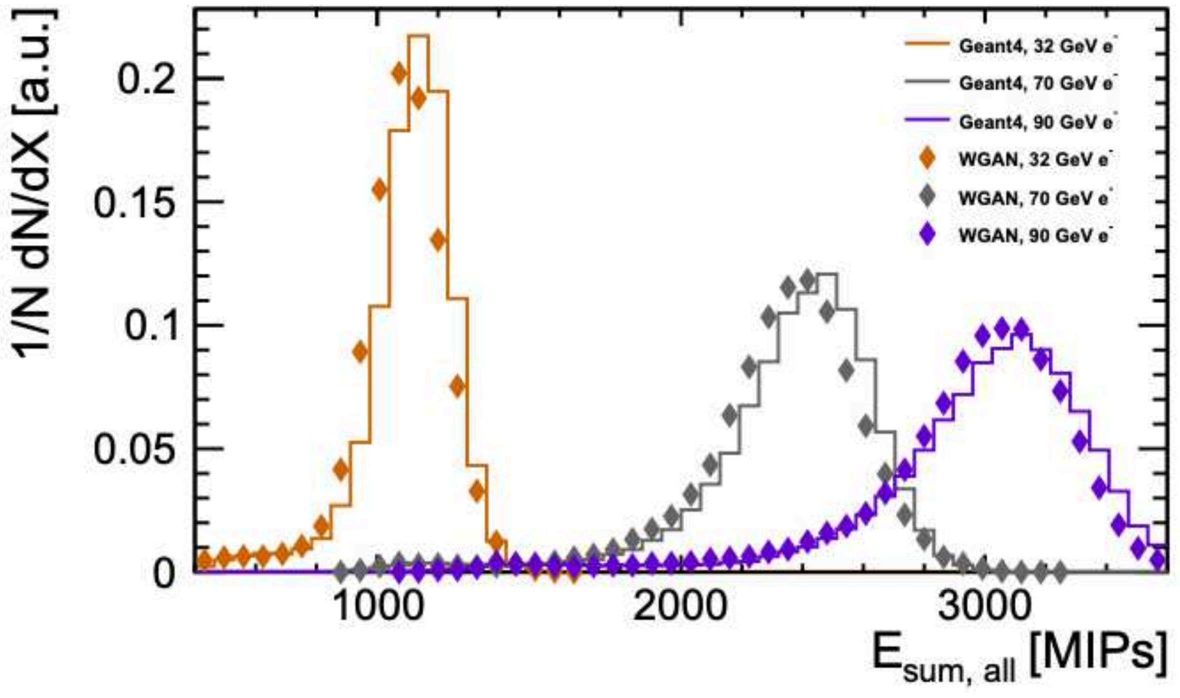
Generative models can be O(10^2-10^5) x quicker than full detector-simulation

Geant4

GAN



Wasserstein GANs have stabler training and good agreement

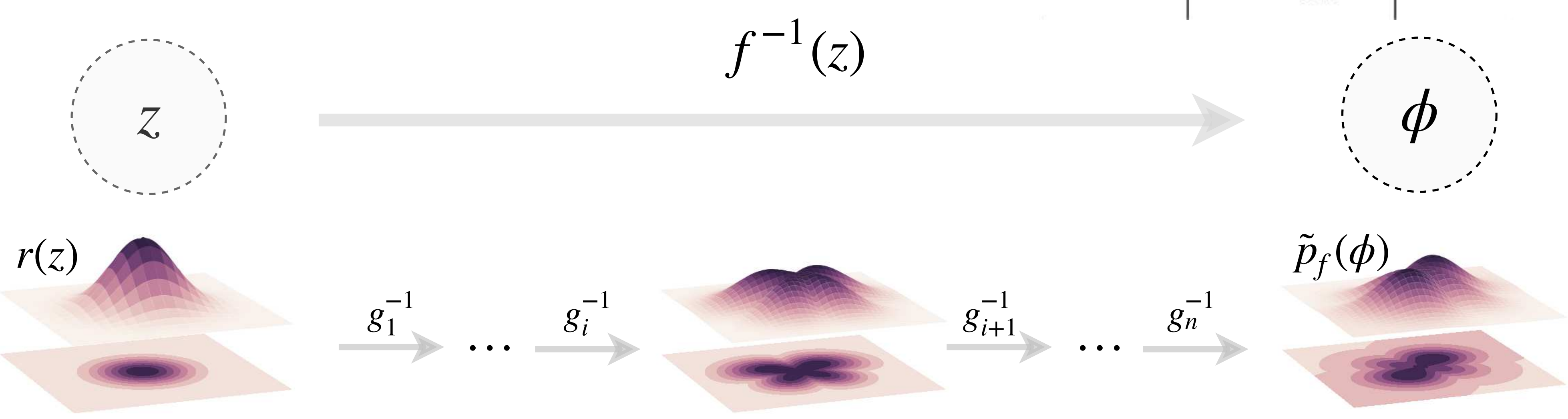


Improved MCMC for Lattice Field Theory

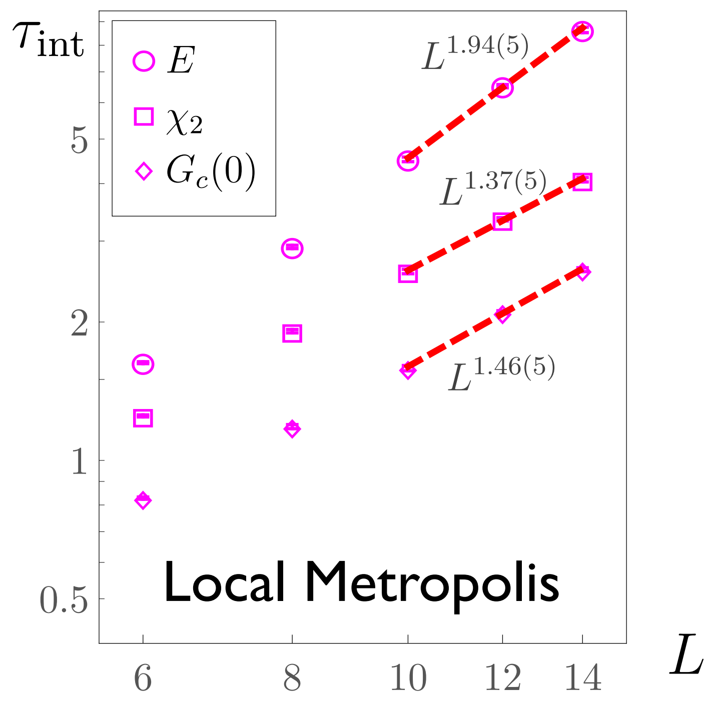
[Albergo, Kanwar, Shanahan, 1904.12072; Talk by G. Kanwar]

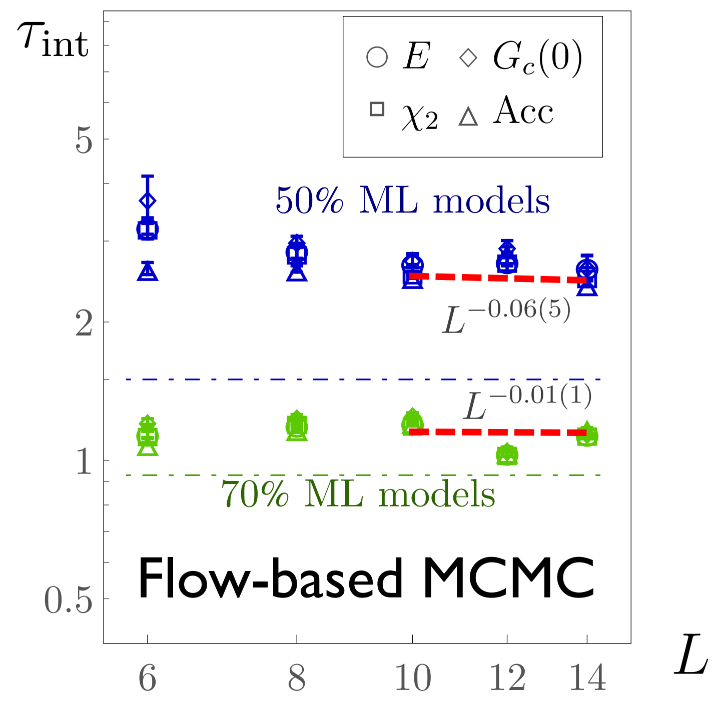
Normalizing flows can be used to sample from complicated distributions

Invertible and tractable Jacobian leads to $\tilde{p}_f(\phi) = \left|\det \frac{\partial f^{-1}(z)}{\partial z}\right|^{-1} r(z)$



Power law growth of autocorrelation time avoided with ML



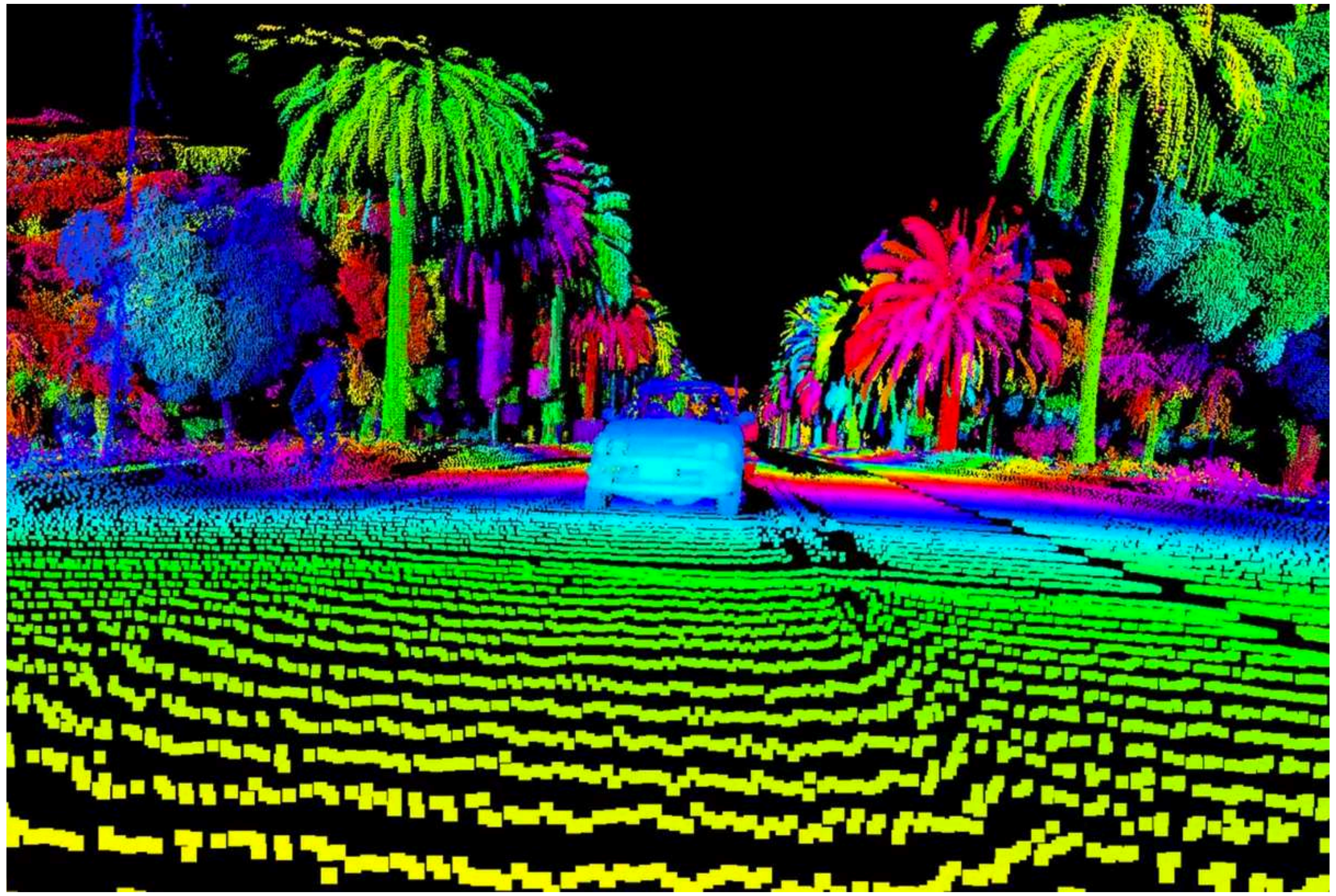


Neural Network Architectures for Particle Physics

Maximally appropriate ML architectures respect symmetries of the underlying data

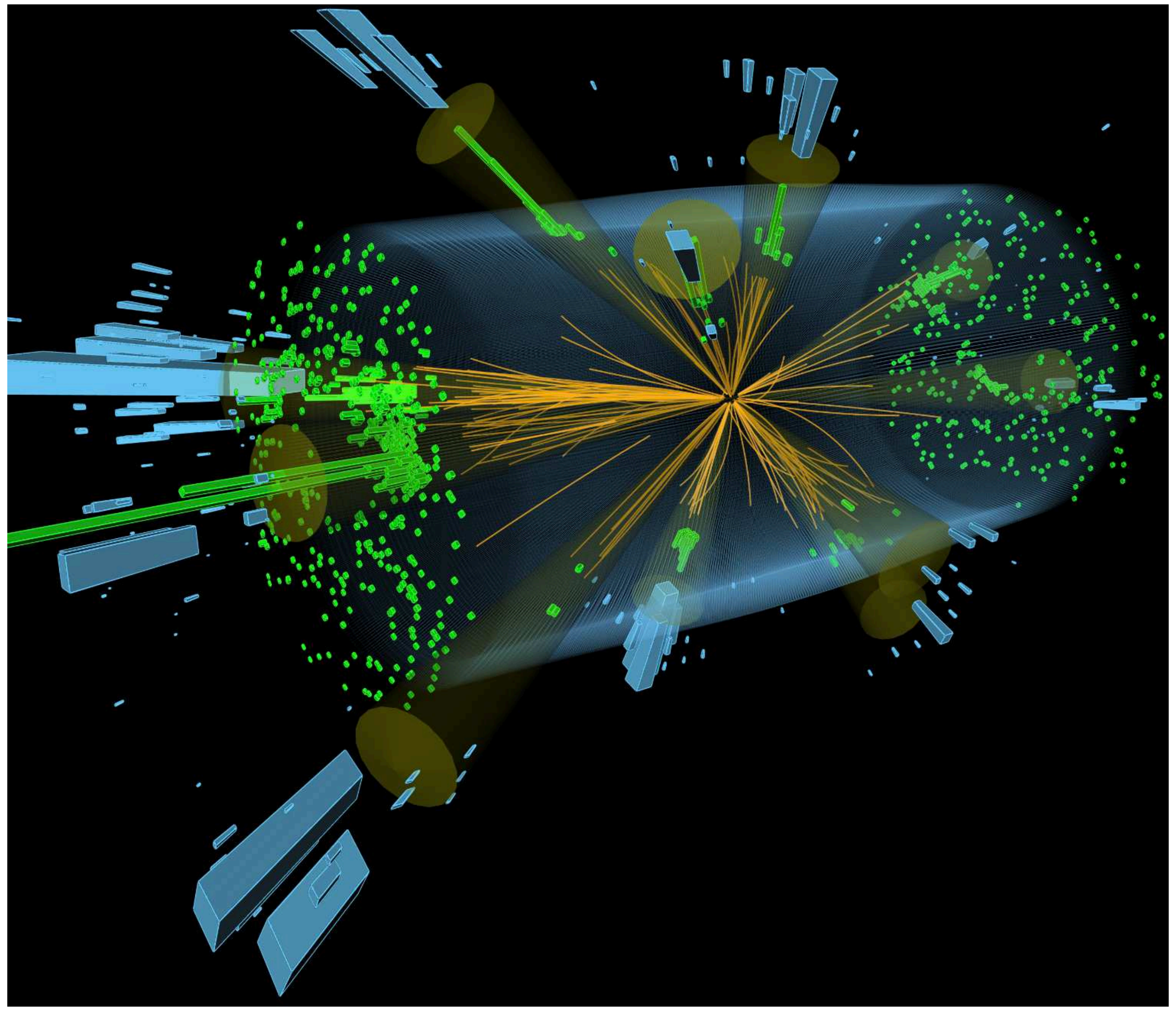
Particle physics events are naturally point clouds (alternatively, images e.g. calorimeters)

Point cloud: "A set of data points in space" –Wikipedia



LIDAR data from self-driving car sensor

An unordered, variable length collection of particles



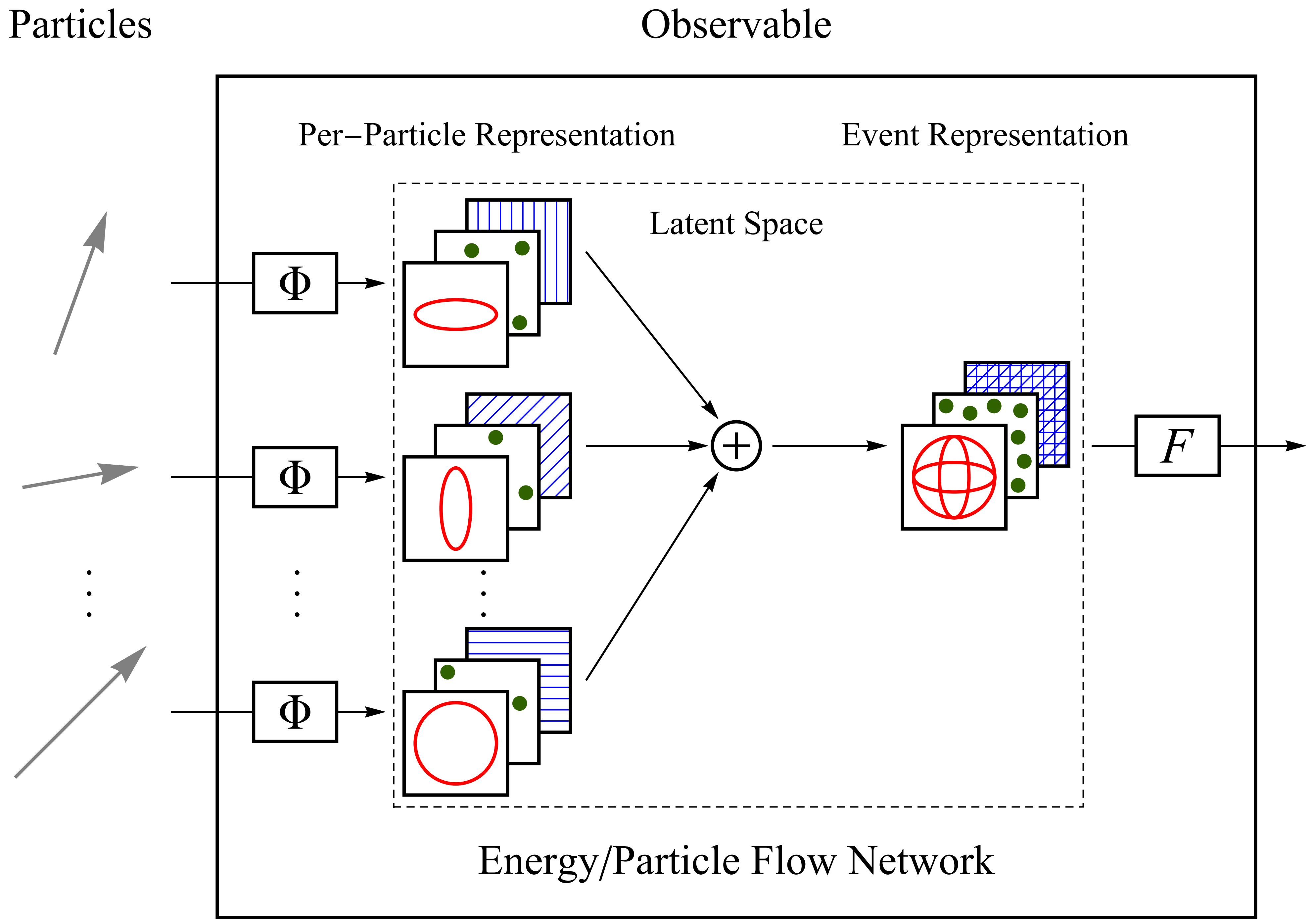
Multi-jet event at CMS

Due to quantum-mechanical indistinguishability

Due to probabilistic nature of event formation

Deep Sets for Particle Jets

[Zaheer, Kottur, Ravanbhakhsh, Póczos, Salakhutdinov, Smola, 1703.06114;
PTK, Metodiev, Thaler, 1810.05165;
EnergyFlow Python Package]



Particle Flow Network (PFN)

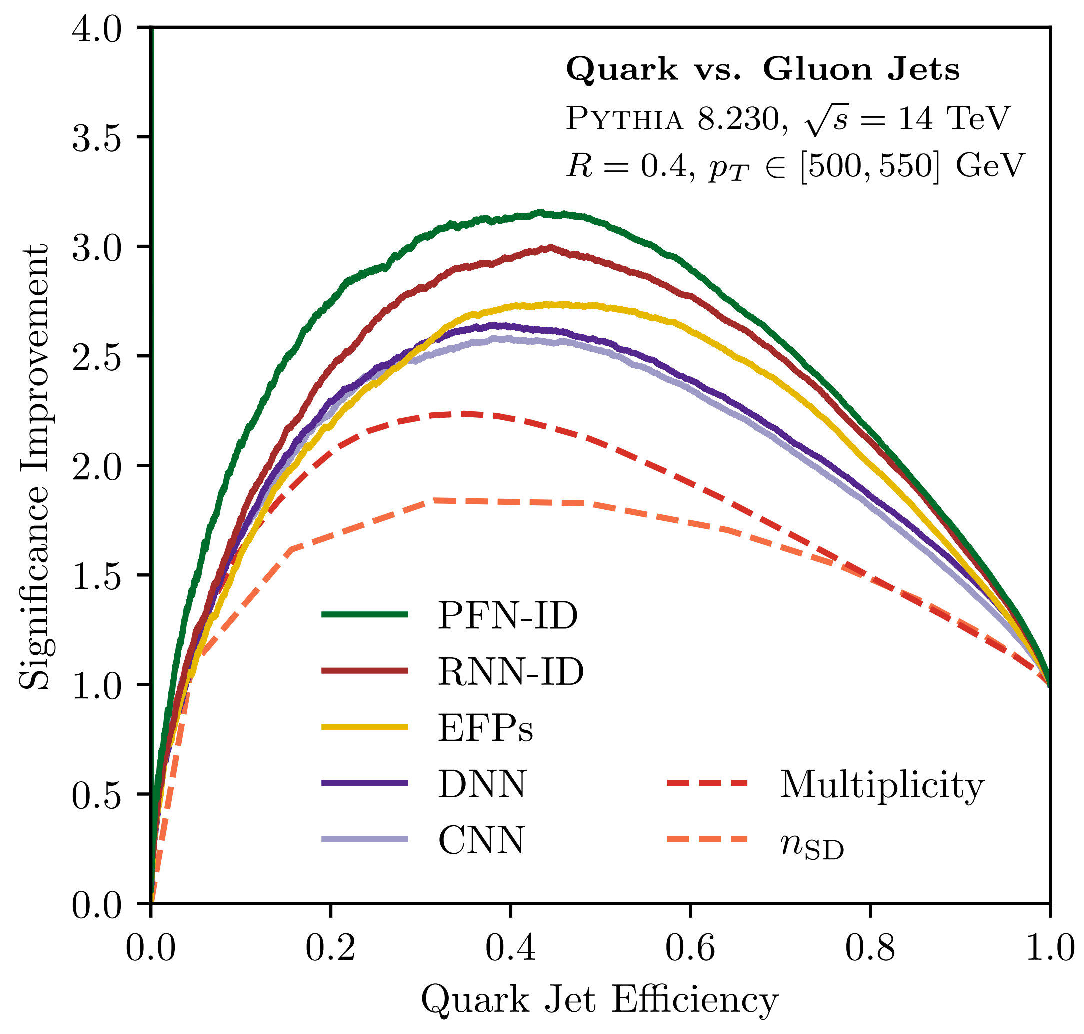
$$\mathrm{PFN}(\{p_1^\mu,\ldots,p_M^\mu\}) = F\left(\sum_{i=1}^{M}\Phi(p_i^\mu)\right)$$

Fully general latent space

Energy Flow Network (EFN)

$$\mathrm{EFN}(\{p_1^\mu,\ldots,p_M^\mu\}) = F\left(\sum_{i=1}^{M} z_i\,\Phi(\hat{p}_i)\right)$$

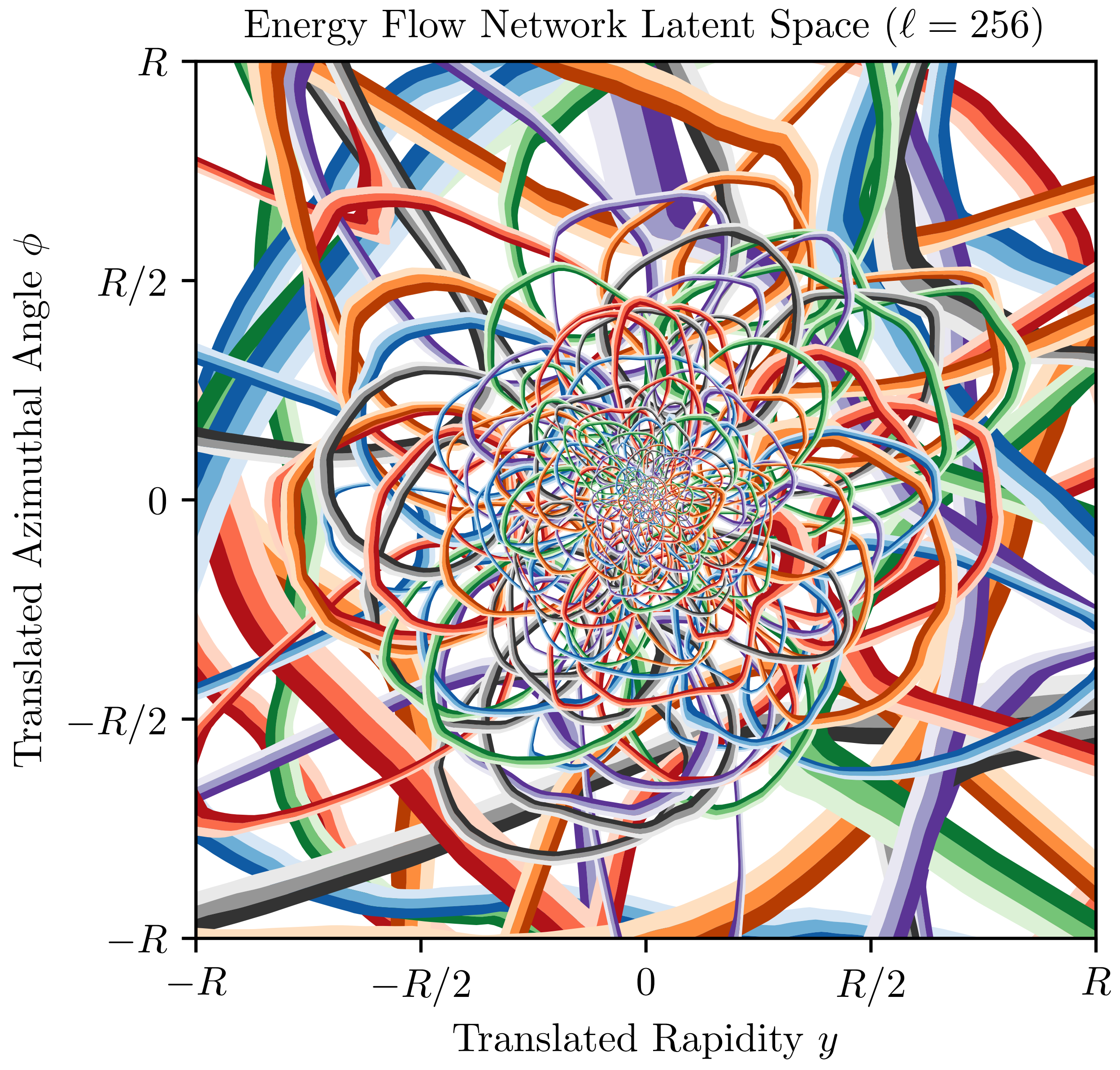
IRC-safe latent space



Improved performance (and training) compared to RNN and CNN

Latent space visualization reveals what the network has learned

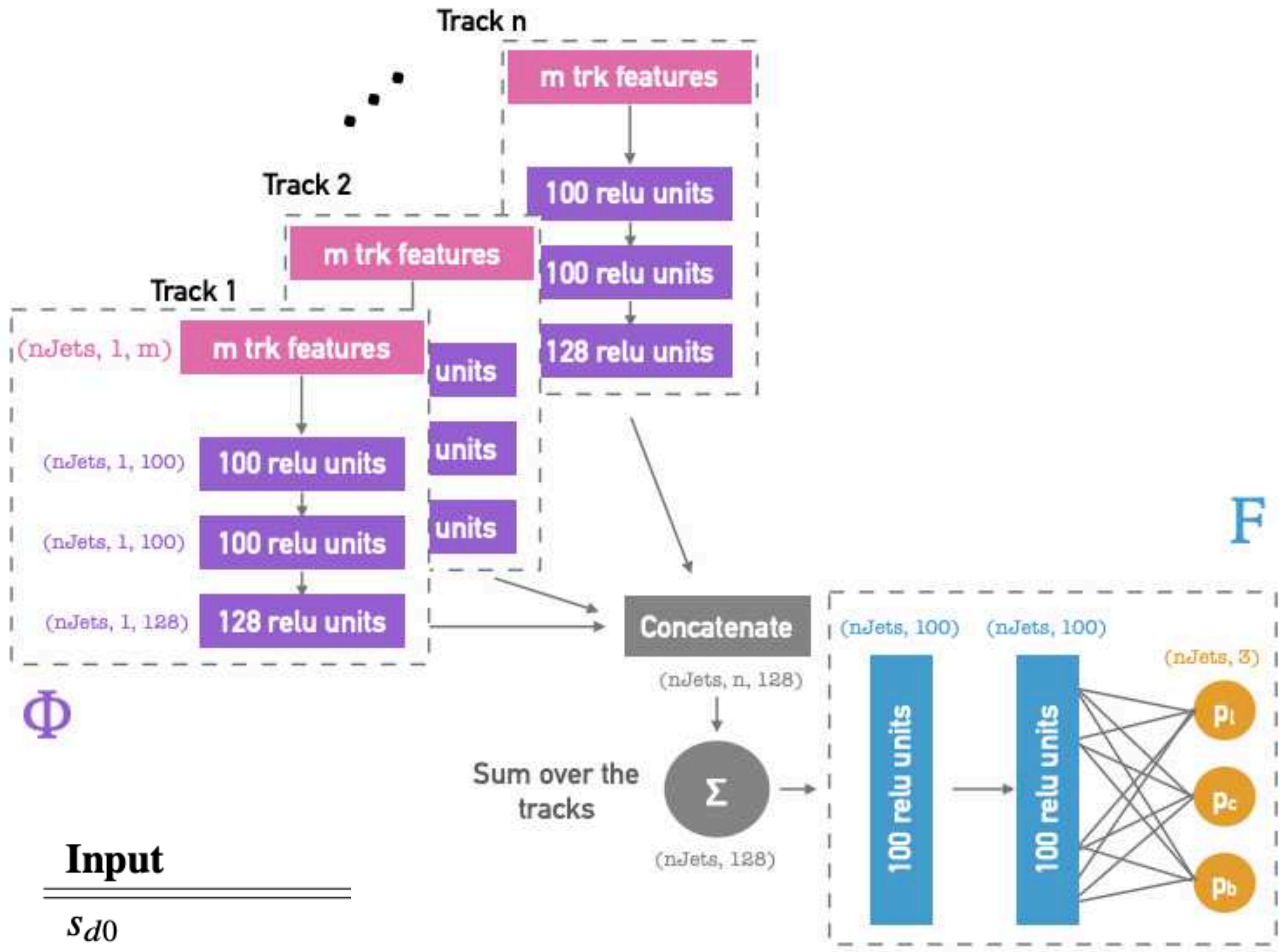
Dynamic pixel sizing related to collinear singularity of QCD!



Other Physics-Inspired Architectures

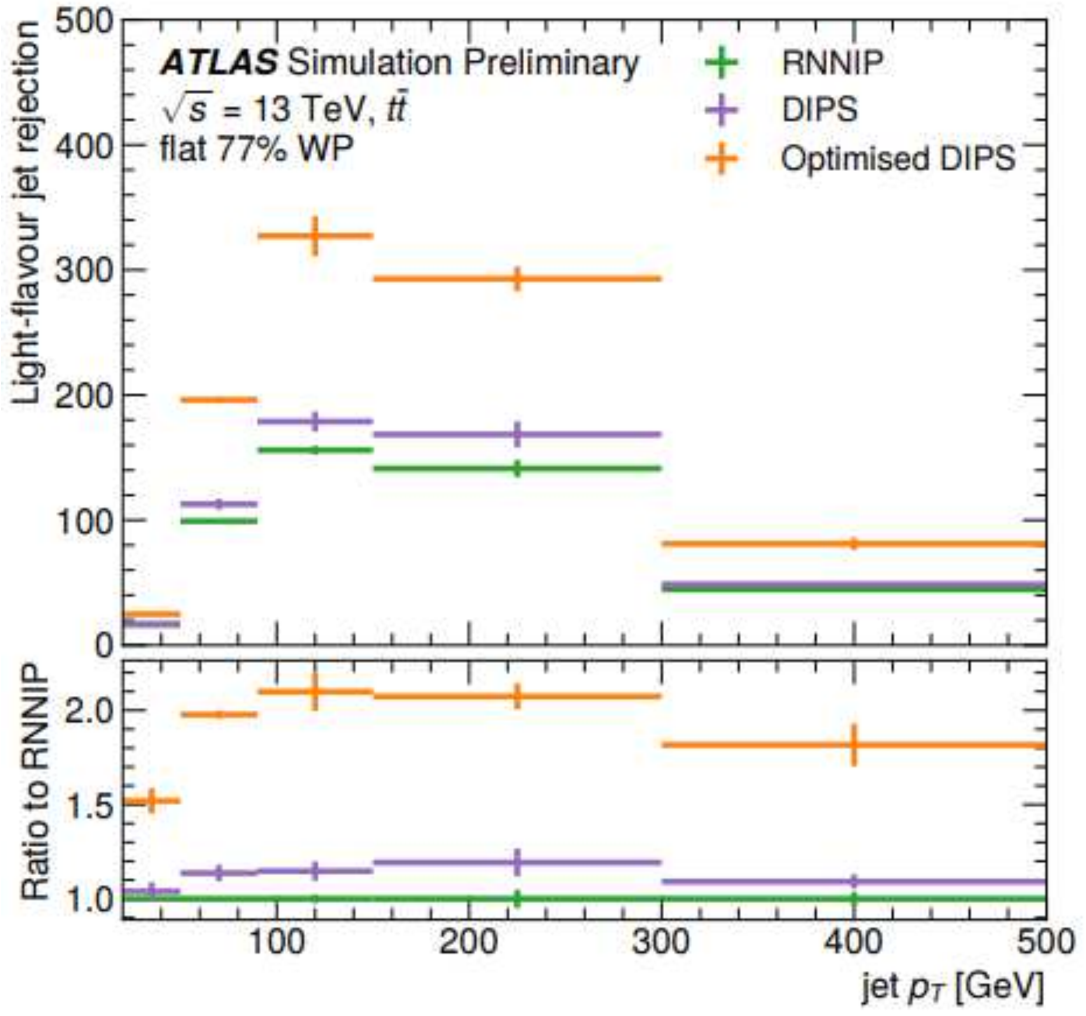
Deep Impact Parameter Sets

[ATL-PHYS-PUB-2020-014]



Input

s_{d0}
s_{z0}
$\log p_{\mathrm{T}}^{frac}$
$\log \Delta R$
IBL hits
PIX1 hits
shared IBL hits
split IBL hits
nPixHits
shared pixel hits
split pixel hits
nSCTHits
shared SCT hits

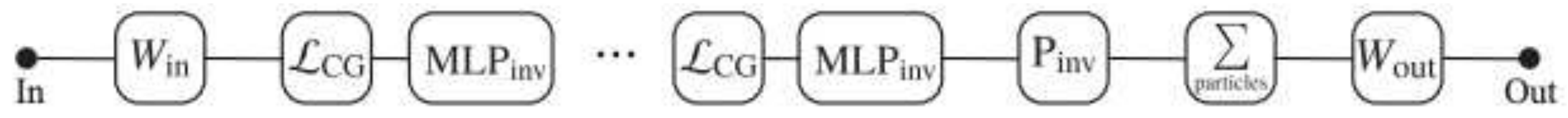


Achieves 2x better flavor tagging than an RNN

“Optimised” includes additional features per track

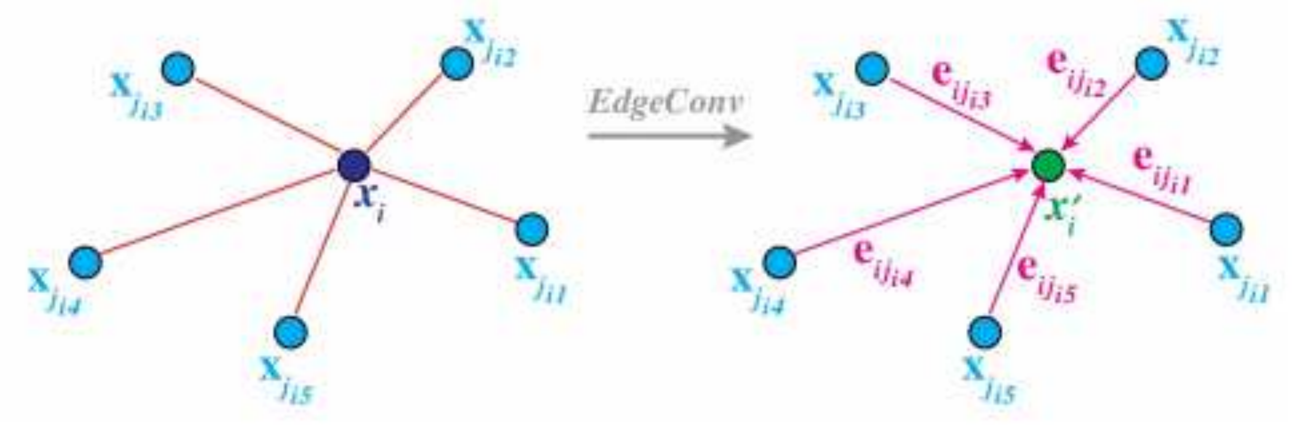
Lorentz Group Equivariant NN

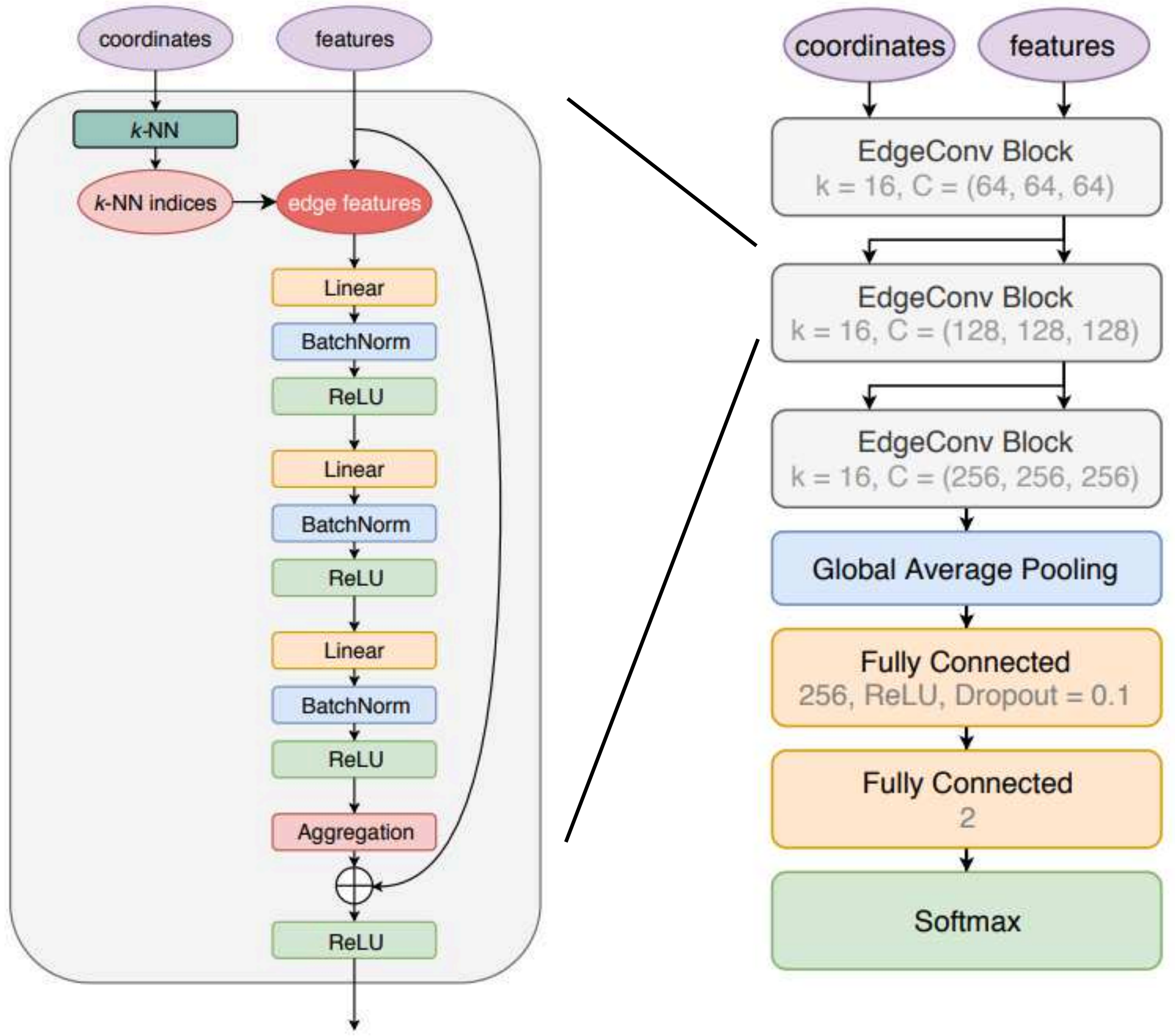
[Bogatskiy, Anderson, Offermann, Roussi, Miller, Kondor, 2006.04780]



Dynamic Graph CNNs (e.g. Particle Net)

[Wang, Sun, Liu, Sarma, Bronstein, Solomon, 1801.07829;
Qu, Gouskos, 1902.08570]





Preserves permutation symmetry while prioritizing relationships between inputs

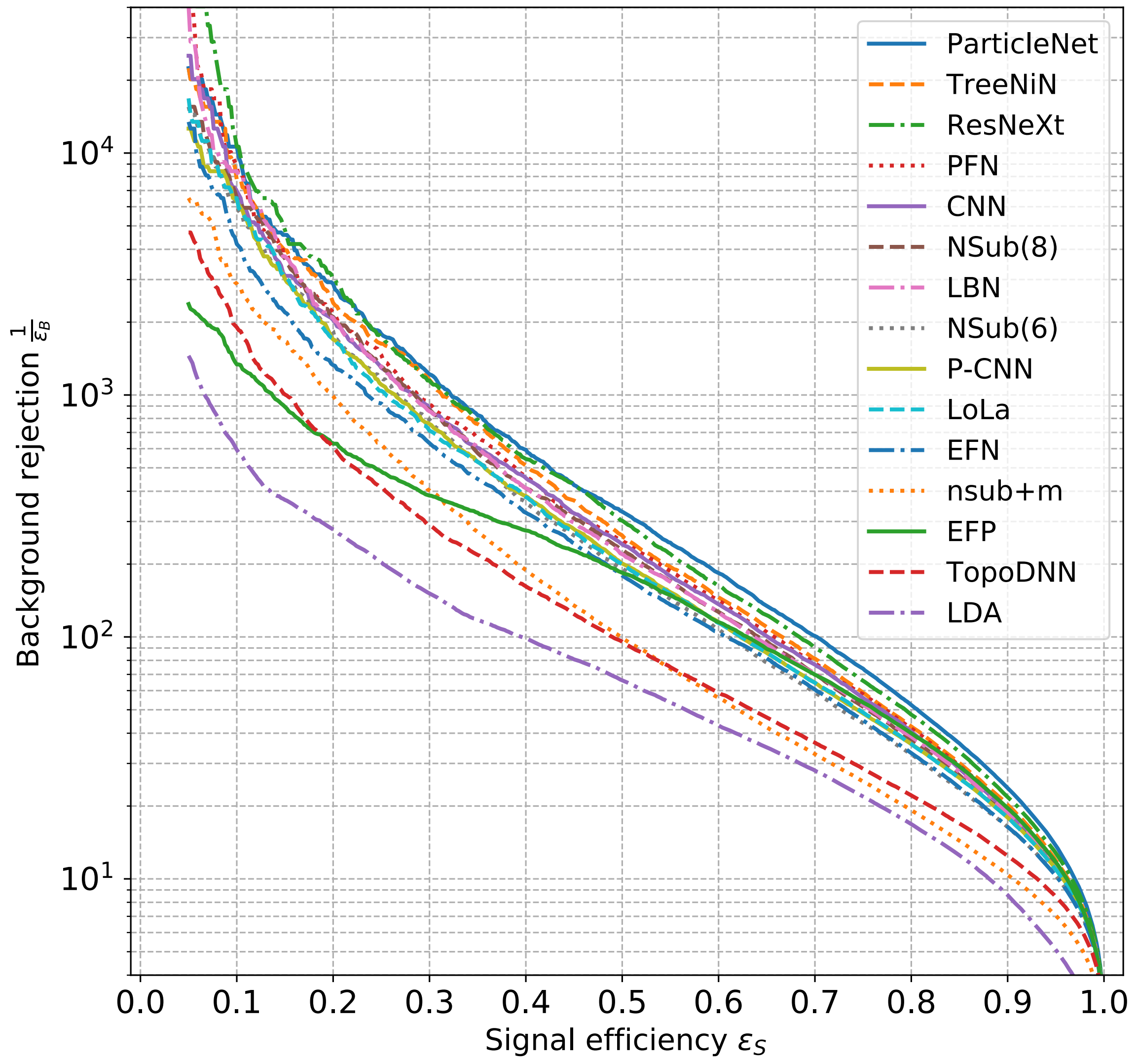
Improved Performance on Classic HEP Tasks with ML

ML optimizes performance

(Jet) Classification

[Kasieczka, Plehn, et al., 1902.09914;
many many other references…]

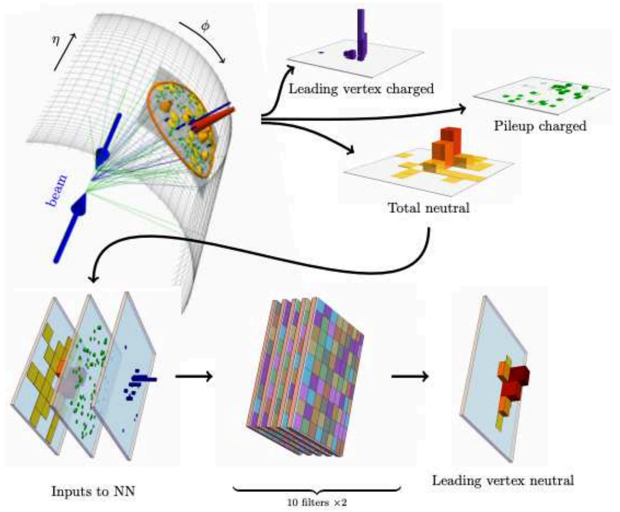
Community top-tagging comparison



Regression (e.g. to remove pileup)

[PTK, Metodiev, Nachman, Schwartz, 1707.08600;
ATL-PHYS-PUB-2019-028;
see also Arjona Martínez, Cerri, Spiropulu, Vlimant, Pierini 1810.07988]

PUMML



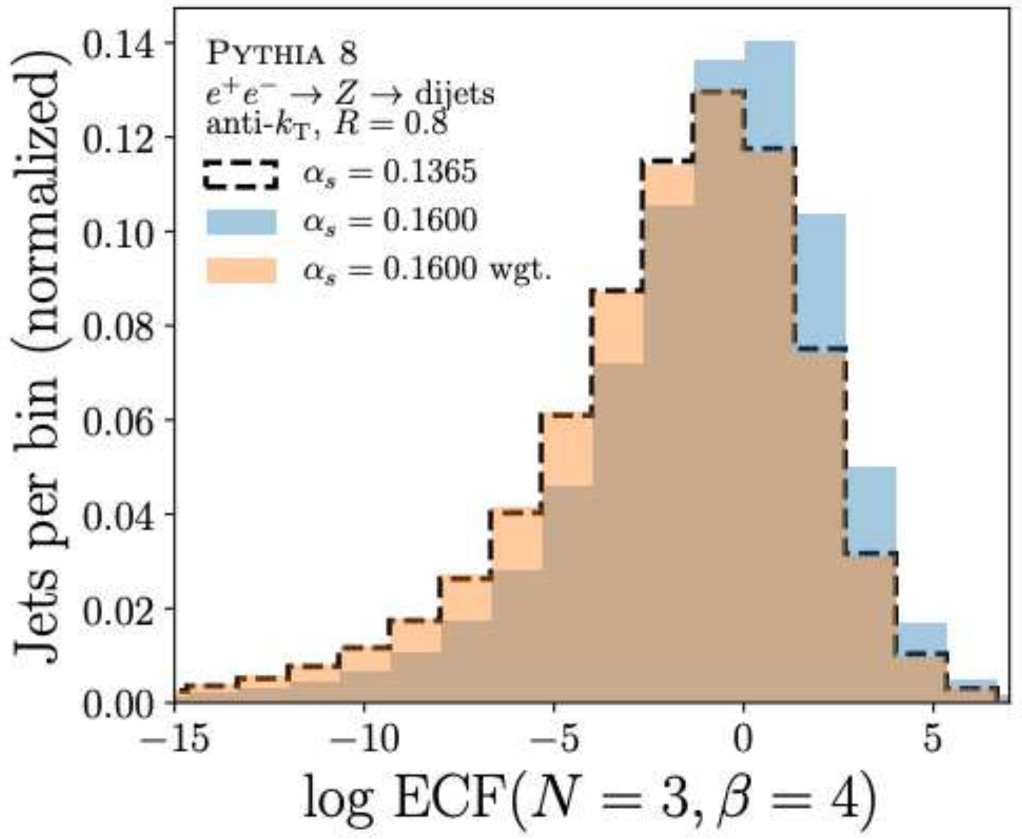
ML can enable an unbinned version of a binned method

(Full) Phase-Space Reweighting

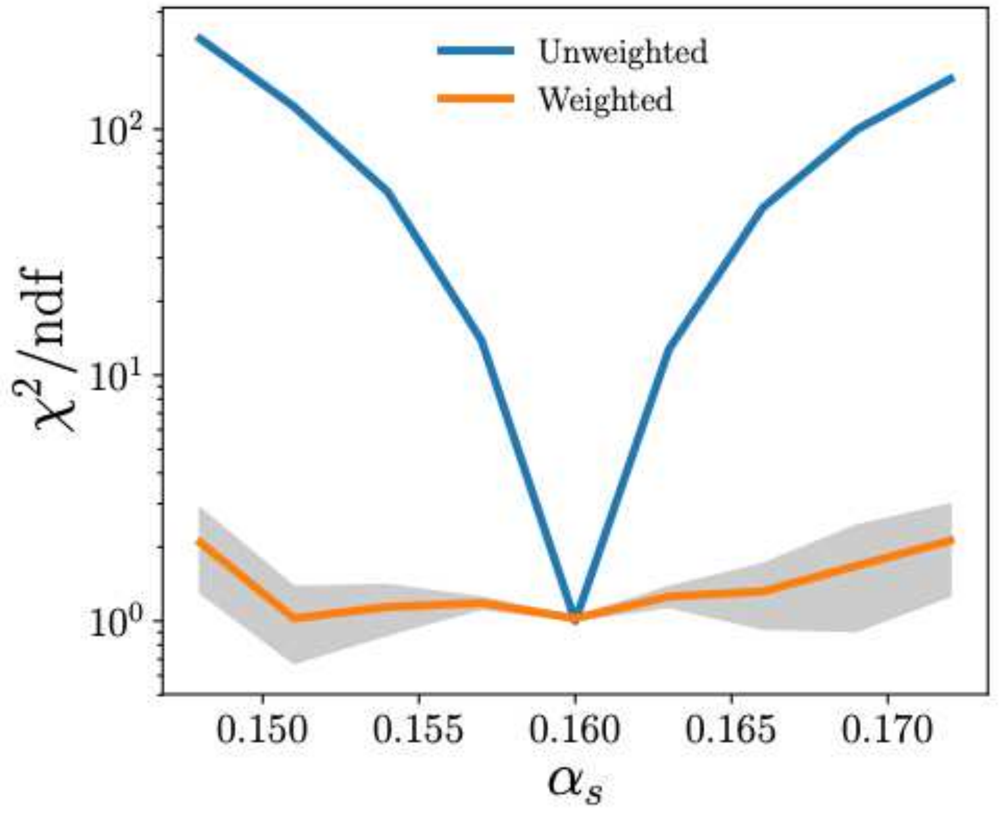
[Cranmer, Pavez, Louppe, 1506.02169;
Andreassen, Nachman, 1907.08209]

Likelihood-free inference via classification and Neyman-Pearson

DCTR uses a single high-dimensional reweighting…



to match ECF distributions …

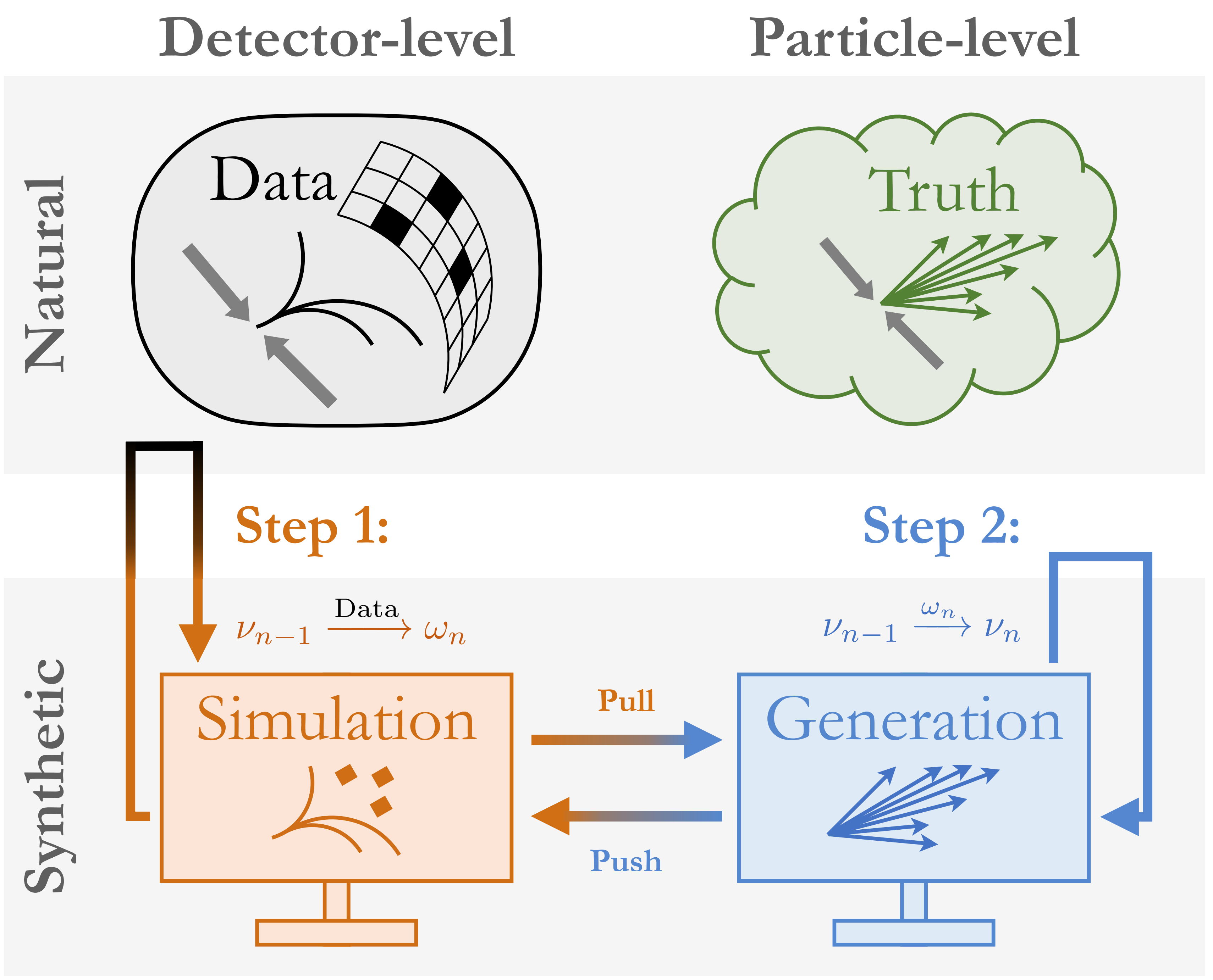


and multiplicity distributions!

High-dimensional reweighting is useful for many tasks …

OmniFold – Unbinned, Full Phase-Space Unfolding

OmniFold weights particle-level Gen to be consistent with Data once passed through the detector



Step 1 – Reweights Sim_{n-1} to data, pulls weights back to particle-level Gen_{n-1}

Step 2 – Reweights Gen_{n-1} to (step 1)-weighted gen_{n-1}, pushes weights to detector-level Sim_n

[Andreassen, PTK, Metodiev, Nachman, Thaler, 1911.09107; PTK talk at ML4Jets 2020]

OmniFold – i.e. continuous IBU

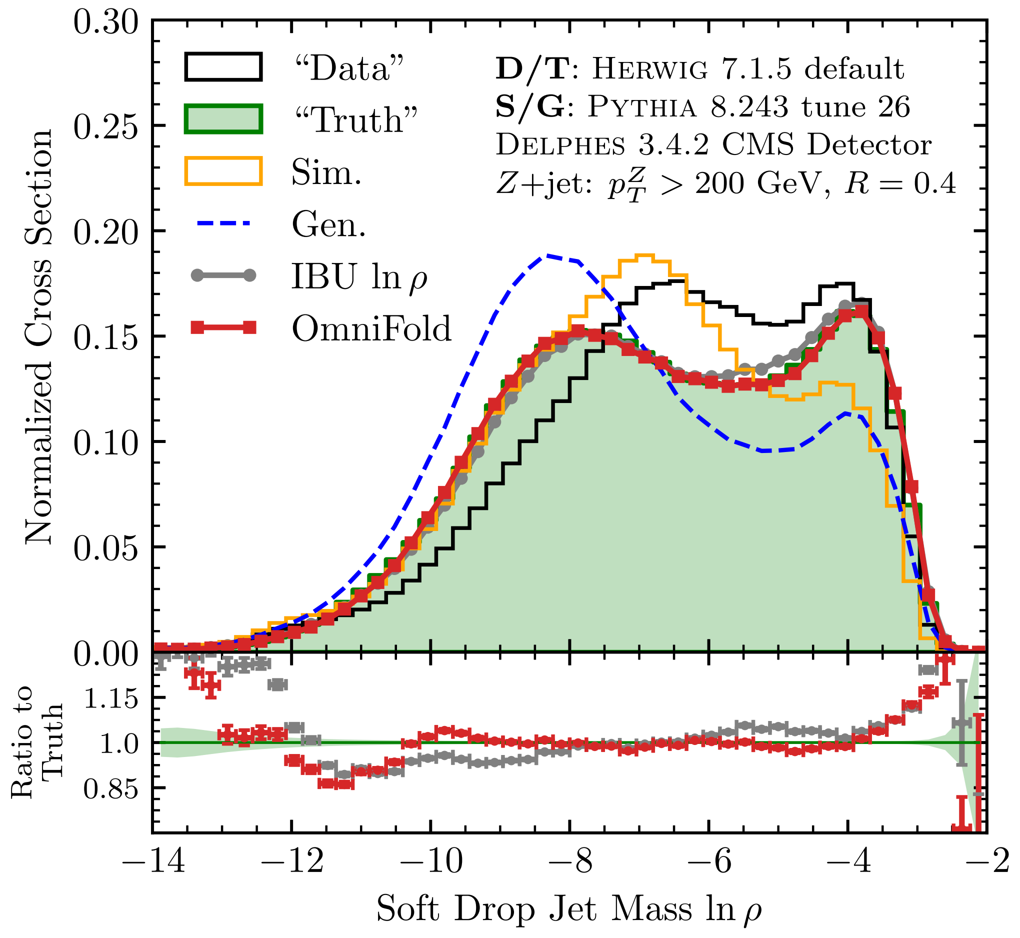
Step 1 – $\omega_n(m) = \nu_{n-1}^{\text{push}} \times L[(1, \text{Data}), (\nu_{n-1}^{\text{push}}, \text{Sim})](m)$

Step 2 – $\nu_n(t) = \nu_{n-1}(t) \times L[(\omega_n^{\text{pull}}, \text{Gen}), (\nu_{n-1}, \text{Gen})](t)$

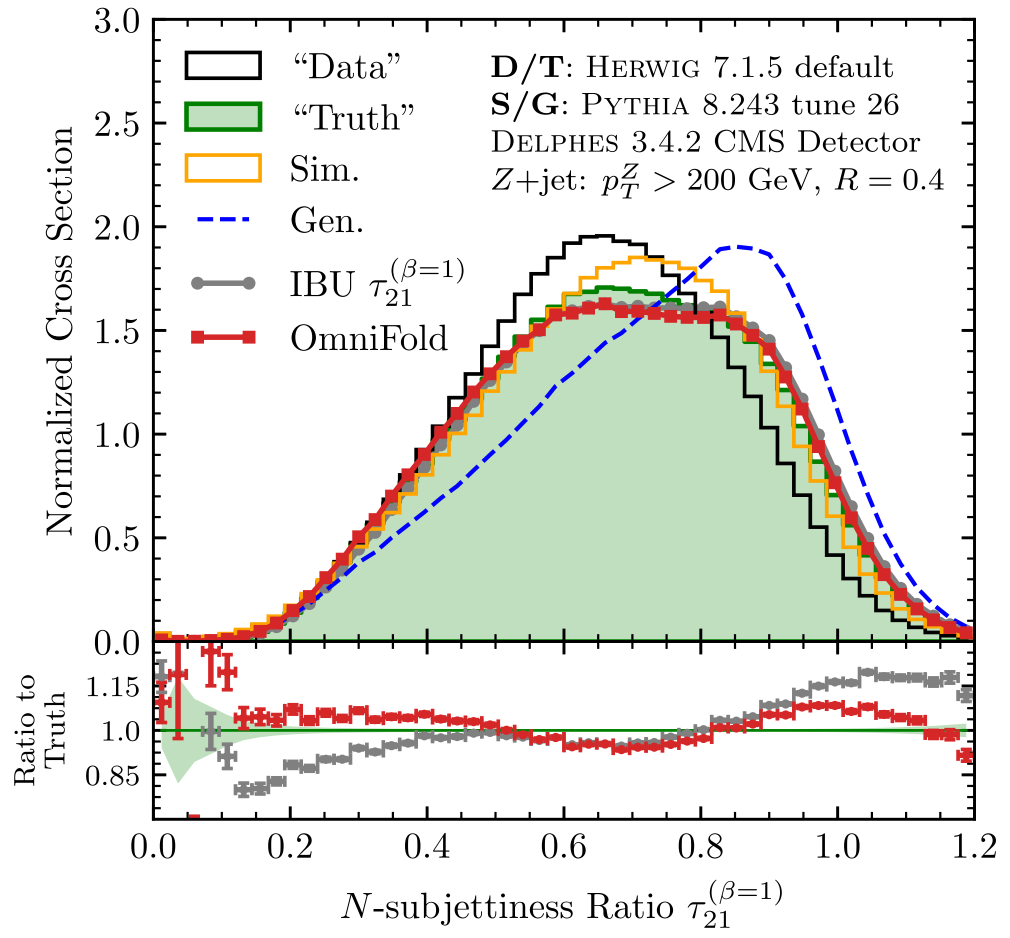
Unfold *any** observable $p_{\text{Gen}}(t)$ using universal weights $\nu_n(t)$

$$\boxed{p_{\text{unfolded}}^{(n)}(t) = \nu_n(t) \times p_{\text{Gen}}(t)}$$

*Observables should be chosen responsibly



IRC safe

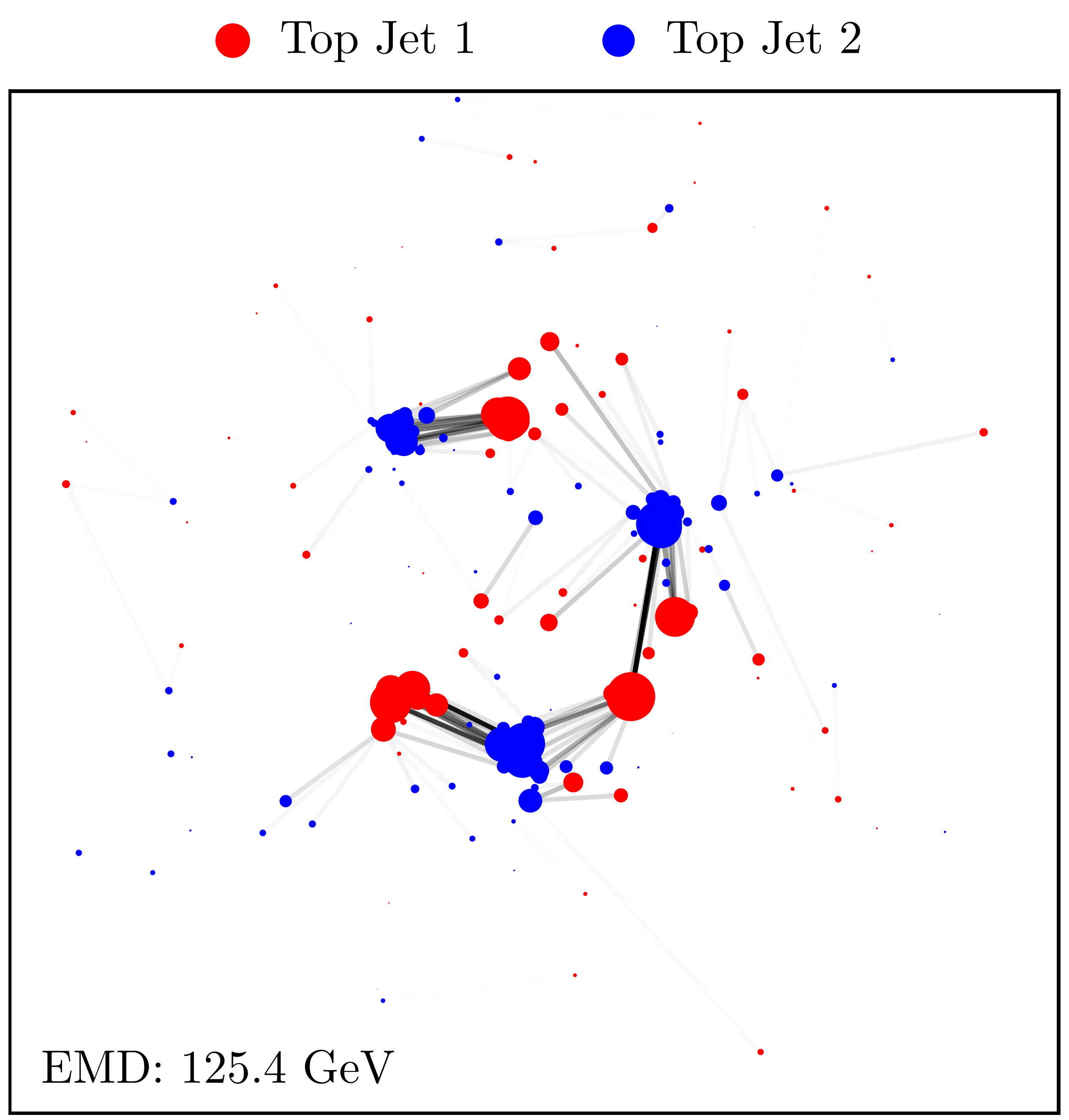


Sudakov safe

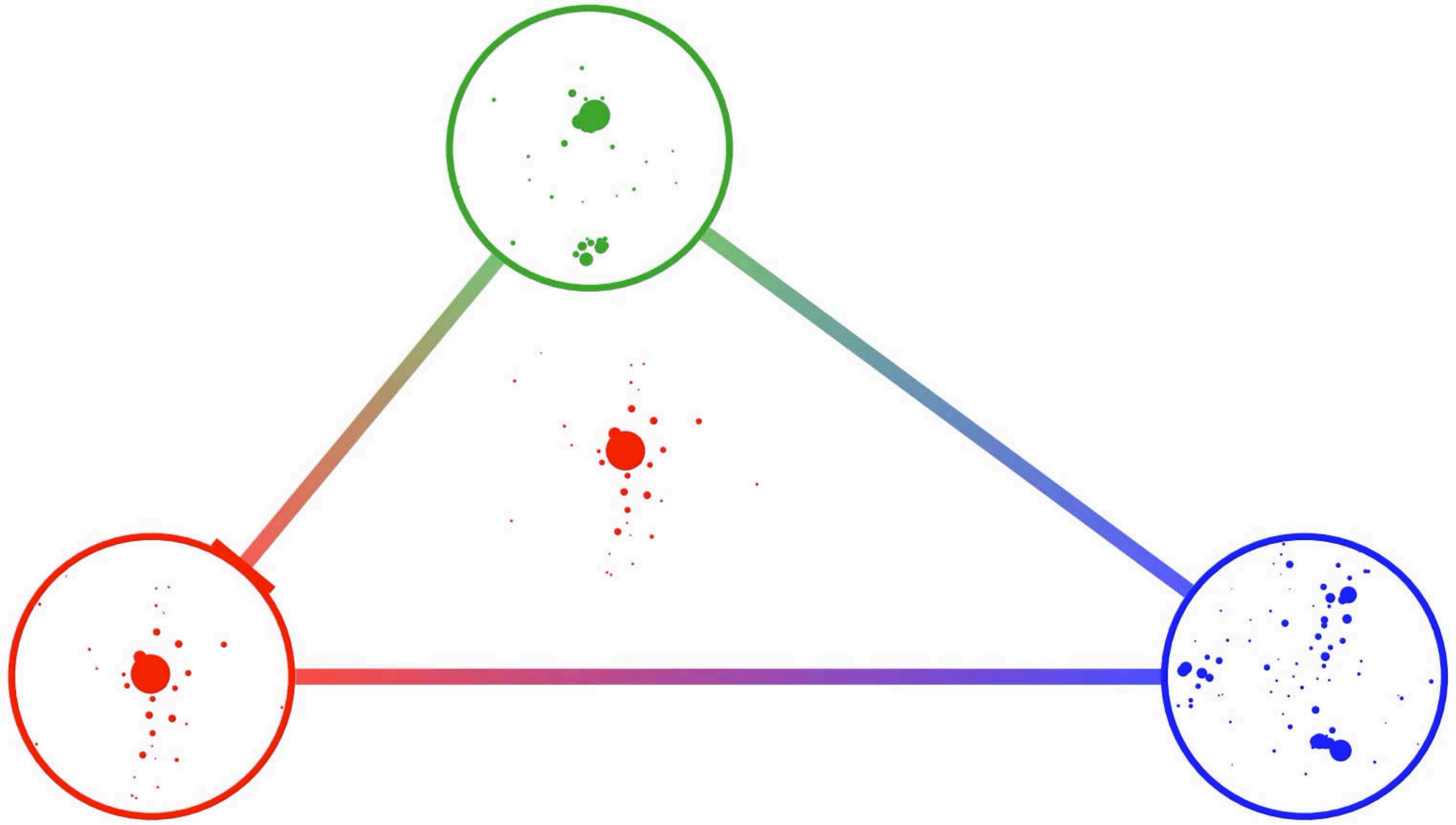
Non-Parametric ML – The Energy Mover's Distance (EMD)

[PTK, Metodiev, Thaler, PRL 2019;
applied on CMS Open Data: PTK, Mastandrea, Metodiev, Naik, Thaler, 1908.08542]

EMD between energy flows defines a metric on the space of events

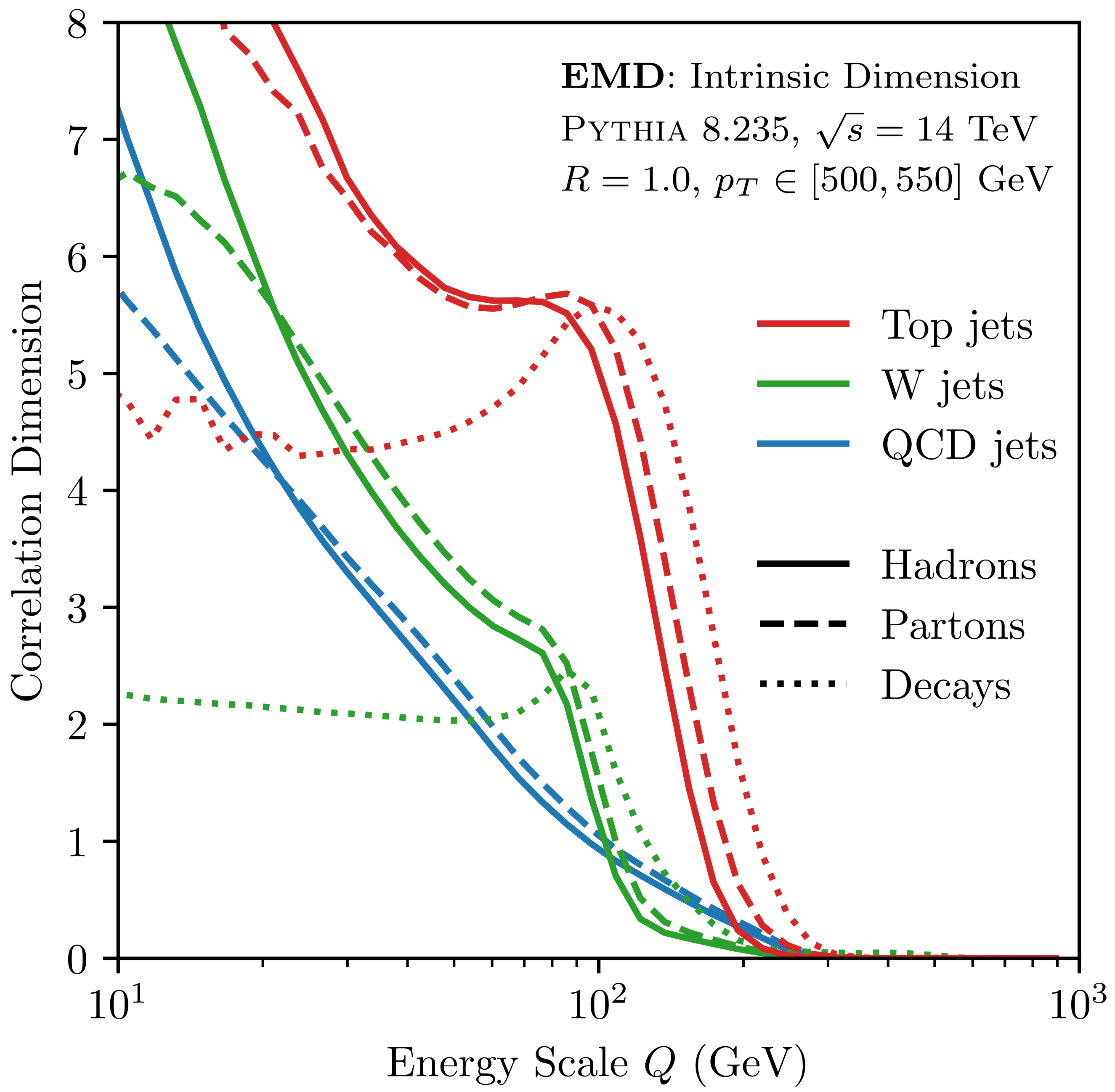


EMD is the work required to rearrange one event into another



Triangle inequality

$$0 \leq \mathrm{EMD}(\mathcal{E}, \mathcal{E}') \leq \mathrm{EMD}(\mathcal{E}, \mathcal{E}'') + \mathrm{EMD}(\mathcal{E}'', \mathcal{E}')$$

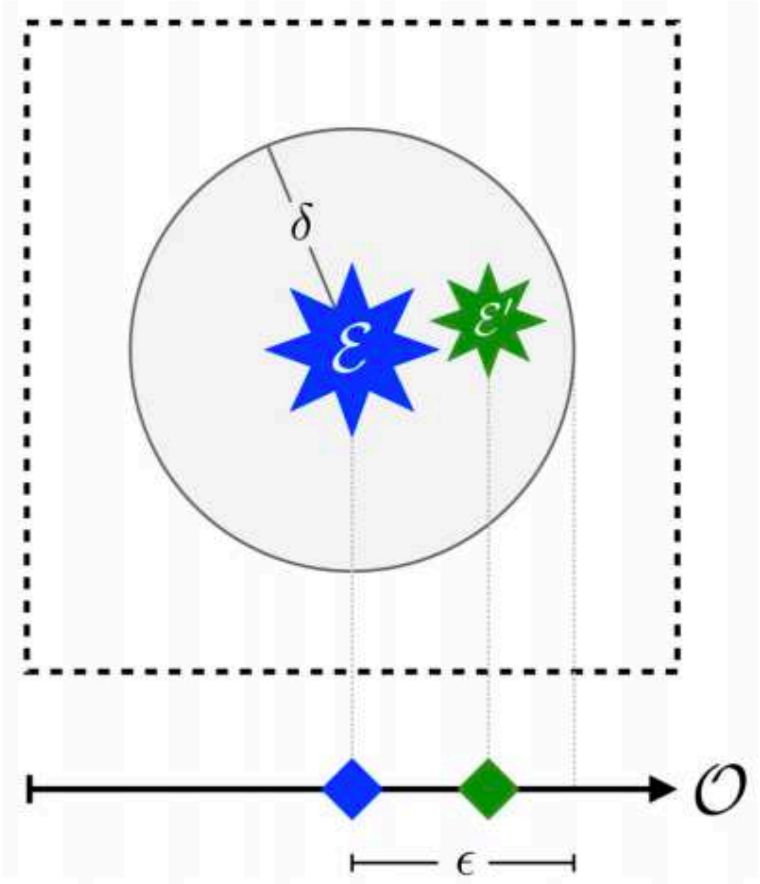


Intrinsic dimensionality of dataset highlights physics at all scales

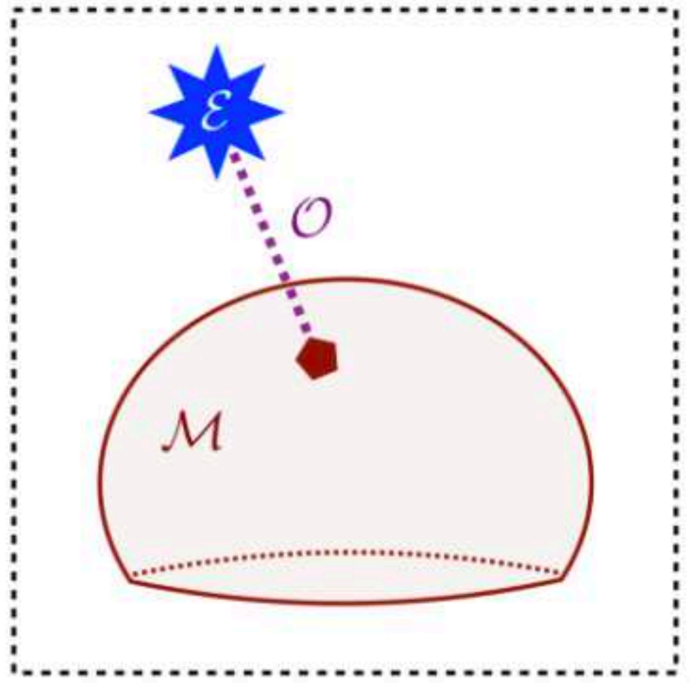
ML Enables New Theoretical Paradigms – Six Decades of Collider Techniques as Geometry

[PTK, Metodiev, Thaler, 2004.04159]

IRC Safety is smoothness in the space of events

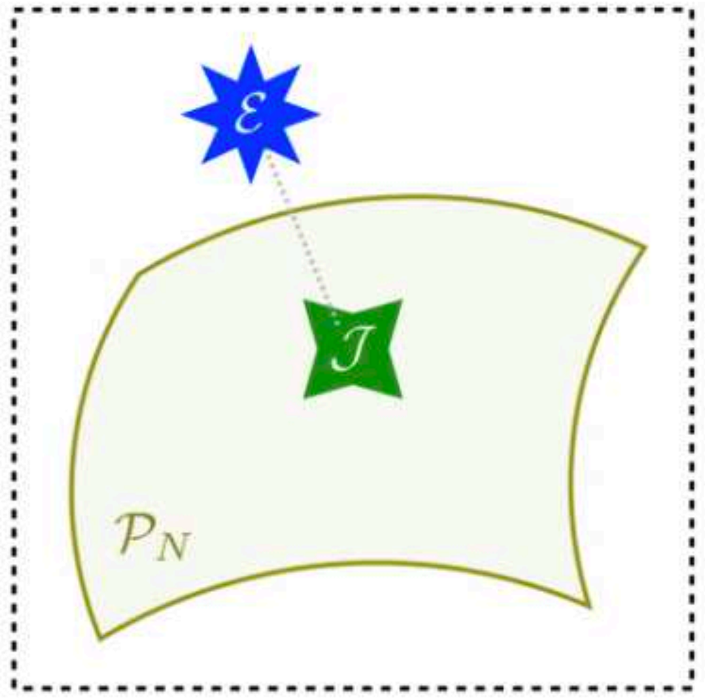


Event shapes are distances from events to manifolds.



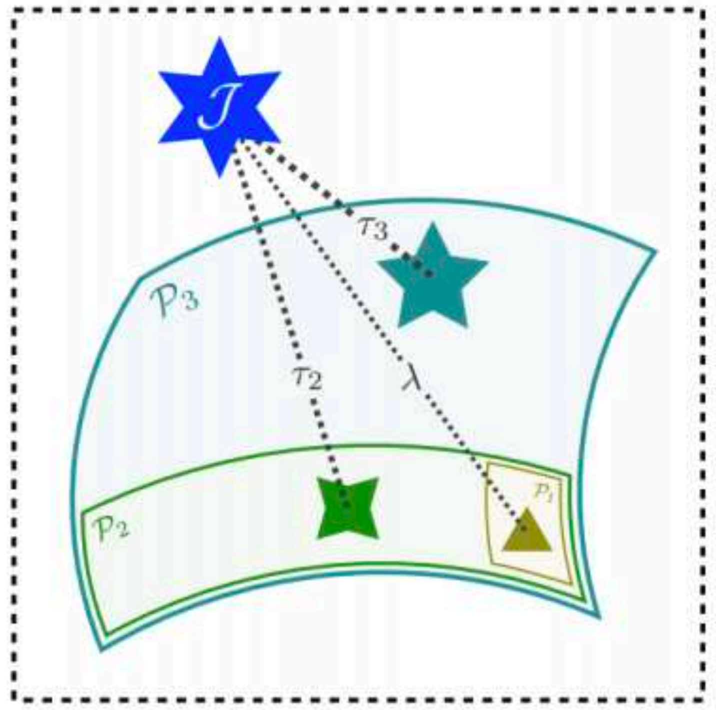
$$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}'\in\mathcal{M}} \text{EMD}_{\beta,R}(\mathcal{E},\mathcal{E}')$$

Jets are projections to few-particle manifolds.



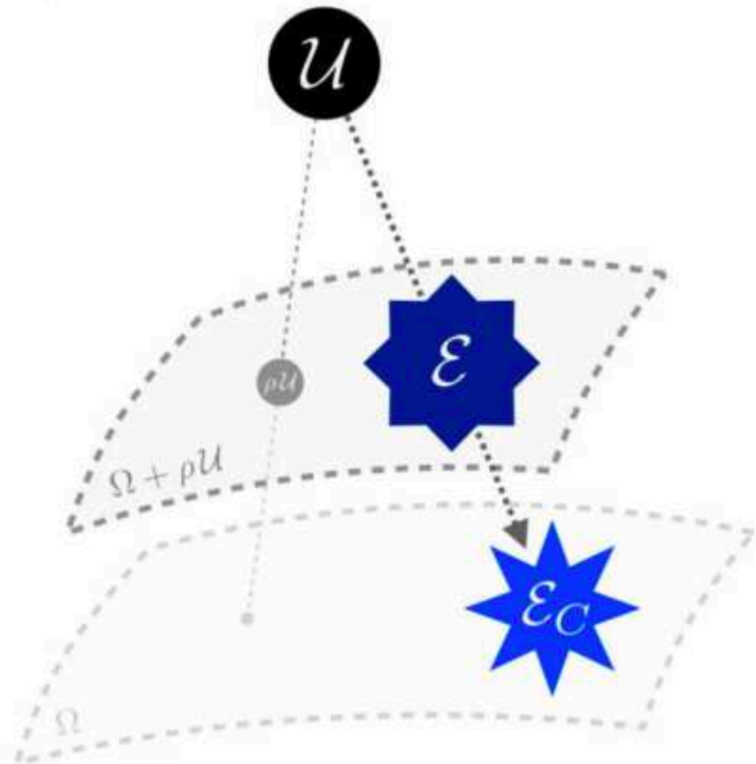
$$\mathcal{J} = \underset{\mathcal{E}'\in\mathcal{P}_N}{\text{argmin}}\, \text{EMD}_{\beta,R}(\mathcal{E},\mathcal{E}')$$

Substructure resolves emissions within the jet.



$$\tau(\mathcal{J}) = \min_{\mathcal{E}'\in\mathcal{P}_N} \text{EMD}_{\beta}(\mathcal{J},\mathcal{E}').$$

Pileup mitigation moves away from uniform radiation.



$$\mathcal{E}_C = \arg\min_{\mathcal{E}'} \text{EMD}(\mathcal{E},\mathcal{E}' + \rho\,\mathcal{U})\,.$$

Taming infinities — Event Shapes — Jet Algorithms — Jet Substructure — Pileup

1960 — 2020

1962-1964 Infrared Safety
[Kinoshita, JMP 1962]
[Lee, Nauenberg, PR 1964]

1977 Thrust, Sphericity
[Farhi, PRL 1977]
[Georgi, Machacek, PRL 1977]

1993 k_T jet clustering
[Ellis, Soper, PRD 1993]
[Catani, Dokshitzer, Seymour, Webber, NPB 1993]

1997-1998 C/A jet clustering
[Wobisch, Wengler, 1998]
[Doskhitzer, Leder, Moretti,Webber, JHEP 1997]

2010-2015 N-(sub)jettiness, XCone
[Stewart, Tackmann, Waalewijn, PRL 2010]
[Thaler, Van Tilburg, JHEP 2011]
[Stewart, Tackmann, Thaler, Vermilion, Wilkason, JHEP 2015]

2014-2019 Constituent Subtraction
[Berta, Spousta, Miller, Leitner, JHEP 2014]
[Berta, Masetti, Miller, Spousta, JHEP 2019]

And many more!

*See backup for more on IRC safety and perturbative calculability

[Slide from talk by Eric Metodiev]

Handling Uncertainties

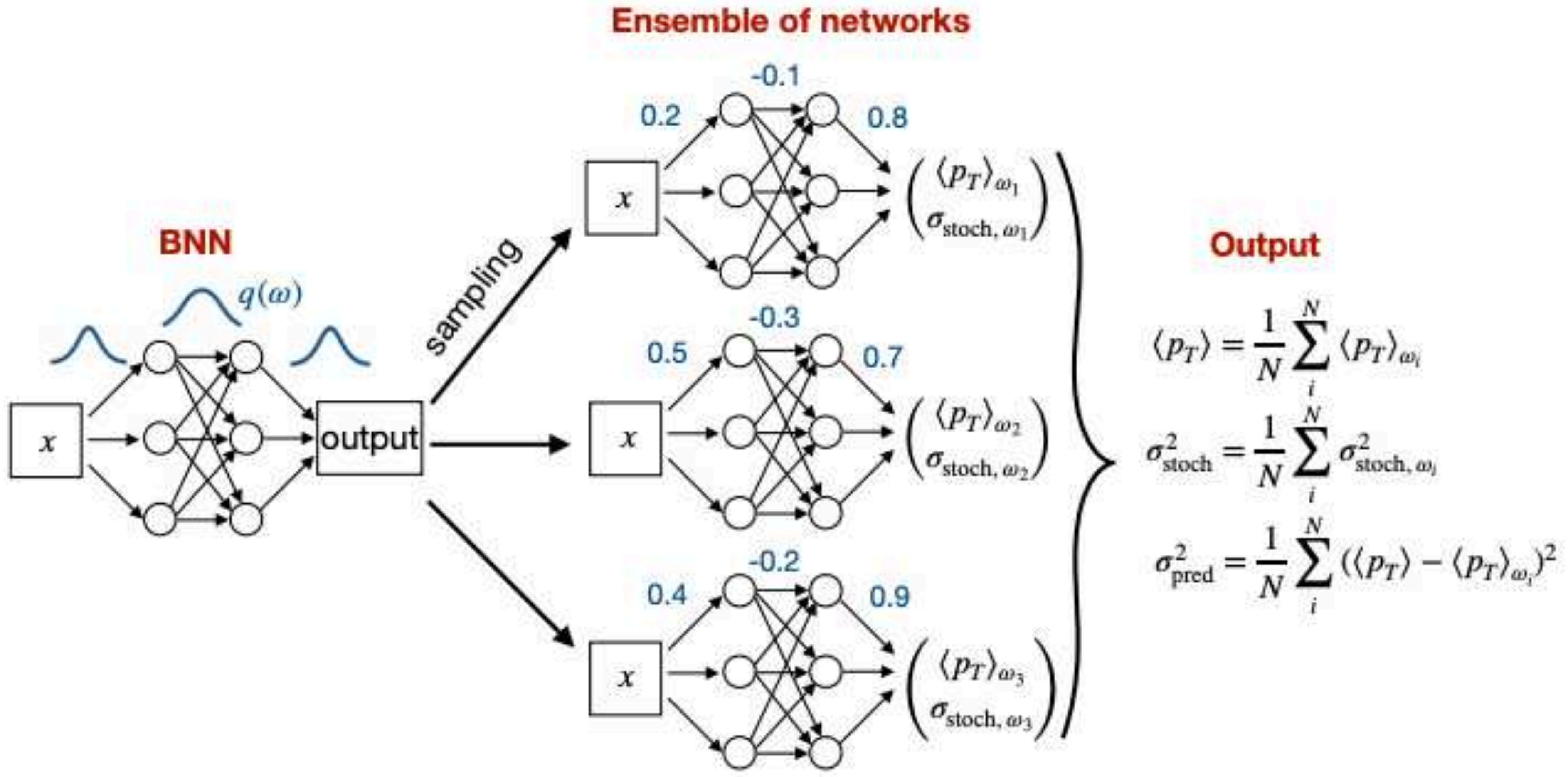
ML can assist in handling uncertainties (but can also create its own difficulties)

Adversarial training to reduce dependence on uncertain regions

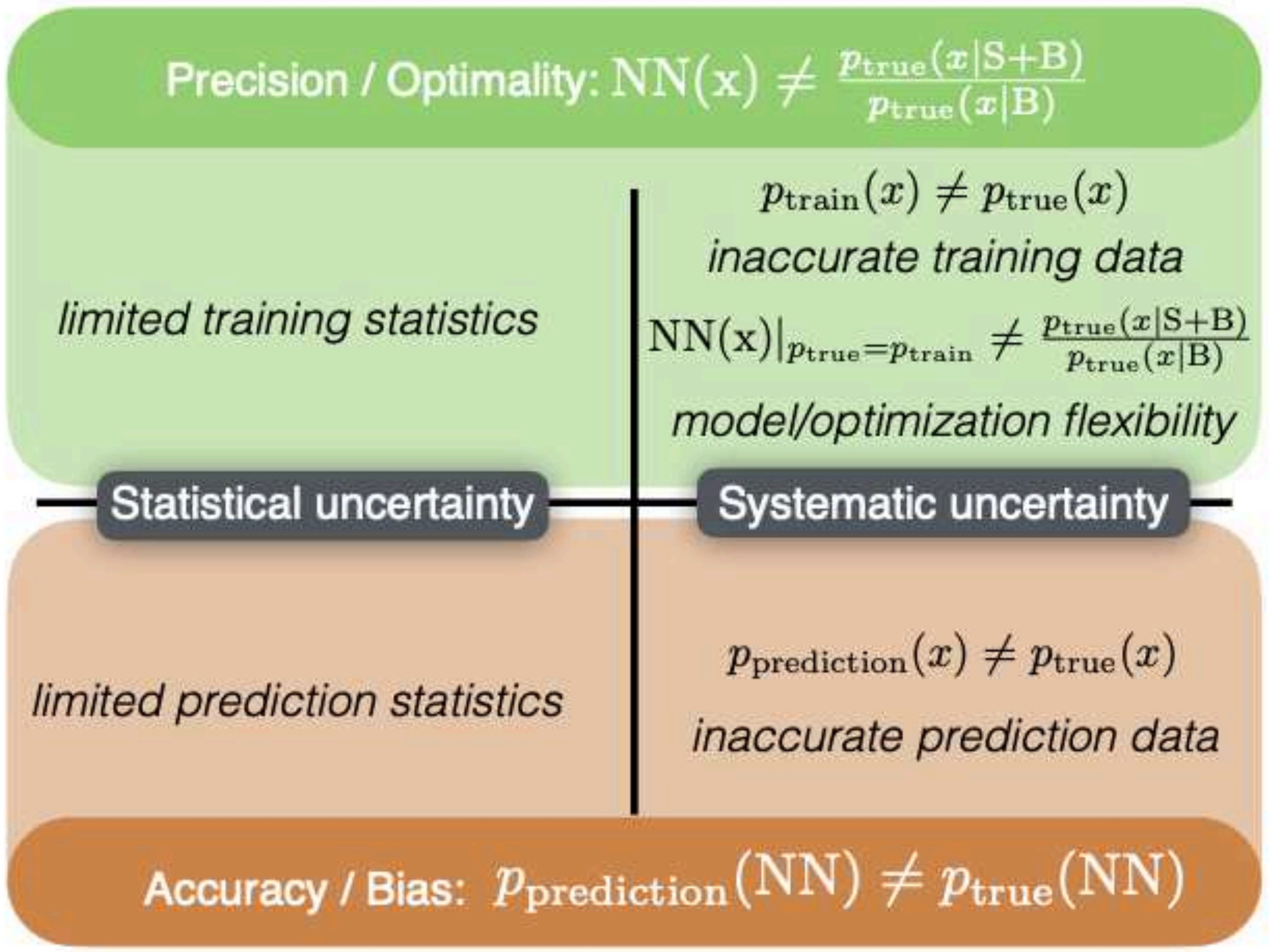
[Englert, Galler, Harris, Spannowsky, 1807.08763]

Bayesian neural networks can estimate some uncertainties

[Bollweg, Haußmann, Kasieczka, Luchmann, Plehn, Thompson, 1904.10004; Kasieczka, Luchmann, Otterpohl, Plehn, 2003.11099]



Sources of uncertainty in a statistical analysis



[Nachman, 1909.03081]

Parametrized models could enable efficient profiling to handle systematic uncertainties

[similar to Baldi, Cranmer, Faucett, Sadowski, Whiteson, 1601.07913]

… And So Much More HEP-Specific ML

Weakly Supervised Learning

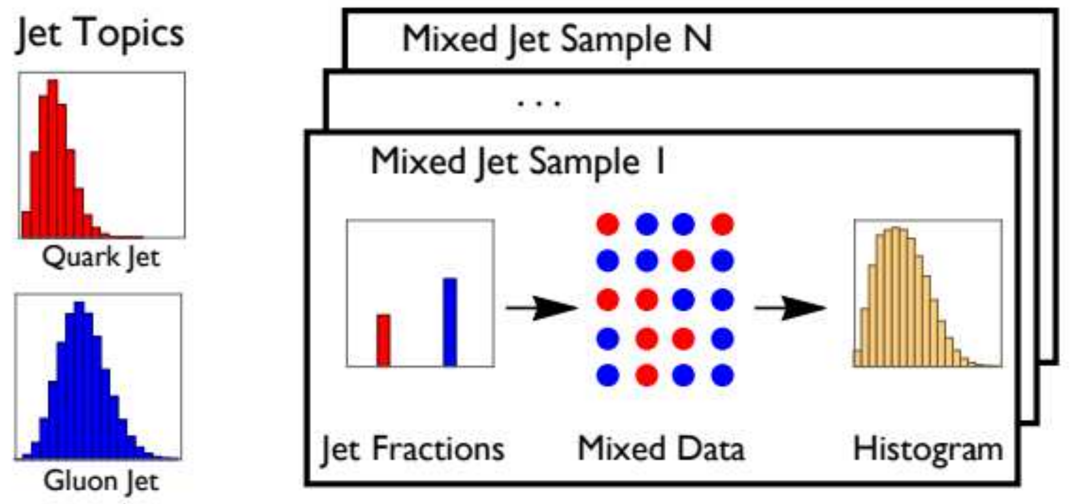
e.g. LLP, CWoLa

[Dery, Nachman, Rubbo, Schwartzman, 1702.00414;
Metodiev, Nachman, Thaler, 1708.02949;
Cohen, Freytsis, Ostdiek, 1706.09451;
PTK, Metodiev, Nachman, Schwartz, 1801.10158]

Topic Modeling

e.g. Jet Topics

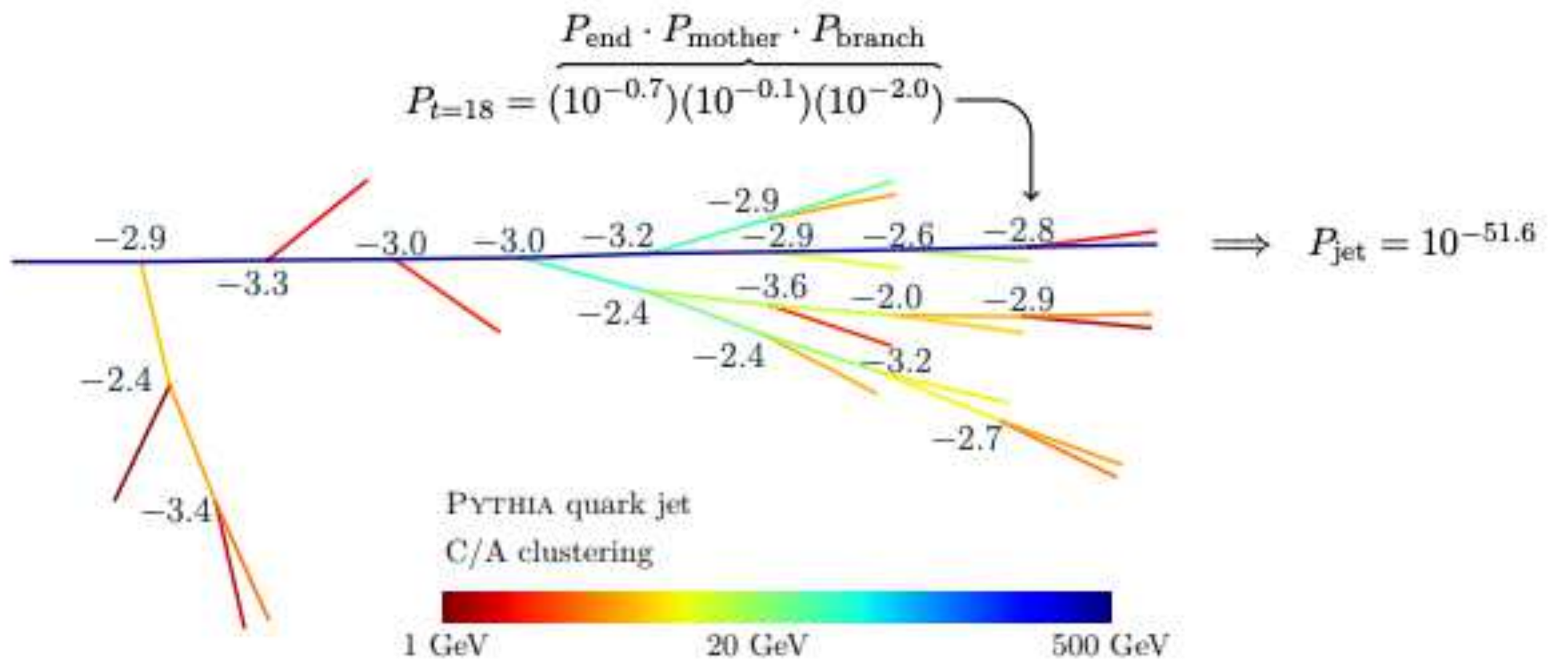
[Metodiev, Thaler, 1802.00008;
PTK, Metodiev, Thaler, 1809.01140]



Unsupervised Learning

e.g. JUNIPR

[Andreassen, Feige, Frye, Schwartz, 1804.09720, 1906.10137]

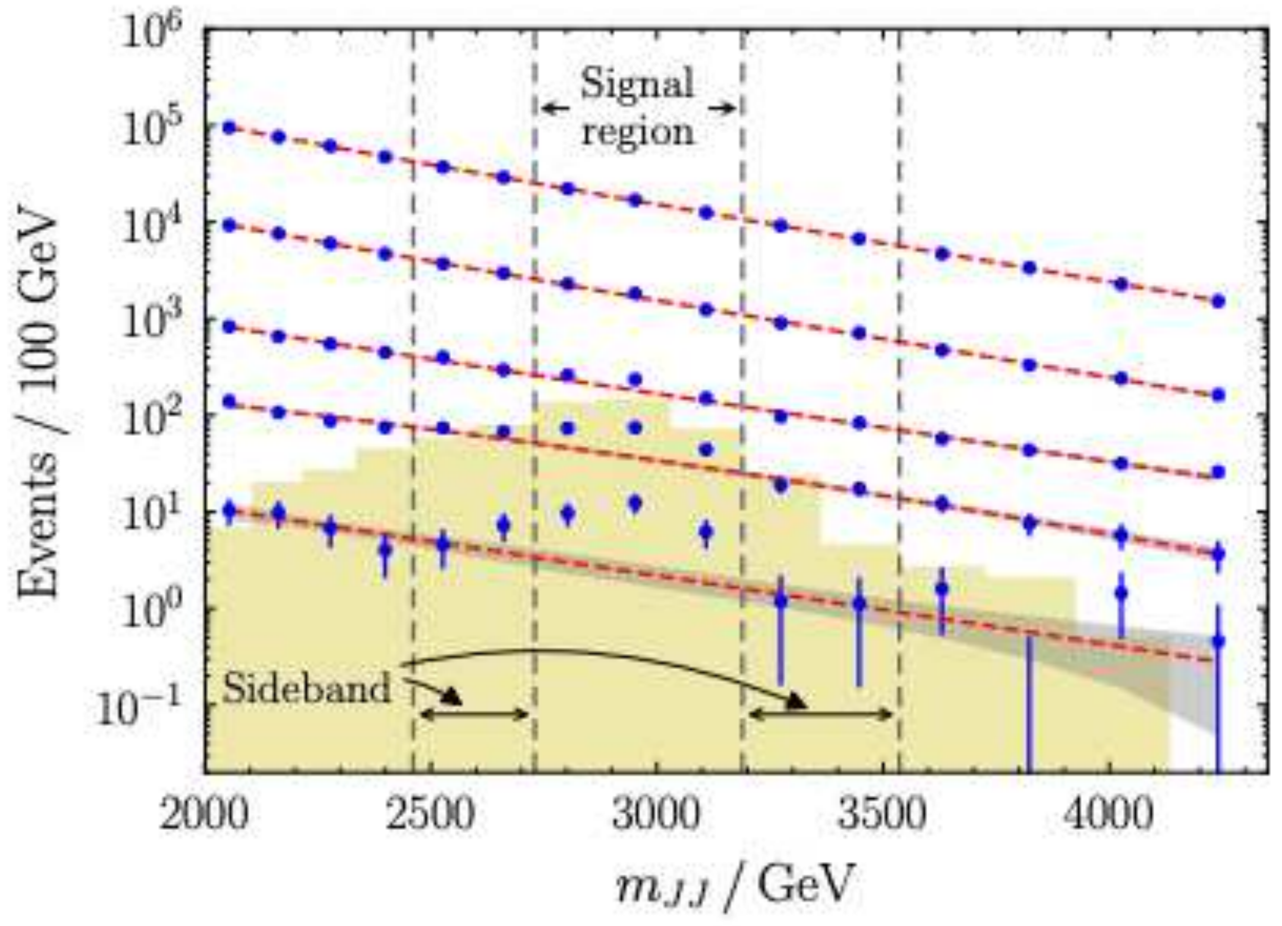


Probabilistic formation of a Pythia jet

Anomaly Detection

tons of recent work

[Collins, Howe, Nachman, 1805.02664;
Farina, Nakai, Shih, 1808.08992;
Heimel, Kasieczka, Plehn, Thompson, 1808.08979;
Cerri, Nguyen, Pierini, Spiropulu, Vlimant, 1811.10276;
Blance, Spannowsky, Waite, 1905.10384;
See LHC Olympics 2020 anomaly detection workshop]

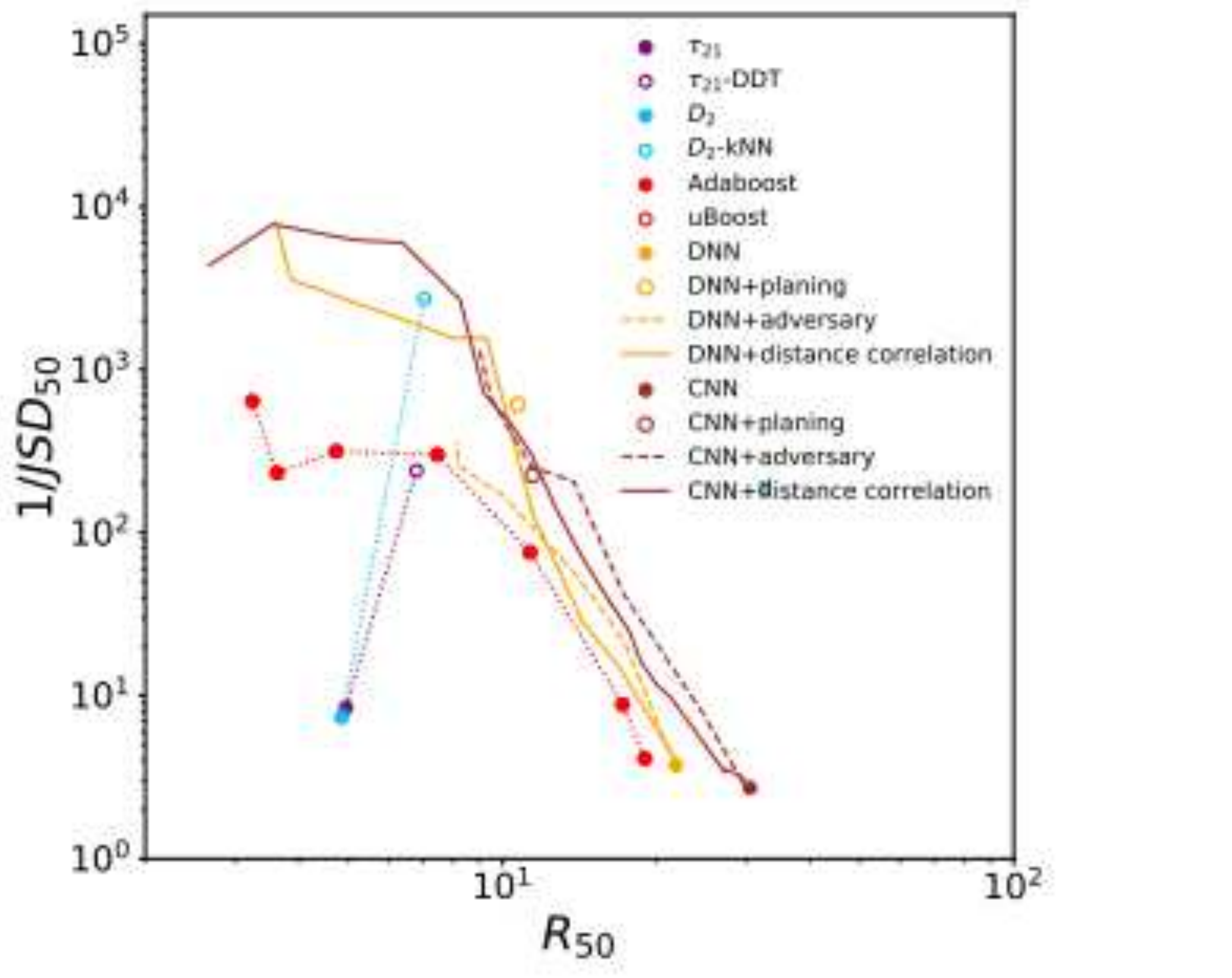


CWoLa hunting to enhance a resonance search

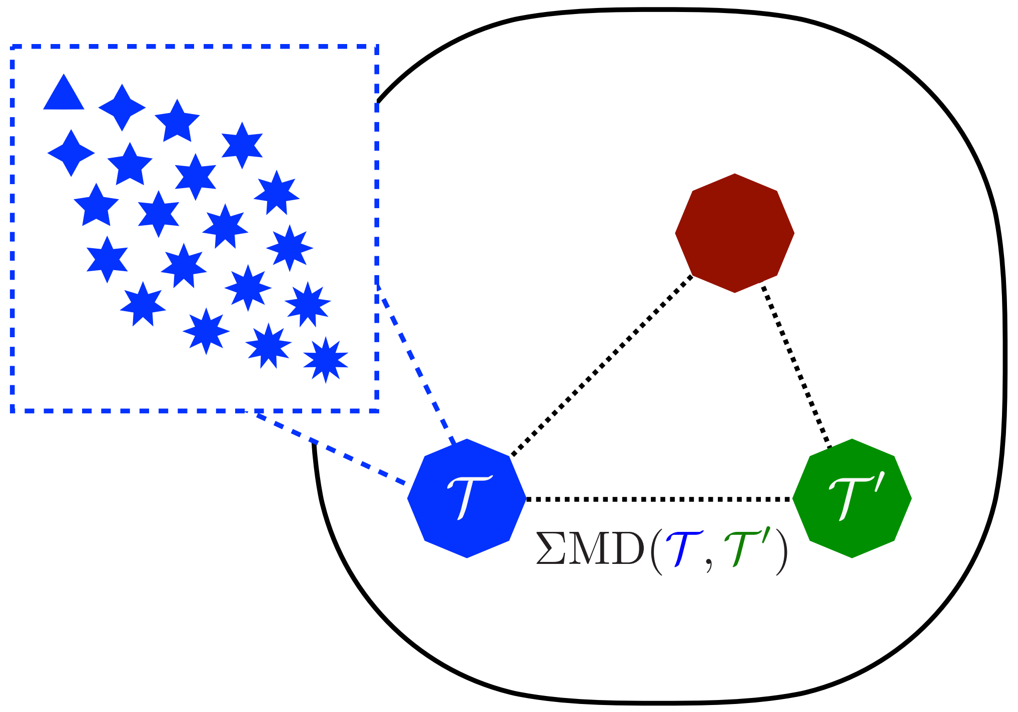
Decorrelation Methods

e.g. DDT, Planing, DisCo

[Dolen, Harris, Marzani, Rappoccio, Tran, 1603.00027;
Chang, Cohen, Ostdiek, 1709.10106;
Kasieczka, Shih, 2001.05310;
Kasieczka, Nachman, Schwartz, Shih, 2007.14400]



Decorrelating ML taggers from a key observable



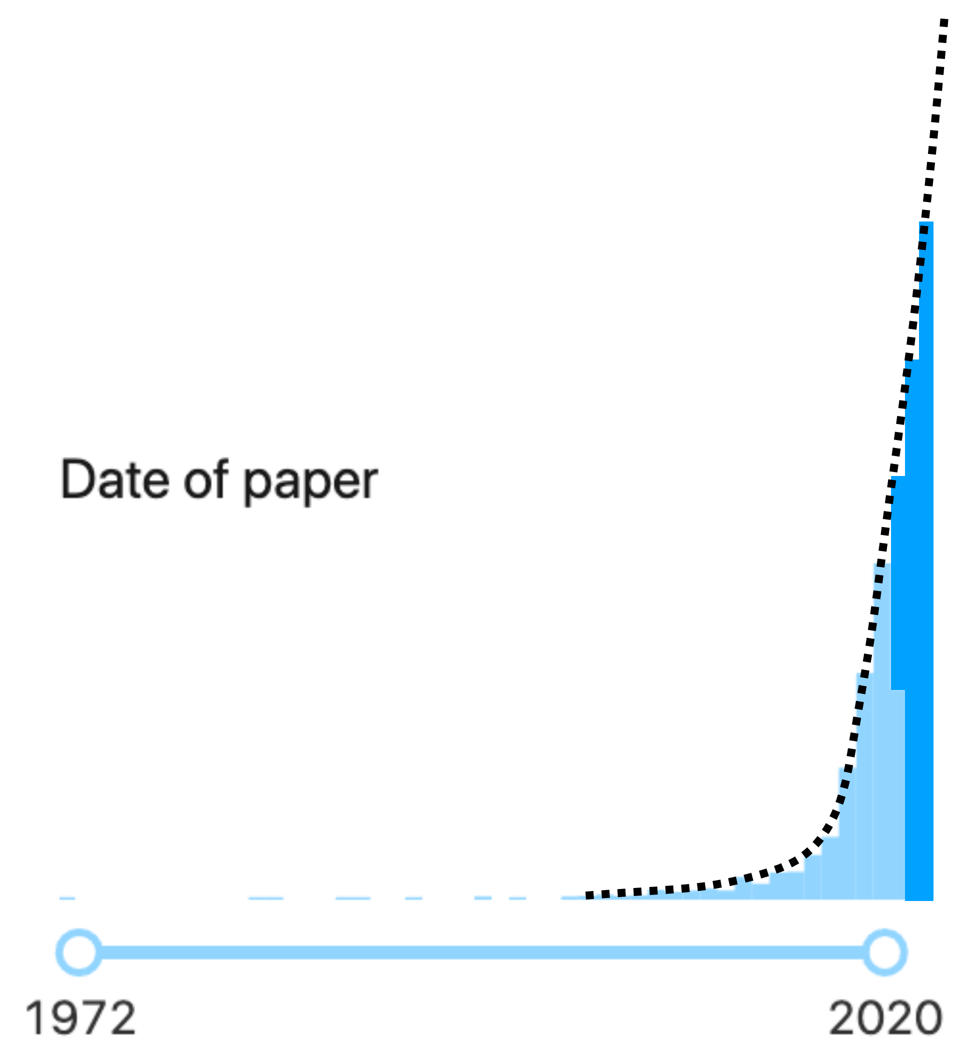
Ubiquity of ML in HEP

Lightning Review

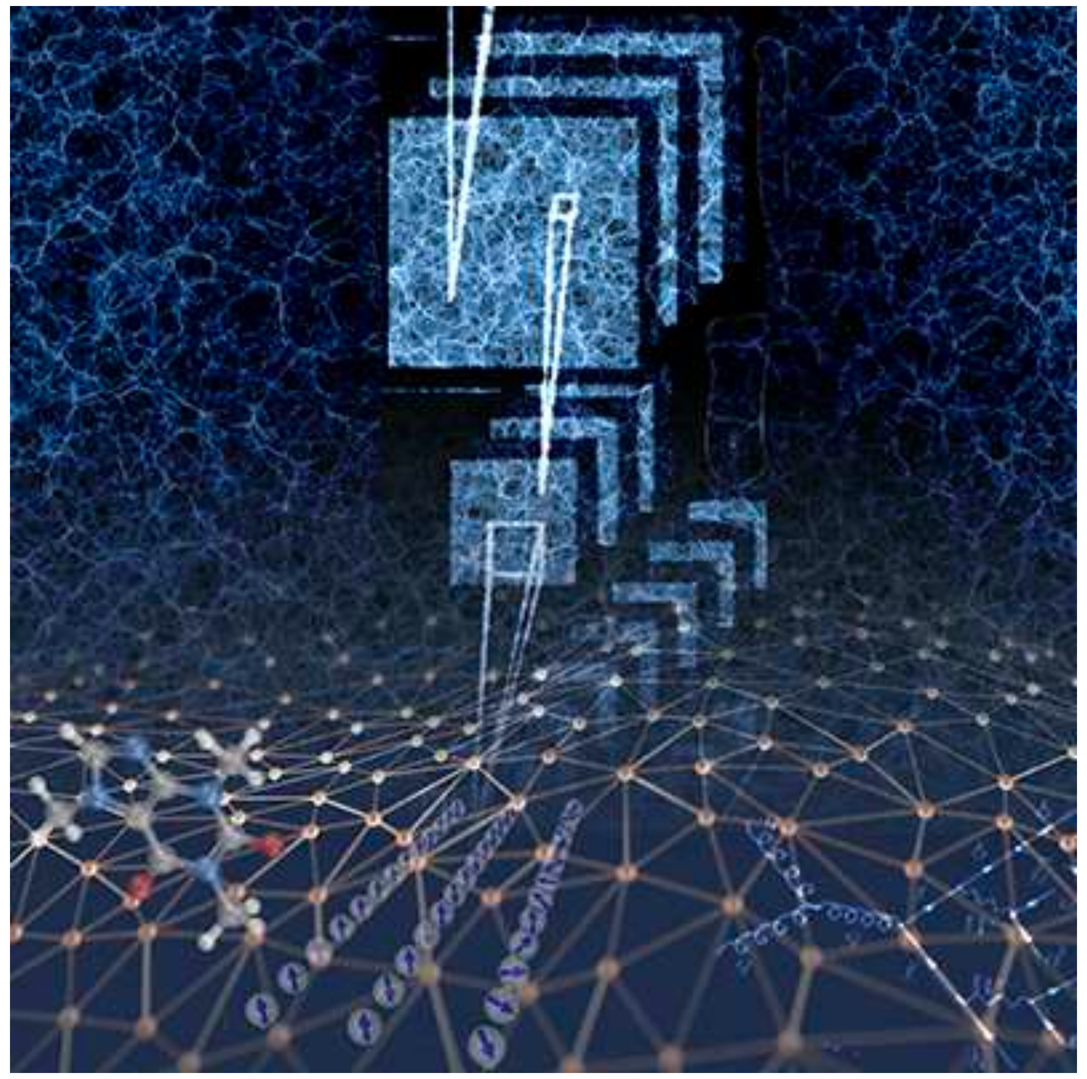
Future Directions

Thoughts on the Future

- *Machine learning will be essential in maximizing HEP potential*
  - We should capitalize on the opportunity to optimize
  - ML is both a computational tool and a useful formalism/language
- *Diversity of applications is impressive and will become more so*
  - Rapidly advancing beyond "hammer and nail" approach for ML in HEP
  - New observables, architectures, paradigms have been developed
- *Collaboration across traditional lines will enable success*
  - Learn from and contribute to the highly-vibrant ML community
  - ML in HEP needs insight from theory and experiment
- *ML strongly overlaps with other computational frontier areas*
  - Software workflows, reproducible analyses, public datasets are critical



HEPML-LivingReview has a thorough and organized list of papers



[Reviews of Modern Physics Cover December 2019 from Machine learning and the physical sciences]

Thank You!

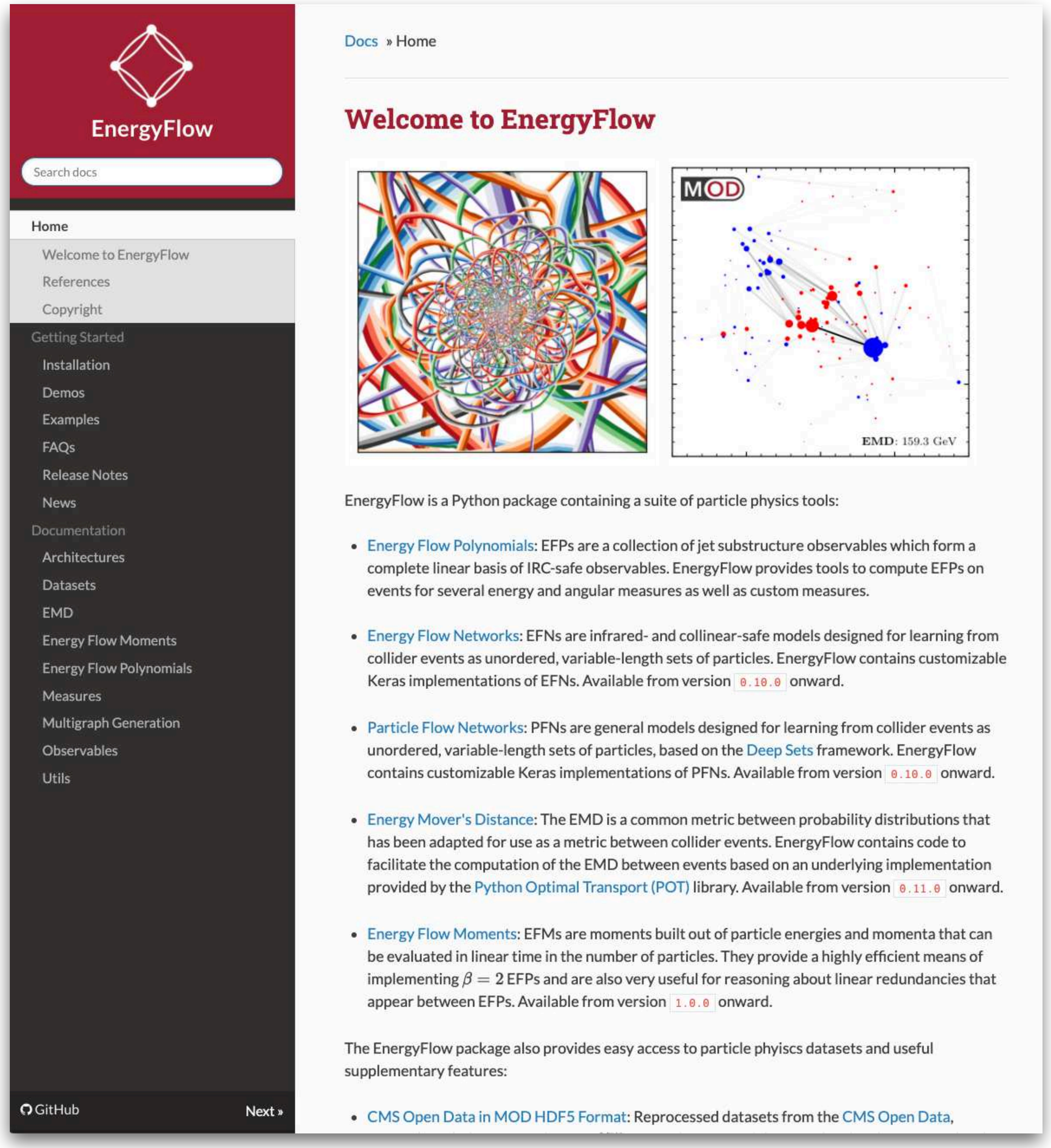
EnergyFlow Python Package

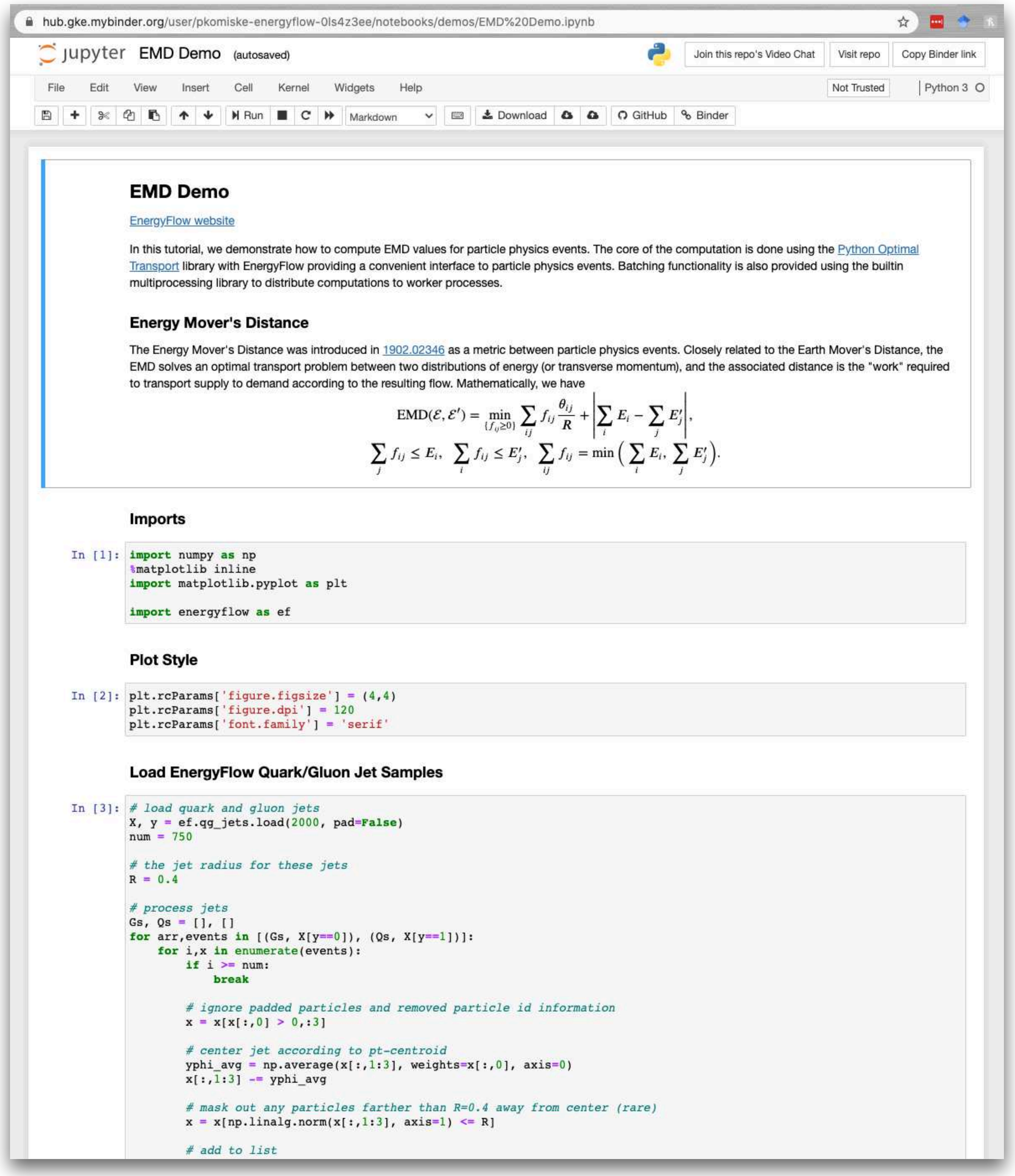
pip3 install energyflow

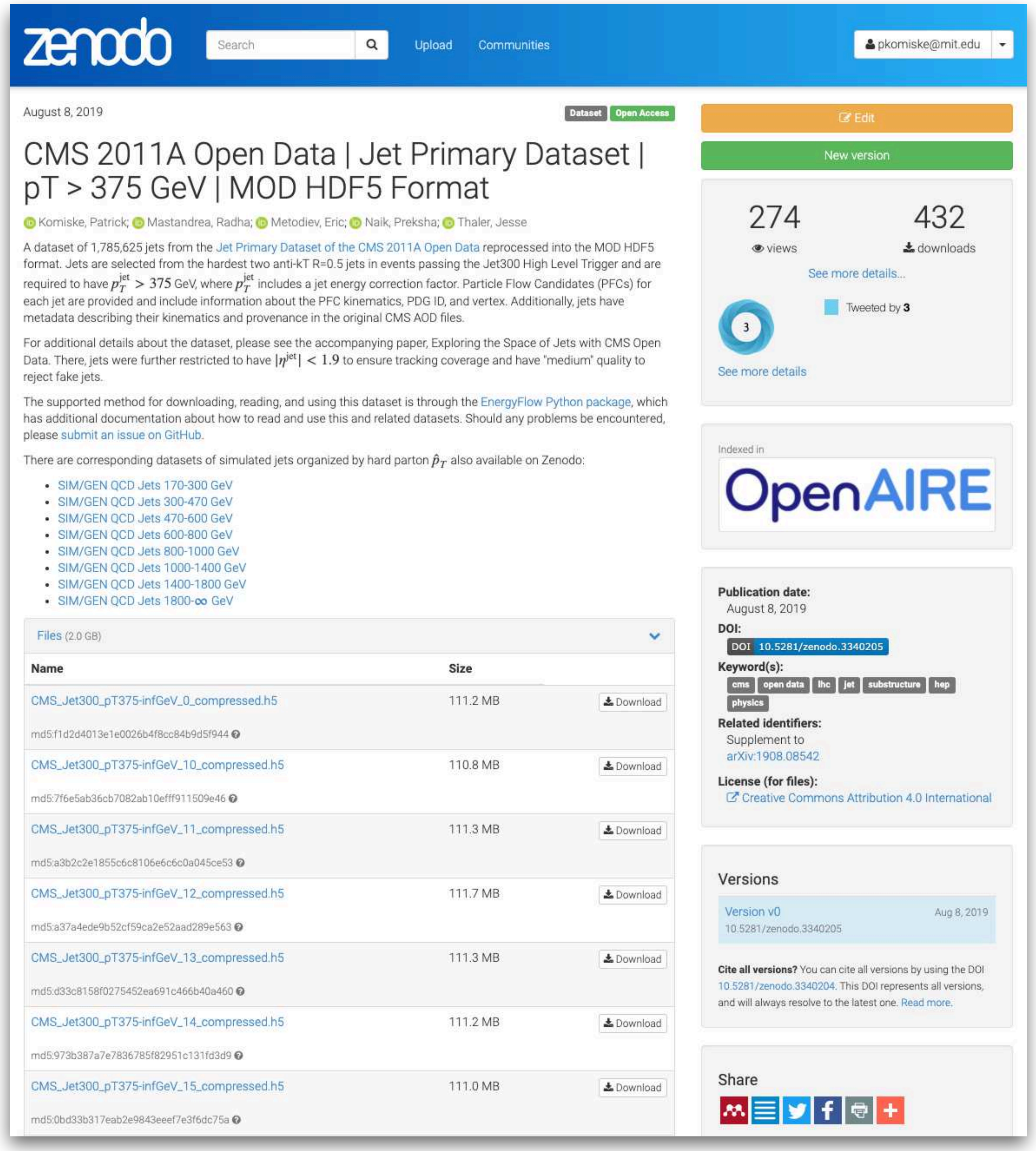
Keras/Tensorflow implementations of Energy/Particle Flow Networks

Interfaces with reprocessed CMS 2011A Jet Primary Dataset hosted on Zenodo

Detailed examples, demos, and documentation







Iterated Bayesian Unfolding (IBU)

Histogram-based unfolding method for a small number of observables

Choose observable(s) and binning at detector-level and particle-level

measured distribution: $m_i = \Pr(\text{measure } i)$ true distribution: $t_j^{(0)} = \Pr(\text{truth is } j)$

Calculate *response matrix* R_{ij} from generated/simulated pairs of events

$R_{ij} = \Pr(\text{measure } i \mid \text{truth is } j)$ R is in general non-square and non-invertible

Calculate new particle-level distribution using Bayes' theorem

$$t_j^{(n)} = \sum_i \Pr(\text{truth}_{n-1} \text{ is } j \mid \text{measure } i) \times \Pr(\text{measure } i) = \sum_i \frac{R_{ij} t_j^{(n-1)}}{\sum_k R_{ik} t_k^{(n-1)}} \times m_i$$

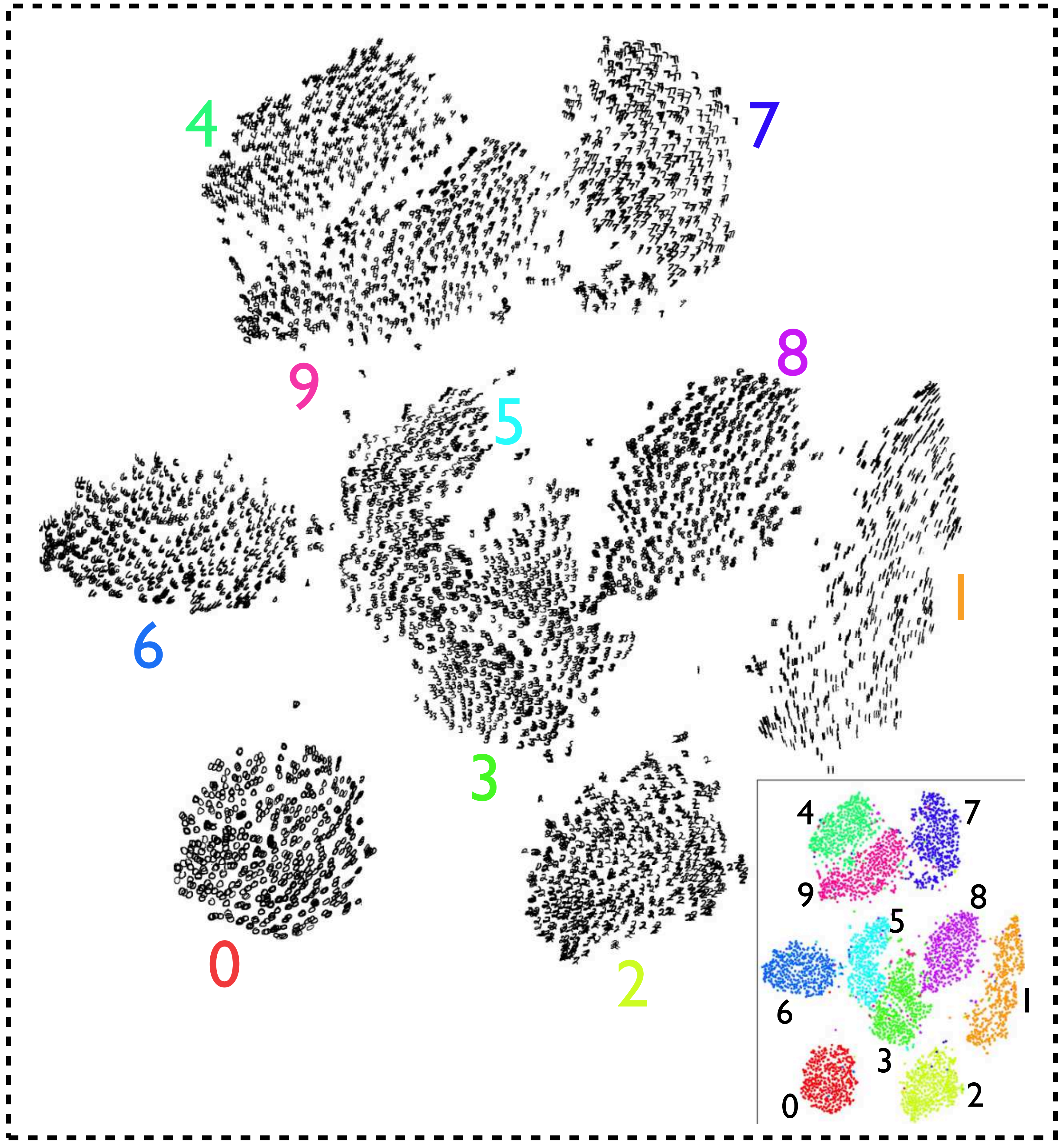
Iterate procedure to remove dependence on prior

[Richardson, 1972; Lucy, 1974; D'Agostini, 1995]

Visualizing Geometry in the Space of Events

t-Distributed Stochastic Neighbor Embedding (t-SNE)

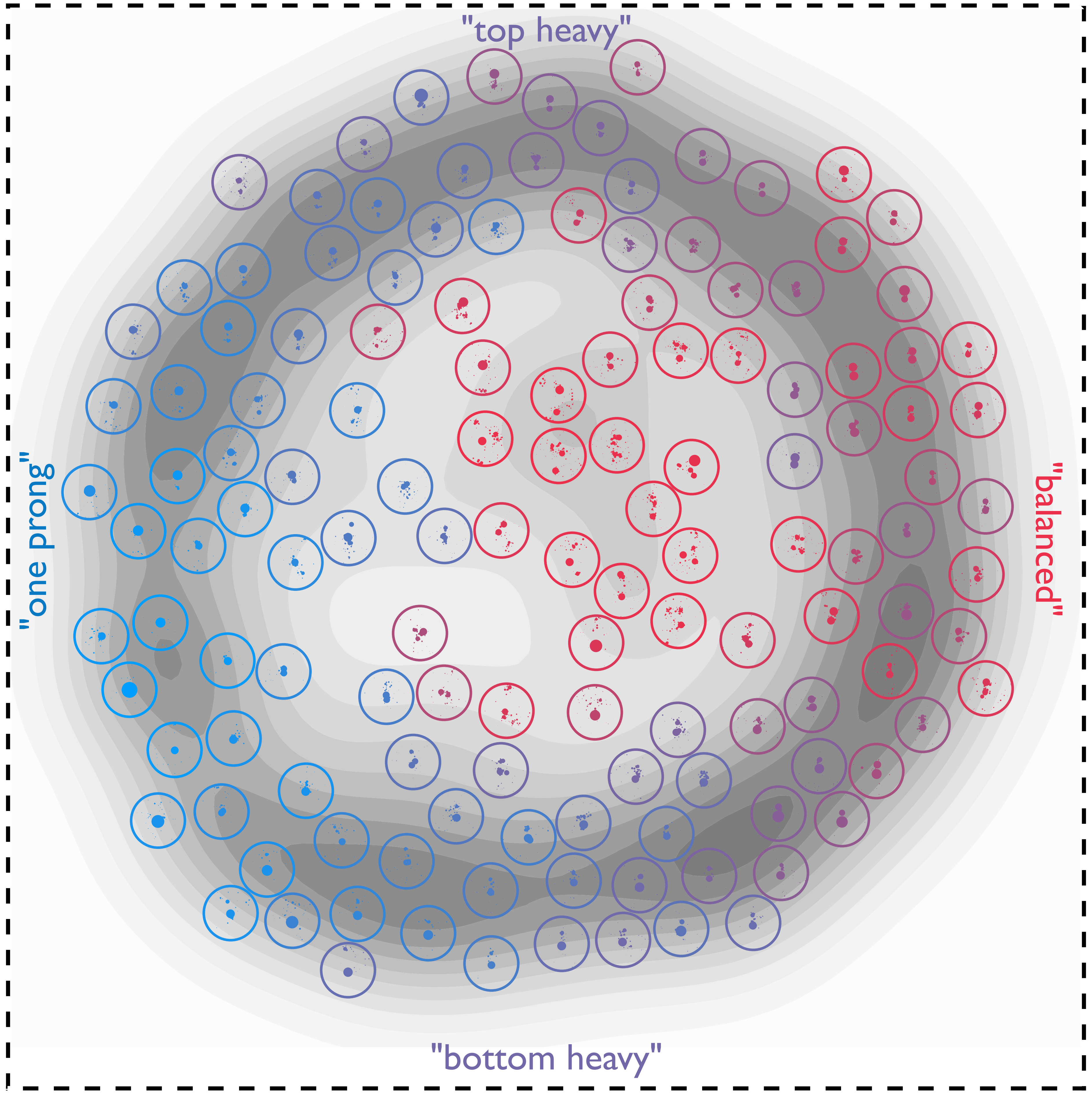
MNIST handwritten digits

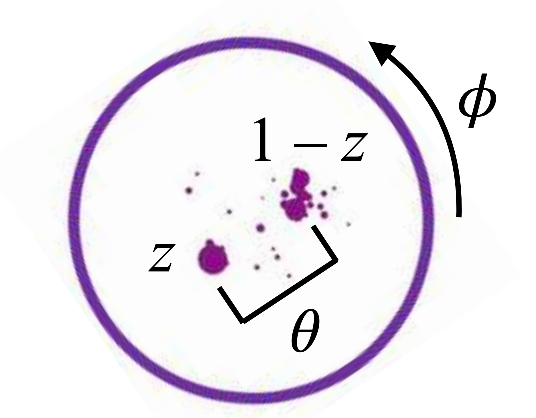


[L. van der Maaten, G. Hinton, JMLR 2008]

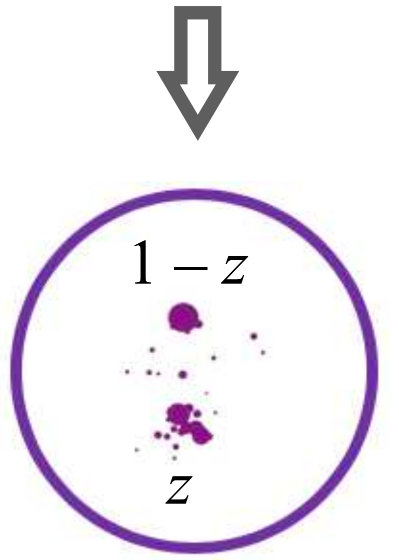
[PTK, Metodiev, Thaler, PRL 2019]

Geometric space of W jets





Constraints: W Mass and ϕ = 0 preprocessing



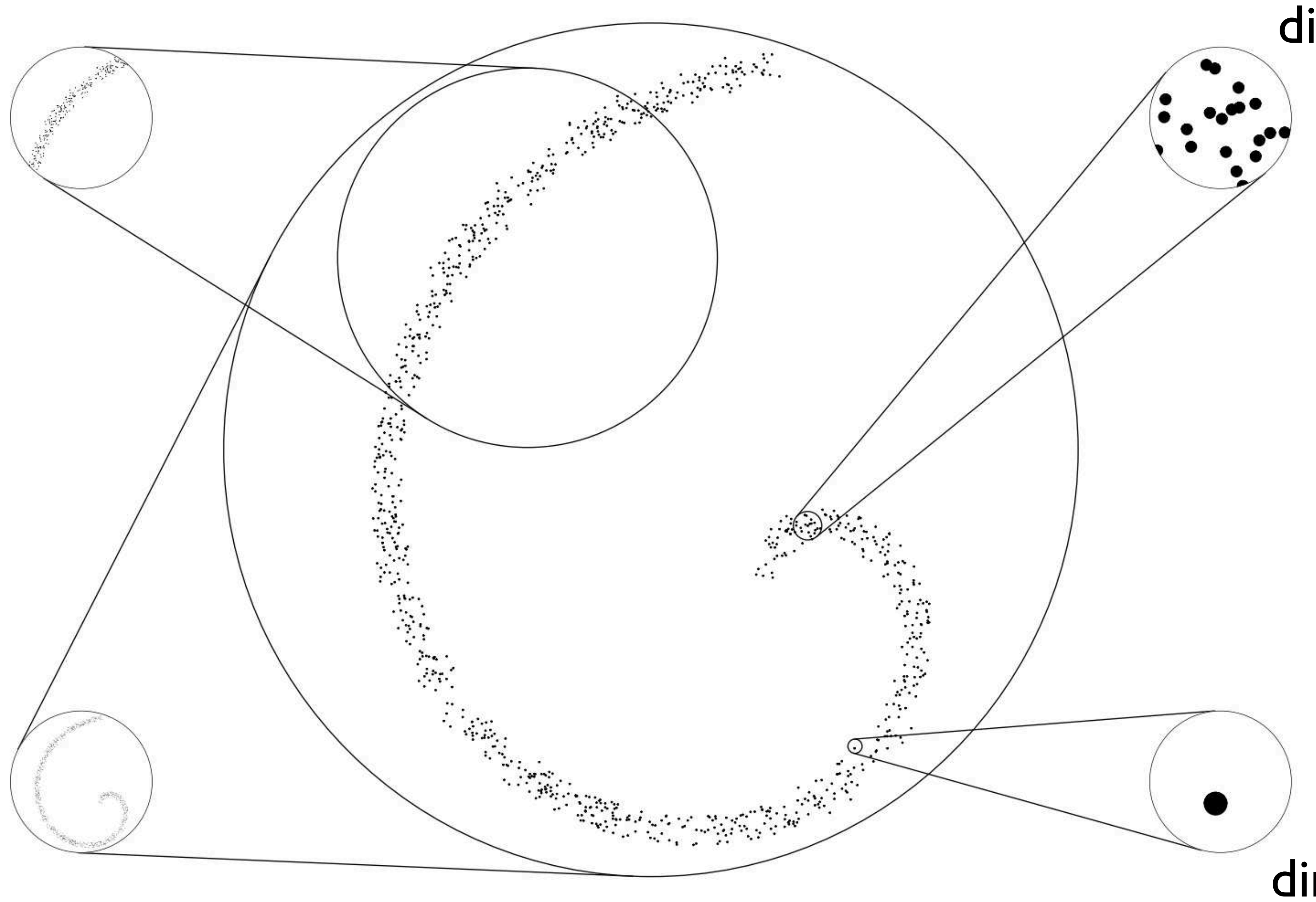
Gray contours represent the density of jets

Each circle is a particular W jet

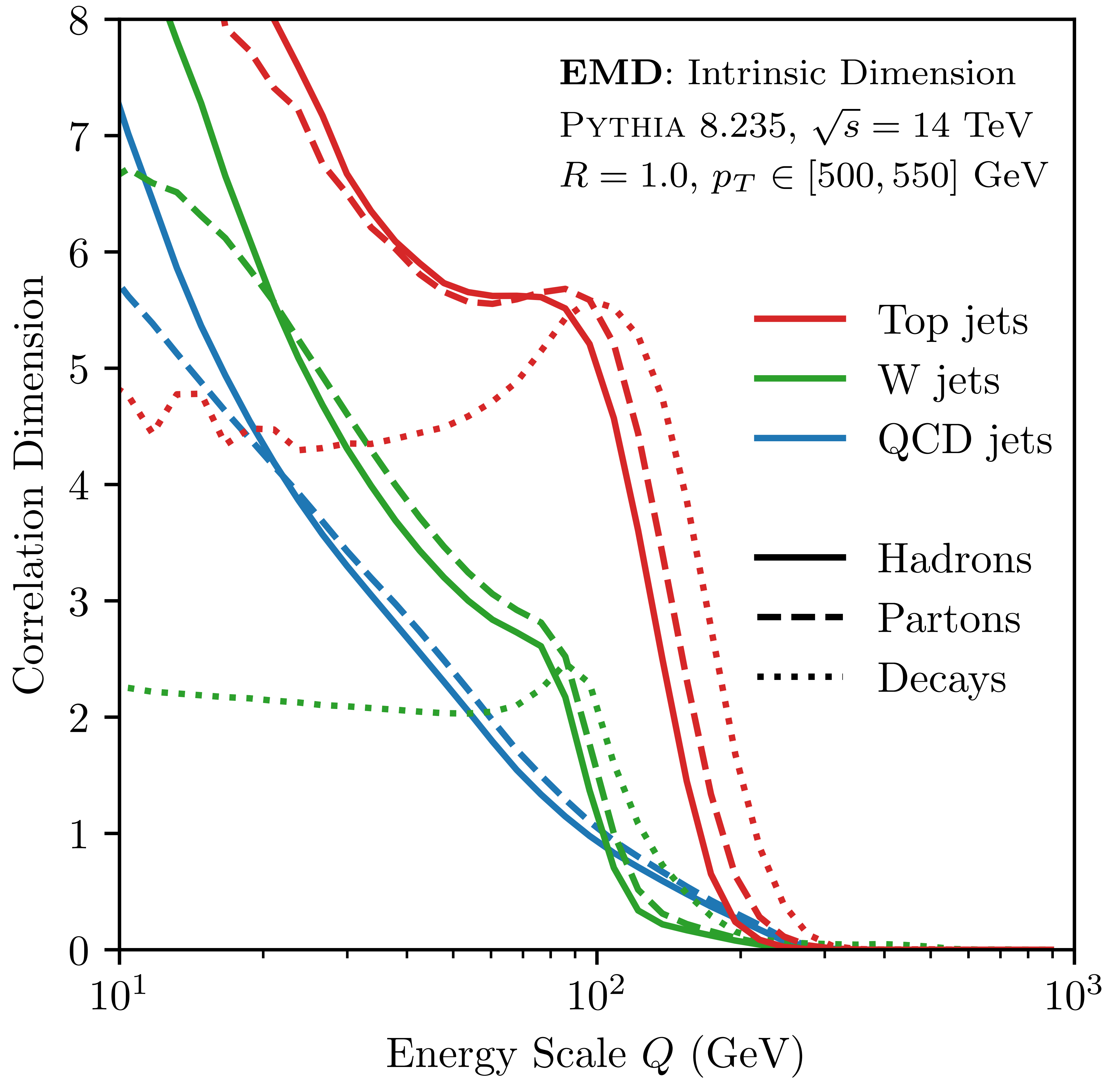
Quantifying Event-Space Manifolds

$$\dim(Q) = Q\frac{\partial}{\partial Q}\ln \sum_i \sum_j \Theta(\text{EMD}(\mathcal{E}_i, \mathcal{E}'_j) < Q)$$

Correlation dimension: *how does the # of elements within a ball of size Q change?*



$$N_{\text{neigh.}}(Q) \propto Q^{\dim} \implies \dim(Q) = Q\frac{d}{dQ}\ln N_{\text{neigh.}}(Q)$$



Correlation dimension lessons:

- Decays are "constant" dim. at low Q
- Complexity hierarchy: QCD < W < Top
- Fragmentation increases dim. at smaller scales
- Hadronization important around 20-30 GeV

[Grassberger, Procaccia, PRL 1983; PTK, Metodiev, Thaler, PRL 2019]

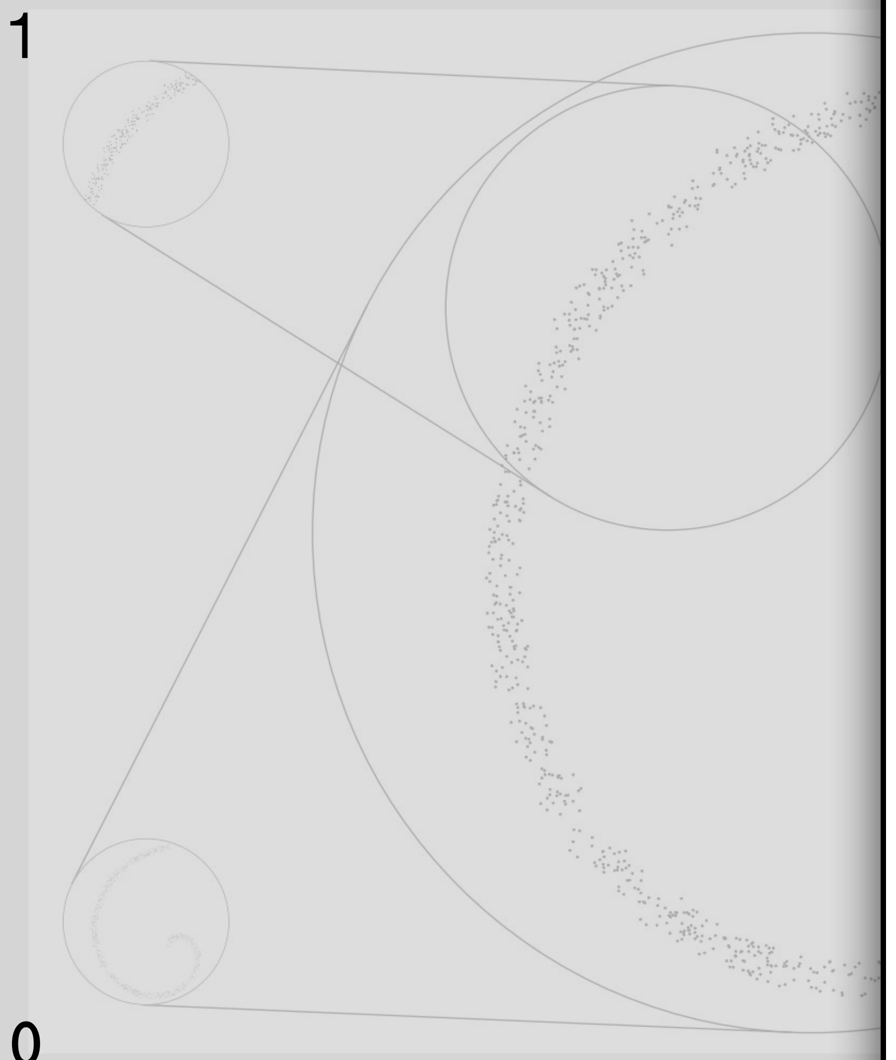
*Preliminary LL calculation for QG jets in backup

Quantifying Event-Space Manifolds

$\sum_j \Theta(\mathrm{EMD}(\mathcal{E}_i, \mathcal{E}'_j) < Q)$

Correlation dimension
elements within a ball

dim ≃ 1

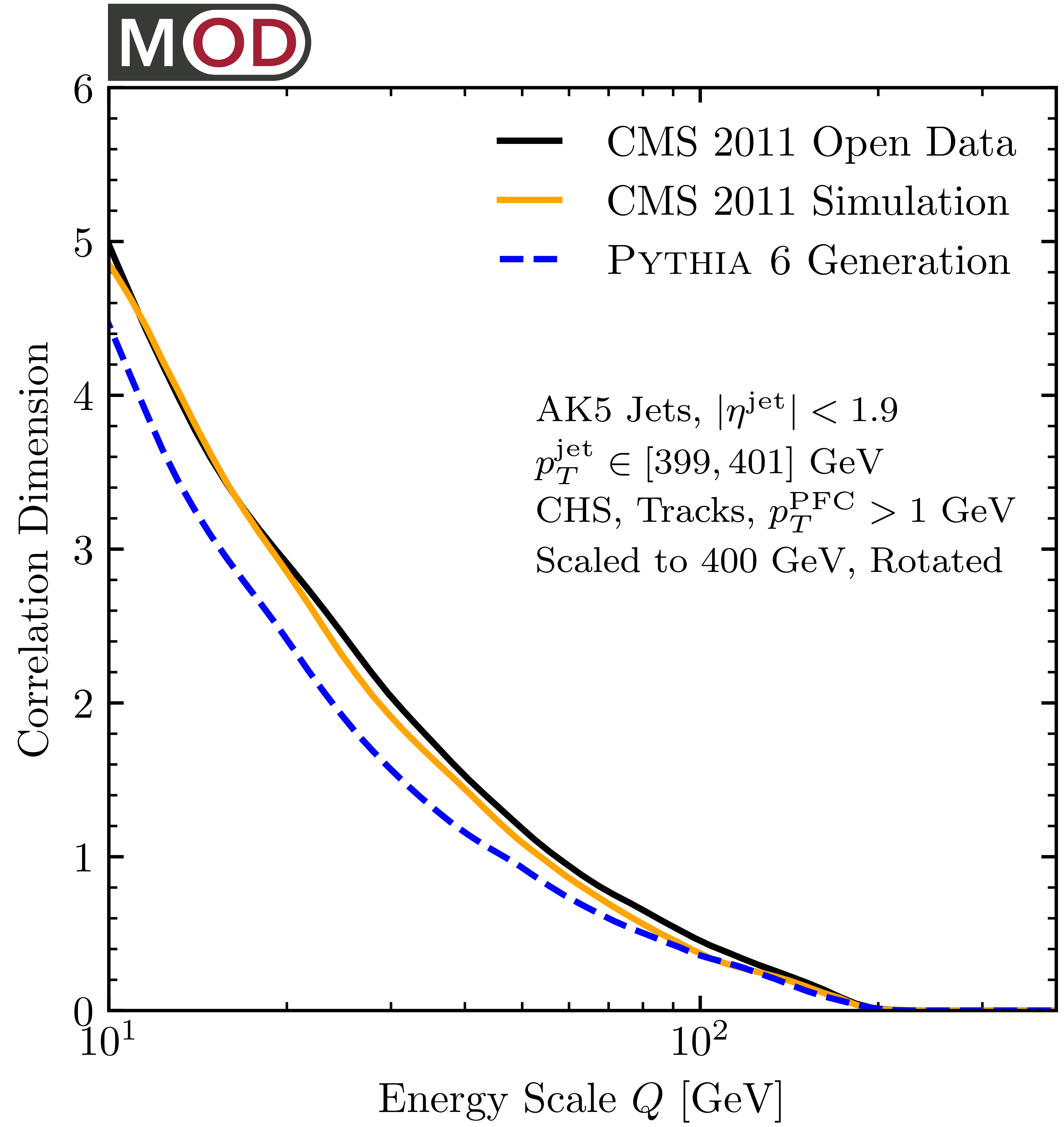


dim → 0

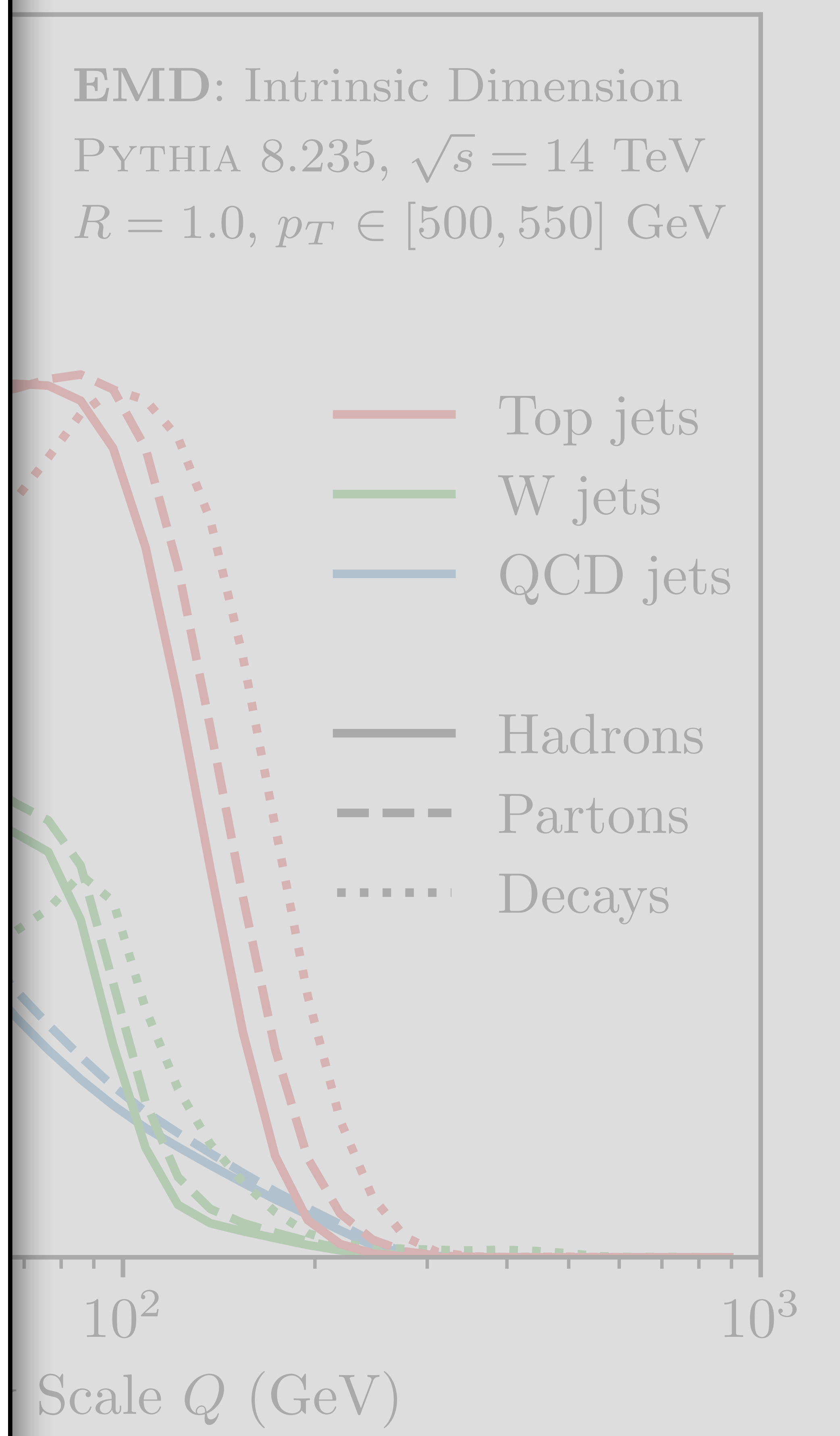
$N_{\text{neigh.}}(Q) \propto Q^{\text{dim}} \implies$ di

Correlation dimens
Decays are "constan
Complexity hierarch
Fragmentation incre
Hadronization impo

… in CMS Open Data



*More in backup

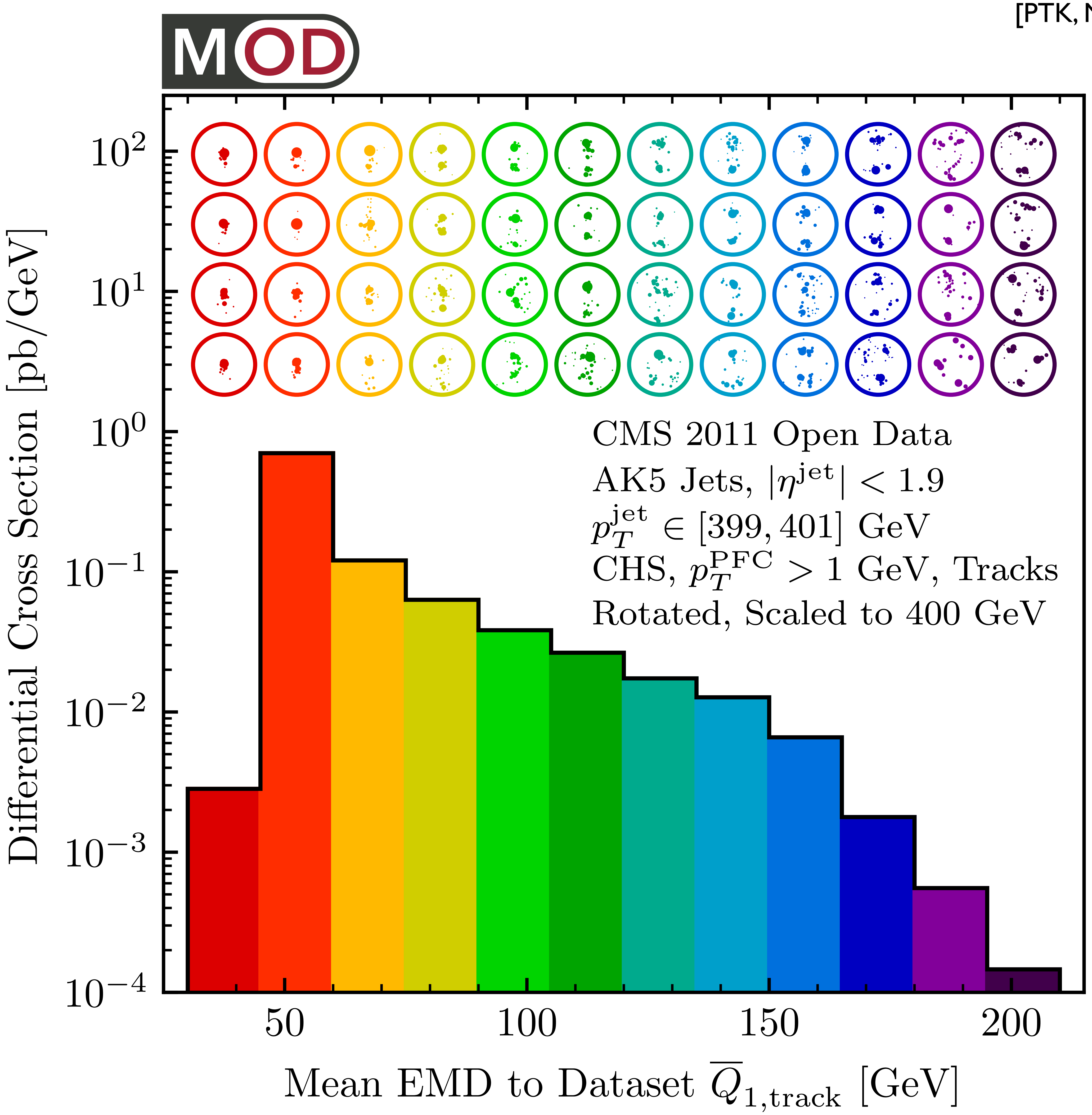


[…ocaccia, PRL 1983; PTK, Metodiev, Thaler, PRL 2019]

*Preliminary LL calculation for QG jets in backup

Visualizing Geometry in CMS Open Data

[PTK, Mastandrea, Metodiev, Naik, Thaler, PRD 2019; code and datasets at energyflow.network]



EMD for anomaly detection

4 medoids in each bin of anomaliness $\bar{Q}_1$

n[th] moment of EMD distribution for a dataset

$$\bar{Q}_n(\mathcal{I}) = \sqrt[n]{\frac{1}{N}\sum_{k=1}^{N}\left(\mathrm{EMD}(\mathcal{I},\mathcal{J}_k)\right)^n}$$

How far does this go?

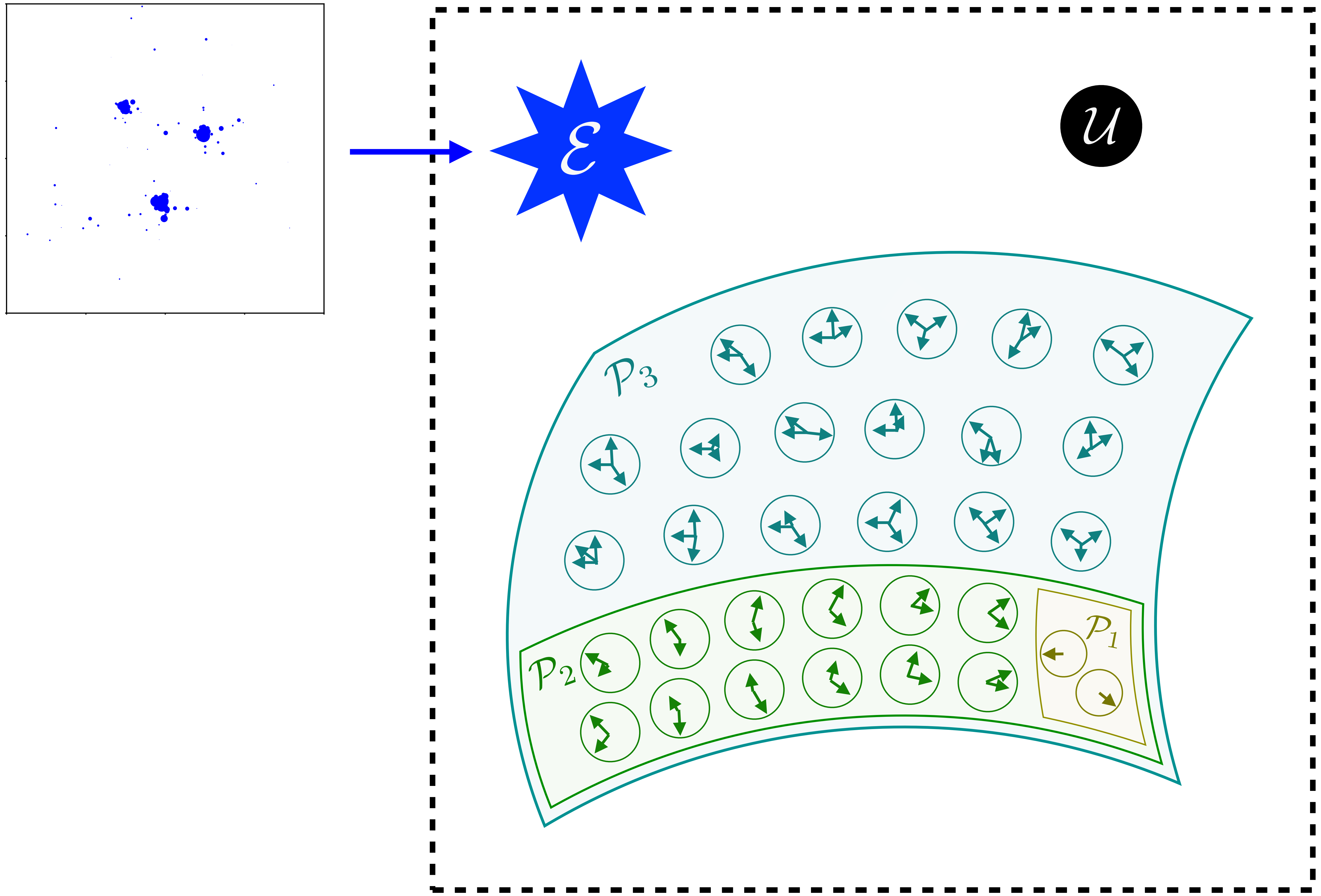
$$\mathcal{V}_k = \frac{1}{N}\sum_{i=1}^{N}\min\left\{\mathrm{EMD}(\mathcal{J}_i,\mathcal{K}_1),\ldots,\mathrm{EMD}(\mathcal{J}_i,\mathcal{K}_k)\right\}$$

k-eventiness? jet from dataset medoids

N-particle Manifolds in the Space of Events

[PTK, Metodiev, Thaler, 2004.04159]

$$\mathcal{P}_N = \text{set of all N-particle configurations} = \left\{ \sum_{i=1}^{N} E_i\, \delta(\hat{n} - \hat{n}_i) \;\middle|\; E_i \geq 0 \right\}$$



$\mathcal{P}_1$: manifold of events with one particle

$\mathcal{P}_2$: manifold of events with two particles

$\mathcal{P}_3$: manifold of events with three particles

$\vdots$

$$\mathcal{P}_N \supset \mathcal{P}_{N-1} \supset \cdots \supset \mathcal{P}_3 \supset \mathcal{P}_2 \supset \mathcal{P}_1$$

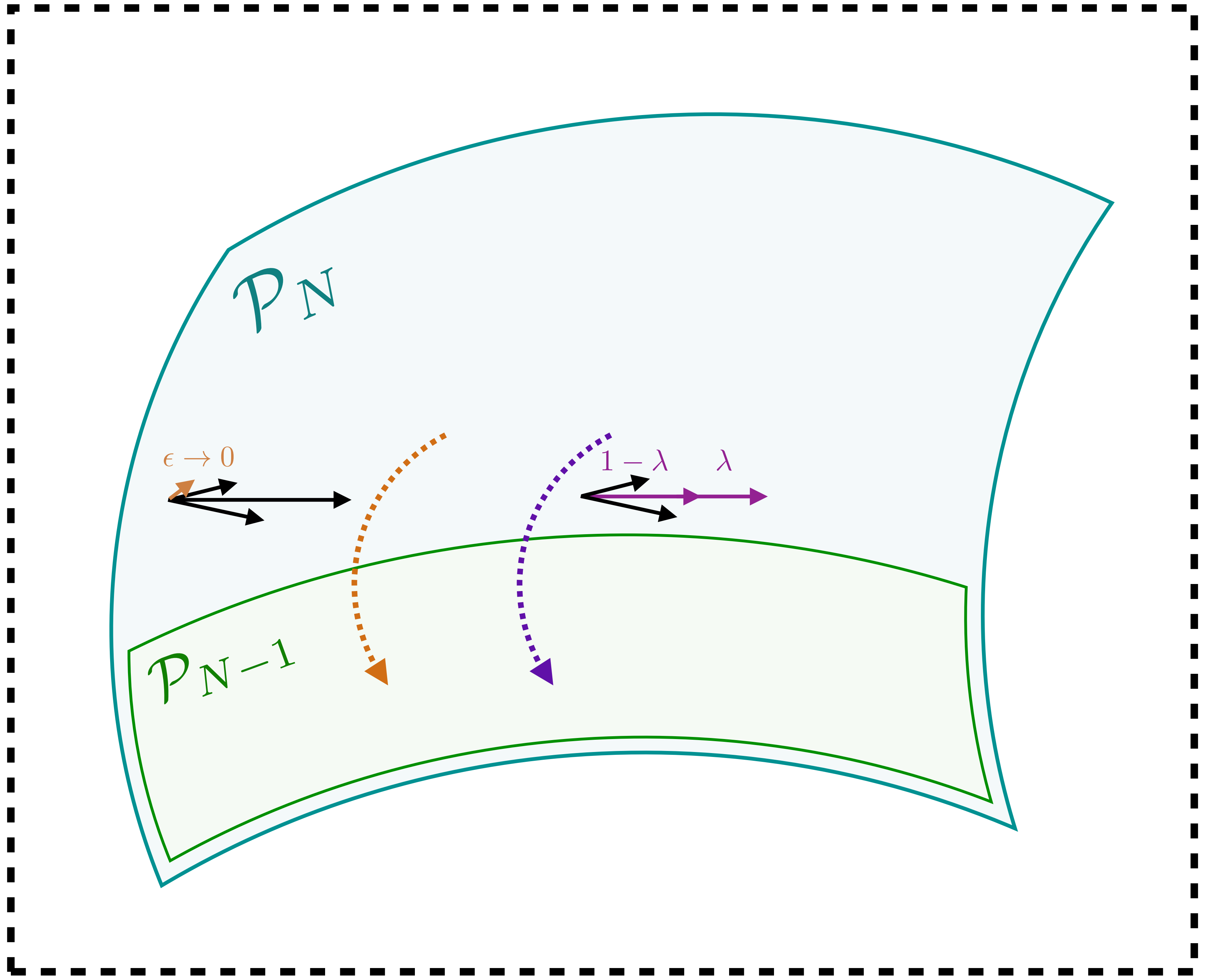
by soft and collinear limits

$\mathcal{U}$ Uniform event, not contained in any P_N

N-particle Manifolds in the Space of Events – Infrared Divergences

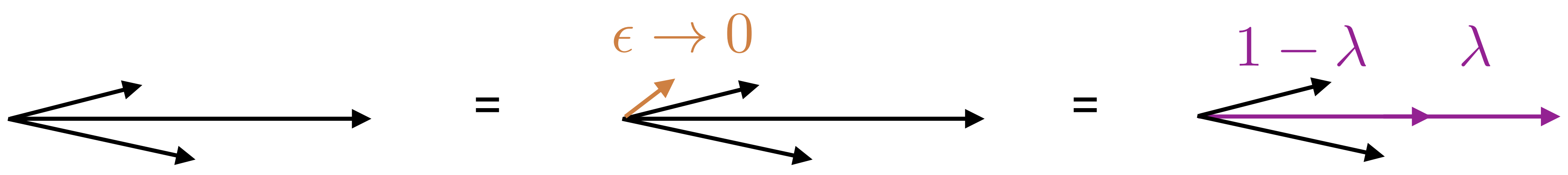
[PTK, Metodiev, Thaler, 2004.04159]

$$\mathcal{P}_N = \text{set of all N-particle configurations} = \left\{ \sum_{i=1}^{N} E_i\, \delta(\hat{n} - \hat{n}_i) \;\middle|\; E_i \geq 0 \right\}$$



$$dP_{i\to ig} \simeq \frac{2\alpha_s}{\pi} C_a \frac{d\theta}{\theta} \frac{dz}{z}$$

Energy flow is unchanged by exact soft/collinear emissions



Functions of energy flow automatically satisfy exact IRC invariance!

Real and virtual divergences appear naturally together

$$\mathcal{P}_N \supset \mathcal{P}_{N-1} \supset \cdots \supset \mathcal{P}_3 \supset \mathcal{P}_2 \supset \mathcal{P}_1$$

by soft and collinear limits

Defining IRC Safety Precisely

[Sterman, Weinberg, PRL 1997; Sterman, PRD 1978; Banfi, Salam, Zanderighi, JHEP 2005]

Infrared and collinear safety is a proxy for perturbative calculability of an observable

Exact IRC invariance

$$\mathcal{O}(p_1^\mu, \ldots, p_M^\mu) = \mathcal{O}(0 p_0^\mu, p_1^\mu, \ldots, p_M^\mu)$$
$$\mathcal{O}(p_1^\mu, \ldots, p_M^\mu) = \mathcal{O}(\lambda p_1^\mu, (1-\lambda) p_1^\mu, \ldots, p_M^\mu)$$

Guarantees observable is well-defined on energy flows

Allows for pathological observables, e.g. pseudo-multiplicity

Smooth IRC invariance

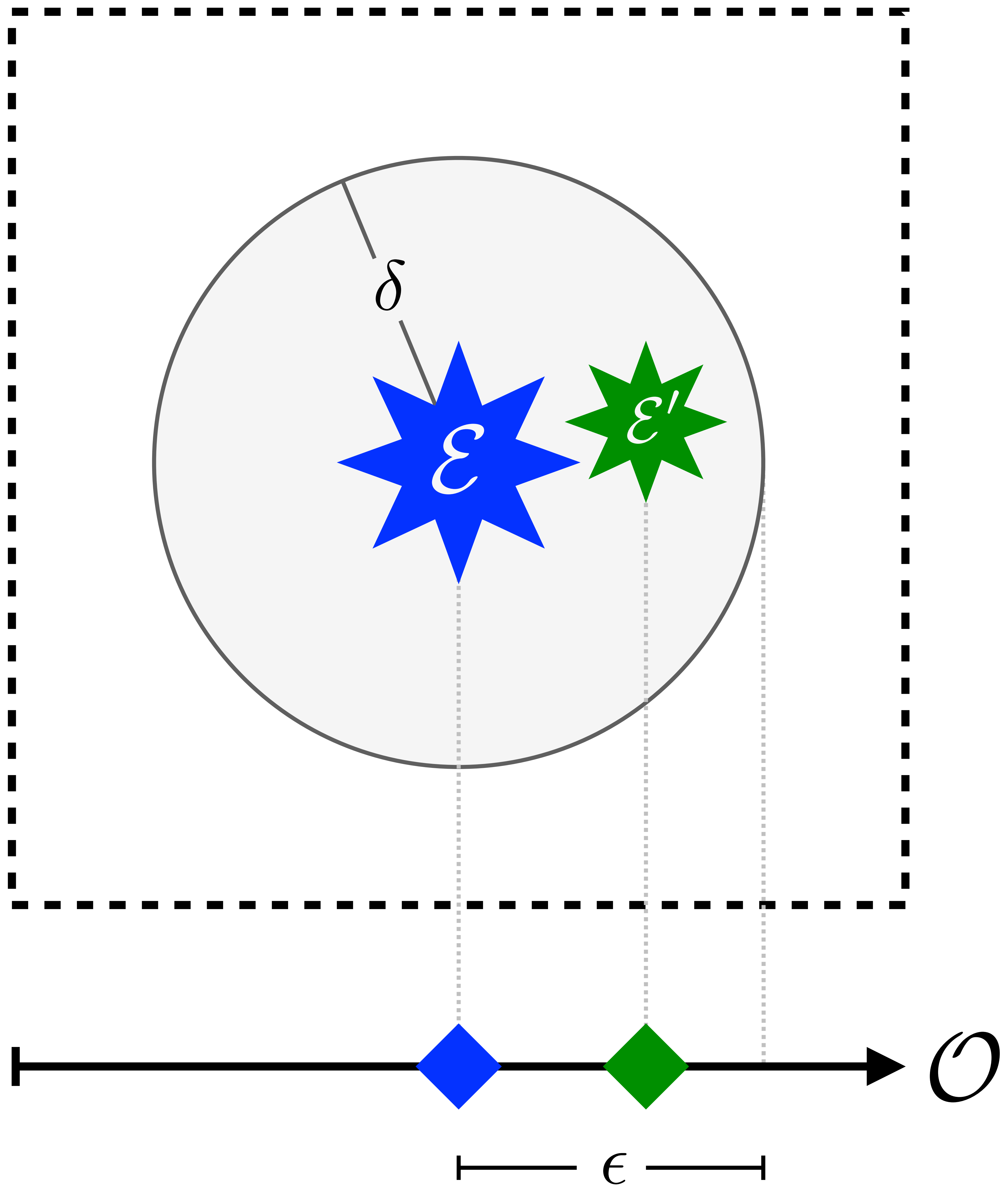
$$\mathcal{O}(p_1^\mu, \ldots, p_M^\mu) = \lim_{\epsilon \to 0} \mathcal{O}(\epsilon p_0^\mu, p_1^\mu, \ldots, p_M^\mu)$$
$$\mathcal{O}(p_1^\mu, \ldots, p_M^\mu) = \lim_{p_0^\mu \to p_1^\mu} \mathcal{O}(\lambda p_0^\mu, (1-\lambda) p_1^\mu, \ldots, p_M^\mu)$$

Eliminates common observables with hard boundaries

All Observables	Comments
Multiplicity $(\sum_i 1)$	IR unsafe and C unsafe
Momentum Dispersion [65] $(\sum_i E_i^2)$	IR safe but C unsafe
Sphericity Tensor [66] $(\sum_i p_i^\mu p_i^\nu)$	IR safe but C unsafe
Number of Non-Zero Calorimeter Deposits	C safe but IR unsafe
Defined on Energy Flows	
Pseudo-Multiplicity $(\min\{N \mid \mathcal{T}_N = 0\})$	Robust to exact IR or C emissions
Infrared & Collinear Safe	
Jet Energy $(\sum_i E_i)$	Disc. at jet boundary
Heavy Jet Mass [67]	Disc. at hemisphere boundary
Soft-Dropped Jet Mass [38, 68]	Disc. at grooming threshold
Calorimeter Activity [69] (N_{95})	Disc. at cell boundary

More EMD Geometry – Continuity in the Space of Events

[PTK, Metodiev, Thaler, 2004.04159]



Classic $\epsilon - \delta$ definition of continuity in a metric space

An observable $\mathcal{O}$ is EMD continuous at an event $\mathcal{E}$ if, for any $\epsilon > 0$, there exists a $\delta > 0$ such that for all events $\mathcal{E}'$:

$$\text{EMD}(\mathcal{E}, \mathcal{E}') < \delta \quad \Longrightarrow \quad |\mathcal{O}(\mathcal{E}) - \mathcal{O}(\mathcal{E}')| < \epsilon.$$

Towards a geometric definition of IRC Safety

IRC Safety = EMD Continuity*

*on all but a negligible set‡ of events

‡a negligible set is one that contains no positive-radius EMD-ball

⋮

Perturbation Theory in the Space of Events

[PTK, Metodiev, Thaler, 2004.04159]

Sudakov safety

[Larkoski, Thaler, JHEP 2014; Larkoski, Marzani, Thaler, PRD 2015]

Some observables have discontinuities on P_N for some N

A resummed IRC-safe companion can mitigate the divergences

$$p(\mathcal{O}_{\text{Sudakov}}) = \int d\mathcal{O}_{\text{Comp.}}\, p(\mathcal{O}_{\text{Sudakov}}|\mathcal{O}_{\text{Comp.}})\, p(\mathcal{O}_{\text{Comp.}})$$

Event geometry suggests N-(sub)jettiness as universal companion

Fixed-order calculability

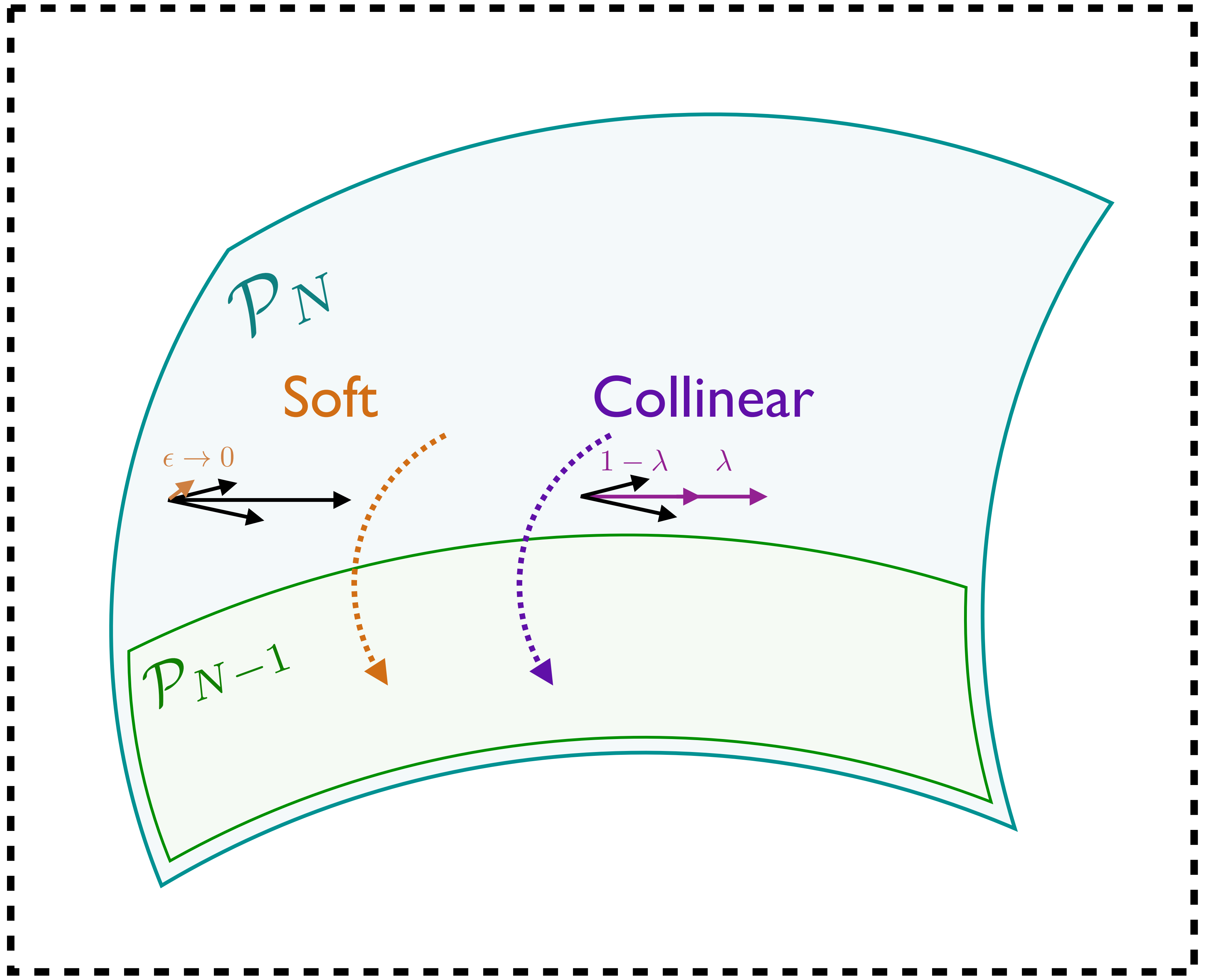
[Sterman, PRD 1979; Banfi, Salam, Zanderighi, JHEP 2005]

Is a statement of integrability on each P_N

EMD continuity must be upgraded to EMD-Hölder continuity on each P_N

$$\lim_{\mathcal{E}\to\mathcal{E}'} \frac{\mathcal{O}(\mathcal{E}) - \mathcal{O}(\mathcal{E}')}{\text{EMD}(\mathcal{E},\mathcal{E}')^c} = 0, \quad c > 0$$

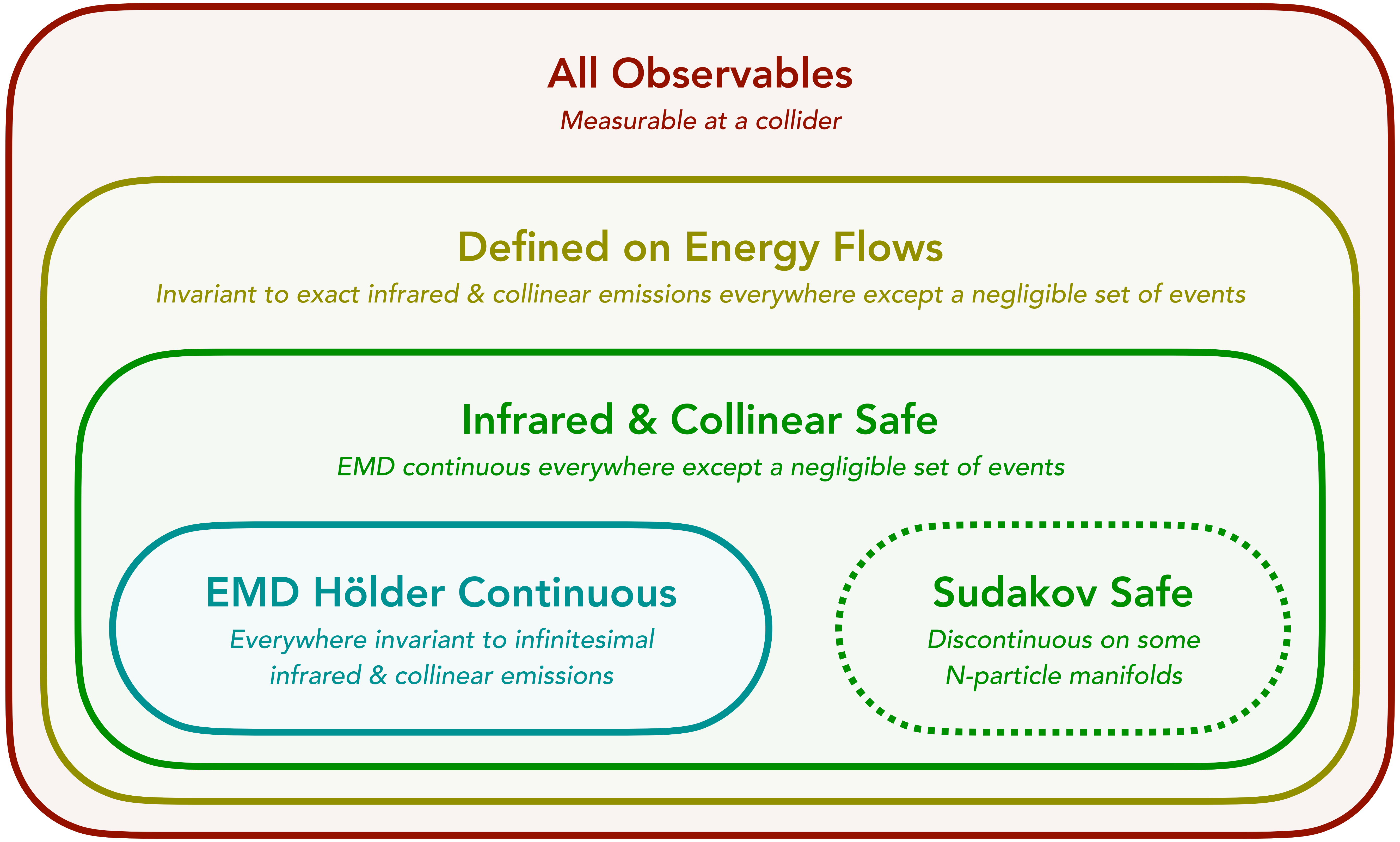
Infrared singularities of massless gauge theories appear on each P_N



Example: $V(\mathcal{E}) = \mathcal{T}_2(\mathcal{E})\left(1 + \frac{1}{\ln E(\mathcal{E})/\mathcal{T}_3(\mathcal{E})}\right)$ is EMD continuous but not EMD Hölder continuous (it is Sudakov safe)

Hierarchy of IRC Safety Definitions

[PTK, Metodiev, Thaler, 2004.04159]



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Sudakov Safe	
Groomed Momentum Fraction [39] (z_g)	Disc. on 1-particle manifold
Jet Angularity Ratios [37]	Disc. on 1-particle manifold
N-subjettiness Ratios [47, 48] (τ_{N+1}/τ_N)	Disc. on N-particle manifold
V parameter [36] (Eq. (2.11))	Hölder disc. on 3-particle manifold
EMD Hölder Continuous Everywhere	
Thrust [40, 41]	
Spherocity [42]	
Angularities [70]	
N-jettiness [44] ($\mathcal{T}_N$)	
C parameter [71–74]	Resummation beneficial at $C = \frac{3}{4}$
Linear Sphericity [72] ($\sum_i E_i n_i^\mu n_i^\nu$)	
Energy Correlators [36, 75–77]	
Energy Flow Polynomials [15, 17]	